



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Hyperbolic Functions (8)

Core Pure Mathematics 1 · Topic 8

5 questions · 35 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

 guidemygrades.com

 @Guidemygrades

 @Guidemygrades

 @Guidemygrades

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
5.2 IMPROPER INTEGRALS		1 Qs · 6 marks	3
1. Q9 <i>also Topic 5</i>	Paper 1: Core Pure Mathematics 1 June2022	6	4
8.1 HYPERBOLIC FUNCTIONS AND GRAPHS		3 Qs · 18 marks	6
2. Q2	Paper 1: Core Pure Mathematics 1 June2022	4	7
3. Q2	Paper 1: Core Pure Mathematics 1 June2025	5	9
4. Q9	Paper 1: Core Pure Mathematics 1 June2025	9	11
8.5 INTEGRATION USING HYPERBOLIC SUBSTITUTIONS		1 Qs · 11 marks	15
5. Q9 <i>also Topic 5</i>	Paper 1: Core Pure Mathematics 1 November2021	11	16



QUESTIONS

5 questions · 35 marks · 3 sub-topics



SUB-TOPIC 5.2

IMPROPER INTEGRALS

Sub-topic 5.2 · 1 question · 6 marks · lowest tariff first



(1)

(3)

(2)



(Total for Question 9 is 6 marks)



SUB-TOPIC 8.1

HYPERBOLIC FUNCTIONS AND GRAPHS

Sub-topic 8.1 · 3 questions · 18 marks · lowest tariff first



2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64 \cosh^4 x - 64 \cosh^2 x - 9 = 0$$

Give your answers in the form $q \ln 2$ where q is rational and in simplest form.

(4)





2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Determine the exact values of x for which

$$\sinh 2x = 3 \sinh x$$

(5)



SUB-TOPIC 8.5

INTEGRATION USING HYPERBOLIC SUBSTITUTIONS

Sub-topic 8.5 · 1 question · 11 marks · lowest tariff first



9. (a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2 - 1}} dx = \frac{1}{2} \left[x\sqrt{x^2 - 1} + \operatorname{arcosh} x \right] + k$$

where k is an arbitrary constant.

(6)

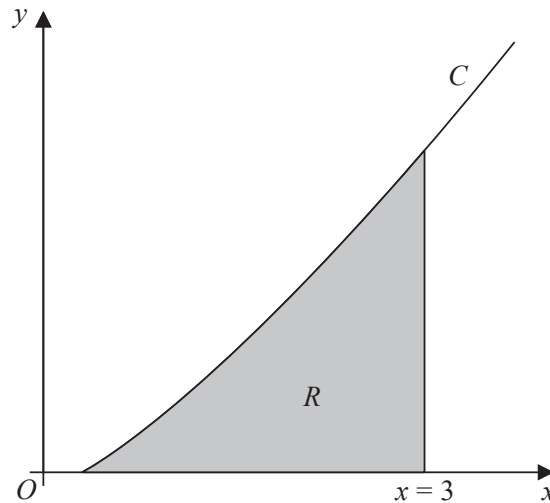


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{4}{15} x \operatorname{arcosh} x \quad x \geq 1$$

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the x -axis and the line with equation $x = 3$

- (b) Using algebraic integration and the result from part (a), show that the area of R is given by

$$\frac{1}{15} \left[17 \ln(3 + 2\sqrt{2}) - 6\sqrt{2} \right] \quad (5)$$

Question 9 continued



Question 9 continued



Question 9 continued



Question 9 continued

(Total for Question 9 is 11 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
9(i) (a)	E. g. - Because the interval being integrated over is unbounded. - cosh x is undefined at the limit of ∞ - the upper limit is infinite	B1	1.2
		(1)	
(i) (b)	$\int_0^\infty \cosh x \, dx = \lim_{t \rightarrow \infty} \int_0^t \cosh x \, dx$ or $\lim_{t \rightarrow \infty} \int_0^t \frac{1}{2}(e^x + e^{-x}) \, dx$	B1	2.5
	$\int_0^t \cosh x \, dx = [\sinh x]_0^t = \sinh t - (-0)$ or $\frac{1}{2} \int_0^t e^x + e^{-x} \, dx = \frac{1}{2} [e^x - e^{-x}]_0^t = \frac{1}{2} [e^t - e^{-t}] - \left[-\frac{1}{2} [e^0 - e^0] \right]$	M1	1.1b
	When $t \rightarrow \infty$ $e^t \rightarrow \infty$ and $e^{-t} \rightarrow 0$ therefore the integral is divergent	A1	2.4
		(3)	
(ii)	$4 \sinh x = p \cosh x \Rightarrow \tanh x = \frac{p}{4}$ or $4 \tanh x = p$ Alternative $\frac{4}{2}(e^x - e^{-x}) = \frac{p}{2}(e^x + e^{-x}) \Rightarrow 4e^x - 4e^{-x} = pe^x + pe^{-x}$ $e^{2x}(4 - p) = p + 4 \Rightarrow e^{2x} = \frac{p + 4}{4 - p}$	M1	3.1a
	$\left\{ -1 < \frac{p}{4} < 1 \Rightarrow \right\} -4 < p < 4$	A1	2.2a
		(2)	
(6 marks)			
(i)(a) B1: For a suitable explanation. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept “Because the upper limit is infinity”, but not “because it is infinity” without reference to what “it” is. Do not accept “the upper limit tends to infinity” or “the integral is unbounded”.			
(i)(b) B1: Writes the integral in terms of a limit as $t \rightarrow \infty$ (or other variable) with limits 0 and “ t ”, or implies the integral is a limit by subsequent working by correct language. M1: Integrates cosh x correctly either as $\sinh x$ or in terms of exponentials and applies correctly the limits of 0 and “ t ”. The bottom limit zero may be implied. No need for the $\lim_{t \rightarrow \infty}$ for this mark but substitution of ∞ is M0. A1: cso States that (as $t \rightarrow \infty$) $\sinh t \rightarrow \infty$ or $e^t \rightarrow \infty$ and $e^{-t} \rightarrow 0$ therefore divergent (or not convergent), or equivalent working. Accept $\sinh t$ is undefined as $t \rightarrow \infty$			
(ii) M1: Divides through by cosh x to find an expression involving $\tanh x$ Alternative: uses the correct exponential definitions and finds an expression for e^{2x} or solves a quadratic in e^{2x} A1: Deduces the correct inequality for p . Note $ p < 4$ is a correct inequality for p .			

Question	Scheme	Marks	AOs
----------	--------	-------	-----

Question	Scheme	Marks	AOs
2	Solves the quadratic equation for $\cosh^2 x$ e.g. $(8 \cosh^2 x - 9)(8 \cosh^2 x + 1) = 0 \Rightarrow \cosh^2 x = \dots$	M1	3.1a
	$\cosh^2 x = \frac{9}{8} \left\{ -\frac{1}{8} \right\}$	A1	1.1b
	$\cosh x = \frac{3}{4}\sqrt{2} \Rightarrow x = \ln \left[\frac{3}{4}\sqrt{2} + \sqrt{\left(\frac{3}{4}\sqrt{2}\right)^2 - 1} \right]$ Alternatively $\cosh x = \frac{3}{4}\sqrt{2} \Rightarrow \frac{1}{2}(e^x + e^{-x}) \Rightarrow e^{2x} - \frac{3}{2}\sqrt{2}e^x + 1 = 0$ $\Rightarrow e^x = \sqrt{2} \text{ or } \frac{\sqrt{2}}{2} \Rightarrow x = \dots$	M1	1.1b
	$x = \pm \frac{1}{2} \ln 2$	A1	2.2a
		(4)	
(4 marks)			
Notes:			
M1: Solves the quadratic equation for $\cosh^2 x$ by any valid means. If by calculator accept for reaching the positive value for $\cosh^2 x$ (negative may be omitted or incorrect) but do not allow for going directly to a value for $\cosh x$. Alternatively score a correct process leading to a value for $\sinh 2x$ or its square (Alt 1) or use of correct exponential form for $\cosh x$ to form and expand to an equation in e^{4x} and e^{2x} (Alt 2). A1: Correct value for $\cosh^2 x$ (ignore negative or incorrect extra roots.). In Alt 1 score for a correct value for $\sinh^2 2x$ or $\sinh 2x$. In Alt 2 score for a correct simplified equation in e^{4x}. M1: For a correct method to achieve at least one value for x (from $\cosh^2 x$). In the main scheme or Alt 1, takes positive square root (if appropriate) and uses the correct formula for $\operatorname{arcosh} x$ or $\operatorname{arsinh} x$ to find a value for x. (No need to see negative square root rejected.) In Alt 2 it is for solving the quadratic in e^{4x} and proceeding to find a value for x. Alternatively uses the exponential definition for $\cosh x$, forms and solves a quadratic for e^x leading to a value for x. A1: Deduces (both) the correct values for x and no others. Must be in the form specified. SC Allow M0A0M1A1 for cases where a calculator was used to get the value for $\cosh x$ with no evidence if a correct method for find both values is shown.			

2 Alt 1	$64 \cosh^2 x (\cosh^2 x - 1) - 9 = 0 \Rightarrow 64 \cosh^2 x \sinh^2 x - 9 = 0$ $\Rightarrow 16 \sinh^2 2x = 9 \Rightarrow \sinh^2 2x = \frac{9}{16}$ $\text{Or } (8 \sinh x \cosh x - 3)(8 \sinh x \cosh x + 3) = 0 \Rightarrow \sinh 2x = \pm \frac{3}{4}$	M1 A1	3.1a 1.1b
	$\sinh 2x = \pm \frac{3}{4} \Rightarrow x = \frac{1}{2} \ln \left[\pm \frac{3}{4} + \sqrt{\frac{9}{16} + 1} \right] \text{ (or use exponentials, or proceed via cosh4x)}$	M1	1.1b
	$x = \pm \frac{1}{2} \ln 2$	A1	2.2a
		(4)	
2 Alt 2	$64 \left(\frac{e^x + e^{-x}}{2} \right)^4 - 64 \left(\frac{e^x + e^{-x}}{2} \right)^2 - 9 = 0 \Rightarrow$ $4(e^{4x} + 4e^{2x} + 6 + 4e^{-2x} + e^{-4x}) - 16(e^{2x} + 2 + e^{-2x}) - 9 = 0$	M1	3.1a
	$4e^{4x} - 17 + 4e^{-4x} = 0$	A1	1.1b
	$(4e^{4x} - 1)(1 - 4e^{-4x}) = 0 \Rightarrow e^{4x} = \dots \Rightarrow x = \dots$	M1	1.1b
	$x = \pm \frac{1}{2} \ln 2$	A1	2.2a
		(4)	



Question	Scheme	Marks	AOs
2	Uses the identity $\sinh 2x = 2 \sinh x \cosh x$ to find a value for $\sinh x$ or $\cosh x$ $2 \sinh x \cosh x = 3 \sinh x \Rightarrow 2 \sinh x \cosh x - 3 \sinh x = 0$ $\sinh x (2 \cosh x - 3) = 0 \Rightarrow \sinh x = \dots$ or $\cosh x = \dots$	M1	3.1a
	$\sinh x = 0 \Rightarrow x = 0$	B1	1.1b
	$\cosh x = \frac{3}{2} \Rightarrow x = \ln \left[\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1} \right]$ Alternatively $\cosh x = \frac{3}{2} \Rightarrow \frac{1}{2}(e^x + e^{-x}) \Rightarrow e^{2x} - 3e^x + 1 = 0$ $\Rightarrow e^x = \frac{3 + \sqrt{5}}{2}$ or $\frac{3 - \sqrt{5}}{2} \Rightarrow x = \ln \dots$	dM1	1.1b
	$x = \ln \left[\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1} \right]$	A1	1.1b
	$x = \pm \ln \left(\frac{3 + \sqrt{5}}{2} \right)$ or $x = \ln \left(\frac{3 \pm \sqrt{5}}{2} \right)$	A1	2.2a
		(5)	
	<u>Alternative method use of exponentials</u> Uses the identities $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$ $\frac{e^{2x} - e^{-2x}}{2} = 3 \left(\frac{e^x - e^{-x}}{2} \right)$ leading to a quartic equation for e^x (Note correct equation is $e^{4x} - 3e^{3x} + 3e^x - 1 = 0$)	M1	3.1a
	$e^x = 1 \Rightarrow x = 0$	B1	1.1b
	Solves quartic to find a value for e^x leading to $x = \ln \dots$	dM1	1.1b
	$x = \ln \left[\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1} \right]$	A1	1.1b
	$x = \pm \ln \left(\frac{3 + \sqrt{5}}{2} \right)$ or $x = \ln \left(\frac{3 \pm \sqrt{5}}{2} \right)$	A1	2.2a
		(5)	
(5 marks)			

Notes:

M1: Uses the identity $\sinh 2x = 2 \sinh x \cosh x$ and proceeds to find a value for $\sinh x$ or $\cosh x$

B1: $x = 0$

dM1: Uses the correct formula for $\operatorname{arcosh} x$ with their value of $\cosh x$ to find a value for x as a natural logarithm or alternatively gives awrt to 3sf. e.g. 0.962, -0.962 . You may need to check their workings.

Alternatively uses the exponential definition for $\cosh x$, forms and solves a quadratic for e^x leading to find a value for x as a natural logarithm. Usual rules apply for solving a quadratic by any means including using a calculator. You may need to check their workings. If correct answers are given without workings, you may award this mark.

A1: Deduces one correct exact value for x . Accept exact unsimplified equivalents.

A1: Deduces both correct exact values for x . Accept exact simplified equivalents. isw

Accept, for example $x = \pm \ln \left(\frac{3}{2} + \frac{\sqrt{5}}{2} \right)$ and $x = \pm \ln \left(\frac{3}{2} + \sqrt{\frac{5}{4}} \right)$

Also accept equivalent answers which you may need to check; there may be several alternatives.

e.g. Accept $x = \ln \left(\frac{2}{3 + \sqrt{5}} \right)$ for $x = \ln \left(\frac{3 - \sqrt{5}}{2} \right)$

Alternative method use of exponentials

M1: Uses correct identities such as $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$ to form a quartic equation for e^x

B1: $x = 0$

dM1: Solve their quartic equation to find a value for x (other than $x = 0$) via a correct method. If correct answers are given from their quartic equation without workings, you may award this mark. You may need to check their workings.

A1: Deduces one correct exact value for x , which may be unsimplified.

A1: Deduces both correct exact values for x isw

Note: Inexact answers for the quartic equation $e^x = 2.618, 0.3819$ leading to awrt 0.962, -0.962 scores maximum M1 B1 dM1 A0 A0

Question	Scheme	Marks	AOs
9(i)(a)	$\sinh x = \left(\frac{e^x - e^{-x}}{2} \right)$ and $\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$ or $\frac{e^x - e^{-x}}{e^x + e^{-x}}$	B1	1.2
	$\frac{3}{4} \sinh x = \tanh x + \frac{1}{5} \Rightarrow \frac{3}{4} \left(\frac{e^x - e^{-x}}{2} \right) = \frac{e^{2x} - 1}{e^{2x} + 1} + \frac{1}{5}$ or $\frac{3}{4} \left(\frac{e^x - e^{-x}}{2} \right) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + \frac{1}{5}$ Leading to a quartic equation for e^x	M1	3.1a
	$15e^{4x} - 48e^{3x} + 32e^x - 15 = 0$ *	A1*	1.1b
		(3)	
(i)(b)	$15(3)^4 - 48(3)^3 + 32(3) - 15 = 0$ therefore $e^x = 3$ is a solution	B1	1.1b
		(1)	
(i)(c)	$(\ln 3, 1)$	B1	1.1b
		(1)	
(ii)	$\int \frac{e^x}{x^2} dx = \lambda e^{\frac{1}{x}}$ $\left\{ u = x^{-1} \Rightarrow \frac{du}{dx} = -x^{-2} \Rightarrow \int x^{-2} e^u \cdot \frac{du}{-x^{-2}} = \int -e^u du \right\}$	M1	3.1a
	$\int \frac{e^x}{x^2} dx = -e^{\frac{1}{x}}$	A1	1.1b
	$\int_{-4}^0 \frac{e^x}{x^2} dx = \lim_{t \rightarrow 0} \left[-e^{\frac{1}{x}} \right]_{-4}^t = \lim_{t \rightarrow 0} \left[\left(-e^{\frac{1}{t}} \right) - \left(-e^{-\frac{1}{4}} \right) \right]$	M1	2.5
	$t \rightarrow 0 \Rightarrow e^{\frac{1}{t}} \rightarrow 0$ therefore $\int_{-4}^0 \frac{e^x}{x^2} dx = e^{-\frac{1}{4}}$ *	A1*	2.4
		(4)	
(9 marks)			

Notes:**(i)(a)****B1:** Recalls the exponential definitions for $\sinh x$ and at least one of $\cosh x$ or $\tanh x$, which may be embedded in their workings.**M1:** Substitutes the correct exponential definitions to form an equation leading to a quartic equation for e^x . Any identities used such as $\sinh 2x = 2 \sinh x \cosh x$ must be correct**A1*:** Correct equation following at least one intermediate stage, with no errors seen. They cannot go directly from their substitution to the given answer. cso**(i) (b)****B1:** Substitutes $e^x = 3$ into each term of the equation, shows $= 0$ and states therefore a solution or writes $(e^x - 3)$ is a factor. Allow a tick, box, QED or appropriate conclusion.

OR

Substitutes $x = \ln 3$ into each term of the equation, shows $= 0$ and states therefore a solution or writes $(e^x - 3)$ is a factor. Allow a tick, box, QED or appropriate conclusion.

OR

Factorises their equation:

$$15e^{4x} - 48e^{3x} + 32e^x - 15 = 0$$

$$\Rightarrow (e^x - 3)(15e^{3x} - 3e^{2x} - 9e^x + 5) = 0$$

and states hence $e^x = 3$ is a solutionDo not award this mark for simply solving a quartic such as $15y^4 - 48y^3 + 32y - 15 = 0$ on the calculator and stating $y = 3$, hence $e^x = 3$ is a solution.**(i)(c)****B1:** States the correct exact coordinates. Allow $x = ..$, $y = ...$. Do not accept decimals.**(ii)****M1:** Use the substitution $u = \pm \frac{1}{x}$ to obtain an integral of the form $\int \lambda e^u du$ or integrates to λe^u Award for a sight of $\pm \lambda e^{\frac{1}{x}}$ as the answer to their integral.Alternatively applies the reverse of the chain rule $\frac{d}{dx} \left(e^{\frac{1}{x}} \right) = \pm \frac{1}{x^2} e^{\frac{1}{x}}$ and integrates to reach $\pm \lambda e^{\frac{1}{x}}$

Answers using integration by parts are unlikely to lead to a correct solution.

A1: Correct integration. Award for correct answer with minimal workings as this can be done by inspection using the reverse of the chain rule. However, withhold this mark for incorrect workings.

M1: Uses correct notation to write the integral as the limit as $t \rightarrow 0$, with the limits of t (or any such variable) and -4 . Applies correctly the limits of -4 and t with correct limit notation, with limits seen substituted the right way round.

Withhold this mark if there is no evidence of using the limiting process. We must see as a minimum $\lim_{t \rightarrow 0}$ at some stage in their work.

A1*: Produces an argument that includes an upper limit that approaches 0.

e.g. States that $t \rightarrow 0 \Rightarrow e^{\frac{1}{t}} \rightarrow 0$ and reaches a value of $e^{-\frac{1}{4}}$

Alternative method: changing limits using $u = \frac{1}{x}$

$$\int_{-4}^0 \frac{e^x}{x^2} dx = \int -e^u du = -e^u \quad \text{M1}$$

$$= \left(\lim_{t \rightarrow 0} \right) \left[-e^u \right]_{\frac{1}{4}}^{\frac{1}{t}} \quad \text{correct limits required} \quad \text{A1}$$

$$= \lim_{t \rightarrow 0} \left[\left(-e^{\frac{1}{t}} \right) - \left(-e^{-\frac{1}{4}} \right) \right] \quad \text{M1}$$

$$t \rightarrow 0 \Rightarrow e^{\frac{1}{t}} \rightarrow 0 \quad \text{therefore} \quad \int_{-4}^0 \frac{e^x}{x^2} dx = e^{-\frac{1}{4}} \quad \text{A1*}$$

Question	Scheme	Marks	AOs
9(a)	$\int \frac{x^2}{\sqrt{x^2-1}} dx \rightarrow \int f(u) du$ Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of u only, including replacing the dx	M1	3.1a
	$\int \frac{\cosh^2 u}{\sqrt{\cosh^2 u - 1}} \sinh u (du)$	A1	1.1b
	Uses correct identities $\cosh^2 u - 1 = \sinh^2 u \text{ and } \cosh 2u = 2\cosh^2 u - 1$ to achieve an integral of the form $A \int (\cosh 2u \pm 1) du \quad A > 0$	M1	3.1a
	Integrates to achieve $A \left(\pm \frac{1}{2} \sinh 2u \pm u \right) (+c) \quad A > 0$	M1	1.1b
	Uses the identity $\sinh 2u = 2 \sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ $\rightarrow \sinh 2u = 2x\sqrt{x^2-1}$	M1	2.1
	$\frac{1}{2} \left[x\sqrt{x^2-1} + \operatorname{arccosh} x \right] + k * \text{cso}$	A1*	1.1b
		(6)	
(b)	Uses integration by parts the correct way around to achieve $\int \frac{4}{15} x \operatorname{arccosh} x dx = Px^2 \operatorname{arccosh} x - Q \int \frac{x^2}{\sqrt{x^2-1}} dx$	M1	2.1
	$= \frac{4}{15} \left(\frac{1}{2} x^2 \operatorname{arccosh} x - \frac{1}{2} \int \frac{x^2}{\sqrt{x^2-1}} dx \right)$	A1	1.1b
	$= \frac{4}{15} \left(\frac{1}{2} x^2 \operatorname{arccosh} x - \frac{1}{2} \left(\frac{1}{2} \left[x\sqrt{x^2-1} + \operatorname{arccosh} x \right] \right) \right)$	B1ft	2.2a
	Uses the limits $x=1$ and $x=3$ the correct way around and subtracts $= \frac{4}{15} \left(\frac{1}{2} (3)^2 \operatorname{arccosh} 3 - \frac{1}{2} \left(\frac{1}{2} \left[3\sqrt{(3)^2-1} + \operatorname{arccosh} 3 \right] \right) \right) - \frac{4}{15} (0)$	dM1	1.1b
	$= \frac{4}{15} \left(\frac{9}{2} \ln(3+\sqrt{8}) - \frac{3\sqrt{8}}{4} - \frac{1}{4} \ln(3+\sqrt{8}) \right)$ $= \frac{1}{15} \left[17 \ln(3+2\sqrt{2}) - 6\sqrt{2} \right] *$	A1*	1.1b
		(5)	
(11 marks)			



Notes:**(a)**

M1: Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of u only. Must have replaced the dx but allow if the du is missing.

A1: Correct integral in terms of u . (Allow if the du is missing.)

M1: Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integrand of the required form

M1: Integrates to achieve the correct form, may be sign errors.

M1: Uses the identities $\sinh 2u = 2\sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ to attempt to find $\sinh 2u$ in terms of x . If using exponentials there must be a full and complete method to attempt the correct form.

A1*: Achieves the printed answer with no errors seen, cso

NB attempts at integration by parts are not likely to make progress – to do so would need to split the integrand as $x \frac{x}{\sqrt{x^2 - 1}}$. If you see any attempts that you feel merit credit, use review.

(b)

M1: Uses integration by parts the correct way around to achieve the required form.

A1: Correct integration by parts

B1ft: Deduces the integral by using the result from part (a). Follow through on their ‘ uv ’

dM1: Dependent on previous method mark. Uses the limits $x = 1$ and $x = 3$ the correct way around and subtracts

A1*cso: Achieves the printed answer with at least one intermediate step showing the evaluation of the arcosh 3, and no errors seen.

