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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Algebra and Functions (4)

Core Pure Mathematics 1 · Topic 4

5 questions · 31 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
4.1 ROOTS OF POLYNOMIALS		1 Qs · 5 marks	3
1. Q6 <i>also Topic 2</i>	Paper 1: Core Pure Mathematics 1 June2025	5	4
4.4 METHOD OF DIFFERENCES		2 Qs · 12 marks	8
2. Q4 <i>also Topic 1</i>	Paper 1: Core Pure Mathematics 1 June2022	7	9
3. Q5	Paper 1: Core Pure Mathematics 1 June2025	5	13
4.6 STANDARD MACLAURIN SERIES		1 Qs · 7 marks	15
4. Q2 <i>also Topic 5</i>	Paper 1: Core Pure Mathematics 1 November2021	7	16
5.4 PARTIAL FRACTIONS WITH QUADRATIC FACTORS		1 Qs · 7 marks	20
5. Q6	Paper 1: Core Pure Mathematics 1 June2022	7	21



QUESTIONS

5 questions · 31 marks · 4 sub-topics



SUB-TOPIC 4.1

ROOTS OF POLYNOMIALS

Sub-topic 4.1 · 1 question · 5 marks · lowest tariff first



6. $f(z) = z^3 + az^2 + bz + c$ where a , b and c are real constants

The roots of the equation $f(z) = 0$ are z_1 , z_2 and z_3

When plotted on an Argand diagram, the points representing these roots form the vertices of a triangle.

Given that

- $z_1 = 2 + 4i$
- the area of the triangle is 12

Determine the two possible functions $f(z)$

(5)





Question 6 continued



Question 6 continued

SUB-TOPIC 4.4

METHOD OF DIFFERENCES

Sub-topic 4.4 · 2 questions · 12 marks · lowest tariff first



4. (a) Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^n \ln\left(\frac{r+1}{r-1}\right) \equiv \ln\left(\frac{n(n+1)}{2}\right)$$

(4)

- (b) Hence find the exact value of

$$\sum_{r=51}^{100} \ln \left(\frac{r+1}{r-1} \right)^{35}$$

Give your answer in the form $a \ln\left(\frac{b}{c}\right)$ where a , b and c are integers to be determined.

(3)







Question 5 continued

SUB-TOPIC 4.6

STANDARD MACLAURIN SERIES

Sub-topic 4.6 · 1 question · 7 marks · lowest tariff first







(Total for Question 2 is 7 marks)



SUB-TOPIC 5.4

PARTIAL FRACTIONS WITH QUADRATIC FACTORS

Sub-topic 5.4 · 1 question · 7 marks · lowest tariff first



6. (a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x + 1)(x^2 + 4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x+1)(x^2+4)} dx = \ln(a\sqrt{2}) + b\pi$$

where a and b are constants to be determined.

(4)

Question 6 continued





Question 6 continued

(Total for Question 6 is 7 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
6	Method 1: using roots		
	$z_2 = 2 - 4i$	B1	1.1b
	$12 = \frac{1}{2}(4 - -4) \times h \Rightarrow h = \dots \Rightarrow z_3 = 2 + h \text{ or } 2 - h$	M1	3.1a
	$(z - (2 - 4i))(z - (2 + 4i))(z - 5)$ or $(z - (2 - 4i))(z - (2 + 4i))(z - (-1))$	dM1	2.1
	$(f(z)) = z^3 - 9z^2 + 40z - 100$ $(f(z)) = z^3 - 3z^2 + 16z + 20$	ddM1 A1	1.1b 2.2a
		(5)	
	Method 2: using sum, pair sum and product of roots		
	$z_2 = 2 - 4i$	B1	1.1b
	$12 = \frac{1}{2}(4 - -4) \times h \Rightarrow h = \dots \Rightarrow z_3 = 2 + h \text{ or } 2 - h$	M1	3.1a
	Sum = $2 + 4i + 2 - 4i + "5" = 9 = -a$ Pair sum = $(2 + 4i)(2 - 4i) + "5"(2 + 4i) + "5"(2 - 4i) = 40 = b$ Product = $"5"(2 + 4i)(2 - 4i) = 100 = -c$ OR Sum = $2 + 4i + 2 - 4i + "(-1)" = 3 = -a$ Pair sum = $(2 + 4i)(2 - 4i) + "(-1)(2 + 4i) + "(-1)(2 - 4i) = 16 = b$ Product = $"(-1)(2 + 4i)(2 - 4i) = -20 = -c$	dM1	2.1
	$(f(z)) = z^3 - 9z^2 + 40z - 100$ $(f(z)) = z^3 - 3z^2 + 16z + 20$	ddM1 A1	1.1b 2.2a
		(5)	
(5 marks)			

Notes:

Method 1:

B1: States the other complex root. May be implied by later working.

M1: Uses the area of the triangle to determine at least one value for the real root. You can award for sight of one correct value of z_3

(Note: $h = 3 \Rightarrow z_3 = 5$ or -1)

dm1: Dependent on the previous method mark. Deduces a correct expression for one of their real roots. Award for $(z - (2 - 4i))(z - (2 + 4i))(z - z_3)$ or equivalent e.g. $(z^2 - 4z + 20)(z - z_3)$

z_3 must be real.

ddm1: Dependent on previous method marks. Multiplies out to find values for the constants a , b and c . Award for the sight of at least 2 correct values of the constants a , b and c in any one of their attempts at the function.

A1: Deduces the correct two functions.

Method 2:

B1: States the other complex root. May be implied by later working.

M1: Uses the area of the triangle to determine at least one value for the real root, You can award for sight of one correct value of z_3 (Note: $h = 3 \Rightarrow z_3 = 5$ or -1)

dm1: Dependent on the previous method mark. Attempts sum, pair sum and product for at least one of their real roots.

ddm1: Dependent on previous method marks. Multiplies out to find values for the constants a , b and c

Award for the sight of at least 2 correct values of the constants a , b and c in any one of their attempts at the function.

A1: Deduces the correct two functions.

Note: There are several methods that can be used to obtain the first method mark:

- Uses vertices $(2, 4)$, $(2, -4)$ and $(x_3, 0)$ to determine at least one value for the real root. Alternatively uses (x_3, y_3) . This may come from a diagram. Do not be concerned about the variables they use.
e.g. $12 = \frac{1}{2} \times 8 \times |x_3 - 2| \Rightarrow z_3 =$ or $x_3 =$
- Uses the formula for the area of a triangle with vertices (x_1, y_1) , (x_2, y_2) , (x_3, y_3) to determine at least one value for the real root. Do not be concerned about the variables they use.

$$\left[\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \right]$$

$$12 = \frac{1}{2} |2(-4 - y_3) + 2(y_3 - 4) + x_3(4 - -4)|$$

$$24 = |-16 + 8x_3|$$

$$\Rightarrow z_3 =$$
 or $x_3 =$
- Uses the shoelace method with no sign errors.

$$\text{e.g. Area} = \frac{1}{2} \begin{vmatrix} 2 & 2 & x_3 & 2 \\ 4 & -4 & 0 & 4 \end{vmatrix}$$

$$12 = \frac{1}{2} |(2 \times -4 + 4x_3) - (2 \times 4 - 4x_3)|$$

$$24 = |-16 + 8x_3|$$

$$\Rightarrow z_3 = \text{ or } x_3 =$$

If you are uncertain in how to apply a method, then please send to review.



Question	Scheme	Marks	AOs
4(a)	Applies $\ln\left(\frac{r+1}{r-1}\right) = \ln(r+1) - \ln(r-1)$ to the problem in order to apply differences.	M1	3.1a
	$\sum_{r=2}^n (\ln(r+1) - \ln(r-1))$ $= (\ln(3) - \ln(1)) + (\ln(4) - \ln(2)) + (\ln(5) - \ln(3)) + \dots$ $+ (\ln(n) - \ln(n-2)) + (\ln(n+1) - \ln(n-1))$	dM1	1.1b
	$\ln(n) + \ln(n+1) - \ln 2$	A1	1.1b
	$\ln\left(\frac{n(n+1)}{2}\right) * \text{cso}$	A1 *	2.1
		(4)	
(b)	$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right) = \sum_{r=2}^{100} \ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} \ln\left(\frac{r+1}{r-1}\right)$ $= \ln\left(\frac{100 \times 101}{2}\right) - \ln\left(\frac{50 \times 51}{2}\right)$	M1	1.1b
	$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35} = 35 \ln\left(\frac{100 \times 101}{2} \div \frac{50 \times 51}{2}\right)$	M1	3.1a
	$= 35 \ln\left(\frac{202}{51}\right)$	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a) M1: Uses the subtraction laws of logs to start the method of differences process. dM1: Demonstrates the method of differences process, should have a minimum of e.g. $r = 2, r = 3, r = 4, r = n - 1$ and $r = n$ shown -- enough to establish <i>at least one cancelling term</i> and <i>all non-disappearing terms</i> though the latter may be implied by correct extraction if only the first few cases are shown. Allow this mark if an extra term for $r = 1$ has been included. A1: Correct terms that do not cancel - must not contradict their list of terms so e.g. if $r = 1$ was included, then A0A0 follows. The $\ln 1$ may be included for this mark. A1*: Achieves the printed answer, with no errors or omissions and must have had a complete list (as per dM1) before extraction (but condone missing brackets on $\ln$ terms). If working with r throughout, they must replace by n to gain the last A, but all other marks are available. NB For attempts at combining log terms instead of using differences, full marks may be awarded for the equivalent steps, but attempts that do not make progress in combining terms will score no marks. (b) Condone a bottom limit of 0 or 1 being used throughout part (b). M1: Attempts to split into (the sum up to 100) – (the sum up to k) where k is 49, 50 or 51 and apply the result of (a) in some way. Condone slips with the power. M1: Having attempted to apply (a), uses difference and power log laws correctly to reach an expression of the required form. A1: Correct answer. Accept equivalents in required form, such as $35 \ln \frac{5050}{1275}$			

Question	Scheme	Marks	AOs
5	$\frac{4}{r^2-1} \equiv \frac{4}{(r-1)(r+1)} \equiv \frac{A}{r-1} + \frac{B}{r+1} \Rightarrow A = \dots, B = \dots$ (NB $A = 2, B = -2$)	M1	3.1a
	$r = 2 \quad \Rightarrow \frac{2}{1} - \frac{2}{3}$	dM1	2.1
	$r = 3 \quad \Rightarrow \frac{2}{2} - \frac{2}{4}$		
	$r = 4 \quad \Rightarrow \frac{2}{3} - \frac{2}{5}$		
	$r = n-1 \quad \Rightarrow \frac{2}{n-2} - \frac{2}{n}$		
	$r = n \quad \Rightarrow \frac{2}{n-1} - \frac{2}{n+1}$		
	$2 + 1 - \frac{2}{n} - \frac{2}{n+1}$	A1	1.1b
	$\frac{3n(n+1) - 2(n+1) - 2n}{n(n+1)} = \frac{3n^2 + 3n - 2n - 2 - 2n}{n(n+1)} = \frac{3n^2 - n - 2}{n(n+1)}$	M1	1.1b
	$\frac{(3n+2)(n-1)}{n(n+1)} \text{ cso}$	A1	2.2a
		(5)	
(5 marks)			

Notes:

M1: A complete strategy to write as partial fractions

dM1: Dependent on previous method mark. Completes the method of differences process, must have a minimum substitution of $r = 2$, $r = 3$, $r = n - 1$ and $r = n$. Condone the use of r for n for this mark.

If pairs of terms are not seen in each substitution for r , this mark can be implied by sight of

$$2 + 1 - \frac{2}{n} - \frac{2}{n+1} \text{ following correct partial fractions.}$$

A1: Correct ‘fractions’ from the beginning and end that do not cancel. Accept exact equivalents.

Must be in terms of n

M1: Combines all their ‘fractions’ (at least two algebraic fractions) over a correct common denominator, leading to a quadratic in the numerator, where brackets may not be expanded. Condone the use of r for n for this mark.

A1: Since the form of the solution is given, only accept $\frac{(3n+2)(n-1)}{n(n+1)}$ and not equivalents, but isw

Do not accept values for p and q written alone.

Note: If they start with $r = 0$, $r = 1$ or $r = 3$, the maximum they can score is M1M0A0M1A0

Similarly, if they use $r = n + 1$ the maximum they can score is M1M0A0M1A0



Question	Scheme	Marks	AOs
2 (a)	$\cos^2 \frac{x}{3} = \left(1 - \frac{\left(\frac{x}{3}\right)^2}{2} + \frac{\left(\frac{x}{3}\right)^4}{24} - \dots \right)^2 = \dots \quad \text{or} \quad \left(1 - \frac{x^2}{18} + \frac{x^4}{1944} - \dots \right)^2 = \dots \quad \text{or}$ $\frac{1}{2} \left(1 \pm \cos \frac{2x}{3} \right) = \frac{1}{2} \left(1 \pm \left(1 - \frac{1}{2} \left(\frac{2x}{3} \right)^2 + \frac{1}{4!} \left(\frac{2x}{3} \right)^4 - \dots \right) \right)$	M1	2.2a
	$= 1 - \frac{x^2}{9} + \frac{1}{243} x^4$	A1	1.1b
		(2)	
(b)	$\int \frac{1 - \frac{x^2}{9} + \frac{1}{243} x^4}{x} = \int \frac{1}{x} - \frac{x}{9} + \frac{1}{243} x^3 = A \ln x + Bx^2 + Cx^4$ where A, B and $C \neq 0$	M1	3.1a
	$\ln x - \frac{x^2}{18} + \frac{1}{972} x^4$	A1ft	1.1b
	$= \text{awrt } 0.98295$	A1	2.2a
		(3)	
(c)	Calculator = awrt 0.98280	B1	1.1b
		(1)	
(d)	E.g. the approximation is correct to 3 d.p.	B1	3.2b
		(1)	
(7 marks)			
Notes:			
(a) M1: Deduces the required series by using the Maclaurin series for $\cos x$, replacing x with $\frac{x}{3}$ and squares, or first applying the double angle identity (allow sign error) and then applying the series for $\cos x$ with $\frac{2x}{3}$. Attempts at finding from differentiation score M0 as the cosine series is required. A1: Correct series			
(b) M1: Divides their series in part (a) by x and integrates to the form $A \ln x + Bx^2 + Cx^4$ A1ft: Correct integration, follow through on their coefficients and need not be simplified. A1: Deduces the definite integral awrt 0.98295			
(c) B1: Correct value.			
(d) B1: Makes a quantitative statement about the accuracy, so e.g. how many decimal places or significant figures it is correct to, or calculates a percentage accuracy to deduce it is reasonable. Do not accept just “underestimate” or similar without quantitative evidence. Allow for a reasonable comment as long as (b) is correct to at least 2 s.f. but (c) must be the correct value.			

Question	Scheme	Marks	AOs
6(a)	$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \Rightarrow 2x^2 + 3x + 6 = A(x^2+4) + (Bx+C)(x+1)$	M1	1.1b
	e.g. $x = -1 \Rightarrow A = \dots$, $x = 0 \Rightarrow C = \dots$, coeff $x^2 \Rightarrow B = \dots$ or Compares coefficients and solves to find values for A , B and C $2 = A + B$, $3 = B + C$, $6 = 4A + C$	dM1	1.1b
	$A = 1$, $B = 1$, $C = 2$	A1	1.1b
		(3)	
(b)	$\int_0^2 \frac{1}{x+1} + \frac{x+2}{x^2+4} dx = \int_0^2 \frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4} dx$ $= \left[\alpha \ln(x+1) + \beta \ln(x^2+4) + \lambda \arctan\left(\frac{x}{2}\right) \right]_0^2$	M1	3.1a
	$= \left[\ln(x+1) + \frac{1}{2} \ln(x^2+4) + \arctan\left(\frac{x}{2}\right) \right]_0^2$	A1	2.1
	$= \left[\ln(3) + \frac{1}{2} \ln(8) + \arctan 1 \right] - \left[\ln(1) + \frac{1}{2} \ln(4) + \arctan(0) \right]$ $= \left[\ln(3) + \frac{1}{2} \ln(8) + \arctan(1) \right] - \left[\frac{1}{2} \ln 4 \right] = \ln\left(\frac{3\sqrt{8}}{2}\right) + \frac{\pi}{4}$	dM1	2.1
	$\ln(3\sqrt{2}) + \frac{\pi}{4}$	A1	2.2a
		(4)	
(7 marks)			
Notes:			
(a) M1: Selects the correct form for partial fractions and multiplies through to form suitable identity or uses a method to find at least one value (e.g. cover up rule). dM1: Full method for finding values for all three constants. Dependent on first M. Allow slips as long as the intention is clear. A1: Correct constants or partial fractions.			
(b) M1: Splits the integral into an integrable form and integrates at least two terms to the correct form. They may use a substitution on the arctan term A1: Fully correct Integration. dM1: Uses the limits of 0 and 2 (or appropriate for a substitution), subtracts the correct way round and combines the ln terms from separate integrals to a single term with evidence of correct ln laws at least once. A1: Correct answer			