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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Polar Coordinates (7)

Core Pure Mathematics 1 · Topic 7

2 questions · 17 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.
Click any row below to jump straight to that sub-topic or question.
On every question page, click ANSWER in the top-right corner to jump to its mark scheme.
Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
7.3 AREAS AND TANGENTS OF POLAR CURVES		2 Qs · 17 marks	3
1. Q3 <i>also Topic 5 & 6</i>	Paper 1: Core Pure Mathematics 1 June2024	8	4
2. Q3	Paper 1: Core Pure Mathematics 1 October2020	9	8

QUESTIONS

2 questions · 17 marks · 1 sub-topics



SUB-TOPIC 7.3

AREAS AND TANGENTS OF POLAR CURVES

Sub-topic 7.3 · 2 questions · 17 marks · lowest tariff first



3.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

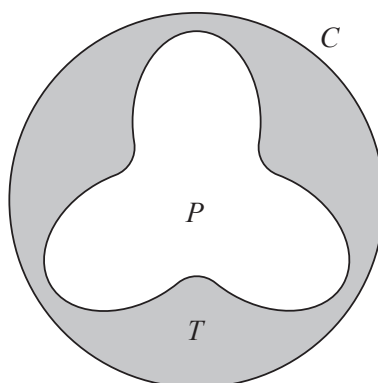


Figure 1

Figure 1 shows the design for a bathing pool.

The pool, P , shown unshaded in Figure 1, is surrounded by a tiled area, T , shown shaded in Figure 1.The tiled area is bounded by the edge of the pool and by a circle, C , with radius 6 m.

The centre of the pool and the centre of the circle are the same point.

The edge of the pool is modelled by the curve with polar equation

$$r = 4 - a \sin 3\theta \quad 0 \leq \theta \leq 2\pi$$

where a is a positive constant.Given that the shortest distance between the edge of the pool and the circle C is 0.5 m,(a) determine the value of a .

(2)

(b) Hence, using algebraic integration, determine, according to the model, the exact area of T .

(6)

Question 3 continued





Question 3 continued

(Total for Question 3 is 8 marks)







(Total for Question 9 is 7 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
3(a)	Max $r = 4 + a = 5.5 \Rightarrow a = \dots$	M1	3.4
	$a = 1.5$	A1	2.2a
		(2)	
(b)	Pool area $= \frac{1}{2} \int_0^{2\pi} (4 - 1.5 \sin 3\theta)^2 d\theta$	M1	3.1a
	$(4 - 1.5 \sin 3\theta)^2 = 16 - 12 \sin 3\theta + 2.25 \sin^2 3\theta$ $= 16 - 12 \sin 3\theta + 2.25 \left(\frac{1 - \cos 6\theta}{2} \right) = 16 - 8a \sin 3\theta + a^2 \left(\frac{1 - \cos 6\theta}{2} \right)$	M1	2.1
	$\int (4 - 1.5 \sin 3\theta)^2 d\theta = 16\theta + 4 \cos 3\theta + \frac{9}{8} \left(\theta - \frac{\sin 6\theta}{6} \right)$ $\left[= 16\theta + \frac{8a}{3} \cos 3\theta + \frac{a^2}{2} \left(\theta - \frac{\sin 6\theta}{6} \right) \right]$	A1ft	1.1b
	$\frac{1}{2} \left[\frac{137}{8} \theta + 4 \cos 3\theta - \frac{3}{16} \sin 6\theta \right]_0^{2\pi} = \dots \left(\frac{137}{8} \pi \right)$	dM1	3.1a
	Area of $T = \pi \times 36 - \frac{137}{8} \pi$	DM1	1.1b
	$= \frac{151}{8} \pi \text{ (m}^2\text{) oe}$	A1	1.1b
		(6)	
(8 marks)			
Notes			
(a) M1: Uses all the information given for the model and realises the maximum value of r is $(4 + a)$ and uses the radius of the circle to find a value for a. A1: Deduces the correct value of a. Note $a = -1.5$ can potentially gain M1A0. (b) Note accept with their a, a made up a or even a itself for the first 5 marks. Note use of $a = -1.5$ can score full marks. M1: Adopts a correct strategy for the area of the pool. This requires the correct use of the polar area formula including the $\frac{1}{2}$. Note the $\frac{1}{2}$ may be implied by choice of limits (e.g. any span of π radians without the half implies an attempt at doubling so the $\frac{1}{2}$ may not appear (even if the symmetry is incorrect).) M1: Squares the bracket, achieving three terms, and applies $\sin^2 3\theta = \frac{\pm 1 \pm \cos 6\theta}{2}$ in order to reach an integrable form. Condone numerical slips when expanding. A1ft: Correct integration in any form ($\frac{1}{2}$ not needed here) (follow through their a). ie as shown in scheme or if gathered it is $\left(32 + a^2 \right) \frac{\theta}{2} + \frac{8a}{3} \cos 3\theta - a^2 \frac{\sin 6\theta}{12}$			



DM1: Depends on previous M. Uses appropriate limits for their integrated function. Allow limits other than 0 and 2π by using symmetry provided the correct multiple is used, e.g. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

followed by doubling (which may be cancelled with the $\frac{1}{2}$) or $-\frac{\pi}{6}$ and $\frac{\pi}{2}$ and multiplying by 3

DM1: Depends on first M. Fully correct strategy for obtaining the area of T . Must have a correct attempt at the circle area (maybe be via integration) and area inside the curve. Symmetry may have been used.

A1: Correct area from fully correct work (all previous marks scored). Units not required. The decimal answer 18.875π is acceptable, but answer must be exact, not a rounded decimal.

Note: $\sin^2 3\theta = \frac{1 + \cos 6\theta}{2}$ use will score a maximum of M1M0A0dM0DM1A0

Note: if $\frac{1}{2} \int_0^{2\pi} (6^2 - (4 - a \sin 3\theta)^2) d\theta$ is used the scheme will follow as above with A1ft for

$10\theta - \frac{8a}{6} \cos 3\theta - \frac{a^2}{4} \left(\theta - \frac{\sin 6\theta}{6} \right)$ and dM1DM1 gained together.

Note: Integrating over 0 to 2π means the trig terms will disappear so watch out for incorrect trig terms in the integration, which will lead to the correct answer but will lose both A marks.



Question	Scheme	Marks	AOs
3	$3(1 - \sin \theta) = 1 + \sin \theta \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \dots$	M1	3.1a
	$\theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6} \right)$	A1	1.1b
	Use of $\frac{1}{2} \int (1 + \sin \theta)^2 d\theta$ or $\frac{1}{2} \int \{3(1 - \sin \theta)\}^2 d\theta$	M1	1.1a
	$\left(\frac{1}{2} \right) \int \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta$ $= \left(\frac{1}{2} \right) \int \left[1 + 2\sin \theta + \sin^2 \theta - 9 + 18\sin \theta - 9\sin^2 \theta \right] d\theta$ or $\int (1 + \sin \theta)^2 d\theta = \int (1 + 2\sin \theta + \sin^2 \theta) d\theta \text{ and}$ $\int 9(1 - \sin \theta)^2 d\theta = 9 \int (1 - 2\sin \theta + \sin^2 \theta) d\theta$	M1 A1	2.1 1.1b
	$\int \sin^2 \theta d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta \Rightarrow$ $\int \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta = 2\sin 2\theta - 12\theta - 20\cos \theta$	M1 A1	3.1a 1.1b
	$A = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta$ or $A = 2 \times \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[(1 + \sin \theta)^2 - 9(1 - \sin \theta)^2 \right] d\theta$ $= \frac{1}{2} \left\{ (-\sqrt{3} - 10\pi + 10\sqrt{3}) - (\sqrt{3} - 2\pi - 10\sqrt{3}) \right\} = \dots$	DM1	3.1a
	$= 9\sqrt{3} - 4\pi$	A1	1.1b
		(9)	
(9 marks)			
Notes			
M1: Realises that the angles at the intersection are required and solves $C_1 = C_2$ to obtain a value for θ A1: Correct value for θ . Must be in radians – if given in degrees you may need to check later to see if they convert to radians before substitution. M1: Evidence selecting the correct polar area formula on either curve M1: Fully expands both expressions for r^2 either as parts of separate integrals or as one complete integral. (Can be scored from incorrect polar area formula, e.g. missing the $\frac{1}{2}$) A1: Correct expansions for both curves (may be unsimplified)			

M1: Selects the correct strategy by applying the correct double angle identity in order to reach an integrable form and attempting the integration of at least one of the curves.

A1: Correct integration (of both integrals if done separately),

FYI: If done separately the correct integrals are

$$\int (1 + \sin \theta)^2 d\theta = \theta - 2 \cos \theta + \frac{1}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) = \frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \text{ and}$$

$$\int 9(1 - \sin \theta)^2 d\theta = 9\theta + 18 \cos \theta + \frac{9}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) = \frac{27}{2} \theta + 18 \cos \theta - \frac{9}{4} \sin 2\theta$$

DM1: Depends on all previous M's. For a fully correct strategy with appropriate limits correctly applied to their integral or integrals and terms combined if necessary. Make sure that if limits of

$\frac{\pi}{6}$ and $\frac{\pi}{2}$ are used that the area is doubled as part of the strategy.

A1: Correct area

