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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Vectors (6)

Core Pure Mathematics 1 · Topic 6

4 questions · 38 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
6.1 EQUATIONS OF LINES IN 3D		1 Qs · 10 marks	3
1. Q7	Paper 1: Core Pure Mathematics 1 June2024	10	4
6.2 EQUATIONS OF PLANES		1 Qs · 8 marks	8
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6.5 INTERSECTIONS AND SHORTEST DISTANCES		1 Qs · 12 marks	13
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4. Q3 <i>also Topic 5 & 7</i>	Paper 1: Core Pure Mathematics 1 June2024	8	19



QUESTIONS

4 questions · 38 marks · 4 sub-topics



SUB-TOPIC 6.1

EQUATIONS OF LINES IN 3D

Sub-topic 6.1 · 1 question · 10 marks · lowest tariff first







SUB-TOPIC 6.2

EQUATIONS OF PLANES

Sub-topic 6.2 · 1 question · 8 marks · lowest tariff first







(Total for Question 7 is 6 marks)



SUB-TOPIC 6.5

INTERSECTIONS AND SHORTEST DISTANCES

Sub-topic 6.5 · 1 question · 12 marks · lowest tariff first







SUB-TOPIC 7.3

AREAS AND TANGENTS OF POLAR CURVES

Sub-topic 7.3 · 1 question · 8 marks · lowest tariff first



3.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

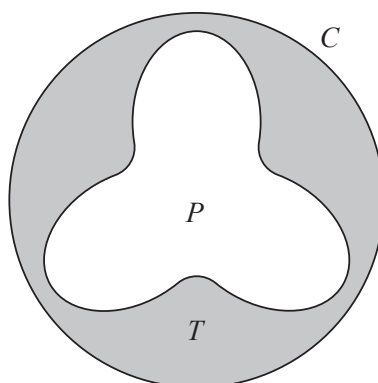


Figure 1

Figure 1 shows the design for a bathing pool.

The pool, P , shown unshaded in Figure 1, is surrounded by a tiled area, T , shown shaded in Figure 1.The tiled area is bounded by the edge of the pool and by a circle, C , with radius 6 m.

The centre of the pool and the centre of the circle are the same point.

The edge of the pool is modelled by the curve with polar equation

$$r = 4 - a \sin 3\theta \quad 0 \leq \theta \leq 2\pi$$

where a is a positive constant.Given that the shortest distance between the edge of the pool and the circle C is 0.5 m,(a) determine the value of a .

(2)

(b) Hence, using algebraic integration, determine, according to the model, the exact area of T .

(6)

Question 3 continued





Question 3 continued

(Total for Question 3 is 8 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
7(a)	$(3\mathbf{i} - 10\mathbf{j} - \mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - 4\mathbf{k}) = 6 - 10 + 4 = 0$ $(3\mathbf{i} - 10\mathbf{j} - \mathbf{k}) \cdot (6\mathbf{i} + \mathbf{j} + 8\mathbf{k}) = 18 - 10 - 8 = 0$	M1	1.1b
	So $3\mathbf{i} - 10\mathbf{j} - \mathbf{k}$ is perpendicular to l	A1	2.1
		(2)	
(b)	$(3\mathbf{i} - 10\mathbf{j} - \mathbf{k}) \cdot (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) = 3 + 20 - 3 = 20$ $3x - 10y - z = 20$	M1	1.1b
		A1	2.5
		(2)	
(c)	$(3\mathbf{i} - 10\mathbf{j} - \mathbf{k}) \cdot (5\mathbf{i} + p\mathbf{j} - 7\mathbf{k}) = 20$ $\Rightarrow 15 - 10p + 7 = 20 \Rightarrow p = \dots$	M1	3.1a
	$p = 0.2$ (oe)	A1	1.1b
		(2)	
(d)	E.g. $1 + 2\lambda = 5 + 6\mu$, $3 - 4\lambda = -7 + 8\mu \Rightarrow \lambda = \dots$ or $\mu = \dots$ $\mu = 0.1$ (or $\lambda = 2.3$) $\Rightarrow A(5.6, 0.3, -6.2)$	M1	1.1b
	$12\mathbf{i} - 11\mathbf{j} + 6\mathbf{k} - (5.6\mathbf{i} + 0.3\mathbf{j} - 6.2\mathbf{k}) = 6.4\mathbf{i} - 11.3\mathbf{j} + 12.2\mathbf{k}$ $(6.4\mathbf{i} - 11.3\mathbf{j} + 12.2\mathbf{k}) \cdot (3\mathbf{i} - 10\mathbf{j} - \mathbf{k}) = 19.2 + 113 - 12.2 = 120$	M1	3.1a
	$120 = \sqrt{6.4^2 + 11.3^2 + 12.2^2} \sqrt{3^2 + 10^2 + 1^2} \cos \alpha \Rightarrow \alpha = \dots$ or e.g. $120 = \sqrt{6.4^2 + 11.3^2 + 12.2^2} \sqrt{3^2 + 10^2 + 1^2} \sin \alpha \Rightarrow \alpha = \dots$	M1	1.1b
	Angle between AB and plane $\theta = 40^\circ$ (awrt)	A1	1.1b
		(4)	
		(10 marks)	
Notes			
(a) M1: Attempts the scalar product between the given vector and the 2 direction vectors (calculation should be shown for at least one). A1: Obtains zero for both with sufficient working shown and concludes perpendicular (each one or states both are). Accept normal to as equivalent to perpendicular to. (b) M1: Attempts the scalar product between the normal vector and $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$. Allow if the vector product is used again from scratch. Allow equivalent approaches such as using the normal to form the equation $3x - 10y - z = d$ and substituting $(1, -2, 3)$ to find d . Maybe be implied by a correct equation if no incorrect working is seen. A1: Correct equation in Cartesian form (allow equivalent Cartesian equations). (c) M1: Attempts the scalar product between the normal vector and $5\mathbf{i} + p\mathbf{j} - 7\mathbf{k}$ (or any point on the line), sets = their 20 and solves a linear equation in p . Alternatively substitutes $x = 5$, $y = p$ and $z = -7$ into their Cartesian equation and solves for p , or any other complete method to find p . A1: For $p = 0.2$ (oe) (d) M1: A complete method to find the coordinates of A . Allow if a slip in transcribing a coordinate occurs – it is a mark for the method. If the correct vectors are clearly intended, give credit. M1: Attempts the vector AB and attempts the scalar product with this and the normal vector. Note that if the correct formulae are seen then allow the method for any value appearing afterwards. M1: Complete method to find the required angle or its complement. A1: For awrt 40°			

Note: Accept alternative vector notations throughout. Where method is not shown for finding coordinates, it may be implied by two correct coordinates.			
Alt (a)	$(2\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \times (6\mathbf{i} + \mathbf{j} + 8\mathbf{k}) = (8 + 4)\mathbf{i} - (16 + 24)\mathbf{j} + (2 - 6)\mathbf{k}$	M1	1.1b
	$= 12\mathbf{i} - 40\mathbf{j} - 4\mathbf{k} = 4(3\mathbf{i} - 10\mathbf{j} - \mathbf{k})$ So $3\mathbf{i} - 10\mathbf{j} - \mathbf{k}$ is perpendicular to l	A1	2.1
		(2)	
Notes			
M1: Attempts the vector product between the direction vectors of the lines, allowing at most one slip in the brackets. A1: Obtains the correct vectors and shows it is a multiple of the required answer and concludes perpendicular.			
Alt II (a)	Normal to plane is $\mathbf{n} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k} \Rightarrow (\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - 4\mathbf{k}) = 0$ and $(\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}) \cdot (6\mathbf{i} + \mathbf{j} + 8\mathbf{k}) = 0$ So $2\alpha + \beta - 4\gamma = 0$, $6\alpha + \beta + 8\gamma = 0$ $\alpha = 3 \Rightarrow \beta = 4\gamma - 6$, $\beta = -8\gamma - 18 \Rightarrow \beta = \dots, \gamma = \dots$	M1	1.1b
	$\gamma = -1, \beta = -10 \Rightarrow 3\mathbf{i} - 10\mathbf{j} - \mathbf{k}$ perpendicular to l	A1	2.1
		(2)	
Notes			
M1: Sets up a normal vector to the plane in terms of variables and attempts the vector product between the direction vectors of the lines and the normal to form simultaneous equations and attempts to solve by choosing one of the values and solving for the other two. A1: Correct values, and obtains the correct vector, or a multiple of it and shows it is a multiple of the required answer and concludes perpendicular.			

Alt (d)	E.g. $1 + 2\lambda = 5 + 6\mu, 3 - 4\lambda = -7 + 8\mu \Rightarrow \lambda = \dots$ or $\mu = \dots$ $\mu = 0.1$ (or $\lambda = 2.3$) $\Rightarrow A(5.6, 0.3, -6.2)$	M1	1.1b
	$\overrightarrow{BX} = m\mathbf{n} \Rightarrow \overrightarrow{OX} = \overrightarrow{OB} + m\mathbf{n} = (12 - 3m)\mathbf{i} - (11 + 10m)\mathbf{j} + (6 - m)\mathbf{k}$ $\Rightarrow 3(12 - 3m) - 10(-11 - 10m) - (6 - m) = 20 \Rightarrow m = \dots \left(-\frac{12}{11}\right)$ $\Rightarrow X = \left(\frac{96}{11}, -\frac{1}{11}, \frac{78}{11}\right)$	M1	3.1a
	$\sin \alpha = \frac{BX}{AB} = \frac{\frac{12}{11} \times \sqrt{3^2 + 10^2 + 1^2}}{\sqrt{6.4^2 + 11.3^2 + 12.2^2}} \Rightarrow \alpha = \dots$	M1	1.1b
	Angle between AB and plane $\theta = 40^\circ$ (awrt)	A1	1.1b
		(4)	
Notes			
M1: A complete method to find the coordinates of A . M1: Forms equation for $\overrightarrow{OX} = \overrightarrow{OB} + m\mathbf{n}$ and proceeds to find where normal to plane through B intersects the plane, or the distance BX ($=m \mathbf{n} $) M1: Complete method to find the required angle or its complement. May find X first to find BX etc, or may use different sides of the triangle. A1: For awrt 40°			

Question	Scheme	Marks	AOs
7(a)	$\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = -2 + 6 - 4 = 0 \text{ and } \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = 4 + 0 - 4 = 0$	M1	1.1b
	Alt: $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \times 1 - 1 \times 0 \\ -(-1 \times 1 - 1 \times 2) \\ -1 \times 0 - 2 \times 2 \end{pmatrix} = \dots$		
	As $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ is perpendicular to both direction vectors (two non-parallel vectors) of Π then it must be perpendicular to Π	A1	2.2a
		(2)	
(b)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \Rightarrow \dots$	M1	1.1a
	$2x + 3y - 4z = 7$	A1	2.2a
		(2)	
(c)	$\frac{ 2(4+t) + 3(-5+6t) - 4(2-3t) - 7 }{\sqrt{2^2 + 3^2 + (-4)^2}} = 2\sqrt{29} \Rightarrow t = \dots$	M1	3.1a
	$t = -\frac{9}{8} \text{ and } t = \frac{5}{2}$	A1	1.1b
	$\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} - \frac{9}{8} \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix} = \dots \text{ or } \mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix} = \dots$	M1	1.1b
	$\left(\frac{23}{8}, -\frac{47}{4}, \frac{43}{8}\right) \text{ and } \left(\frac{13}{2}, 10, -\frac{11}{2}\right)$	A1	2.2a
		(4)	
(8 marks)			
Notes:			
(a) M1: Attempts the scalar product of each direction vector and the vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. Some numerical calculation is required, just “= 0” is insufficient. Alternatively, attempts the cross product (allow sign slips) with the two direction vectors. A1: Shows that both scalar products = 0 (minimum $-2 + 6 - 4 = 0$ and $4 - 4 = 0$) and makes a minimal conclusion with no erroneous statements. If using cross product, the calculation must be correct, and a minimal conclusion given with no erroneous statements.			

(b)

M1: Applies $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \Rightarrow \dots$

A1: $2x + 3y - 4z = 7$

(c)

M1: A fully correct method for finding a value of t . Other methods are possible, but must be valid and lead to a value of t . Examples of other methods:

- $2\sqrt{29} = \pm \left(\frac{2(4+t) + 3(-5+6t) - 4(2-3t)}{\sqrt{2^2 + 3^2 + (-4)^2}} - \frac{7}{\sqrt{29}} \right)$ using plane parallel to l through origin

and shortest distance from plane to origin.

- $2(4+t) + 3(-5+6t) - 4(2-3t) = 7 \Rightarrow t = t_i$ (t at intersection of line and plane) and

$$\sin \theta = \frac{(2, 3, -4)^T \cdot (1, 6, -3)^T}{\sqrt{29}\sqrt{46}} \text{ (sine of angle between line and plane) followed by}$$

$$\sin \theta = \frac{2\sqrt{29}}{k\sqrt{46}} \Rightarrow k = \dots \Rightarrow t = t_i \pm k$$

A1: Correct values for t . Both are required.

M1: Uses a value of t to find a set of coordinates for A .

A1: Both correct sets of coordinates for A .



Question	Scheme	Marks	AOs
5(a)	$2(\lambda - 5) + 3(-3\lambda - 4) - 2(5\lambda + 3) = 6 \Rightarrow \lambda = \dots(-2)$ $\lambda = "-2" \Rightarrow x = \dots$ or $y = \dots$ or $z = \dots$ or e.g. $2x + 3(-3x - 15 - 4) - 2(5x + 25 + 3) = 6 \Rightarrow x = \dots$	M1	1.1b
	$(-7, 2, -7)$	A1	1.1b
		(2)	
(b)	E.g. $\mathbf{r} = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2t \\ 3t \\ -2t \end{pmatrix}$ meets the plane when $2(-5 + 2t) + 3(-4 + 3t) - 2(3 - 2t) = 6 \Rightarrow t = \dots$	M1	3.1a
	$t = 2 \Rightarrow$ mirror point is $\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \times 4 \\ 3 \times 4 \\ -2 \times 4 \end{pmatrix} = \dots$	M1	1.1b
	$= \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}$	A1	1.1b
	$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 3 - (-7) \\ 8 - 2 \\ -5 - (-7) \end{pmatrix} = \dots$	ddM1	1.1b
	$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix} *$	A1*	2.1
		(5)	
(b) Alternative for first 2 marks:			
	Distance from $(-5, -4, 3)$ to plane is $\frac{ 2 \times -5 + 3 \times -4 - 2 \times 3 - 6 }{\sqrt{2^2 + 3^2 + 2^2}} = 2\sqrt{17}$	M1	3.1a
	$\begin{vmatrix} 2k \\ 3k \\ -2k \end{vmatrix} = 4\sqrt{17} \Rightarrow 4k^2 + 9k^2 + 4k^2 = 16 \times 17 \Rightarrow k = 4$ $k = 4 \Rightarrow$ mirror point is $\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \times 4 \\ 3 \times 4 \\ -2 \times 4 \end{pmatrix} = \dots$	M1	1.1b

**Do not penalise the omission of “ $\mathbf{r} =$ ” more than once in this question
so penalise only once and on its first occurrence.**

Notes:

(a) **Correct answer only scores no marks.**

M1: Substitutes the parametric form of the line into the plane and solves for their parameter and uses this to find at least one coordinate. Alternatively substitutes line equation into the plane equation to obtain and solve an equation in one variable.

A1: Correct point. Accept as $x = -7$, $y = 2$, $z = -7$ or as a vector.

(b)

M1: Identifies a point on l_1 and uses the equation of the line through this point perpendicular to the plane to find the parameter where it intersects the plane. May use a different starting point.

M1: Uses twice their parameter to find the image point of their starting point in the plane.

A1: Correct image point.

ddM1: Finds the equation of the line between their intersection point from (a) and their image point.

May be implied if they use e.g. $3\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$ as the position vector.

Alternatively, checks that the image point satisfies the equation for the other line.

Depends on both previous method marks.

A1*: Fully correct work with conclusion that hence the given equation gives the required line.

Must be as printed with “ $\mathbf{r} =$ ” but condone use of a different parameter e.g. t , λ etc.

If they checked that the image point satisfied the equation of line 2 they would need to then say e.g. that both lines pass through $(-7, 2, -7)$ and make a minimal conclusion.

Alternative for first 2 marks in (b):

M1: Identifies a point on l_1 and finds the perpendicular distance from this point to the plane. May use a different starting point.

M1: Uses twice their distance to find the image point of their starting point in the plane.



(c)	Line joining mirror points intersects plane at $\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \times 2 \\ 3 \times 2 \\ -2 \times 2 \end{pmatrix}$, so equation of line is $\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + s \begin{pmatrix} -1 - (-7) \\ 2 - 2 \\ -1 - (-7) \end{pmatrix} = \dots$	M1	3.1a
	$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + s \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix}$ oe e.g. $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	A1	2.5
		(2)	
Alternative 1 to (c) (Not on spec)			
	Normal to Π_2 : $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 5 \\ 10 & 6 & 2 \end{vmatrix} = \begin{pmatrix} -36 \\ 48 \\ 36 \end{pmatrix}$ Direction of l_2 : $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 3 \\ 2 & 3 & -2 \end{vmatrix} = \begin{pmatrix} -17 \\ 0 \\ -17 \end{pmatrix}$ equation of line is $\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + s \begin{pmatrix} -17 \\ 0 \\ -17 \end{pmatrix}$ oe	M1	3.1a
		A1	2.5
		(2)	
Alternative 2 to (c) (Not on spec)			
	Normal to Π_2 : $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 5 \\ 10 & 6 & 2 \end{vmatrix} = \begin{pmatrix} -36 \\ 48 \\ 36 \end{pmatrix}$ $(-3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}) \cdot (-7\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}) = 8$ Π_2 is $3x - 4y - 3z = -8$ then e.g. solves simultaneously with Π_1 and $x = \lambda$ to give $y = 2, z = \lambda$ So equation of line is $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ oe	M1	3.1a
		A1	2.5
		(2)	
Alternative 3 to (c) (Not on spec)			
	As alternative 2 to find the equation of plane 2: $3x - 4y - 3z = -8$ Then solves simultaneously with plane 1 to give e.g. $y = 2, x = z$ Hence $\mathbf{r} = \begin{pmatrix} s \\ 2 \\ s \end{pmatrix}$ oe	M1	3.1a
		A1	2.5
		(2)	



Notes:

(c)

M1: A complete method to find the required equation. E.g. finds a second point on the common line and uses this and the point from (a) to find the direction and then forms the vector equation of the required line. May use earlier work to find the second point or start again.

A1: Correct equation formed, accept with any parameter which is not x , y or z .

Must include “ $\mathbf{r} =$ ” unless this was already penalised in part (b).

Alternative 1:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 and then attempting the vector product of this with the normal to Π_1 to find the direction of the common line and forms the vector equation with a point on the line.

The normal vectors may also be found using scalar products e.g.

$$\mathbf{n} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) = 0, (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (10\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}) = 0 \Rightarrow x = \dots, y = \dots$$

A1: Correct equation formed, accept with any parameter which is not x , y or z .

Must include “ $\mathbf{r} =$ ” unless already penalised in (b).

Alternative 2:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above and finds the equation of Π_2 by using a point on the line and then solves this with Π_1 by eliminating one of the variables to form the vector equation of the line of intersection.

A1: Correct equation formed, accept with any parameter which is not x , y or z .

Must include “ $\mathbf{r} =$ ” unless already penalised in (b).

Alternative 3:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above and finds the equation of Π_2 by using a point on the line and then solves this simultaneously with Π_1 to form the vector equation of the line of intersection.

A1: Correct equation formed, accept with any parameter which is not x , y or z .

Must include “ $\mathbf{r} =$ ” unless already penalised in (b).

There will be other methods in part (c)
Generally the method mark requires a complete attempt
to find the equation of the line of intersection.



(d)	Line from (c) must lie in plane, so $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is perpendicular to $\begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = 0 \Rightarrow 1 \times 1 + 0 \times 1 + 1 \times a = 0 \Rightarrow a = \dots$	M1	3.1a
	$a = -1$	A1	1.1b
	$b = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 2$	A1	2.2a
		(3)	
Alternative 1 to (d):			
	$\begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b \Rightarrow -7 + 2 - 7a = b$ $\Rightarrow a = \dots$ or $b = \dots$ $\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b \Rightarrow -1 + 2 - a = b$	M1	3.1a
	$a = -1$ or $b = 2$	A1	1.1b
	$a = -1$ and $b = 2$	A1	2.2a
		(2)	
Alternative 2 to (d) (Not on spec):			
	Normal to Π_2 : $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 5 \\ 10 & 6 & 2 \end{vmatrix} = \begin{pmatrix} -36 \\ 48 \\ 36 \end{pmatrix}$ $\begin{vmatrix} 3 & -4 & -3 \\ 2 & 3 & -2 \\ 1 & 1 & a \end{vmatrix} = 0 \Rightarrow 3(3a+2) + 4(2a+2) - 3(-1) = 0 \Rightarrow a = \dots$	M1	3.1a
	$a = -1$	A1	1.1b
	$a = -1$ and $b = 2$	A1	2.2a
		(2)	
Alternative 3 to (d):			
	$\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ lies in $\Pi_3 \Rightarrow s + 2 + as = b$ $(a+1)s + 2 = b \Rightarrow a = \dots$	M1	3.1a
	$a = -1$	A1	1.1b
	$a = -1$ and $b = 2$	A1	2.2a
		(2)	

	Alternative 4 to (d):		
	Normal to Π_2 : $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 5 \\ 10 & 6 & 2 \end{vmatrix} = \begin{pmatrix} -36 \\ 48 \\ 36 \end{pmatrix}$ $(-3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}) \cdot (-7\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}) = 8$ Π_2 is $3x - 4y - 3z = -8$ then solves simultaneously with Π_1 and Π_3 and uses consistency e.g. $\begin{aligned} 3x - 4y - 3z &= -8 \\ 2x + 3y - 2z &= 6 \\ x + y + az &= b \end{aligned}$ $x + y + az = b \Rightarrow x = b - y - az$ $3b - 3y - 3az - 4y - 3z = -8 \Rightarrow 7y + (3a + 3)z = 3b + 8$ $2b - 2y - 2az + 3y - 2z = 6 \Rightarrow y - (2a + 2)z = 6 - 2b$ Consistency $\Rightarrow -14a - 14 = 3 + 3a \Rightarrow a = \dots \text{ or } 3b + 8 = 42 - 14b \Rightarrow b = \dots$	M1	3.1a
	$a = -1$ or $b = 2$	A1	1.1b
	$a = -1$ and $b = 2$	A1	2.2a

(12 marks)

Notes:

(d)

M1: Realises the direction of (c) is perpendicular to the normal to Π_3 and applies the dot product = 0 to find a .

A1: Correct value for a

A1: Deduces the value of b using their a and one of their points on the line.

Alternative 1:

M1: Uses their point from (a) and another point on the line of intersection and substitutes both points into the given equation and solves the resulting equations for a or b .

A1: Correct value for a or b .

A1: Correct value for a and b .

Alternative 2:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above (may have been found earlier) and then attempts the determinant of the matrix of normal vectors = 0 and solves for a

A1: Correct value for a .

A1: Deduces the correct value of b . E.g. by using their a with a point on the line.

Alternative 3:

M1: Substitutes their line from part (c) into the equation for Π_3 and compares coefficients to establish a value for a .



A1: Correct value for a .

A1: Deduces the correct value of b .

Alternative 4:

M1: Finds the equation for l_2 and then solves all 3 equations using the fact they are consistent leading to a value for a or a value for b .

A1: Correct value for a or b .

A1: Correct value for a and b .

There will be other methods in part (d)
Generally the method mark requires a complete attempt
to find the value of a or b .



Question	Scheme	Marks	AOs
3(a)	Max $r = 4 + a = 5.5 \Rightarrow a = \dots$	M1	3.4
	$a = 1.5$	A1	2.2a
		(2)	
(b)	Pool area $= \frac{1}{2} \int_0^{2\pi} (4 - 1.5 \sin 3\theta)^2 d\theta$	M1	3.1a
	$(4 - 1.5 \sin 3\theta)^2 = 16 - 12 \sin 3\theta + 2.25 \sin^2 3\theta$ $= 16 - 12 \sin 3\theta + 2.25 \left(\frac{1 - \cos 6\theta}{2} \right) = 16 - 8a \sin 3\theta + a^2 \left(\frac{1 - \cos 6\theta}{2} \right)$	M1	2.1
	$\int (4 - 1.5 \sin 3\theta)^2 d\theta = 16\theta + 4 \cos 3\theta + \frac{9}{8} \left(\theta - \frac{\sin 6\theta}{6} \right)$ $\left[= 16\theta + \frac{8a}{3} \cos 3\theta + \frac{a^2}{2} \left(\theta - \frac{\sin 6\theta}{6} \right) \right]$	A1ft	1.1b
	$\frac{1}{2} \left[\frac{137}{8} \theta + 4 \cos 3\theta - \frac{3}{16} \sin 6\theta \right]_0^{2\pi} = \dots \left(\frac{137}{8} \pi \right)$	dM1	3.1a
	Area of $T = \pi \times 36 - \frac{137}{8} \pi$	DM1	1.1b
	$= \frac{151}{8} \pi \text{ (m}^2\text{) oe}$	A1	1.1b
		(6)	
(8 marks)			
Notes			
(a) M1: Uses all the information given for the model and realises the maximum value of r is $(4 + a)$ and uses the radius of the circle to find a value for a. A1: Deduces the correct value of a. Note $a = -1.5$ can potentially gain M1A0. (b) Note accept with their a, a made up a or even a itself for the first 5 marks. Note use of $a = -1.5$ can score full marks. M1: Adopts a correct strategy for the area of the pool. This requires the correct use of the polar area formula including the $\frac{1}{2}$. Note the $\frac{1}{2}$ may be implied by choice of limits (e.g. any span of π radians without the half implies an attempt at doubling so the $\frac{1}{2}$ may not appear (even if the symmetry is incorrect).) M1: Squares the bracket, achieving three terms, and applies $\sin^2 3\theta = \frac{\pm 1 \pm \cos 6\theta}{2}$ in order to reach an integrable form. Condone numerical slips when expanding. A1ft: Correct integration in any form ($\frac{1}{2}$ not needed here) (follow through their a). ie as shown in scheme or if gathered it is $\left(32 + a^2 \right) \frac{\theta}{2} + \frac{8a}{3} \cos 3\theta - a^2 \frac{\sin 6\theta}{12}$			

dm1: Depends on previous M. Uses appropriate limits for their integrated function. Allow limits other than 0 and 2π by using symmetry provided the correct multiple is used, e.g. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

followed by doubling (which may be cancelled with the $\frac{1}{2}$) or $-\frac{\pi}{6}$ and $\frac{\pi}{2}$ and multiplying by 3

DM1: Depends on first M. Fully correct strategy for obtaining the area of T . Must have a correct attempt at the circle area (maybe be via integration) and area inside the curve. Symmetry may have been used.

A1: Correct area from fully correct work (all previous marks scored). Units not required. The decimal answer 18.875π is acceptable, but answer must be exact, not a rounded decimal.

Note: $\sin^2 3\theta = \frac{1 + \cos 6\theta}{2}$ use will score a maximum of M1M0A0dM0DM1A0

Note: if $\frac{1}{2} \int_0^{2\pi} (6^2 - (4 - a \sin 3\theta)^2) d\theta$ is used the scheme will follow as above with A1ft for

$10\theta - \frac{8a}{6} \cos 3\theta - \frac{a^2}{4} \left(\theta - \frac{\sin 6\theta}{6} \right)$ and dM1DM1 gained together.

Note: Integrating over 0 to 2π means the trig terms will disappear so watch out for incorrect trig terms in the integration, which will lead to the correct answer but will lose both A marks.