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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Matrices (3)

Core Pure Mathematics 1 · Topic 3

4 questions · 34 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
3.1 MATRIX ARITHMETIC		1 Qs · 9 marks	3
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QUESTIONS

4 questions · 34 marks · 3 sub-topics



SUB-TOPIC 3.1

MATRIX ARITHMETIC

Sub-topic 3.1 · 1 question · 9 marks · lowest tariff first







SUB-TOPIC 3.3

LINEAR TRANSFORMATIONS

Sub-topic 3.3 · 1 question · 6 marks · lowest tariff first



1. The transformation P is an enlargement, centre the origin, with scale factor k , where $k > 0$
The transformation Q is a rotation through angle θ degrees anticlockwise about the origin.
The transformation P followed by the transformation Q is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$$

- (a) Determine

- (i) the value of k ,
- (ii) the smallest value of θ

(4)

A square S has vertices at the points with coordinates $(0, 0)$, $(a, -a)$, $(2a, 0)$ and (a, a) where a is a constant.

The square S is transformed to the square S' by the transformation represented by $\mathbf{M}$.

- (b) Determine, in terms of a , the area of S'

(2)





(Total for Question 1 is 6 marks)



SUB-TOPIC 3.6

INVERSE MATRICES

Sub-topic 3.6 · 2 questions · 19 marks · lowest tariff first



8. A colony of small mammals is being studied.
In the study, the mammals are divided into 3 categories

N (newborns)	0 to less than 1 month old
J (juveniles)	1 to 3 months old
B (breeders)	over 3 months old

- (a) State one limitation of the model regarding the division into these categories.

(1)

A model for the population of the colony is given by the matrix equation

$$\begin{pmatrix} N_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_n \\ J_n \\ B_n \end{pmatrix}$$

where a and b are constants, and N_n , J_n and B_n are the respective numbers of the mammals in each category n months after the start of the study.

At the start of the study the colony has breeders only, with no newborns or juveniles.

According to the model, after 2 months the number of newborns is 48 and the number of juveniles is 40

- (b) (i) Determine the number of mammals in the colony at the start of the study.

- (ii) Show that $a = 0.8$

(4)

- (c) Determine, in terms of b ,

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1}$$

(3)

Given that the model predicts approximately 1015 mammals **in total** at the start of a particular month, and approximately 596 **newborns**, 464 **juveniles** and 437 **breeders** at the start of the next month,

- (d) determine the value of b , giving your answer to 2 decimal places.

(3)

It is decided to monitor the number of **newborn** males and females as a part of the study. Assuming that 42% of newborns are male,

- (e) refine the matrix equation for the model to reflect this information, giving a reason for your answer.

(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)

(2)











MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
4(i) (a)	It is possible as the number of columns of matrix A matches the number of rows of matrix B .	B1	2.4
	(b) It is not possible as matrix A and matrix B have different dimensions o.e. different number of columns	B1	2.4
		(2)	
(ii) (a)	$\lambda = 5$	B1	2.2a
	$a = 1, b = 2$	B1	2.2a
(b)	Inverse matrix $= \frac{1}{5} \begin{pmatrix} 0 & 5 & 0 \\ 2 & 12 & -1 \\ -1 & -11 & 3 \end{pmatrix}$	B1 ft	3.1a
		(3)	
(iii)	A complete method to find the determinant of the matrix and set equal to zero.	M1	1.1b
	Determinant $= 1(\sin \theta \sin 2\theta - \cos \theta \cos 2\theta) - 1(0) + 1(0) = 0$	A1	1.1b
	Uses compound angle formula to achieve $\cos 3\theta = 0$ leading to $\theta = \dots$ or use of $\sin 2q = 2\sin q \cos q$ and $\cos 2q = 1 - 2\sin^2 q$ (e.g. to achieve $\cos q(4\sin^2 q - 1) = 0$) leading to $\theta = \dots$ or use of $\sin 2q = 2\sin q \cos q$ and $\cos 2q = 2\cos^2 q - 1$ (e.g. to achieve $4\cos^3 q - 3\cos q = 0$) leading to $\theta = \dots$	M1	3.1a
	$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$	A1	1.1b
		(4)	
(9 marks)			
Notes:			
(i)(a) B1: Comments that the number of columns of matrix A (2) equals the number of rows of matrix B (2) therefore it is possible. Accept other terminology that is clear in intent e.g. “length of A ” and “height of B ”			
(b) B1: Comments that matrix A and matrix B have different dimensions therefore it is not possible.			
(ii)(a) B1: Deduces the correct value for $\lambda = 5$ B1: Deduces the correct values for a and b			
(b) B1ft: Identifies and applies a correct method find the inverse matrix. May multiply from the given equation, in which case follow through on their value of lambda. Alternatively, award for a correct matrix found by calculator or long hand having found a and b and using these values in the matrix.			

(iii)

M1: A complete method to find the determinant of the matrix and sets it equal to 0

A1: Correct equation

M1: Uses appropriate correct trig identities to solve the equation and finds a value for q

A1: All three correct values $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ and no others in the range.



Question	Scheme	Marks	AOs
1(a)	$\det \mathbf{M} = -4 \times -4 - 4\sqrt{3} \times -4\sqrt{3} = \dots \Rightarrow k = \sqrt{\det \mathbf{M}} = \dots$	M1	3.1a
	Way 1 $k = 8$	A1	1.1b
	$\Rightarrow \mathbf{Q} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \Rightarrow \cos \theta = -\frac{1}{2} \Rightarrow \theta = \dots$	M1	1.1b
	$(\cos \theta < 0, \sin \theta > 0 \Rightarrow \text{Quadrant 2 so}) \quad \theta = 120^\circ$	A1	1.1b
		(4)	
	Way 2 $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = k \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$	M1	3.1a
	Achieves both the equations $k \cos \theta = -4$ and $k \sin \theta = 4\sqrt{3}$	A1	1.1b
	$\frac{k \sin \theta}{k \cos \theta} = \frac{4\sqrt{3}}{-4} \Rightarrow \tan \theta = -\sqrt{3} \Rightarrow \theta = \dots$	M1	1.1b
	$\theta = 120^\circ$ and $k = 8$	A1	1.1b
		(4)	
(b)	Area of $S' = \text{area of } S \times k^2$ (The area of the square $S = 2a^2$)	M1	1.1b
	Area of $S' = 128a^2$	A1ft	2.2a
		(2)	
(6 marks)			
Notes:			
(a) Way 1 M1: A full method to find k such as attempting the square root of the determinant of $\mathbf{M}$. It is immediately deducible so the method may be implied by $k = 8$. A1: $k = 8$ M1: A full method to find a value of θ using their k, no need to justify quadrant. Only one equation needed for this mark. Allow if a radians answer is given. May be implied by a correct angle. A1: Correct angle in degrees. Way 2 M1: Multiplies the correct matrix representing transformation Q by the matrix representing transformation P and sets equal to matrix $\mathbf{M}$. Allow for the matrices either way round as the transformations commute. No need to see the identity matrix, just multiplying through by k is sufficient. A1: Both correct equations. Note that if a correct value of k is found, this A is scored under Way 1. M1: Solves their simultaneous equations to find a value for θ (or k) A1: $\theta = 120^\circ$ and $k = 8$			
(b) M1: Complete method to find the area of S': 'their k^2' $\times$ 'their $2a^2$'. Must be an attempt at the area of S but it need not be correct. A1ft: Deduces the correct area for S', follow through their value of k			

Question	Scheme	Marks	AOs
8(a)	Accept E.g. • 1 month is too old for “newborn” • The mammals might not start breeding at exactly 3 months old • The mammals will stop breeding beyond a certain age • Being over 3 months old doesn’t necessarily mean the mammal can breed • Some mammals over 3 months may be infertile so will not be breeders • Some juveniles might be breeders But not • The size of the categories is different • There might be overlap • The exact age of mammals might not be known • The numbers in each category will be different • Breeding age is different for different species	B1	3.5b
		(1)	

(a)

B1: Any valid limitation – see scheme for some examples. Must refer to a feature of the categories given.

(b)(i)	$\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^2 \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} 2k \\ 0 \\ 0.96k \end{pmatrix}$ or $\begin{pmatrix} 0 & 0.96 & 1.92 \\ ab & b^2 & 2a \\ 0.48a & 0.48b + 0.96 \times 0.48 & 0.96^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix} = \begin{pmatrix} 1.92k \\ 2ak \\ 0.9216k \end{pmatrix}$	M1	3.4
	$48 = 2 \times 0.96k \Rightarrow k = \dots$		
	$k = 25$ so 25 mammals at the start of the study		
(ii)	$40 = 2ka \Rightarrow a = 0.8^*$	A1*	1.1b
		(4)	

(b)(i)

M1: Attempts to use the given information to set up a matrix equation and find the numbers of mammals after one month e.g.

$$\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_0 \\ J_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} 2B_0 \\ aN_0 + bJ_0 \\ 0.48J_0 + 0.96B_0 \end{pmatrix}$$

or attempts to square the matrix to find the number of mammals after two months e.g.

$$\begin{pmatrix} 0 & 0.96 & 1.92 \\ ab & b^2 & 2a \\ 0.48a & 0.48b + 0.96 \times 0.48 & 0.96^2 \end{pmatrix} \begin{pmatrix} N_0 \\ J_0 \\ B_0 \end{pmatrix} = \dots$$

dM1: Forms an equation, in their variable for number of breeders at the start, setting their number of newborns after 2 months equal to 48 and solves for their variable to find the initial number of breeders.

A1: For identifying 25 mammals at the start of the study. Allow 25 mammals or just 25 or e.g. $B_0 = 25$ so ignore how they label it just look for 25

Note that in some cases work may be minimal e.g.

$$\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix} = \begin{pmatrix} N_1 \\ J_1 \\ B_1 \end{pmatrix} \Rightarrow 0.96B_0 = B_1, \quad \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_1 \\ J_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} 48 \\ 40 \\ B_2 \end{pmatrix} \Rightarrow 2B_1 = 48$$

$$B_1 = 24 = 0.96B_0 \Rightarrow B_0 = 25$$

(ii)

A1*: For correctly showing $a = 0.8$. Must see the correct work to establish the correct value or equivalent by verification with a minimal conclusion e.g.

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 25 \end{pmatrix} = \begin{pmatrix} 50 \\ 0 \\ 24 \end{pmatrix} = \begin{pmatrix} 48 \\ 40 \\ B_2 \end{pmatrix} \Rightarrow 0.8 \times 50 = 40 \text{ Hence true }^*$$



(c)	$\det \begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} = 0 - 0 + 2(0.48 \times 0.8 - 0) = 0.768$	B1	2.2a
	$\text{adj} \begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} = \begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$	M1	1.1b
	$= \begin{pmatrix} 1.25b & 1.25 & -\frac{125}{48}b \\ -1 & 0 & \frac{25}{12} \\ 0.5 & 0 & 0 \end{pmatrix}$ oe e.g. $\frac{1}{0.768} \begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$ or e.g. $\frac{125}{96} \begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$	A1	1.1b
		(3)	

(c)

B1: Deduces correct determinant for the matrix. Allow equivalents e.g. $\frac{96}{125}$ May be implied.

M1: Recognisable attempt at the adjoint matrix. Look for at least 3 non-zero entries correct.

A1: Correct inverse. Accept awrt $-2.6b$ for the upper right entry and awrt 2.08 for middle right entry, or accept with determinant still outside. Apply isw once a correct answer is seen.



(d)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.25b & 1.25 & -\frac{125}{48}b \\ -1 & 0 & \frac{25}{12} \\ 0.5 & 0 & 0 \end{pmatrix} \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$	M1	3.1b
	$\text{Total} = 1.25b \times 596 + 1.25 \times 464 - \frac{125}{48}b \times 437 - 596 + \frac{25}{12} \times 437 + 0.5 \times 596$		
	$\Rightarrow 1015 = x + y + z = 745b + 580 - 1138b - 596 + 910.4 + 298 \Rightarrow b = \dots$	dM1	3.4
	$b = \text{awrt } 0.45$	A1	1.1b
		(3)	

(d) Alternative:			
	$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix} \Rightarrow \begin{aligned} 2z &= 596 \\ 0.8x + by &= 464 \\ 0.48y + 0.96z &= 437 \end{aligned}$	M1	3.1b
	$\Rightarrow z = 298, y = \frac{3773}{12} (314.4\dots)$		
	$x + y + z = 1015 \Rightarrow x = \dots \frac{4831}{12} (402.5\dots)$		
	$0.8x + by = 464 \Rightarrow 0.8 \times \frac{4831}{12} + b \times \frac{3773}{12} = 464 \Rightarrow b = \dots$	dM1	3.4
	$b = \text{awrt } 0.45$	A1	1.1b
		(3)	

(d)

M1: Attempts (their inverse matrix) $\times \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ correctly and adds the 3 expressions together to find

the total in terms of b .

M1: Sets their total = 1015 and solves for b .

A1: awrt 0.45

Alternative:

M1: Uses the original matrix with $a = 0.8$ and $\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ to form 3 equations in their variables and b

and uses these and the 1015 to find the number of Newborns.

M1: Uses their values in the y component and solves for b .

A1: awrt 0.45



(e)	Let NM_n be newborn males and NF_n be newborn females in month n $\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0.84 \\ 0 & 0 & 0 & 1.16 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$ or e.g. $\begin{pmatrix} NF_{n+1} \\ NM_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1.16 \\ 0 & 0 & 0 & 0.84 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NF_n \\ NM_n \\ J_n \\ B_n \end{pmatrix}$	M1 A1ft	3.5c 3.3
		(2)	

(13 marks)

Notes:

(e)

M1: Defines new variables for male and female newborns (accept if a clear notation is used if not defined) and sets up a 4×4 matrix with structure shown, or male and female rows swapped, with the correct 0 entries in at least 4 places.

A1ft: Fully correct matrix system shown, accepting anything (including 0) for the unknown spaces shown – but must have all the 0's **and** upper right entries correct. Accept b or their value of b in place of 0.45



Question	Scheme	Marks	AOs
1(a)	$\det(\mathbf{M}) = 3(3a - a) - 6(a^2 - 2) = 0$	M1	1.2
	$6a^2 - 6a - 12 = 0 \Rightarrow a = \dots$	dM1	1.1b
	$a = -1, 2$	A1	1.1b
		(3)	
(b)	Sets determinant = -108 , solves a 3TQ to find a negative value of a $-6a^2 + 6a + 12 = -108 \Rightarrow a = \dots$ or $6a^2 - 6a - 12 = 108 \Rightarrow a = \dots$	M1	3.1a
	$a = -4$	A1	2.2a
	$\begin{pmatrix} \frac{2}{27} & -\frac{2}{9} & -\frac{1}{18} \\ \frac{7}{54} & \frac{1}{9} & \frac{1}{36} \\ -\frac{5}{54} & -\frac{2}{9} & -\frac{11}{36} \end{pmatrix}$	A1	1.1b
		(3)	
(6 marks)			

Notes:

(a)

M1: Finds the determinant of the matrix **M** and sets = 0Allow for sight of $\pm 3(3a - a) \pm 6(a^2 - 2) \pm 0(a^2 - 6) = 0$ and do not allow sign slips in their minors.

Need not be simplified

dM1: Solves their 3TQ. Usual rules apply for solving a quadratic by any means including using a calculator.**A1:** Both correct values for a .**Note:** Correct answers without a method can be awarded M1dM1A1

Alternatively, the determinant can be found using the vector product, for example:

$$\Delta = 0$$

$$\Delta = a \cdot b \times c$$

$$0 = \begin{pmatrix} 3 \\ a \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 3 \\ a \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ a \end{pmatrix}$$

$$0 = \begin{pmatrix} 3 \\ a \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3a - a \\ -6a \\ 6 \end{pmatrix}$$

$$0 = \begin{pmatrix} 3 \\ a \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2a - a \\ -6a \\ 6 \end{pmatrix}$$

$$0 = 6a - 6a^2 + 12$$

If you are uncertain, please send to review

(b)

M1: A complete method to find a value for a . Sets **their** determinant equal to -108 , solves a 3TQ and proceeds to find at least one negative value of a .They must use their determinant equal to -108 and not for example a changed quadratic from (a) equal to -108 . Usual rules apply for solving a quadratic by any means including using a calculator.**A1:** Deduces $a = -4$ **only**, which could be implied by a correct matrix.**A1:** Correct matrix. Accept exact equivalents but not answers rounded to decimal places. Isw once a correct matrix is seen.

$$\text{e.g. Accept } \frac{1}{108} \begin{pmatrix} 8 & -24 & -6 \\ 14 & 12 & 3 \\ -10 & -24 & -33 \end{pmatrix} \quad \text{or} \quad -\frac{1}{108} \begin{pmatrix} -8 & 24 & 6 \\ -14 & -12 & -3 \\ 10 & 24 & 33 \end{pmatrix}$$

