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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Proof (1)

Core Pure Mathematics 1 · Topic 1

6 questions · 42 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

6 questions · 42 marks · 3 sub-topics



SUB-TOPIC 1.1

INDUCTION: SUMS OF SERIES

Sub-topic 1.1 · 3 questions · 23 marks · lowest tariff first



4. Prove that, for $n \in \mathbb{Z}$, $n \geq 0$

$$\sum_{r=0}^n \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where a, b and c are integers to be found.

(5)





(Total for Question 4 is 5 marks)



$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(4n^2-1)$$




6. (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^n (3r+1)(r+2) = n(n+2)(n+3) \quad (6)$$

(ii) Prove by induction that for all positive **odd** integers n

$$f(n) = 4^n + 5^n + 6^n$$

is divisible by 15

(6)





(Total for Question 6 is 12 marks)



SUB-TOPIC 1.3

INDUCTION: MATRICES

Sub-topic 1.3 · 1 question · 5 marks · lowest tariff first



4. Prove by induction that for $n \in \mathbb{N}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & -2n \\ 0 & 1 \end{pmatrix}$$

(5)



(Total for Question 4 is 5 marks)



SUB-TOPIC 1.5

OTHER PROOFS

Sub-topic 1.5 · 2 questions · 14 marks · lowest tariff first



(1)

$$\int_1^{\infty} \frac{1}{x(2x+5)} dx = a \ln b$$

(6)

(Total for Question 2 is 7 marks)







(Total for Question 4 is 7 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
4	$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{A}{r+1} + \frac{B}{r+2} + \frac{C}{r+3} \Rightarrow A=..., B=..., C=...$ $\left(\text{NB } A=\frac{1}{2} \quad B=-1 \quad C=\frac{1}{2} \right)$	M1	3.1a
	$r=0 \quad \frac{1}{2} \left[\frac{1}{1} - \frac{2}{2} + \frac{1}{3} \right] \text{ or } \frac{1}{2 \cdot 1} - \frac{1}{2} + \frac{1}{2 \cdot 3} \text{ or } \frac{1}{2} - \frac{1}{2} + \frac{1}{6}$	M1	2.1
	$r=1 \quad \frac{1}{2} \left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right] \text{ or } \frac{1}{2 \cdot 2} - \frac{1}{3} + \frac{1}{2 \cdot 4} \text{ or } \frac{1}{4} - \frac{1}{3} + \frac{1}{8}$		
	$r=n-1 \quad \frac{1}{2} \left[\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} \right] \text{ or } \frac{1}{2 \cdot n} - \frac{1}{n+1} + \frac{1}{2 \cdot n+2}$ $\text{or } \frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2n+4}$		
	$r=n \quad \frac{1}{2} \left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \right] \text{ or } \frac{1}{2 \cdot n+1} - \frac{1}{n+2} + \frac{1}{2 \cdot n+3}$ $\text{or } \frac{1}{2n+2} - \frac{1}{n+2} + \frac{1}{2n+6}$		
	$\frac{1}{2} - \frac{1}{2} + \frac{1}{4} + \frac{1}{2(n+2)} - \frac{1}{n+2} + \frac{1}{2(n+3)}$ $\text{or } \frac{1}{4} - \frac{1}{2(n+2)} + \frac{1}{2(n+3)}$	A1	1.1b
	$= \frac{n^2 + 5n + 6 + 2n + 6 - 4n - 12 + 2n + 4}{4(n+2)(n+3)}$	M1	1.1b
	$= \frac{(n+1)(n+4)}{4(n+2)(n+3)}$	A1	2.2a
		(5)	
(5 marks)			



Question	Scheme	Marks	AOs
6	If $n = 1$ $\sum_{r=1}^n (2r-1)^2 = (2-1)^2 = 1$ and $\frac{1}{3}n(4n^2-1) = \frac{1}{3}(1)(4(1)^2-1) = 1$ (LHS=RHS) so true for $n = 1$	B1	2.4
	(Assume true for $n = k$ so $\sum_{r=1}^k (2r-1)^2 = \frac{1}{3}k(4k^2-1)$ then) $\sum_{r=1}^{k+1} (2r-1)^2 = \frac{1}{3}k(4k^2-1) + (2(k+1)-1)^2$	M1	2.1
	E.g. $= \frac{1}{3}(2k+1)(2k^2+5k+3)$ or $= \frac{4}{3}k^3 - \frac{1}{3}k + 4k^2 + 4k + 1$	dM1	1.1b
	$\frac{1}{3}(k+1)(2k+3)(2k+1)$ or $\frac{1}{3}(k+1)(4k^2+8k+3)$ or $\frac{4k^3}{3} + 4k^2 + \frac{11k}{3} + 1$	A1	1.1b
	$= \frac{1}{3}(k+1)(4(k+1)^2-1)$ Or see notes	A1	2.2a
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n .	A1	2.4
		(6)	
(6 marks)			
Notes			
B1: Demonstrates the statement is true for $n = 1$. Accept as minimum $(1)^2=1$ and $\frac{1}{3}(3)=1$ for the check (both sides must clearly be evaluated as 1), and for the conclusion “true for $n=1$” or broken “if/when $n=1$ <check> (hence) true/shown/tick” – must be part of the initial check (not just in the conclusion). M1: Assume the result for $n = k$ and attempts to add the $(k+1)^{\text{th}}$ term to the result for the sum to k terms. An assumption may be clearly made, but accept tacit assumptions. Allow if there are minor slips in the expressions if the intent is clear. dM1: Makes progress towards proving the inductive step by either factorising the common factor $(2k+1)$ or expanding fully. There may be variations so score for equivalent progress. A1: Either for a correct factorised form with $(k+1)$ as a factor or a fully correct expansion with gathered terms. Must have come from correct work from the inductive step, not backwards worked from the $n = k + 1$ expression. A1: Completes the inductive steps by obtaining the correct expression in terms of $(k+1)$ with suitable intermediate step e.g. clear factorisation of $(k+1)$ first, OR by expanding the required expression for $n = k + 1$ to achieve equal expressions. Must have been completely correct work, no errors. A1: Depends on the MMAA marks having been scored with at least an attempt to check one side of the $n = 1$ case having been made. Correct complete conclusion. Must include the notions of “true for $n = 1$”, “true for $n = k$ implies true for $n = k + 1$” and “hence true for all n” though the exact wording will vary. The conclusion should be given at the end with the exception that the “true for $n = 1$” may be stated with the initial check. Note: stating “true for $n = k$ and $n = k + 1$” will be A0. Note: Accept use of n instead of k for all except the final A mark. Note: Attempts at using summation formulae without induction score no marks.			

Question	Scheme	Marks	AOs
6(i)	When $n = 1$, $\sum_{r=1}^1 (3r+1)(r+2) = 4 \times 3 = 12$ $1(1+2)(1+3) = 12$ (so the statement is true for $n = 1$)	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^k (3r+1)(r+2) = k(k+2)(k+3)$	M1	2.4
	$\sum_{r=1}^{k+1} (3r+1)(r+2) = k(k+2)(k+3) + (3k+4)(k+3)$	M1	2.1
	$= (k+3)(k^2 + 5k + 4)$	A1	1.1b
	$\sum_{r=1}^{k+1} (3r+1)(r+2) = (k+1)(k+3)(k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1} (3r+1)(r+2) = (\underline{k+1})(\underline{k+1+2})(\underline{k+1+3})$ If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n .	A1	2.4
		(6)	
(ii) Way 1	When $n = 1$, $4^1 + 5^1 + 6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^k + 5^k + 6^k$ is divisible by 15	M1	2.4
	$f(k+2) = 4^{k+2} + 5^{k+2} + 6^{k+2}$	M1	2.1
	$= 16 \times 4^k + 16 \times 5^k + 16 \times 6^k + 9 \times 5^k + 20 \times 6^k$ $= 16f(k) + 45 \times 5^{k-1} + 120 \times 6^{k-1}$	A1 A1	1.1b 1.1b
	E.g As 15 divides $f(k)$, 45 and 120, so 15 divides $f(k+1)$. If true for $n = k$ then true for $n = k + 2$, true for $n = 1$ so true for all positive odd integers n	A1	2.4
		(6)	
(ii) Way 2	When $n = 1$, $4^1 + 5^1 + 6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^k + 5^k + 6^k$ is divisible by 15	M1	2.4
	$f(k+2) - f(k) = 4^{k+2} + 5^{k+2} + 6^{k+2} - 4^k - 5^k - 6^k$	M1	2.1
	$= 15 \times 4^k + 24 \times 5^k + 35 \times 6^k$ $= 15f(k) + 45 \times 5^{k-1} + 120 \times 6^{k-1}$	A1	1.1b
	$f(k+2) = 16f(k) + 45 \times 5^{k-1} + 120 \times 6^{k-1}$	A1	1.1b
	E.g $f(k+2) = 16f(k) + 15(3 \times 5^{k-1} + 8 \times 6^{k-1})$ so if true for $n = k$ then true for $n = k + 2$, true for $n = 1$ so true for all positive odd integers n	A1	2.4
		(6)	

(ii) Way 3	When $n = 1$, $4^1 + 5^1 + 6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = 2k + 1$ so $f(2k+1) = 4^{2k+1} + 5^{2k+1} + 6^{2k+1}$ is divisible by 15	M1	2.4
	$f(2k+3) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3}$ or $f(2k+3) - f(2k+1) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3} - 4^{2k+1} - 5^{2k+1} - 6^{2k+1}$	M1	2.1
	$f(2k+3) = 16 \times 4^{2k+1} + 25 \times 5^{2k+1} + 36 \times 6^{2k+1}$ $= 16(4^{2k+1} + 5^{2k+1} + 6^{2k+1}) + 9 \times 5^{2k+1} + 20 \times 6^{2k+1}$ OR $f(2k+3) - f(2k+1) = 16 \times 4^{2k+1} + 25 \times 5^{2k+1} + 36 \times 6^{2k+1} - 4^{2k+1} - 5^{2k+1} - 6^{2k+1}$ $= 15 \times 4^{2k+1} + 120 \times 5^{2k} + 210 \times 6^{2k}$	A1	1.1b
	$f(2k+3) = 16(4^{2k+1} + 5^{2k+1} + 6^{2k+1}) + 45 \times 5^{2k} + 120 \times 6^{2k}$ OR $f(2k+3) = f(2k+1) + 15 \times 4^{2k+1} + 120 \times 5^{2k} + 210 \times 6^{2k}$	A1	1.1b
	$f(2k+3) = 16(4^{2k+1} + 5^{2k+1} + 6^{2k+1}) + 15(3 \times 5^{2k} + 8 \times 6^{2k})$ OR $f(2k+3) = f(2k+1) + 15(4^{2k+1} + 8 \times 5^{2k} + 14 \times 6^{2k})$ and <u>If true for $n = 2k+1$ then true for $n = 2k+3$, true for $n = 1$ so true for all positive odd integers n</u>	A1	2.4
		(6)	
(12 marks)			
Notes			
(i) B1: Shows the statement is true for $n = 1$ by evaluating both sides. There is no need for statement “hence true for $n = 1$ ” for this mark but if they never state this the final A will be forfeited. Look for a minimum of $4 \times 3 = 12$ for the LHS and $1 \times 3 \times 4$ for the RHS. If only one side is explicitly evaluated, it is B0, but all other marks may be gained. M1: Makes an assumption statement that assumes the result is true for $n = k$ M1: Makes the inductive step by attempting to add the $(k + 1)^{\text{th}}$ term to the assumed result. Attempts at using the standard summation formulae score M0, as the question requires induction. A1: Correct expression with at least one correct linear factor A1: Obtains a fully correct factorised expression. May be as in scheme or in terms of $k + 1$. A1: Correct complete conclusion with all ideas conveyed at the end or as a narrative and the sum to $k+1$ expressed in terms of $k + 1$ (or with the expression in term of $k + 1$ stated earlier – it must be seen at some stage). Allow slips in notation if the intent is correct. Depends on all except the B mark, though an attempt at checking the $n = 1$ case must have been made. (ii) Way 1 B1: Shows that $f(1) = 15$ M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then ... etc.) M1: Attempts $f(k + 2)$			

A1: Correctly obtains $16f(k)$ **or** $45 \times 5^{k-1} + 120 \times 6^{k-1}$

A1: Reaches a correct expression for $f(k+2)$ in terms of $f(k)$

A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow “true for all n ” where n represents natural numbers, as this is incorrect.

Way 2

B1: Shows that $f(1) = 15$

M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then ... etc.)

M1: Attempts $f(k+2) - f(k)$ or equivalent work

A1: Achieves a correct expression for $f(k+2) - f(k)$ in terms of $f(k)$

A1: Reaches a correct expression for $f(k+2)$ in terms of $f(k)$

A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow “true for all n ” where n represents natural numbers, as this is incorrect.

Way 3

B1: Shows that $f(1) = 15$

M1: Makes a statement that assumes the result is true for some odd value of n (Assume (true for) $n = 2k + 1$ is sufficient (or may use $2k - 1$ – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then ... etc.)

M1: Attempts $f(2k+3)$ (like Way 1), or $f(2k+3) - f(2k+1)$ or equivalent work (like Way 2) It must be the correct $f(2k+1)$ used, in the latter case (subtracting $4^k + 5^k + 6^k$ instead of $4^{2k+1} + 5^{2k+1} + 6^{2k+1}$ is M0)

A1: Reaches $16(4^{2k+1} + 5^{2k+1} + 6^{2k+1}) + 9 \times 5^{2k+1} + 20 \times 6^{2k+1}$ or suitable equivalent OR achieves a correct expression for $f(2k+3) - f(2k+1)$ in terms of $f(2k+1)$ where factors of 15 are apparent.

A1: Reaches $16(4^{2k+1} + 5^{2k+1} + 6^{2k+1}) + 45 \times 5^{2k} + 120 \times 6^{2k}$ or a suitable equivalent OR a correct expression for $f(2k+3)$ of form $Af(2k+1) + 15(\dots)$ (though the $15(\dots)$ could be separate multiples of 15 where the 15 need not yet be extracted).

A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow “true for all n ” where n represents natural numbers, as this is incorrect, but if n represents odd numbers then allow.

Note: $4^{2k} = 16^k$ etc may be used throughout.

Note: Way 3 should apply to cases where the question is rephrased in terms of $4^{2n+1} + 5^{2n+1} + 6^{2n+1}$ for all n (rather than odd n).

Accept use of alternative equivalent language throughout.



Question	Scheme	Marks	AOs
4	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & -2n \\ 0 & 1 \end{pmatrix}$		
	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 1 & -2 \times 1 \\ 0 & 1 \end{pmatrix} \text{ (so true when } n = 1 \text{)}$	B1	2.2a
	$\text{(Assume true for } n = k, \text{ then)}$ $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix} \text{ or } = \begin{pmatrix} 1 & -2-2k \\ 0 & 1 \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 1 & -2(k+1) \\ 0 & 1 \end{pmatrix}$	A1	2.2a
	Hence result is true for $n = k + 1$. As <u>true for $n = 1$</u> and have shown <u>if true for $n = k$ then it is true for $n = k + 1$</u> , so it is <u>true for all n</u> .	A1	2.5
		(5)	
(5 marks)			

Notes:

B1: Shows true for $n = 1$.

Need to see $n = 1$ **substituted** into rhs. The minimum for B1 would be $\begin{pmatrix} 1 & -2 \times 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

There is no need to state “True for $n = 1$ ”

M1: (Assumes for $n = k$ and) multiplies original matrix by k th power matrix either way round. Note that the assumption statement is not needed for this mark (but see below) so just look for:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

A1: Achieves a correct unsimplified matrix.

A1: Reaches correct form for the matrix with the $k + 1$ factored out with no errors and the correct unsimplified matrix seen previously. Note that the result may be proved by equivalence (see below).

A1: Correct conclusion **with assumption made** (which may be implied in their conclusion if they say “if true for $n = k$ then...”). This mark is dependent on all the previous marks apart from the B mark and is gained by conveying all the underlined points.

Allow this mark to score as long as all the underlined points are seen as narrative in their solution.

There must be the assumption statement somewhere or the “if...then...” idea in the conclusion. If awarded for the assumption statement condone e.g. true for $n = k$ in the conclusion.

The conclusion must convey the “if true for $n = k$ then true for $n = k + 1$ ” idea and not e.g. true for k , true for $k + 1$, true for 1 therefore...but see the previous note.

For the “true for all n ” part condone e.g. “true for n ”, “true for all integers after 1”, “true for $\square^+$ ”,

But do **not** allow “true for all values”, “true for all real numbers”



Q4 Extra Notes:

1. For candidates who use n instead of k throughout withhold the final mark if the work is otherwise correct.

2. For equivalence proofs, this would be minimally acceptable:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \times 1 \\ 0 & 1 \end{pmatrix} \quad \mathbf{B1}$$

“Need to prove” oe e.g. “target is”, “ $n = k + 1$ ”, “need” etc. $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix}$

Note that there must be a reference to $k + 1$ as above or e.g. (Target $\Rightarrow$) $\begin{pmatrix} 1 & -2(k+1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix} \quad \mathbf{M1A1A1}$$

Hence result is true for $n = k + 1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k + 1$, so it is true for all n . **A1**

Without the “Need to prove...” oe the response would score B1M1A1 and then A1 if the factorised form was shown or equivalence shown and then A1 for the correct conclusion.

3. Allow e.g. “correct” for “true” in the conclusion



Question	Scheme	Marks	AOs
2(a)	E.g. - Because the interval being integrated over is unbounded - Accept because the upper limit is infinity - Accept because a limit is required to evaluate it	B1	2.4
		(1)	
(b)	$\frac{1}{x(2x+5)} = \frac{A}{x} + \frac{B}{2x+5} \Rightarrow A = \dots, B = \dots$	M1	3.1a
	$\frac{1}{x(2x+5)} = \frac{1}{5x} - \frac{2}{5(2x+5)}$	A1	1.1b
	$\int \frac{1}{5x} - \frac{2}{5(2x+5)} dx = \frac{1}{5} \ln x - \frac{1}{5} \ln(2x+5)$	A1ft	1.1b
	$\frac{1}{5} \ln x - \frac{1}{5} \ln(2x+5) = \frac{1}{5} \ln \frac{x}{(2x+5)}$	M1	2.1
	$\lim_{x \rightarrow \infty} \left\{ \frac{1}{5} \ln \frac{x}{2x+5} \right\} = \frac{1}{5} \ln \frac{1}{2}$	B1	2.2a
	$\Rightarrow \int_1^{\infty} \frac{1}{x(2x+5)} dx = \frac{1}{5} \ln \frac{1}{2} - \frac{1}{5} \ln \frac{1}{7} = \frac{1}{5} \ln \frac{7}{2}$	A1	1.1b
		(6)	
(7 marks)			
Notes			
(a) B1: For a suitable explanation with no contrary reasoning. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept “Because the upper limit is infinity”. Do not award if there are erroneous statements e.g. referring to as $x = 0$ the integrand is not defined. Do not accept “because one of the limits is undefined” unless they state they mean ∞ . Do not accept “it is undefined when $x = \infty$ ” without reference to “it” being the upper limit. (b) M1: Selects the correct form for partial fractions and proceeds to find values for A and B A1: Correct constants or partial fractions A1ft: $\int \frac{p}{x} + \frac{q}{2x+5} dx = p \ln x + \frac{q}{2} \ln(2x+5)$ Note that $\frac{1}{5} \ln 5x - \frac{1}{5} \ln(10x+25)$ is correct. M1: Combines logs correctly. May see $-\frac{1}{5} \ln \left(\frac{2x+5}{x} \right) = -\frac{1}{5} \ln \left(2 + \frac{5}{x} \right)$ B1: Correct upper limit for $x \rightarrow \infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) A1: Deduces the correct value for the improper integral in the correct form			



Question	Scheme	Marks	AOs
(b) Way 2	$\frac{1}{x(2x+5)} = \frac{1}{2\left(x^2 + \frac{5}{2}x\right)} = \frac{1}{2} \times \frac{1}{\left(x + \frac{5}{4}\right)^2 - \frac{25}{16}}$	M1 A1	3.1a 1.1b
	$\int \frac{1}{x(2x+5)} dx = \frac{1}{2} \times \frac{2}{5} \ln \left \frac{x + \frac{5}{4} - \frac{5}{4}}{x + \frac{5}{4} + \frac{5}{4}} \right = \frac{1}{5} \ln \left \frac{2x}{2x+5} \right $	M1 A1ft	2.1 1.1b
	$\lim_{x \rightarrow \infty} \left\{ \frac{1}{5} \ln \frac{2x}{2x+5} \right\} = \frac{1}{5} \ln \frac{2}{2} = 0$	B1	2.2a
	$\Rightarrow \int_1^{\infty} \frac{1}{x(2x+5)} dx = 0 - \frac{1}{5} \ln \frac{2}{7} = \frac{1}{5} \ln \frac{7}{2}$	A1	1.1b
		(6)	
Notes			
Note the method marks as MAMABA, and should be entered in this order on ePEN. M1: Expands the denominator and completes the square. A1: Correct expression M1: For $\frac{1}{(x+p)^2 - a^2} \rightarrow k \ln \left \frac{x+p-a}{x+p+a} \right$ A1ft: $\frac{1}{2} \frac{1}{(x+a)^2 - a^2} \rightarrow \frac{1}{2a} \ln \left \frac{x}{x+2a} \right$ with their a (may be simplified as in scheme). B1: Correct upper limit for $x \rightarrow \infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) Note in this method the upper limit evaluates to zero. A1: Deduces the correct value for the improper integral in the correct form. Accept $-\frac{1}{5} \ln \frac{2}{7}$			

Question	Scheme	Marks	AOs
4(a)	Applies $\ln\left(\frac{r+1}{r-1}\right) = \ln(r+1) - \ln(r-1)$ to the problem in order to apply differences.	M1	3.1a
	$\sum_{r=2}^n (\ln(r+1) - \ln(r-1))$ $= (\ln(3) - \ln(1)) + (\ln(4) - \ln(2)) + (\ln(5) - \ln(3)) + \dots$ $+ (\ln(n) - \ln(n-2)) + (\ln(n+1) - \ln(n-1))$	dM1	1.1b
	$\ln(n) + \ln(n+1) - \ln 2$	A1	1.1b
	$\ln\left(\frac{n(n+1)}{2}\right) * \text{cso}$	A1 *	2.1
		(4)	
(b)	$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right) = \sum_{r=2}^{100} \ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} \ln\left(\frac{r+1}{r-1}\right)$ $= \ln\left(\frac{100 \times 101}{2}\right) - \ln\left(\frac{50 \times 51}{2}\right)$	M1	1.1b
	$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35} = 35 \ln\left(\frac{100 \times 101}{2} \div \frac{50 \times 51}{2}\right)$	M1	3.1a
	$= 35 \ln\left(\frac{202}{51}\right)$	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a) M1: Uses the subtraction laws of logs to start the method of differences process. dM1: Demonstrates the method of differences process, should have a minimum of e.g. $r = 2, r = 3, r = 4, r = n - 1$ and $r = n$ shown -- enough to establish <i>at least one cancelling term</i> and <i>all non-disappearing terms</i> though the latter may be implied by correct extraction if only the first few cases are shown. Allow this mark if an extra term for $r = 1$ has been included. A1: Correct terms that do not cancel - must not contradict their list of terms so e.g. if $r = 1$ was included, then A0A0 follows. The $\ln 1$ may be included for this mark. A1*: Achieves the printed answer, with no errors or omissions and must have had a complete list (as per dM1) before extraction (but condone missing brackets on $\ln$ terms). If working with r throughout, they must replace by n to gain the last A, but all other marks are available. NB For attempts at combining log terms instead of using differences, full marks may be awarded for the equivalent steps, but attempts that do not make progress in combining terms will score no marks. (b) Condone a bottom limit of 0 or 1 being used throughout part (b). M1: Attempts to split into (the sum up to 100) – (the sum up to k) where k is 49, 50 or 51 and apply the result of (a) in some way. Condone slips with the power. M1: Having attempted to apply (a), uses difference and power log laws correctly to reach an expression of the required form. A1: Correct answer. Accept equivalents in required form, such as $35 \ln \frac{5050}{1275}$			