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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Differential Equations (9)

Core Pure Mathematics 1 · Topic 9

11 questions · 125 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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9.1 FIRST ORDER DES: INTEGRATING FACTOR		2 Qs · 18 marks	3
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QUESTIONS

11 questions · 125 marks · 6 sub-topics



SUB-TOPIC 9.1

FIRST ORDER DES: INTEGRATING FACTOR

Sub-topic 9.1 · 2 questions · 18 marks · lowest tariff first



3. (a) Determine the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = e^{2x} \cos^2 x$$

giving your answer in the form $y = f(x)$

(3)

Given that $y = 3$ when $x = 0$

- (b) determine the smallest positive value of x for which $y = 0$

(3)



Question 3 continued

(Total for Question 3 is 6 marks)





SUB-TOPIC 9.2

GENERAL AND PARTICULAR SOLUTIONS IN CONTEXT

Sub-topic 9.2 · 3 questions · 32 marks · lowest tariff first







Question 8 continued

(Total for Question 8 is 9 marks)



7. A sample of bacteria in a sealed container is being studied.

The number of bacteria, P , in thousands, is modelled by the differential equation

$$(1+t) \frac{dP}{dt} + P = t^{\frac{1}{2}} (1+t)$$

where t is the time in hours after the start of the study.

Initially, there are exactly 5000 bacteria in the container.

- (a) Determine, according to the model, the number of bacteria in the container 8 hours after the start of the study.

(6)

- (b) Find, according to the model, the rate of change of the number of bacteria in the container 4 hours after the start of the study.

(4)

- (c) State a limitation of the model.

(1)



Question 7 continued



Question 7 continued



(Total for Question 7 is 11 marks)

6. Water is flowing into and out of a large tank.

Initially the tank contains 10 litres of water.

The rate of flow of the water is modelled so that

- there are V litres of water in the tank at time t minutes after the water begins to flow
- water enters the tank at a rate of $\left(3 - \frac{4}{1 + e^{0.8t}}\right)$ litres per minute
- water leaves the tank at a rate proportional to the volume of water remaining in the tank

Given that when $t = 0$ the volume of water in the tank is decreasing at a rate of 3 litres per minute, use the model to

(a) show that the volume of water in the tank at time t satisfies

$$\frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - 0.4V \quad (3)$$

(b) Determine $\frac{d}{dt}(\arctan e^{0.4t})$

(2)

Hence, by solving the differential equation from part (a),

(c) determine an equation for the volume of water in the tank at time t .

Give your answer in simplest form as $V = f(t)$

After 10 minutes, the volume of water in the tank was 8 litres.

(d) Evaluate the model in light of this information. (1)

Question 6 continued



Question 6 continued



Question 6 continued

(Total for Question 6 is 12 marks)



SUB-TOPIC 9.3

DIFFERENTIAL EQUATIONS IN KINEMATICS

Sub-topic 9.3 · 1 question · 9 marks · lowest tariff first



Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 9 marks)



SUB-TOPIC 9.5

SECOND ORDER NON-HOMOGENEOUS DES

Sub-topic 9.5 · 2 questions · 22 marks · lowest tariff first



10.

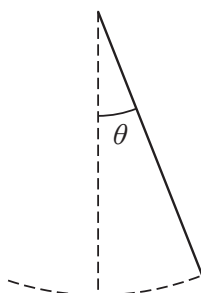


Figure 3

The motion of a pendulum, shown in Figure 3, is modelled by the differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

where θ is the angle, in radians, that the pendulum makes with the downward vertical, t seconds after it begins to move.

(a) (i) Show that a particular solution of the differential equation is

$$\theta = \frac{1}{12}t \sin 3t \quad (4)$$

(ii) Hence, find the general solution of the differential equation.

(4)

Initially, the pendulum

- makes an angle of $\frac{\pi}{3}$ radians with the downward vertical
- is at rest

Given that, 10 seconds after it begins to move, the pendulum makes an angle of α radians with the downward vertical,

(b) determine, according to the model, the value of α to 3 significant figures.

(4)

Given that the true value of α is 0.62

(c) evaluate the model.

(1)

The differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

models the motion of the pendulum as moving with forced harmonic motion.

(d) Refine the differential equation so that the motion of the pendulum is simple harmonic motion.

(1)



Question 10 continued



Question 10 continued



Question 10 continued

(Total for Question 10 is 14 marks)



4. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{3x}$$

giving your answer in the form $y = f(x)$

(5)

Given that $y = 5$ and $\frac{dy}{dx} = 12$ when $x = 0$

- (b) determine the particular solution of the differential equation.

(3)

Question 4 continued

SUB-TOPIC 9.8

DAMPED OSCILLATIONS

Sub-topic 9.8 · 1 question · 12 marks · lowest tariff first







(Total for Question 6 is 12 marks)



SUB-TOPIC 9.9

COUPLED FIRST ORDER SYSTEMS

Sub-topic 9.9 · 2 questions · 32 marks · lowest tariff first







(Room for Question 6 is to the right)







(Total for Question 5 is 17 marks)

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
3(a)	$\frac{dy}{dx} + y \tan x = e^{2x} \cos x$ $\text{IF} = e^{\int \tan x \, dx} = e^{\ln \sec x} = \sec x \Rightarrow \sec x \frac{dy}{dx} + y \sec x \tan x = e^{2x}$ $\Rightarrow y \sec x = \int e^{2x} \, dx$	M1	3.1a
	$y \sec x = \frac{1}{2} e^{2x} (+c)$	A1	1.1b
	$y = \left(\frac{1}{2} e^{2x} + c \right) \cos x$	A1	1.1b
		(3)	
(b)	$x = 0, y = 3 \Rightarrow c = \dots \{2.5\}$	M1	3.1a
	$y = \left(\frac{1}{2} e^{2x} + \frac{5}{2} \right) \cos x = 0 \Rightarrow \cos x = 0 \Rightarrow x = \dots$	M1	1.1b
	$x = \frac{\pi}{2}$	A1	1.1b
		(3)	
(6 marks)			
Notes:			
(a) M1: Finds the integrating factor and attempts the solution of the differential equation. Look for I.F. = $e^{\int \tan x \, dx} \Rightarrow y \times \text{'their I.F.'} = \int e^{2x} \cos x \times \text{'their I.F.'} \, dx$ A1: Correct solution condone missing + c A1: Correct general solution, Accept equivalents of the form $y = f(x)$, such as $y = \frac{e^{2x}}{2 \sec x} + \frac{c}{\sec x}$			
(b) M1: Uses $x = 0 \quad y = 3$ to find the constant of integration. Allow if done as part of part (a) and allow for their answer to (a) as long as it has a constant of integration to find. M1: Sets $y = 0$ in an equation of the form $y = (Ae^{2x} + c) \cos x$ (oe) where A is 1, 2 or $\frac{1}{2}$, with their c or constant c and makes a valid attempt to solve the equation to find a value for x. (Allow even if the constant of integration has not been found). A1: Depends on both M's. Awrt 1.57 or $\frac{\pi}{2}$ only. There must have been an attempt to find the constant of integration, but allow from a correct answer to (a) as long as a positive value for c has been found (can be scored from implicit form).			

Question	Scheme	Marks	AOs
8(a)	$(\sin 1)(273.2) - 213 \cos 1 = A(\sin 2)(\sin 1) \Rightarrow A = 150$	B1	3.3
		(1)	
(b)	$\sin t \frac{dx}{dt} - x \cos t = 150 \sin 2t \sin t \Rightarrow \frac{dx}{dt} - x \frac{\cos t}{\sin t} = 150 \sin 2t$ $IF = e^{-\int \frac{\cos t}{\sin t} dx} = e^{-\ln \sin t} = \operatorname{cosec} t$ $\Rightarrow \operatorname{cosec} t \frac{dx}{dt} - x \operatorname{cosec} t \cot t = 150 \frac{\sin 2t}{\sin t}$ oe or $x \operatorname{cosec} t = \int 150 \frac{\sin 2t}{\sin t} dt$ oe	M1	3.1b
	Uses $\sin 2t = 2 \sin t \cos t$ to arrange into an integrable form and then integrates $x \operatorname{cosec} t = 300 \int \cos t dt \Rightarrow x \operatorname{cosec} t = \dots$	dM1	3.1a
	$x \operatorname{cosec} t = 300 \sin t \{+c\}$	A1	1.1b
	Uses $t = 1, x = 213$ to find the value of c $\frac{213}{\sin 1} = "300" \sin 1 + c$ $\Rightarrow c = \dots$	M1	3.4
	$x = "300" \sin^2 t + "0.687" \sin t$ or $f(t) = "300" \sin^2 t + "0.687" \sin t$	A1	1.1b
		(5)	
(c)	$300 \sin^2 t + 0.687 \sin t = 0$ $\sin t (300 \sin t + 0.687) = 0$ $\Rightarrow \sin t = \dots$	M1	3.4
	$t = \pi$ or $t = \pi - \sin^{-1}\left(-\frac{0.687}{300}\right)$ or $\pi + \sin^{-1}\left(\frac{0.687}{300}\right)$ or awrt $t = 3.14$	M1	1.1b
	13:08 or 13:09 or 1.08 (pm) or 1.09 (pm)	A1	3.2a
		(3)	
(d)	$300 \sin^2 t + 0.687 \sin t = 300.68$ $300 \sin^2 t + 0.687 \sin t - 300.68 = 0$ $\Rightarrow \sin t = \dots$	M1	3.4
	$t = \text{awrt } 1.57$	A1	1.1b

		(2)	
(e)	The model suggests that the time to go up the hill and the time to go down will be the similar. In reality the times will be different.	B1ft	3.5b
		(1)	
(12 marks)			



Notes:

(a)

B1: $A = \text{awrt } 150$

Note $A = 150.0436\dots$

You may mark (b) (c) and (d) together so do not be concerned about the labelling.

(b)

M1: Finds an integrating factor of the form $e^{\pm \int \frac{\cos t}{\sin t} dx} = e^{\pm \ln \sin t}$ or $\operatorname{cosec} t$ (or $\sin t$) and proceeds to multiply through by their IF or

Look for

$$I.F. = e^{\pm \int \frac{\cos t}{\sin t} dt} \Rightarrow x \times \text{'their } I.F.\text{'} = \int A \sin 2t \times \text{'their } I.F.\text{' } dt$$

Where they must be using a numerical A

dm1: Dependent on the previous method mark. Uses the identity $\sin 2t = 2 \sin t \cos t$ to arrange RHS into an integrable form with a correct integrating factor of $\operatorname{cosec} t$ and then attempts to integrate using their numerical A . Condone a sign slip on their RHS.

$$x \operatorname{cosec} t = 2A \int \cos t dt \Rightarrow x \operatorname{cosec} t = \pm \alpha \sin t \quad (+c)$$

A1: Correct general solution, condone missing $+c$

M1: Using the model, $t=1$, $x=213$ to find the value of their constant c . Substitution if not seen is implied by sight of 213 for x and at least one correct value for $\sin t$. They must use radians and not degrees; you may need to check.

A1: Correct particular solution of $x = 300 \sin^2 t + 0.687 \sin t$

$c = \text{awrt } 0.69$ (when using $A=150$)

Alternatively:

When using answers for A which have not been rounded to 3sf, the equation will instead be

$$x = 300.0872\dots \sin^2 t + 0.613 \sin t \quad \text{in which case allow } c = \text{awrt } 0.61$$

(c)

M1: Sets $x=0$ and solves a quadratic equation of the form $\lambda \sin^2 t + \mu \sin t = 0$, where $\lambda, \mu \neq 0$ finding at least one value for $\sin t$

M1: Solves a quadratic equation of the form $2A \sin^2 t + c \sin t = 0$ where $A, c > 0$ to find a correct value of t . May be implied by a value of t awrt 3.14

Award for $t = \pi$ but accept the use of $\arcsin\left(\frac{-\mu}{\lambda}\right) + \pi$ or, where they must add π to a **positive** value of t . They must also be working in radians.

A1: Correct time awrt 13:08 or 13:09 or 1.08 (pm), 1.09 (pm), which comes from a **correct** equation

(d)

M1: Solves their 3TQ equation of the form $2A \sin^2 t + c \sin t = 300.68$ where $A, c > 0$ to find an answer for $\sin t$. You may need to check the value for $\sin t$ or t obtained from their quadratic. A correct equation followed by $t = \text{awrt } 1.57$ is sufficient.

A1: $t = \text{awrt } 1.57$ isw. Answer must come from a **correct** equation. Do not allow an answer of $\frac{\pi}{2}$ which may have been obtained from $\sin t = 1$ even if it proceeds to 1.57



Here it asks for the value of t , so do not accept a time, for example 11.42am

Alternatively uses

$$300.0872 \sin^2 t + 0.613 \sin t - 300.68 = 0 \Rightarrow \sin t = \dots t = \text{awrt } 1.56$$

(e)

B1ft: They must have stated times or values for t in (c) and (d) for this mark.

We need a comparison consistent with their **times** from (c) and (d) **and** reality.

Their values may not be correct, but their statement should be consistent with their times.

You may ignore incorrect statements once a correct statement is given, provided their incorrect statement does not contradict their correct statement.

e.g.

Accept

- The time to go up and down is the same, which is not realistic
- The times are similar which is unlikely
- It should take longer to go up the hill
- In reality it will take longer to go up the hill than down the hill (because of the slope)

Condone

- It takes longer to go up the hill than down the hill
- It takes the same amount of time to go up and down
- In the model it takes them longer to go down the hill
- The rambler may not walk **up and down** at the same speed, or rate

Reject

- the rambler may need to take a rest
- the model is inaccurate or invalid for certain values of t
- the rambler may walk down a different path or route
- the rambler is modelled as a particle
- the rambler walks at a constant velocity
- the rambler may not walk at the same speed/pace for the entire journey

If you are uncertain please send to review



Question	Scheme		Marks	AOs
8(a)	Volume of paint = 30 litres therefore Rate of paint out = $3 \times \frac{r}{30}$ litres per second		M1	3.3
	$\frac{dr}{dt} = 2 - \frac{r}{10}$		A1	1.1b
			(2)	
(b)	Rearranges $\frac{dr}{dt} + \frac{r}{10} = 2$ and attempts integrating factor IF = $e^{\int \frac{1}{10} dt} = \dots$	Separates the variables $\int \frac{1}{20-r} dr = \frac{1}{10} dt$ $\Rightarrow \dots$	M1	3.1a
	$re^{\frac{t}{10}} = \int 2e^{\frac{t}{10}} dt \Rightarrow re^{\frac{t}{10}} = \lambda e^{\frac{t}{10}} (+c)$	Integrates to the form $\lambda \ln(20-r) = \frac{1}{10} t (+c)$	M1	1.1b
	$re^{\frac{t}{10}} = 20e^{\frac{t}{10}} + c$	$-\ln(20-r) = \frac{1}{10} t + c$	A1ft	1.1b
	$t = 0, r = 10 \Rightarrow c = \dots$		M1	3.4
	$r = \frac{20e^{\frac{t}{10}} - 10}{e^{\frac{t}{10}}} = 15$ rearranges to achieve $e^{\frac{t}{10}} = \alpha$ and solves to find a value for t or $r = 20 - 10e^{-\frac{t}{10}} = 15$ rearranges to achieve $e^{-\frac{t}{10}} = \beta$ and solves to find a value for t	$-\ln(20-15) = \frac{1}{10} t - \ln 10$ Leading to a value for t	M1	3.4
	$t = \text{awrt } 7 \text{ seconds}$		A1	2.2b
			(6)	
(c)	The model predicts 7 seconds but it actually takes 9 seconds so (over) 2 seconds out (over 20%), therefore it is not a good model		B1ft	3.5a
			(1)	
(9 marks)				
Notes:				
(a)				
M1: Clearly identifies that Rate of paint out = $3 \times \frac{r}{\text{their volume}}$. It is a “show that” question so there must be clearly reasoning. Just answer with no reasoning scores M0.				
A1: Puts all the components together to form the correct differential equation.				

(b)

M1: Identifies as a first order differential equation and finds the integrating factor or separates the variables and integrates. Allow if there are sign slips in rearranging (e.g. to $\frac{dr}{dt} - \frac{r}{10} = 2$) or in the integrating factor and allow with their value for a or with a as an unknown.

M1: Multiplies through by the IF and attempts to integrate or integrates to the form

$$\lambda \ln(2a - r) = \frac{1}{a}t + c \text{ oe}$$

A1ft: Correct integration, including constant of integration. Follow through on their value of a , but not sign slips from rearrangement. So allow for $re^{\frac{t}{a}} = 2ae^{\frac{t}{a}} + c$ or $-\ln(2a - r) = \frac{1}{a}t + c$ oe with a or their a .

M1: Uses the initial conditions to find the constant of integration. Must see substitution or can be implied by the correct value for their equation. Allow for finding in terms of a if separation of variables used.

M1: Sets $r = 15$, achieves $e^{\frac{t}{10}} = \alpha > 0$ or $e^{-\frac{t}{10}} = \beta > 0$ as appropriate and solves to find a value for t . Separates the variable method sets $r = 15$ and rearranges to find a value for t . **Note:** For this mark a value of a is needed, but need not be the correct one.

A1cso: $t = \text{awrt } 7$ seconds from fully correct work.

(c)

B1ft: See scheme, follow through on their answer to part (b). Accept any reasonable comparative comment but must have a reason, not just a statement of good or not good. So e.g. look for finding the difference between their answer and 9, or the percentage difference. If their answer is close to 9, then accept a conclusion of being a good model if a suitable reason is given. May substitute 9 into their equation and obtain a value to compare with 15 and make a similar conclusion.



Question	Scheme	Marks	AOs
7(a)	$(1+t)\frac{dP}{dt} + P = t^{\frac{1}{2}}(1+t) \Rightarrow \frac{dP}{dt} + \frac{P}{1+t} = t^{\frac{1}{2}}$	B1	1.1b
	$I = e^{\int \frac{1}{1+t} dt} = 1+t \Rightarrow P(1+t) = \int t^{\frac{1}{2}}(1+t) dt = \dots$	M1	3.1b
	$P(1+t) = \frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + c$	A1	1.1b
	$t = 0, P = 5 \Rightarrow c = 5$	M1	3.4
	$P = \frac{\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + 5}{(1+t)} = \frac{\frac{2}{3}8^{\frac{3}{2}} + \frac{2}{5}8^{\frac{5}{2}} + 5}{9} = \dots$	M1	1.1b
	$= 10\,277 \text{ bacteria (allow awrt } 10\,300)$	A1	2.2b
	(6)		
(b)	$P = \frac{\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + 5}{(1+t)} \Rightarrow \frac{dP}{dt} = \frac{(1+t)\left(t^{\frac{1}{2}} + t^{\frac{3}{2}}\right) - \left(\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + 5\right)}{(1+t)^2}$ Alt: $P + (1+t)\frac{dP}{dt} = t^{\frac{1}{2}} + t^{\frac{3}{2}} \Rightarrow \frac{dP}{dt} = \frac{t^{\frac{1}{2}} + t^{\frac{3}{2}} - \frac{\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + 5}{(1+t)}}{(1+t)}$	M1 A1ft	3.4 1.1b
	$\left(\frac{dP}{dt}\right)_{t=1} = \frac{dP}{dt} = \frac{5 \times 10 - \left(\frac{16}{3} + \frac{64}{5} + 5\right)}{(5)^2} = \frac{403}{375}$	dM1	3.1a
	$\frac{403}{375} \times 1000 = \frac{3224}{3} (= \text{awrt } 1070) \text{ bacteria per hour}$	A1	3.2a
	(4)		
	(b) Alternative:		
	$P = \frac{\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + 5}{(1+t)} = \frac{\frac{16}{3} + \frac{64}{5} + 5}{(1+4)}$	M1	3.4
	$= \frac{347}{75}$	A1ft	1.1b
	$(1+t)\frac{dP}{dt} + P = t^{\frac{1}{2}}(1+t) \Rightarrow 5\frac{dP}{dt} + \frac{347}{75} = 2 \times 5 \Rightarrow \frac{dP}{dt} = \frac{403}{375}$	dM1	3.1a
	$\frac{403}{375} \times 1000 = \frac{3224}{3} (= 1075) \text{ bacteria per hour}$	A1	3.2a
	(4)		
	(c)		
	E.g. • The number of bacteria increases indefinitely which is not realistic	B1	3.5b
	(1)		
(11 marks)			

Notes

(a)

B1: A correct rearrangement (may be implied by subsequent work). Alternatively, recognises the LHS as a derivative and writes $(1+t)\frac{dP}{dt} + P = \frac{d}{dt}(P(1+t)) \left(= t^{\frac{1}{2}}(1+t) \right)$ (may be implied).

M1: Uses the model to find the integrating factor (or recognise the derivative) and attempts the solution of the differential equation to achieve $P \times \text{their IF} = \int \text{their } t^{\frac{1}{2}} \times \text{their IF } dt = \dots$ but do not be too concerned with the mechanics of integrating the RHS but it must be attempted.

A1: Correct solution

M1: Interprets the initial conditions to find the constant of integration. Must be using $t = 0$ and $P = 5$ in an equation with a constant of integration, but their equation may have come from incorrect work. This is correctly interpreting the initial conditions and attempting to use them.

M1: Uses their solution to the problem to find the population after 8 hours. Must be using their solution, but allow for any equations which arise from an attempt at solving the differential equation.

A1: **cs0** Correct number of bacteria (accept awrt 10 300) from a correct equation

(b)

M1: Realises the need to differentiate the model and uses an appropriate method to find the derivative. Allow the M for attempts at implicit differentiation with $(1+t)P = \dots$ Trivialised differentiation from incorrect work is M0.

A1ft: Correct differentiation of the correct answer to (a) up to the constant of integration to obtain dP/dt in terms of t (if implicit differentiation is used, they must get to a function in terms of t only, or revert to the Alternative method). Follow through on their c in an otherwise correct equation from (a).

M1: Uses $t = 4$ in their dP/dt (allow from any attempts at the derivative) to obtain a value for dP/dt .

A1: Correct answer, allow 1075 or answers rounding down to 1070 with correct units. Accept as 1.07 thousand bacteria per hour.

(NB If 5000 is used in (a) instead of 5, the answer here would be -198.725)

Alternative:

M1: Substitutes $t = 4$ into their P

A1ft: Correct value for P . Follow through on their constant of integration from part (a), but the rest of the equation must be correct.

M1: Uses $t = 4$ and their P to find a value for dP/dt

A1: Correct answer allow 1075 or answers rounding down to 1070 with correct units. Accept as 1.07 thousand bacteria per hour.

(c)

B1: Suggests a suitable limitation which must refer to the model. Allow for a sensible comment even if they have no equation for the model

Do not allow answers such as “the model does not take account of external factors such as temperature” as we do not know what factors the model does take account of.



Question	Scheme	Marks	AOs
6(a)	$\frac{dV}{dt} = 3 - \frac{4}{1+e^{0.8t}} \pm kV$ (where k is constant)	M1	3.3
	$t = 0, V = 10, \frac{dV}{dt} = -3 \Rightarrow -3 = 3 - \frac{4}{1+1} - 10k \Rightarrow k = \dots$	dM1	3.4
	$\Rightarrow 10k = 4 \Rightarrow k = \frac{2}{5} \Rightarrow \frac{dV}{dt} = 3 - \frac{4}{1+e^{0.8t}} - 0.4V^*$	A1*	2.1
		(3)	
(b)	$\frac{d}{dt}(\arctan e^{0.4t}) = \frac{1}{1+(e^{0.4t})^2} \times ke^{0.4t}$	M1	1.1b
	$\frac{d}{dt}(\arctan e^{0.4t}) = \frac{2e^{0.4t}}{5(1+e^{0.8t})} \text{ oe}$	A1	1.1b
		(2)	
Alternative to part (b):			
	$y = \arctan e^{0.4t} \Rightarrow \tan y = e^{0.4t} \Rightarrow \sec^2 y \frac{dy}{dx} = 0.4e^{0.4t}$	M1	1.1b
	$\frac{dy}{dx} = \frac{0.4e^{0.4t}}{\sec^2 y} = \frac{0.4e^{0.4t}}{1+\tan^2 y} = \frac{0.4e^{0.4t}}{1+(e^{0.4t})^2}$	A1	1.1b
		(2)	
(c)	$\frac{dV}{dt} + 0.4V = 3 - \frac{4}{1+e^{0.8t}} \Rightarrow I.F. \left(= e^{\int 0.4 dt} \right) = e^{0.4t}$	B1	2.2a
	$e^{0.4t}V = \int 3e^{0.4t} - \frac{4e^{0.4t}}{1+e^{0.8t}} dt$	M1	1.1b
	$= Ae^{0.4t} - B \arctan(e^{0.4t}) (+c)$	M1	1.1b
	$e^{0.4t}V = 7.5e^{0.4t} - 10 \arctan(e^{0.4t}) (+c)$	A1	1.1b
	$V = 10, t = 0 \Rightarrow 10 = 7.5 - 10 \arctan 1 + c \Rightarrow c = \dots$	M1	3.4
	$V = 7.5 - 10e^{-0.4t} \arctan(e^{0.4t}) + 2.5(\pi + 1)e^{-0.4t}$	A1	2.1
		(6)	
(d)	E.g. $V(10) \approx 7.4$ litres so the model is not very accurate as it predicts approximately 7.5% below the actual level.	B1ft	3.5a
		(1)	
(12 marks)			

Notes**(a)****M1:** Sets up the correct equation for the model using the information in the question.**dm1:** Uses the initial conditions to find the constant of proportionality for flow out.

Condone use of $\frac{dV}{dt} = +3$. **Depends on the first mark.**

A1*: Correct equation shown from correct work proceeding via $10k = 4$ to find k .**Attempts in (a) using verification score no marks:**

E.g. $\frac{dV}{dt} = 3 - \frac{4}{1+e^{0.8t}} - \frac{2}{5}V \Rightarrow -3 = 3 - \frac{4}{2} - 0.4V \Rightarrow V = \frac{4}{0.4} = 10$

(b)**M1:** Differentiates to achieve the form shown. Allow $k = 1$ **A1:** Correct derivative in any form. Need not be simplified.**Alternative:**

M1: Takes tan of both sides and differentiates implicitly and reaches $\frac{1}{1+(e^{0.4t})^2} \times ke^{0.4t}$. Allow $k = 1$.

A1: Correct derivative in any form. Need not be simplified.**(c)**

B1: Deduces the correct integrating factor for the equation. May be implied by sight of $\frac{d}{dt}(e^{0.4t}V) = \dots$ or equivalent work.

M1: Fully multiplies through by their integrating factor and integrates the LHS (look for

$I.F. \times V = \int I.F. \times \left(3 - \frac{4}{1+e^{0.8t}}\right) dt$ though condone missing dt .

M1: Attempts the integral of the RHS.

Award for $\int \alpha e^{0.4t} dt = \beta e^{0.4t} \quad \alpha \neq \beta$ or $\int \frac{\alpha e^{0.4t}}{1+e^{0.8t}} dt = \beta \arctan e^{0.4t}, \quad \beta \neq 0$

A1: Correct integration, need not be simplified. Allow if the $+ c$ is missing for this mark.**M1:** Attempts to find their constant – which must have been treated correctly from point of integration.

Note that this is not formally dependent but there must have been an attempt to integrate.

A1: Correct answer. The question says “simplest form” but allow equivalent expressions e.g.

$V = 7.5 - \frac{10 \arctan(e^{0.4t})}{e^{0.4t}} + \frac{5\pi}{2e^{0.4t}} + \frac{5}{2e^{0.4t}}$ but do not allow inexact values for the constants.

(d)**B1ft:** Evaluates V where $V > 0$ at $t = 10$ and makes an appropriate comment.

For the evaluation, allow if a value of V is obtained even if there is no evidence of substitution provided that it is clear that $t = 10$ has not been substituted into something that is not V . So you do not need to check their value.

For the tolerance you may need to use your own judgement but a general guide is:

$0 < V < 7$ Not a good model
 $7 \text{ ,, } V < 7.7 \text{ or } 8.3 \text{ ,, } V < 9$ Allow good **or** poor model
 $7.7 \text{ ,, } V < 8.3$ Good model
 $V \dots 9$ Not a good model



Question	Scheme	Marks	AOs
5(a)	$(t+2)\frac{dv}{dt} + 3v = k(t+2) - 3 \Rightarrow \frac{dv}{dt} + \frac{3v}{t+2} = k - \frac{3}{t+2}$ $I = e^{\int \frac{3}{t+2} dt} = (t+2)^3$	M1	3.1b
	$\Rightarrow v(t+2)^3 = \int \left(k(t+2)^3 - 3(t+2)^2 \right) dt = \dots$	M1	1.1b
	$v(t+2)^3 = \frac{k}{4}(t+2)^4 - (t+2)^3 (+c)$ $\left[\text{Alt } (t+2)^3 v = \frac{kt^4}{4} + 2kt^3 - t^3 + 6kt^2 - 6t^2 + 8kt - 12t(+c) \right]$	A1	1.1b
	$t = 0, v = 0 \Rightarrow c = 8 - 4k$	M1	3.4
	$v(t+2)^3 = \frac{k}{4}(t+2)^4 - (t+2)^3 + 8 - 4k$ $\Rightarrow v = \frac{k}{4}(t+2) - 1 + \frac{4(2-k)}{(t+2)^3} *$	A1*	2.1
		(5)	
(b)	$v = 4, t = 2 \Rightarrow 4 = k - 1 + \frac{4(2-k)}{64} \Rightarrow k = \dots (5.2)$	M1	3.1b
	$\Rightarrow v = \frac{"5.2"}{4}(5+2) - 1 + \frac{4(2-"5.2")}{(5+2)^3} = \dots$	dM1	3.4
	Velocity is 8.06 ms^{-1} (awrt)	A1	3.2a
		(3)	
(c)	E.g. - The model suggests the velocity increases indefinitely which is unlikely - The raindrop will reach the ground/doesn't fall forever so not valid - The raindrop will reach a terminal velocity after a finite time so not good model	B1	3.5a
		(1)	
(9 marks)			

Notes

(a)

M1: Begins to solve the problem with a correct process to find the integrating factor for the model. Look for an attempt at $e^{\int \frac{A}{t+2} dt} = \dots$ where A is a constant (positive or negative). For attempts at IF by recognition they must achieve form $A(t+2)^3$

M1: Attempts the solution of the differential equation using a correct method with their integrating factor. The dt may be implied. They must attempt the integration as part of the method, though it need not be correct – any changed function will do. So score for

$v \times$ their IF = their attempt at integrating their IF $\times \left(k - \frac{3}{t+2} \right)$ allowing for errors expanding this bracket.

A1: Correct solution. The $+ c$ may be missing for this mark.

M1: Interprets the initial conditions to find the constant of integration – must have a constant of integration for this mark and the correct initial conditions must be clear.

A1*: Obtains the printed answer with no errors and at least one intermediate line between

$v(t+2)^3 = \frac{k}{4}(t+2)^4 - (t+2)^3 + c$ and the stated answer. Allow terms in different order, but the $8 - 4k$ must be factorised.

(b)

M1: Uses the given conditions to establish the value of k in the model

dM1: Uses their value of k and the equation of the model with $t = 5$ to find a value for v . If a value of k has been found accept “ $t = 5 \quad v = \dots$ ” with any value for an attempt.

A1: Correct value **including units** (allow awrt 8.06 ms^{-1}). Accept exact answer $5531/686$

(c)

B1: Evaluates the model by making a suitable comment – see scheme for examples. Accept answers relating to the size of velocity increasing beyond what is reasonable or answers that allude to the situation not being able to continue for ever. Accept if symbols are used, e.g. “as $t \rightarrow \infty, v \rightarrow \infty$ which cannot happen” but must include a comment on validity, not just a statement about what happens (e.g. $t \rightarrow \infty, v \rightarrow \infty$ with no comment).

Do not accept e.g. “ v does not tend to a value” as this is just a statement about what happens. If contrary information is given then award B0.

Some examples of borderline answers seen.

“Not valid as rain is limited to how fast its velocity is therefore can’t be big.” Scores B1

“Due to gravitational force and not huge height between cloud and ground, the model would be inaccurate for large period of time, because the velocity of raindrops would be a big number, which is unlikely.” B1

“Not valid for very large value of t as raindrop will not move that fast.” B0

“Raindrop has very little mass so effect due to gravity is lower. Unlikely to have a 1000 ms^{-1} raindrop. Not very valid model for large values of t ” B1

“The water droplet wouldn’t always follow the model in cases where it hits a surface” B0

“No valid as raindrop will hit ground soon. Speed can be infinitely big which is not plausible.” B1

“Would not be valid as the raindrop likely has already hit the ground.” B1

You may need to use your judgement on whether their comments are acceptable.



10(a)(i)	$\frac{d\theta}{dt} = \alpha \sin 3t + \beta t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \delta \cos 3t + \gamma t \sin 3t$	Let $\theta = \lambda t \sin 3t$ $\frac{d\theta}{dt} = \alpha \sin 3t + \beta t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \delta \cos 3t + \gamma t \sin 3t$	M1	1.1b
	$\frac{d\theta}{dt} = \frac{1}{12} \sin 3t + \frac{1}{4} t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \frac{1}{4} \cos 3t + \frac{1}{4} \cos 3t - \frac{3}{4} t \sin 3t$ $= \frac{1}{2} \cos 3t - \frac{3}{4} t \sin 3t$	$\frac{d\theta}{dt} = \lambda \sin 3t + 3\lambda t \cos 3t$ and $\frac{d^2\theta}{dt^2} = 3\lambda \cos 3t + 3\lambda \cos 3t - 9\lambda t \sin 3t$ $= 6\lambda \cos 3t - 9\lambda t \sin 3t$	A1	1.1b
	$\frac{1}{2} \cos 3t - \frac{3}{4} t \sin 3t + 9\left(\frac{1}{12} t \sin 3t\right)$ $= \dots$	$6\lambda \cos 3t - 9\lambda t \sin 3t + 9(\lambda t \sin 3t)$ $= \frac{1}{2} \cos 3t \Rightarrow \lambda = \dots$	dM1	3.4
	$= \frac{1}{2} \cos 3t$ so PI is $\theta = \frac{1}{12} t \sin 3t$ $*$	$\theta = \frac{1}{12} t \sin 3t *$	A1*	2.1
			(4)	
(a)(ii)	$m^2 + 9 = 0 \Rightarrow m = \pm 3i$		M1	1.1b
	$\theta = A \cos 3t + B \sin 3t$		A1	1.1b
	$(\theta =) CF + PI$		dM1	1.1b
	$\theta = A \cos 3t + B \sin 3t + \frac{1}{12} t \sin 3t$		A1	1.1b
			(4)	
(b)	$t = 0, \theta = \frac{\pi}{3} \Rightarrow A = \dots \left\{ \frac{\pi}{3} \right\}$		M1	3.4
	$t = 0, \frac{d\theta}{dt} = -3A \sin 3t + 3B \cos 3t + \frac{1}{12} \sin 3t + \frac{1}{4} t \cos 3t = 0$ $\Rightarrow B = \dots \{0\}$		M1	3.4
	$\alpha = \frac{\pi}{3} \cos(3 \times 10) + \frac{1}{12} (10) \sin(3 \times 10) = \dots$		ddM1	1.1b
	$\alpha = \pm \text{awrt } 0.662$		A1	3.4
			(4)	
(c)	0.662 is close to 0.62 so a good model (at $t = 10$)		B1ft	3.5a
			(1)	
(d)	$\frac{d^2\theta}{dt^2} + 9\theta = 0$ oe		B1	3.5c
			(1)	

(14 marks)

Notes:**(a)(i) Note: mark (a) as a whole****M1:** Differentiates the given PI twice using the product rule to achieve the required form.

Alternatively, uses a correct form for the PI and differentiates twice using the product rule to achieve the required form. A correct form may involve other terms with coefficients that will be zero, e.g.

 $\theta = \lambda t \sin 3t + \mu t \cos 3t$ is fine. Also allow e.g. $\theta = \lambda t \sin \omega t$ **A1:** Correct derivatives.**dm1:** Depends on first M, substitutes into the given differential equation and attempts to simplify. In the Alt they must go on to find value for λ .**A1*:** Achieves $\frac{1}{2} \cos 3t$ and makes a minimal conclusion (e.g. //). Alternatively reaches the correct PI.**(a)(ii)****M1:** Uses the model to form and solve the auxiliary equation. Accept $m^2 + 9 = 0 \rightarrow m = \pm 3i$ or ± 3 **A1:** Correct complementary function. Must be in terms of t but allow recovery if initially in terms of x but changed later.**dm1:** Dependent on the previous method mark. Finds the general solution by adding the particular integral to the complementary function.**A1:** Correct general solution including " $\theta =$ ", which may be recovered in part (b).**(b)****M1:** Uses the initial conditions of the model, $t = 0$, $\theta = \frac{\pi}{3}$ to find a value for a constant.**M1:** Differentiates the general solution and uses the initial conditions of the model $t = 0$, $\frac{d\theta}{dt} = 0$ to find a value for the other constant.**ddM1:** Dependent on both previous method marks. Substitutes $t = 10$ into their particular solution. If not substitution is seen, accept any value as the attempt as long as they have found all relevant constants.**A1:** Accept awrt ± 0.662 **(c)****B1ft:** Makes a quantitative comparison of the size of their answer to part (b) with 0.62 and makes conclusion (e.g. good model). Follow through on their answer to (b) and draws an appropriate conclusion about the model. Accept "not reasonable" as long as it is supported with evidence but there must be some instructive comparison and a conclusion about the model - not just stating how much it is out. The reason given must be correct.

Accept e.g. a correct percentage error with reasonable conclusion, or statement approximately equal with conclusion.

Do not accept e.g. "does not agree to 1 s.f." or "out by 0.6" as these lack context. Do not accept arguments based solely on a difference in sign, they must be referring to the relative size of angle.

(d)**B1:** Refines the model, accept any constant on the right hand side.

Question	Scheme	Marks	AOs
4(a)	$m^2 - 4m + 4 = 0 \Rightarrow m = \dots \{2\}$	M1	1.1b
	$(y) = Ae^{2x} + Bxe^{2x}$	A1	1.1b
	PI $y = \lambda e^{3x}, \frac{dy}{dx} = 3\lambda e^{3x}$ and $\frac{d^2y}{dx^2} = 9\lambda e^{3x}$ $9\lambda e^{3x} - 4(3\lambda e^{3x}) + 4(\lambda e^{3x}) = 2e^{3x}$ leading to $\lambda = \dots$	M1	3.1a
	$(y) = CF + PI$	dM1	1.1b
	$y = Ae^{2x} + Bxe^{2x} + 2e^{3x}$	A1	1.1b
		(5)	
(b)	$x = 0, y = 5 \Rightarrow 5 = A + 2 \Rightarrow A = \dots$	M1	1.1b
	$\frac{dy}{dx} = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x} + 6e^{3x}$ and uses their value of A and $x = 0, \frac{dy}{dx} = 12$ to find the value of B $12 = 6 + B + 6 \Rightarrow B = \dots$	dM1	3.1a
	$y = 3e^{2x} + 2e^{3x}$	A1	1.1b
		(3)	
(8 marks)			

Notes:**(a)**

M1: Forms the correct auxiliary equation, leading to finding a value for m .

Allow the use of any variable instead of m for this mark.

A1: Correct complementary function, oe such as $y = (A + Bx)e^{2x}$

Do not need $y =$ here but **must** be in terms of x , but may be recovered later.

M1: A complete method to find the particular integral. Uses the correct form $y = \lambda e^{3x}$, differentiates twice and substitutes correctly into the differential equation to find a value for $\lambda (= 2)$. May use a different variable for x here.

dM1: Dependent on the previous method mark. Finds the general solution by adding the particular integral (of the correct form) to their complementary function (which may not be of the correct form). Do not need $y =$ here. May use a different variable for x but their variable must be consistent.

A1: Correct general solution. **Must** have $y =$ here and **must** be in terms of x

(b)

M1: For a general solution of the form $Ae^{\alpha x} + Bxe^{\alpha x} + \text{"their PI"}$, uses the initial conditions, $x = 0, y = 5$ to find a value for a constant.

(The correct form will be $Ae^{mx} + Bxe^{mx} + \lambda e^{3x}$)

dM1: Dependent on the previous method mark.

Differentiates the general solution to achieve an expression of the form $mAe^{mx} + Be^{mx} + mBxe^{mx} + 3\lambda e^{3x}$ using their m from part (a) where $m \neq 1$

Then uses the initial conditions to find a value for the other constant.

A1: Correct particular solution. Must have $y =$ here and must be in terms of x



Question	Scheme	Marks	AOs
6(a)	$5k(13.6) + 2k(0) + 17(-20) = 0 \Rightarrow k = \dots$	M1	3.3
	$k = 5$	A1	1.1b
		(2)	
(b)	Solves their $25m^2 + 10m + 17 = 0 \Rightarrow m = \dots$	M1	3.1b
	$m = -0.2 \pm 0.8i$	A1	1.1b
	$x = e^{-0.2t} (A \cos 0.8t + B \sin 0.8t)$	A1ft	1.1b
	$t = 0, x = -20 \Rightarrow A = \dots (= -20)$	M1	3.4
	$\frac{dx}{dt} = -0.2e^{-0.2t} (A \cos 0.8t + B \sin 0.8t) + e^{-0.2t} (-0.8A \sin 0.8t + 0.8B \cos 0.8t)$	M1	1.1b
	$t = 0 \frac{dx}{dt} = 0 \Rightarrow -0.2A + 0.8B = 0 \Rightarrow B = \dots (= -5)$	dM1	3.4
	$x = e^{-0.2t} (-20 \cos 0.8t - 5 \sin 0.8t)$ o.e.	A1	1.1b
		(7)	
(c)	Vertical height $= 30 + [e^{-0.2 \times 15} (-20 \cos(0.8 \times 15) - 5 \sin(0.8 \times 15))]$	M1	3.4
	Vertical height = awrt 29.3 m	A1	2.2b
		(2)	
(d)	For example It is unlikely that the rope will remain taut The model predicts the tourist will continue to move up and down, (but in fact they will lose momentum) The tourist is modelled as a particle	B1	3.5b
		(1)	
(12 marks)			
Notes:			
(a) M1: Substitutes $\frac{d^2x}{dt^2} = 13.6$ $\frac{dx}{dt} = 0$ and $x = -20$ into the differential equation to find a value for k . Allow if there are sign slips but must be attempting the values in the correct places. A1: Correct value $k = 5$			
(b) M1: Forms and solves the auxiliary equation. A1: Correct solution to the auxiliary equation (not follow through). A1ft: Correct complementary function for their solutions to their auxiliary equation. (Follow through on distinct real, repeated or complex roots.)			

M1: Uses the information from the model $t = 0 \quad x = -20$ to find a constant or equation linking two constants in their equation.

M1: Differentiates an expression of the form $e^{kt} (A \cos \lambda_1 t + B \sin \lambda_2 t)$ using the product rule to find an expression for the velocity.

dM1: Uses the information from the model, $t = 0 \quad \frac{dx}{dt} = 0$ to find and solve another equation for the constants.

A1: Correct equation for displacement.

(c)

M1: Finds the height above the river by finding the displacement after 15 seconds and adding 30

A1: Vertical height = awrt 29.3 m

(d)

B1: Any suitable comment relating to the given model or the outcomes of it. See scheme for examples. Do not accept just “air resistance has not been considered” as the question does not say this was ignored. However, if a valid consequence of what including air resistance would mean to the model, then the mark may be awarded.

Question	Scheme	Marks	AOs
8(a)	$\frac{d^2x}{dt^2} = \frac{dx}{dt} + \frac{dy}{dt} = \frac{dx}{dt} + 3y - 2x$	M1	1.1b
	$= \frac{dx}{dt} + 3\left(\frac{dx}{dt} - x\right) - 2x$	M1	2.1
	$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 0^*$	A1*	1.1b
		(3)	
(b)	$m^2 - 4m + 5 = 0 \Rightarrow m = \dots$	M1	3.4
	$m = 2 \pm i$	A1	1.1b
	$x = e^{at} (A \cos \beta t + B \sin \beta t)$	M1	3.4
	$x = e^{2t} (A \cos t + B \sin t)$	A1	1.1b
		(4)	
(c)	$y = \frac{dx}{dt} - x = e^{2t} (B \cos t - A \sin t + 2A \cos t + 2B \sin t) - e^{2t} (A \cos t + B \sin t)$	M1	3.4
	$y = e^{2t} ((A+B) \cos t + (B-A) \sin t)$	A1	1.1b
		(2)	
(d)	$A = 100, 275 = A + B \Rightarrow B = 175$	M1	3.3
	$x = y \Rightarrow 100 \cos t + 175 \sin t = 275 \cos t + 75 \sin t \Rightarrow \tan t = \dots$	dM1	3.1a
	$\tan t = 1.75$	A1	1.1b
	$T = 24 \tan^{-1}(1.75) = \dots$	M1	3.2a
	$= 25.24$	A1	1.1b
		(5)	
(e)	E.g. • Both populations become negative for some times which is not possible	B1	3.5b
		(1)	
(15 marks)			
Notes			
Note: Accept the dot notation $\dot{x}$ etc, as equivalents for the derivatives wrt t throughout. (a) M1: Differentiates the first equation and substitutes for $\frac{dy}{dt}$ from the second equation. M1: Proceeds to an equation in x and $\frac{dx}{dt}$ only by substituting for y . A1*: Achieves the printed answer with no errors. (b) M1: Uses the model to form and solve the Auxiliary Equation. A1: Correct roots of the AE. M1: Uses the model to form a Complementary Function appropriate for their roots. Accept complex index form. A1: Correct General Solution. Accept complex index form. Must include the x			

(c)

M1: Uses the model and their answer to part (b) to give y in terms of t . Must involve an attempt at the product rule. Alternatively, they may repeat the whole process of parts (a) and (b) again on y – score for a full process leading to the solution for y in terms of t only.

A1: Correct simplified equation though may be with expanded brackets but like terms should be gathered (ie the $A\cos t$ and $B\sin t$ terms). If starting over, allow if names of constants are the same as for (b) (ie treat independently) but they will lose later marks. Accept complex index form.

(d)

M1: Realises the need to use the initial conditions to establish the values of their constants. But must have consistent constants (ie four different constants if they did part (c) from scratch).

Solutions with complex indices are unlikely to achieve this mark. If unsure, use the Review system to consult your team leader.

dM1: Sets $x = y$ and collects and reaches $\tan t = \dots$. Alternatively may convert into sine or cosine and reach an equivalent equation.

A1: Correct equation – accept equivalent if other trig approaches are used. Must come from correct work. Note $\sin t = \frac{7}{\sqrt{65}}$ and $\cos t = \frac{4}{\sqrt{65}}$ are the other ratios.

M1: Solves their $\tan t = \dots$ and multiplies by 24. Must be working in radians. If degrees mode is used score M0.

A1: Correct value. Ignore units.

(e)

B1: Must have at least one correct general equation for x or y for this mark to be awarded.

Rote answers about generic models without evidence to support any claims should not be given credit.

Suggests a suitable correct limitation of the model. This **must** focus on the fact the equations produce negative values. Award if a correct answer is given and ignore extra comments.

E.g. “gives negative values” would be a minimal acceptable answer.

Note: Answers about unlimited growth that imply the populations increase indefinitely or about scarcity of resource **without** an accompanying correct statement are B0.

Question	Scheme	Marks	AOs
5(a)	$\frac{d^2x}{dt^2} = -5\frac{dx}{dt} + 10\frac{dy}{dt}$ oe e.g. $\frac{dy}{dt} = \frac{1}{10}\left(\frac{d^2x}{dt^2} + 5\frac{dx}{dt}\right)$	B1	1.1b
	$\frac{d^2x}{dt^2} = -5\frac{dx}{dt} + 10(-2x + 3y - 4)$ $= -5\frac{dx}{dt} - 20x + \frac{30}{10}\left(\frac{dx}{dt} + 5x + 30\right) - 40$ Or $\frac{1}{10}\left(\frac{d^2x}{dt^2} + 5\frac{dx}{dt}\right) = -2x + \frac{3}{10}\left(30 + 5x + \frac{dx}{dt}\right) - 4$	M1	2.1
	$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 50^*$	A1*	1.1b
		(3)	
(b)	$m^2 + 2m + 5 = 0 \Rightarrow m = \dots$	M1	3.4
	$m = -1 \pm 2i$	A1	1.1b
	$m = \alpha \pm \beta i \Rightarrow x = e^{\alpha t} (A \cos \beta t + B \sin \beta t) = \dots$	M1	3.4
	$x = e^{-t} (A \cos 2t + B \sin 2t)$	A1	1.1b
	PI: Try $x = k \Rightarrow 5k = 50 \Rightarrow k = 10$	M1	3.4
	GS: $x = e^{-t} (A \cos 2t + B \sin 2t) + 10$	A1ft	1.1b
		(6)	
(c)	$\frac{dx}{dt} = e^{-t} (2B \cos 2t - 2A \sin 2t) - e^{-t} (A \cos 2t + B \sin 2t)$	B1ft	1.1b
	$(y =) \frac{1}{10} \left(\frac{dx}{dt} + 5x + 30 \right) = \dots$	M1	3.4
	$y = \frac{1}{10} e^{-t} ((4A + 2B) \cos 2t + (4B - 2A) \sin 2t) + 8$	A1	1.1b
		(3)	
(d)	$t = 0, x = 2 \Rightarrow 2 = A + 10 \Rightarrow A = -8$	M1	3.1b
	$t = 0, y = 5 \Rightarrow 5 = \frac{1}{10} (2B - 32) + 8 \Rightarrow B = 1$	M1	3.3
	$x = e^{-t} (\sin 2t - 8 \cos 2t) + 10$	A1	2.2a
	$y = e^{-t} (2 \sin 2t - 3 \cos 2t) + 8$	A1	2.2a
		(4)	
(e)	E.g When $t > 8$, the amount of compound X and the amount of compound Y remain (approximately) constant at 10 and 8 respectively, which suggests that the chemical reaction has stopped. This supports the scientist's claim.	B1	3.5a
		(1)	
(17 marks)			

Notes

(a)

B1: Differentiates the first equation with respect to t correctly. May have rearranged to make y the subject first. The dot notation for derivatives may be used.

M1: Uses the second equation to eliminate y to achieve an equation in x , $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$.

A1*: Achieves the printed answer with no errors.

(b)

M1: Uses the model to form and attempts to solve the auxiliary equation (Accept a correct equation followed by two values for m as an attempt to solve.)

A1: Correct roots of the AE

M1: Uses the model to form the complementary function. Must be in terms of t only (not x)

A1: Correct CF

M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI

A1ft: Combines their CF (which need not be correct) with the correct PI to give x in terms of t so look for $x = \text{their CF} + 10$

(c)

B1ft: Correct differentiation of their x . Follow through their $e^{at} (A \cos \beta t + B \sin \beta t)$

M1: Uses the model and their answer to part (b) to find an expression for y in terms of t

A1: Correct equation. Mark the final answer but there is not need for terms to be gathered but must have $y = \dots$

(d)

M1: Realises the need to use the initial conditions in the equation for x

M1: Realises the need to use the initial conditions in the equation for y to find both unknown constants - must have equations from which both unknowns can be found. Alternatively, a complete method using $\frac{dx}{dt}$ to find the second constant is made.

A1: Deduces the correct equation for x

A1: Deduces the correct equation for y . For this equation constants should have been gathered.

(e)

B1: Allow for any appropriate comment with valid supporting reason. They must have equations of the correct form from (d). The coefficients may be incorrect, but they must have positive limits for each of x and y .

Both x and y should be considered (see below for exception), and a reason and some comment about the suitability of the model made (though you may allow implicit conclusions). E.g.

- for values of $t > 8$, the amounts of compounds X and Y present settle at 10 and 8 without really varying, which supports the claim.
- $\frac{dx}{dt} \approx 0$ and $\frac{dy}{dt} \approx 0$ when $t = 8$, so neither are changing, which supports the claim.
- As t gets large x and y tend to limits to 10 and 8 neither will be zero, hence the claim is not supported.
- $x = 10.0$ (awrt) and $y = 8.00$ (awrt) when $t = 8$, since neither is zero it is likely the reaction is still continuing so the claim is not supported.

Exception: Allow a reason that states the model assumes that the reaction continues indefinitely, so the claim is not supported. (The reaction stopping would require a change in the model.)



Do NOT allow an answer that only considers x or y . E.g. $x = 10$ when $t = 8$ so the model is not supported is B0 since there is no consideration that y may be zero and hence end the reaction.

Alt for (c) and (d) restarting:

B1: Correct second order equation for y formed: $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 40$

M1: Full method to obtain the general solution: they may recognise the similarity to the equation in x and jump straight to finding the PI, or may form the aux equation etc again, but look for an attempt that combines a (correctly formed) CF and a PI. For this mark allow if the constants used are the same as those for the equation in x .

A1: Correct solution for y with different constants than those for x , though allow recovery if they realise in (d) that they need different constants.

For (d)

M1: As main scheme, allow for using the initial conditions in one equation to make a start finding the constants.

M1: For a full method to obtain all four constants – if the same constant were used for both equations in (c) (inconsistently) then this mark cannot be scored. A full method here would, for instance, require finding $\frac{dx}{dt}$ and using this along with the given initial equations and initial

conditions to find the second constants for each equation.

A1: One correct equation with SC of being qualified by the first M only if a full method to find both constants for just one equation is made (so M1M0A1A0 is possible in this case).

A1: Both equations correct.