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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Calculus (5)

Core Pure Mathematics 1 · Topic 5

8 questions · 71 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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4.6 STANDARD MACLAURIN SERIES		1 Qs · 7 marks	3
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QUESTIONS

8 questions · 71 marks · 7 sub-topics



SUB-TOPIC 4.6

STANDARD MACLAURIN SERIES

Sub-topic 4.6 · 1 question · 7 marks · lowest tariff first







(Total for Question 2 is 7 marks)



SUB-TOPIC 5.1

VOLUMES OF REVOLUTION

Sub-topic 5.1 · 2 questions · 24 marks · lowest tariff first



8. (a) Given

$$z^n + \frac{1}{z^n} = 2 \cos n\theta \quad n \in \mathbb{N}$$

show that

$$32 \cos^6 \theta \equiv \cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10 \quad (5)$$

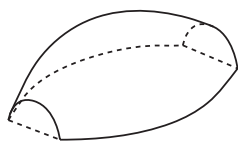


Figure 1

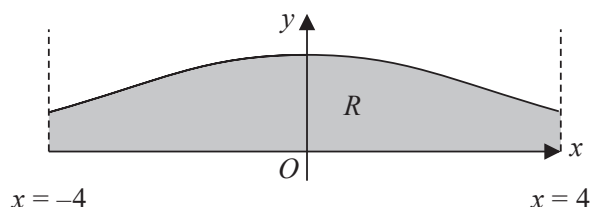


Figure 2

Figure 1 shows a solid paperweight with a flat base.

Figure 2 shows the curve with equation

$$y = H \cos^3 \left(\frac{x}{4} \right) \quad -4 \leq x \leq 4$$

where H is a positive constant and x is in radians.

The region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = -4$, the line with equation $x = 4$ and the x -axis.

The paperweight is modelled by the solid of revolution formed when R is rotated 180° about the x -axis.

Given that the maximum height of the paperweight is 2 cm,

(b) write down the value of H . (1)

(c) Using algebraic integration and the result in part (a), determine, in cm^3 , the volume of the paperweight, according to the model. Give your answer to 2 decimal places.

[Solutions based entirely on calculator technology are not acceptable.] (5)

(d) State a limitation of the model. (1)





(Total for Question 6 is 12 marks)



10. (a) Given that, for $n \in \mathbb{N}$

$$z^n + \frac{1}{z^n} = 2 \cos n\theta$$

$$z^n - \frac{1}{z^n} = 2i \sin n\theta$$

show that

$$8 \sin^4 \theta \equiv \cos 4\theta - 4 \cos 2\theta + 3 \quad (5)$$



Figure 1

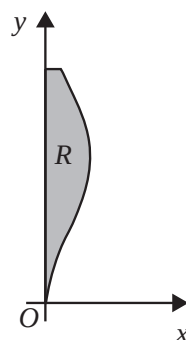


Figure 2

Figure 1 shows the central vertical cross-section of a solid wooden ornament.

Figure 2 shows the curve with equation

$$x = \sin^2\left(\frac{1}{2}y\right) \quad 0 \leq y \leq \frac{8\pi}{5}$$

The region R , shown shaded in Figure 2, is bounded by the curve, the line with

equation $y = \frac{8\pi}{5}$ and the y -axis.

The ornament is modelled by the solid of revolution formed when R is rotated 360° about the y -axis. The units are centimetres.

- (b) Using algebraic integration and the result in part (a), determine, in cm^3 , the volume of wood needed to make the ornament, according to the model. Give your answer to 2 significant figures.

[Solutions based entirely on calculator technology are not acceptable.]

(5)

Given that

- the density of the wood is 0.85 g/cm^3
- the mass of the ornament is 6 grams

- (c) comment on the suitability of the model.

(2)







SUB-TOPIC 5.2

IMPROPER INTEGRALS

Sub-topic 5.2 · 1 question · 6 marks · lowest tariff first



(1)

(3)

(2)



(Total for Question 9 is 6 marks)



SUB-TOPIC 5.3

MEAN VALUE OF A FUNCTION

Sub-topic 5.3 · 1 question · 7 marks · lowest tariff first



(Total for Question 5 is 7 marks)

SUB-TOPIC 5.4

PARTIAL FRACTIONS WITH QUADRATIC FACTORS

Sub-topic 5.4 · 1 question · 8 marks · lowest tariff first



7. (a) Express

$$\frac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)}$$

in partial fractions.

(4)

(b) Hence, show that

$$\int_0^1 \frac{2x^3 + 10x^2 + 9x + 22}{(x+2)(x^2+3)} dx = 2 + \ln\left(\frac{27}{4}\right) - \frac{1}{6\sqrt{3}}$$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

SUB-TOPIC 5.6

INTEGRATION USING TRIG SUBSTITUTIONS

Sub-topic 5.6 · 1 question · 11 marks · lowest tariff first



9. (a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2 - 1}} dx = \frac{1}{2} \left[x\sqrt{x^2 - 1} + \operatorname{arcosh} x \right] + k$$

where k is an arbitrary constant.

(6)

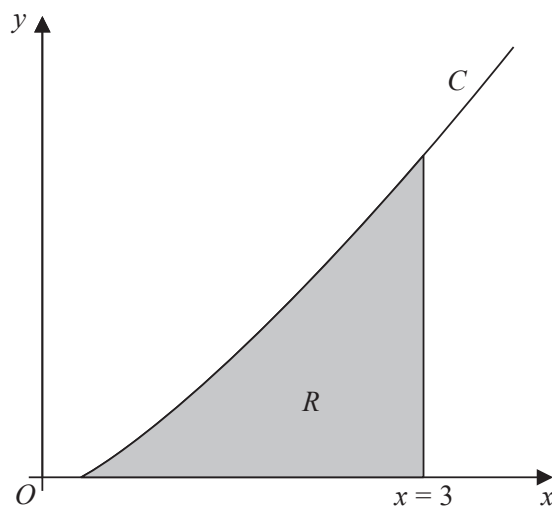


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{4}{15} x \operatorname{arcosh} x \quad x \geq 1$$

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the x -axis and the line with equation $x = 3$

- (b) Using algebraic integration and the result from part (a), show that the area of R is given by

$$\frac{1}{15} \left[17 \ln(3 + 2\sqrt{2}) - 6\sqrt{2} \right] \quad (5)$$

Question 9 continued

Question 9 continued



Question 9 continued





SUB-TOPIC 7.3

AREAS AND TANGENTS OF POLAR CURVES

Sub-topic 7.3 · 1 question · 8 marks · lowest tariff first



3.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

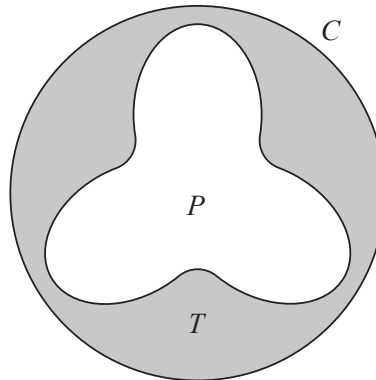


Figure 1

Figure 1 shows the design for a bathing pool.

The pool, P , shown unshaded in Figure 1, is surrounded by a tiled area, T , shown shaded in Figure 1.The tiled area is bounded by the edge of the pool and by a circle, C , with radius 6 m.

The centre of the pool and the centre of the circle are the same point.

The edge of the pool is modelled by the curve with polar equation

$$r = 4 - a \sin 3\theta \quad 0 \leq \theta \leq 2\pi$$

where a is a positive constant.Given that the shortest distance between the edge of the pool and the circle C is 0.5 m,(a) determine the value of a .

(2)

(b) Hence, using algebraic integration, determine, according to the model, the exact area of T .

(6)





MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
2 (a)	$\cos^2 \frac{x}{3} = \left(1 - \frac{\left(\frac{x}{3}\right)^2}{2} + \frac{\left(\frac{x}{3}\right)^4}{24} - \dots \right)^2 = \dots \quad \text{or} \quad \left(1 - \frac{x^2}{18} + \frac{x^4}{1944} - \dots \right)^2 = \dots \quad \text{or}$ $\frac{1}{2} \left(1 \pm \cos \frac{2x}{3} \right) = \frac{1}{2} \left(1 \pm \left(1 - \frac{1}{2} \left(\frac{2x}{3} \right)^2 + \frac{1}{4!} \left(\frac{2x}{3} \right)^4 - \dots \right) \right)$	M1	2.2a
	$= 1 - \frac{x^2}{9} + \frac{1}{243} x^4$	A1	1.1b
		(2)	
(b)	$\int \frac{1 - \frac{x^2}{9} + \frac{1}{243} x^4}{x} = \int \frac{1}{x} - \frac{x}{9} + \frac{1}{243} x^3 = A \ln x + Bx^2 + Cx^4$ where A, B and $C \neq 0$	M1	3.1a
	$\ln x - \frac{x^2}{18} + \frac{1}{972} x^4$	A1ft	1.1b
	$= \text{awrt } 0.98295$	A1	2.2a
		(3)	
(c)	Calculator = awrt 0.98280	B1	1.1b
		(1)	
(d)	E.g. the approximation is correct to 3 d.p.	B1	3.2b
		(1)	
(7 marks)			
Notes:			
(a) M1: Deduces the required series by using the Maclaurin series for $\cos x$, replacing x with $\frac{x}{3}$ and squares, or first applying the double angle identity (allow sign error) and then applying the series for $\cos x$ with $\frac{2x}{3}$. Attempts at finding from differentiation score M0 as the cosine series is required. A1: Correct series			
(b) M1: Divides their series in part (a) by x and integrates to the form $A \ln x + Bx^2 + Cx^4$ A1ft: Correct integration, follow through on their coefficients and need not be simplified. A1: Deduces the definite integral awrt 0.98295			
(c) B1: Correct value.			
(d) B1: Makes a quantitative statement about the accuracy, so e.g. how many decimal places or significant figures it is correct to, or calculates a percentage accuracy to deduce it is reasonable. Do not accept just “underestimate” or similar without quantitative evidence. Allow for a reasonable comment as long as (b) is correct to at least 2 s.f. but (c) must be the correct value.			

Question	Scheme	Marks	AOs
8(a)	$\left(z + \frac{1}{z}\right)^6 = 64 \cos^6 \theta$	B1	2.1
	$\left(z + \frac{1}{z}\right)^6 = z^6 + 6(z^5)\left(\frac{1}{z}\right) + 15(z^4)\left(\frac{1}{z^2}\right) + 20(z^3)\left(\frac{1}{z^3}\right)$ $+ 15(z^2)\left(\frac{1}{z^4}\right) + 6(z)\left(\frac{1}{z^5}\right) + \left(\frac{1}{z^6}\right)$	M1	2.1
	$= \left[z^6 + \frac{1}{z^6}\right] + 6\left[z^4 + \frac{1}{z^4}\right] + 15\left[z^2 + \frac{1}{z^2}\right] + 20$	A1	1.1b
	Uses $z^n + \frac{1}{z^n} = 2 \cos n \theta$ $\{64 \cos^6 \theta\} = 2 \cos 6 \theta + 12 \cos 4 \theta + 30 \cos 2 \theta + 20$	M1	2.1
	$32 \cos^6 \theta = \cos 6 \theta + 6 \cos 4 \theta + 15 \cos 2 \theta + 10 * \text{cso}$	A1 *	1.1b
		(5)	
(b)	$H = 2$	B1	3.3
		(1)	
(c)	$\text{vol} = \left\{\frac{1}{2}\right\} \pi \int \left(2 \cos^3 \left(\frac{x}{4}\right)\right)^2 dx$	B1ft	3.4
	$\text{vol} = \{2\pi\} \int \cos^6 \left(\frac{x}{4}\right) dx$ $= \{2\pi\} \int \frac{1}{32} \left(\cos \left(\frac{6x}{4}\right) + 6 \cos \left(\frac{4x}{4}\right) + 15 \cos \left(\frac{2x}{4}\right) + 10\right) dx = \dots$	M1	1.1b
	$= \{2\pi\} \left[\frac{1}{32} \left(\frac{2}{3} \sin \left(\frac{3x}{2}\right) + 6 \sin(x) + 30 \sin \left(\frac{x}{2}\right) + 10x \right) \right]$	A1	1.1b
	$= 2 \times 2\pi \left[\frac{1}{32} \left(\frac{2}{3} \sin \left(\frac{3}{2} \times 4\right) + 6 \sin(4) + 30 \sin \left(\frac{4}{2}\right) + (10 \times 4) \right) - 0 \right] = \dots$ or = $2\pi \left[\frac{1}{32} \left(\frac{2}{3} \sin \left(\frac{3}{2} \times 4\right) + 6 \sin(4) + 30 \sin \left(\frac{4}{2}\right) + (10 \times 4) \right) - \frac{1}{32} \left(\frac{2}{3} \sin \left(\frac{3}{2} \times -4\right) + 6 \sin(-4) + 30 \sin \left(-\frac{4}{2}\right) + (10 \times -4) \right) \right]$...	dM1	3.4
	$= 24.56$	A1	1.1b
		(5)	

(d)	The equation of the curve may not be suitable The measurements may not be accurate The paperweight may not be smooth	B1	3.5b
		(1)	
(12 marks)			
Notes:			
(a) B1: Correct identity or equivalent rearrangement. This can appear anywhere in the proof. M1: Attempts the expansion of $\left(z + \frac{1}{z}\right)^6$ must have at least 3 correct terms. Combining the powers when expanding is fine. A1: Correct expansion with z terms simplified, need not be rearranged. (So a correct expansion will score M1A1.) M1: Uses $z^n + \frac{1}{z^n} = 2 \cos n \theta$ to write the expression in terms multiple angles of $\cos 6 \theta$, $\cos 4 \theta$ and $\cos 2 \theta$. Pairing of terms must be seen. A1*: Achieves the printed answer with no errors or omissions. Cso For approaches using De Moivre B0M1A1M0A0 may be scored if the binomial expansions is attempted (and correct for the A).			
(b) B1: See scheme			
(c) Note: The question instructs use of algebraic integration and part (a), so answer only can score at most B1 for implied correct formula. B1ft: Correct expression for the volume of the paperweight or the solid formed through 360° rotation, stated or implied, ignore limits. No need to expand, but must be applied, not just a formula in y, though allow a correct formula followed by correct integral if the π disappears. Follow through their H M1: Uses the result in part (a) to express the volume in an integrable form and attempts to integrate. Note use of θ instead of x is permissible for this mark. Allow if one term is missing or miscopied. A1: Correct integration in terms of x. Ignore π, their H^2 and the $\frac{1}{32}$. Note if θ has been used it is A0 unless a correct substitution method has been implied as the coefficients will be incorrect. dM1: Dependent on previous method mark and must have reached and integral of the correct form - in terms of x with correct arguments allowing for one slip. Finds the required volume using either $\pi \int_0^4 y^2 dx$ or $\frac{1}{2} \pi \int_{-4}^4 y^2 dx$ and applies their limits - accept any value following a valid attempt at the integration as an attempt at applying limits. A1: cao 24.56			
(d) B1: States an appropriate limitation. See scheme for some examples. The limitation should refer to the paperweight, not to paper. Do not accept “it does not take into account thickness of material” as it is a solid, not a shell, being modelled. Award the mark for a correct reason if two reasons are given and one is incorrect.			

Question	Scheme	Marks	AOs
10(a)	$\left(z - \frac{1}{z}\right)^4 = (2i\sin\theta)^4 = 16\sin^4\theta$	B1	1.1b
	$\left(z - \frac{1}{z}\right)^4 = z^4 + 4\left(z^3\right)\left(-\frac{1}{z}\right) + 6\left(z^2\right)\left(-\frac{1}{z}\right)^2 + 4\left(z\right)\left(-\frac{1}{z}\right)^3 + \left(-\frac{1}{z}\right)^4$	M1	2.1
	$= \left[z^4 + \frac{1}{z^4}\right] - 4\left[z^2 + \frac{1}{z^2}\right] + 6$	A1	1.1b
	Uses $z^n + \frac{1}{z^n} = 2\cos n\theta$ $\{16\sin^4\theta\} \equiv 2\cos 4\theta - 8\cos 2\theta + 6$	M1	2.1
	$8\sin^4\theta \equiv \cos 4\theta - 4\cos 2\theta + 3$ * cso	A1 *	1.1b
		(5)	
(b)	$\text{vol} = \pi \int \left(\sin^2\left(\frac{1}{2}y\right)\right)^2 dy$	B1	3.4
	$\text{vol} = \{\pi\} \int \sin^4\left(\frac{1}{2}y\right) (dy)$	M1	1.1b
	$= \{\pi\} \int \frac{1}{8}(\cos(2y) - 4\cos(y) + 3) (dy) = \dots$		
	$= \{\pi\} \left[\frac{1}{8} \left(\frac{1}{2} \sin(2y) - 4\sin(y) + 3y \right) \right]$	A1	1.1b
	$= \pi \left[\frac{1}{8} \left(\frac{1}{2} \sin\left(2 \times \frac{8\pi}{5}\right) - 4\sin\left(\frac{8\pi}{5}\right) + \left(3 \times \frac{8\pi}{5}\right) \right) - 0 \right]$ $= \dots$	dM1	3.4
	awrt 7.3 (cm ³)	A1	1.1b
		(5)	
(c)	Mass = “7.3” x 0.85 $= \dots$	M1	2.2b
	Mass = 6.2 (grams) therefore a good model	A1ft	3.5a
		(2)	
	Alternative 1		
	Volume = $6 \div 0.85 = \dots$	M1	2.2b
	Volume = 7.1 (cm ³) therefore a good model	A1ft	3.5a
		(2)	



	Alternative 2		
	Density = $6 \div 7.3$	M1	2.2b
	Density = $0.82 \text{ (g/cm}^3\text{)}$ therefore a good model	A1ft	3.5a
		(2)	
(12 marks)			



Notes:**(a)****B1:** See scheme. This can appear anywhere in the proof.Accept $2^4 \sin^4 \theta$ for $16 \sin^4 \theta$, but not $(2 \sin \theta)^4$

Alternatively, they may instead substitute $\sin \theta = \frac{1}{2i} \left(z - \frac{1}{z} \right)$ into the expression $8 \sin^4 \theta$. This can be implied but must come from correct work. This can appear anywhere in their proof.

e.g. $8 \sin^4 \theta = 8 \left[\frac{1}{2i} \left(z - \frac{1}{z} \right) \right]^4$ or $8 \sin^4 \theta = 8 \left[\left(z - \frac{1}{z} \right) \right]^4 \frac{1}{2^4}$

and allow $8 \sin^4 \theta = \frac{1}{2} \left[\left(z - \frac{1}{z} \right) \right]^4$

M1: Finds the expansion of $\left(z - \frac{1}{z} \right)^4$ which may be unsimplified. All five terms must be present.

Condone sign slips only

A1: Correct expansion, with terms grouped.

M1: Uses $z^n + \frac{1}{z^n} = 2 \cos n\theta$ to write in terms of $\cos 4\theta$ and $\cos 2\theta$

A1*: Achieves the printed answer with no errors or omissions. Cso Follows A0.**(b)**

B1: Correct formula $\pi \int \left(\sin^2 \left(\frac{1}{2} y \right) \right)^2 dy$ (but not $\text{vol} = \pi \int x^2 dy$) used to find a volume, stated or

implied, ignore limits. If there is a missing π or dy in their integral, then withhold this mark only. However, this mark may be awarded if π and dy are seen together later in their integral.

Do not award the following marks if algebraic integration is not used. For example finding an answer of 7.3 without algebraic integration will obtain M0A0dM0A0

M1: Uses the result in part (a) to express the volume in an integrable form and attempts to integrate.

Award for an integral of the form $\int \frac{1}{8} (A \cos(2y) + B \cos(y) + C) (dy)$ with at least one term integrated correctly. Do not be concerned if they use a different variable such as θ for y .

Special Case:**If they have not used part (a) and instead use the double angle formulae:**

$$\sin^2 \alpha = \frac{1}{2} - \frac{1}{2} \cos 2\alpha \quad \text{and} \quad \sin^4 \alpha = \left(\frac{1}{2} - \frac{1}{2} \cos 2\alpha \right)^2 \quad \text{with} \quad \alpha = \frac{1}{2} y$$

In this case they must obtain an exact integral equivalent to $\int \frac{1}{8} (\cos(2y) - 4 \cos(y) + 3) (dy)$ and proceed to integrate at least one term correctly.

A1: Correct integration. May be in terms of another variable such as θ . Ignore π 

dM1: Dependent on previous method mark. Finds the required volume using $\pi \int_0^{\frac{8\pi}{5}} x^2 dy$ and applies

their limits to their integral and subtracts the correct way round.

If there are no limits seen substituted, then a correct final answer implies the correct use of limits and the inclusion of π . This is provided they have already achieved an integrated expression of

$\frac{1}{8} \left(\frac{1}{2} \sin(2y) - 4 \sin(y) + 3y \right)$ or with correct limits seen, possibly on their integral.

If their integration is incorrect, then there must be evidence of substituting both limits in each of their terms and subtracting. Allow the omission of subtracting zero provided their integration would produce zero for the lower limit.

A1: awrt 7.3

(c)

M1: Finds the mass of the ornament by multiplying their volume by 0.85

A1ft: Draws an appropriate conclusion about the suitability of the model, comparing their two masses.

If the masses differ by 10% then they must conclude it is a good model.

If the masses differ between 10% and 20% then they may conclude it is either a good model or poor model.

If the masses differ by greater than 20% then they must conclude it is a poor model.

Alternative 1

M1: Finds the volume of the ornament by dividing the mass of 6 grams by 0.85 g/cm^3

A1ft: Draws an appropriate conclusion about the suitability of the model, comparing their two volumes.

If the volumes differ by 10% then they must conclude it is a good model.

If the volumes differ between 10% and 20% then they may conclude it is either a good model or poor model.

If the volumes differ by greater than 20% then they must conclude it is a poor model.

Alternative 2

M1: Finds the density of the ornament by dividing the mass of 6 grams by 7.3 g/cm^3

A1ft: Draws an appropriate conclusion about the suitability of the model, comparing their two densities.

If the densities differ by 10% then they must conclude it is a good model.

If the densities differ between 10% and 20% then they may conclude it is either a good model or poor model.

If the densities differ by greater than 20% then they must conclude it is a poor model.

Question	Scheme	Marks	AOs
9(i) (a)	E. g. - Because the interval being integrated over is unbounded. - cosh x is undefined at the limit of ∞ - the upper limit is infinite	B1	1.2
		(1)	
(i) (b)	$\int_0^\infty \cosh x \, dx = \lim_{t \rightarrow \infty} \int_0^t \cosh x \, dx$ or $\lim_{t \rightarrow \infty} \int_0^t \frac{1}{2}(e^x + e^{-x}) \, dx$	B1	2.5
	$\int_0^t \cosh x \, dx = [\sinh x]_0^t = \sinh t - (-0)$ or $\frac{1}{2} \int_0^t e^x + e^{-x} \, dx = \frac{1}{2} [e^x - e^{-x}]_0^t = \frac{1}{2} [e^t - e^{-t}] - \left[-\frac{1}{2} [e^0 - e^0] \right]$	M1	1.1b
	When $t \rightarrow \infty$ $e^t \rightarrow \infty$ and $e^{-t} \rightarrow 0$ therefore the integral is divergent	A1	2.4
		(3)	
(ii)	$4 \sinh x = p \cosh x \Rightarrow \tanh x = \frac{p}{4}$ or $4 \tanh x = p$ Alternative $\frac{4}{2}(e^x - e^{-x}) = \frac{p}{2}(e^x + e^{-x}) \Rightarrow 4e^x - 4e^{-x} = pe^x + pe^{-x}$ $e^{2x}(4 - p) = p + 4 \Rightarrow e^{2x} = \frac{p + 4}{4 - p}$	M1	3.1a
	$\left\{ -1 < \frac{p}{4} < 1 \Rightarrow \right\} -4 < p < 4$	A1	2.2a
		(2)	
(6 marks)			
(i)(a) B1: For a suitable explanation. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept “Because the upper limit is infinity”, but not “because it is infinity” without reference to what “it” is. Do not accept “the upper limit tends to infinity” or “the integral is unbounded”.			
(i)(b) B1: Writes the integral in terms of a limit as $t \rightarrow \infty$ (or other variable) with limits 0 and “ t ”, or implies the integral is a limit by subsequent working by correct language. M1: Integrates cosh x correctly either as $\sinh x$ or in terms of exponentials and applies correctly the limits of 0 and “ t ”. The bottom limit zero may be implied. No need for the $\lim_{t \rightarrow \infty}$ for this mark but substitution of ∞ is M0. A1: cso States that (as $t \rightarrow \infty$) $\sinh t \rightarrow \infty$ or $e^t \rightarrow \infty$ and $e^{-t} \rightarrow 0$ therefore divergent (or not convergent), or equivalent working. Accept $\sinh t$ is undefined as $t \rightarrow \infty$			
(ii) M1: Divides through by cosh x to find an expression involving $\tanh x$ Alternative: uses the correct exponential definitions and finds an expression for e^{2x} or solves a quadratic in e^{2x} A1: Deduces the correct inequality for p . Note $ p < 4$ is a correct inequality for p .			

Question	Scheme	Marks	AOs
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Question	Scheme	Marks	AOs
5(i)	$\int 2e^{\frac{1}{2}x} dx = -4e^{\frac{1}{2}x}$	B1	1.1b
	$\int_1^{\infty} 2e^{\frac{1}{2}x} dx = \lim_{t \rightarrow \infty} \left[\left(-4e^{\frac{1}{2}t} \right) - \left(-4e^{\frac{1}{2}} \right) \right]$	M1	2.1
	$= 4e^{\frac{1}{2}}$	A1	1.1b
		(3)	
(ii)(a)	Mean temperature $= \frac{1}{24} \int_0^{24} \left(8 - 5 \sin\left(\frac{\pi}{12}t\right) - \cos\left(\frac{\pi}{6}t\right) \right) dt$	B1	1.2
	$= \frac{1}{24} \left[\left(8t + \frac{60}{\pi} \cos\left(\frac{\pi}{12}t\right) - \frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right) \right) \right]_0^{24} = \frac{1}{24} [\dots]$	M1	1.1b
	$= \frac{1}{24} \left[\left(8(24) + \frac{60}{\pi} - \frac{6}{\pi} \times 0 \right) - \left(\frac{60}{\pi} \right) \right] = 8 * \text{cso}$	A1*	2.1
		(3)	
(ii)(b)	E.g. increase the value of the constant 8 / adapt the constant 8 to a function which takes values greater than 8.	B1	3.5c
		(1)	
(7 marks)			
Notes:			
(i) B1: Correct integration. M1: Attempt to integrate to a form $\lambda e^{\frac{1}{2}x}$ where $\lambda \neq 2$, and applies correct limits with some consideration of the infinite limit given (e.g. with the limit statement). Only allow with ∞ used as the limit if subsequent work shows the term is zero. A1: Correct value			
(ii)(a) B1: Recalls the correct formula for finding the mean value of a function. You may see the division by “24” only at the end. No integration is necessary, just a correct statement with an integral. M1: Integrates to a form $\alpha t + \beta \cos\left(\frac{\pi}{12}t\right) + \delta \sin\left(\frac{\pi}{6}t\right)$ and uses the limits of 0 and 24 (the correct way around). If no explicit substitution is seen, accept any value following the integral as an attempt. Answers from a calculator with no correct integral seen score M0 as the question requires calculus to be used. A1*cso: Achieves 8 with no errors seen following a full attempt at the substitution. Must have seen some evidence of the limits used, minimum required for substitution is $\left[\left(8(24) + \frac{60}{\pi} \right) - \left(\frac{60}{\pi} \right) \right]$.			
(ii)(b) B1: Accept any reasonable adaptation to the equation that will increase the mean value. E.g. as in scheme, or introduce another positive term, or decrease the constant 5 etc. It must be clear which constant they are referring to in their reason, not just “increase the constant”.			

Question	Scheme	Marks	AOs
7(a)	$\frac{2x^3+10x^2+9x+22}{(x+2)(x^2+3)} = A + \frac{B}{x+2} + \frac{Cx+D}{x^2+3}$	M1	1.1a
	$2x^3+10x^2+9x+22 = A(x+2)(x^2+3) + B(x^2+3) + (Cx+D)(x+2)$ Correct method to find at least 3 of the values A, B, C and D for example $x = -2 \Rightarrow B = \dots \{28 = 7B\}$ $\text{coeff } x^3 \Rightarrow A = \dots \{2 = A\}$ $\text{coeff } x^2 \Rightarrow C = \dots \{10 = 2A + B + C\}$ $\text{coeff } x \Rightarrow D = \dots \{9 = 3A + 2C + D\}$ $x = 0 \Rightarrow D = \dots \{22 = 6A + 3B + 2D\}$	dM1	3.1a
	$A = 2$	B1	1.1b
	$2 + \frac{4}{x+2} + \frac{2x-1}{x^2+3}$	A1	1.1b
		(4)	
	Alternative: long division $\begin{array}{r} 2 \\ x^3 + 2x^2 + 3x + 6 \overline{) 2x^3 + 10x^2 + 9x + 22} \\ \underline{2x^3 + 4x^2 + 6x + 12} \\ 6x^2 + 3x + 10 \\ 2 + \frac{\dots}{2x^3 + 10x^2 + 9x + 22} \\ \frac{6x^2 + 3x + 10}{(x+2)(x^2+3)} = \frac{P}{x+2} + \frac{Qx+R}{x^2+3} \end{array}$	M1	1.1a
	$6x^2 + 3x + 10 = P(x^2+3) + (Qx+R)(x+2)$ Correct method to find at least two of the values P, Q and R for example $x = -2 \Rightarrow P = \dots \{28 = 7P\}$ $\text{coeff } x^2 \Rightarrow Q = \dots \{6 = P + Q\}$ $x = 0 \Rightarrow R = \dots \{10 = 3P + 2R\}$ $\text{coeff } x \Rightarrow R = \dots \{3 = 2Q + R\}$	dM1	3.1a
	$\begin{array}{r} 2 \\ x^3 + 2x^2 + 3x + 6 \overline{) 2x^3 + 10x^2 + 9x + 22} \\ \underline{2x^3 + 4x^2 + 6x + 12} \\ 6x^2 + 3x + 10 \\ \text{or states } 2 + \frac{\dots}{2x^3 + 10x^2 + 9x + 22} \end{array}$	B1	1.1b

	or states $A = 2$		
	$2 + \frac{4}{x+2} + \frac{2x-1}{x^2+3}$	A1	1.1b
		(4)	
(b)	$\dots + \int \frac{4}{x+2} + \frac{2x-1}{x^2+3} dx = \dots + \int \frac{4}{x+2} + \frac{2x}{x^2+3} - \frac{1}{x^2+3} dx$ $= \alpha \ln(x+2) + \beta \ln(x^2+3) + \lambda \arctan\left(\frac{x}{\sqrt{3}}\right)$	M1	3.1a
	$= 2x + 4\ln(x+2) + \ln(x^2+3) - \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right)$	A1	2.1
	$= \left[2(1) + 4\ln(1+2) + \ln(1^2+3) - \frac{1}{\sqrt{3}} \arctan\left(\frac{1}{\sqrt{3}}\right) \right] -$ $\left[2(0) + 4\ln(0+2) + \ln(0^2+3) - \frac{1}{\sqrt{3}} \arctan\left(\frac{0}{\sqrt{3}}\right) \right]$ $= \left[2 + 4\ln(3) + \ln(4) - \frac{1}{\sqrt{3}} \arctan\left(\frac{1}{\sqrt{3}}\right) \right] - [4\ln 2 + \ln 3] = \dots$	dM1	1.1b
	$2 + \ln\left(\frac{27}{4}\right) - \frac{\pi}{6\sqrt{3}} \text{ * cso}$	A1*	2.1
		(4)	
(8 marks)			



Notes:
(a)

M1: Selects the correct form for partial fractions.

Must be of the form $A + \frac{B}{x+2} + \frac{Cx+D}{x^2+3}$ so do not award if their A is not present.

Allow if they set up an expression of the form $A + \frac{\mu x + B}{x+2} + \frac{Cx+D}{x^2+3}$ or $A + \mu x + \frac{B}{x+2} + \frac{Cx+D}{x^2+3}$ provided they then go on to show their $\mu = 0$

dm1: Dependent on having the correct form for the partial fractions.

Complete method for finding the value of at least 3 constants in $A + \frac{B}{x+2} + \frac{Cx+D}{x^2+3}$

Allow slips, provided their intention is clear.

B1: Correct value for A. This is independent of any method and should be awarded regardless of an incorrect use of partial fractions.

A1: Correct solution written as partial fractions, but not for listing values for constants.

Alternative: long division

M1: Selects the correct form for partial fractions, (following an attempt at algebraic long division which led to an integer quotient, which may or may not be 2, and their remainder must be a 3TQ)

Their partial fractions must be of the form $\frac{P}{x+2} + \frac{Qx+R}{x^2+3}$

dm1: Complete method for finding the value of at least 2 constants. Dependent on having the correct form for the partial fractions. Allow slips, provided their intention is clear.

B1: Correct constant 2, may be seen in their long division and is likely to appear at the beginning of their workings. This is independent of any method and should be awarded regardless of any incorrect use of partial fractions.

A1: Correct solution written as partial fractions, but not for listing values for constants.

(b)

M1: Rewrites the integral (without the constant term) into an integrable form and integrates to the

correct form $\alpha \ln(x+2) + \beta \ln(x^2+3) + \lambda \arctan\left(\frac{x}{\sqrt{3}}\right)$, where α, β and λ are non-zero constants.

A1: Fully correct integration for the **whole** expression.

dm1: Uses the limits of 0 and 1, subtracts the correct way round and combines their ln terms correctly.

A1*: Correct answer **cs0** (but can be written in any order)



Question	Scheme	Marks	AOs
9(a)	$\int \frac{x^2}{\sqrt{x^2-1}} dx \rightarrow \int f(u) du$ Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of u only, including replacing the dx	M1	3.1a
	$\int \frac{\cosh^2 u}{\sqrt{\cosh^2 u - 1}} \sinh u (du)$	A1	1.1b
	Uses correct identities $\cosh^2 u - 1 = \sinh^2 u \text{ and } \cosh 2u = 2\cosh^2 u - 1$ to achieve an integral of the form $A \int (\cosh 2u \pm 1) du \quad A > 0$	M1	3.1a
	Integrates to achieve $A \left(\pm \frac{1}{2} \sinh 2u \pm u \right) (+c) \quad A > 0$	M1	1.1b
	Uses the identity $\sinh 2u = 2 \sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ $\rightarrow \sinh 2u = 2x\sqrt{x^2-1}$	M1	2.1
	$\frac{1}{2} \left[x\sqrt{x^2-1} + \operatorname{arccosh} x \right] + k * \text{cso}$	A1*	1.1b
		(6)	
(b)	Uses integration by parts the correct way around to achieve $\int \frac{4}{15} x \operatorname{arccosh} x dx = Px^2 \operatorname{arccosh} x - Q \int \frac{x^2}{\sqrt{x^2-1}} dx$	M1	2.1
	$= \frac{4}{15} \left(\frac{1}{2} x^2 \operatorname{arccosh} x - \frac{1}{2} \int \frac{x^2}{\sqrt{x^2-1}} dx \right)$	A1	1.1b
	$= \frac{4}{15} \left(\frac{1}{2} x^2 \operatorname{arccosh} x - \frac{1}{2} \left(\frac{1}{2} \left[x\sqrt{x^2-1} + \operatorname{arccosh} x \right] \right) \right)$	B1ft	2.2a
	Uses the limits $x=1$ and $x=3$ the correct way around and subtracts $= \frac{4}{15} \left(\frac{1}{2} (3)^2 \operatorname{arccosh} 3 - \frac{1}{2} \left(\frac{1}{2} \left[3\sqrt{(3)^2-1} + \operatorname{arccosh} 3 \right] \right) \right) - \frac{4}{15} (0)$	dM1	1.1b
	$= \frac{4}{15} \left(\frac{9}{2} \ln(3+\sqrt{8}) - \frac{3\sqrt{8}}{4} - \frac{1}{4} \ln(3+\sqrt{8}) \right)$ $= \frac{1}{15} \left[17 \ln(3+2\sqrt{2}) - 6\sqrt{2} \right] *$	A1*	1.1b
		(5)	
(11 marks)			

Notes:**(a)**

M1: Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of u only. Must have replaced the dx but allow if the du is missing.

A1: Correct integral in terms of u . (Allow if the du is missing.)

M1: Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integrand of the required form

M1: Integrates to achieve the correct form, may be sign errors.

M1: Uses the identities $\sinh 2u = 2\sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ to attempt to find $\sinh 2u$ in terms of x . If using exponentials there must be a full and complete method to attempt the correct form.

A1*: Achieves the printed answer with no errors seen, cso

NB attempts at integration by parts are not likely to make progress – to do so would need to split the integrand as $x \frac{x}{\sqrt{x^2 - 1}}$. If you see any attempts that you feel merit credit, use review.

(b)

M1: Uses integration by parts the correct way around to achieve the required form.

A1: Correct integration by parts

B1ft: Deduces the integral by using the result from part (a). Follow through on their ‘ uv ’

dM1: Dependent on previous method mark. Uses the limits $x = 1$ and $x = 3$ the correct way around and subtracts

A1*cso: Achieves the printed answer with at least one intermediate step showing the evaluation of the arcosh 3, and no errors seen.



Question	Scheme	Marks	AOs
3(a)	Max $r = 4 + a = 5.5 \Rightarrow a = \dots$	M1	3.4
	$a = 1.5$	A1	2.2a
		(2)	
(b)	Pool area $= \frac{1}{2} \int_0^{2\pi} (4 - 1.5 \sin 3\theta)^2 d\theta$	M1	3.1a
	$(4 - 1.5 \sin 3\theta)^2 = 16 - 12 \sin 3\theta + 2.25 \sin^2 3\theta$ $= 16 - 12 \sin 3\theta + 2.25 \left(\frac{1 - \cos 6\theta}{2} \right) = 16 - 8a \sin 3\theta + a^2 \left(\frac{1 - \cos 6\theta}{2} \right)$	M1	2.1
	$\int (4 - 1.5 \sin 3\theta)^2 d\theta = 16\theta + 4 \cos 3\theta + \frac{9}{8} \left(\theta - \frac{\sin 6\theta}{6} \right)$ $\left[= 16\theta + \frac{8a}{3} \cos 3\theta + \frac{a^2}{2} \left(\theta - \frac{\sin 6\theta}{6} \right) \right]$	A1ft	1.1b
	$\frac{1}{2} \left[\frac{137}{8} \theta + 4 \cos 3\theta - \frac{3}{16} \sin 6\theta \right]_0^{2\pi} = \dots \left(\frac{137}{8} \pi \right)$	dM1	3.1a
	Area of $T = \pi \times 36 - \frac{137}{8} \pi$	DM1	1.1b
	$= \frac{151}{8} \pi \text{ (m}^2\text{) oe}$	A1	1.1b
		(6)	
(8 marks)			
Notes			
(a) M1: Uses all the information given for the model and realises the maximum value of r is $(4 + a)$ and uses the radius of the circle to find a value for a. A1: Deduces the correct value of a. Note $a = -1.5$ can potentially gain M1A0. (b) Note accept with their a, a made up a or even a itself for the first 5 marks. Note use of $a = -1.5$ can score full marks. M1: Adopts a correct strategy for the area of the pool. This requires the correct use of the polar area formula including the $\frac{1}{2}$. Note the $\frac{1}{2}$ may be implied by choice of limits (e.g. any span of π radians without the half implies an attempt at doubling so the $\frac{1}{2}$ may not appear (even if the symmetry is incorrect).) M1: Squares the bracket, achieving three terms, and applies $\sin^2 3\theta = \frac{\pm 1 \pm \cos 6\theta}{2}$ in order to reach an integrable form. Condone numerical slips when expanding. A1ft: Correct integration in any form ($\frac{1}{2}$ not needed here) (follow through their a). ie as shown in scheme or if gathered it is $\left(32 + a^2 \right) \frac{\theta}{2} + \frac{8a}{3} \cos 3\theta - a^2 \frac{\sin 6\theta}{12}$			

dM1: Depends on previous M. Uses appropriate limits for their integrated function. Allow limits other than 0 and 2π by using symmetry provided the correct multiple is used, e.g. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

followed by doubling (which may be cancelled with the $\frac{1}{2}$) or $-\frac{\pi}{6}$ and $\frac{\pi}{2}$ and multiplying by 3

DM1: Depends on first M. Fully correct strategy for obtaining the area of T . Must have a correct attempt at the circle area (maybe be via integration) and area inside the curve. Symmetry may have been used.

A1: Correct area from fully correct work (all previous marks scored). Units not required. The decimal answer 18.875π is acceptable, but answer must be exact, not a rounded decimal.

Note: $\sin^2 3\theta = \frac{1 + \cos 6\theta}{2}$ use will score a maximum of M1M0A0dM0DM1A0

Note: if $\frac{1}{2} \int_0^{2\pi} (6^2 - (4 - a \sin 3\theta)^2) d\theta$ is used the scheme will follow as above with A1ft for

$10\theta - \frac{8a}{6} \cos 3\theta - \frac{a^2}{4} \left(\theta - \frac{\sin 6\theta}{6} \right)$ and dM1DM1 gained together.

Note: Integrating over 0 to 2π means the trig terms will disappear so watch out for incorrect trig terms in the integration, which will lead to the correct answer but will lose both A marks.