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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Matrices (3)

Core Pure Mathematics 2 · Topic 3

7 questions · 64 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

7 questions · 64 marks · 6 sub-topics



SUB-TOPIC 3.1

MATRIX ARITHMETIC

Sub-topic 3.1 · 1 question · 10 marks · lowest tariff first







(Total for Question 3 is 10 marks)



SUB-TOPIC 3.3

LINEAR TRANSFORMATIONS

Sub-topic 3.3 · 1 question · 12 marks · lowest tariff first





Question 3 continued



SUB-TOPIC 3.4

INVARIANT POINTS AND LINES

Sub-topic 3.4 · 1 question · 5 marks · lowest tariff first



2.

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ 5 & 3 \end{pmatrix}$$

The matrix $\mathbf{A}$ represents the linear transformation M .

Prove that, for the linear transformation M , there are no invariant lines.

(5)

(Total for Question 2 is 5 marks)



SUB-TOPIC 3.6

INVERSE MATRICES

Sub-topic 3.6 · 1 question · 11 marks · lowest tariff first







Question 7 continued

(Total for Question 7 is 11 marks)



SUB-TOPIC 3.7

SOLVING SIMULTANEOUS EQUATIONS WITH MATRICES

Sub-topic 3.7 · 1 question · 8 marks · lowest tariff first



Question 2 continued



Question 2 continued



Question 2 continued

(Total for Question 2 is 8 marks)



SUB-TOPIC 3.8

GEOMETRIC INTERPRETATION OF SIMULTANEOUS EQUATIONS

Sub-topic 3.8 · 2 questions · 18 marks · lowest tariff first



where k is a constant

- (4)

- (3)

- (7)

Question 6 continued





(Total for Question 6 is 14 marks)



5. Three planes are defined by the following equations

$$2x - y + z = 3$$

$$x + py - 3z = q$$

$$3x + y - 2z = 4$$

where p and q are constants.

Given that the planes form a sheaf, determine

(i) the value of p

(ii) the value of q

(4)

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
3(a)	$\mathbf{M}^2 + 11\mathbf{M} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \Rightarrow \begin{pmatrix} 34 & 5k-10 \\ 6k-12 & k^2+30 \end{pmatrix} + \begin{pmatrix} -22 & 55 \\ 66 & 11k \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$	M1	1.1b
	$\Rightarrow a = 12$	A1	2.2a
	$5k - 10 + 55 = 0 \Rightarrow 5k = -45 \Rightarrow k = -9^*$ or $6k - 12 + 66 = 0 \Rightarrow 6k = -54 \Rightarrow k = -9^*$ or $k^2 + 11k + 30 = 12 \Rightarrow k^2 + 11k + 18 = 0 \Rightarrow k = -2, -9^*,$ $k \neq -2$ as $5 \times -2 - 10 + 55 \neq 0$ or $6 \times -2 - 12 + 66 \neq 0$	A1*	2.1
		(3)	
	Alternative Using $k = -9$ $\mathbf{M}^2 + 11\mathbf{M} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \Rightarrow \begin{pmatrix} 34 & -55 \\ -66 & 111 \end{pmatrix} + \begin{pmatrix} -22 & 55 \\ 66 & -99 \end{pmatrix} = \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$	M1	1.1b
	$\Rightarrow a = 12$	A1	2.2a
	Conclusion: therefore $k = -9$	A1*	2.1
		(3)	
(b)	$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix} \Rightarrow \begin{cases} -2x + 5(mx+c) = X \\ 6x - 9(mx+c) = mX+c \end{cases}$	M1	1.1b
	$\Rightarrow 6x - 9mx - 9c = -2mx + 5m^2x + 5mc + c$	M1	3.1a
	$\left\{ \Rightarrow (5m^2 + 7m - 6)x + (5m + 10)c = 0 \right\}$	A1	1.1b
	$\Rightarrow 5m^2 + 7m - 6 = 0 \left\{ \Rightarrow (m+2)(5m-3) \right\} \Rightarrow m = -2, \frac{3}{5}$	M1	1.1b
	$m = \frac{3}{5} \Rightarrow 5m + 10 \neq 0$ so need $c = 0$ hence $y = \frac{3}{5}x$ is a fixed line	A1	2.2a
	$m = -2 \Rightarrow 5m + 10 = 0$ so c can be anything, so $y = -2x + c$ for any c is fixed.	A1	2.2a
		(6)	
(c)	$((0, c) \rightarrow (5c, -9c)$ so need $c = 0$, $(1, m) \rightarrow (-2 + 5m, 6 - 9m)$ so need or $5m = 3$ hence $y = \frac{3}{5}x$ contains fixed points.	B1	3.2a

	or $\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x \\ \frac{3}{5}x \end{pmatrix} = \begin{pmatrix} -2x + 3x \\ 6x - \frac{27}{5}x \end{pmatrix} = \begin{pmatrix} x \\ \frac{3}{5}x \end{pmatrix} \Rightarrow y = \frac{3}{5}x$ or $\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{cases} -2x + 5y = x \\ 6x - 9y = y \end{cases} \Rightarrow y = \frac{3}{5}x$		
		(1)	

(10 marks)

Notes:
(a)
M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of a deduced

A1*: Correct work to show $k = -9$. If off diagonals are used no further justification is needed (they are “given” the result is true). If the bottom right entry is used there must be a valid reason for rejecting -2 as a solution (ie checking the off diagonal).

Alternative: Using $k = -9$
M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of a deduced

A1*: Draws the conclusion that $k = -9$
(b)
M1: Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

M1: Eliminates “ X ” to get a linear equation in “ x ”.

A1: Correct equation need not be simplified isw

M1: Solves their quadratic equation in m by any valid means including calculator

A1: Deduces $y = \frac{3}{5}x$ is a fixed line (where $c = 0$). If the value for m here is wrong, allow this A for $y = -2x$ if the general case for the final A is not scored.

A1: Deduces $y = -2x + c$ is a fixed line where c can be any value. Must include all the lines.


Note: $y = \frac{3}{5}x$ and $y = -2x$ scores final A1 A0

Special Case 1 M1M1A0M1A1A0

$$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} X \\ mX \end{pmatrix} \Rightarrow \begin{cases} -2x + 5(mx) = X \\ 6x - 9(mx) = mX \end{cases}$$

M1: Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

M1: Eliminates “X” to get a linear equation in “x”. $\Rightarrow 6x - 9mx = -2mx + 5m^2x$

A0: Incorrect equation.

M1: Solves their quadratic equation in m by any valid means.

$$\Rightarrow 5m^2 + 7m - 6 = 0 \Rightarrow (m + 2)(5m - 3) \Rightarrow m = -2, \frac{3}{5}$$

A1: Deduces $y = \frac{3}{5}x$ is a fixed line (where $c = 0$). If the value for m here is wrong, allow this A for $y = -2x$ if the general case for the final A is not scored.

A0: Incorrect equation

Special Case 2 Finding the line of invariant points M1M0A0M0A1A0

$$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{cases} -2x + 5y = x \\ 6x - 9y = y \end{cases}$$

M1: Sets up the matrix equation for a line of invariant points and extracts the simultaneous equations from the matrix equation.

M0 A0 M0

A1: Deduces $y = \frac{3}{5}x$ is a fixed line

A0: Incorrect equation

Outside the specification

M1: Find the eigenvalues $\begin{pmatrix} -2-\lambda & 5 \\ 6 & -9-\lambda \end{pmatrix} \Rightarrow (2-\lambda)(-9-\lambda) - 6 \times 5 = 0$ leading to a 3TQ and solves to find a value for λ , $\lambda^2 + 11\lambda - 12 = 0 \Rightarrow \lambda = \dots \{1, -12\}$

M1: Uses one of their eigenvalues to find an equation $\begin{pmatrix} -2 & 5 \\ 6 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -12 \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \dots \{y = -2x\}$ or



A1: One correct equation

M1: Uses both of their eigenvalues to find an equation

A1: Deduces $y = \frac{3}{5}x$

A1: Deduces $y = -2x + c$

(c)

B1: Identifies $y = \frac{3}{5}x$ is a line of fixed points with reason. Allow if $c = 0$ is assumed. See scheme for possible reason.



Question	Scheme	Marks	AOs
3(a)	$n=1, \text{LHS} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^1 = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}, \text{RHS} = \begin{pmatrix} 1 & 5(2^1-1) \\ 0 & 2^1 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k, \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^k = \begin{pmatrix} 1 & 5(2^k-1) \\ 0 & 2^k \end{pmatrix}$ then for $n = k+1$ $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 5(2^k-1) \\ 0 & 2^k \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$ or $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 5(2^k-1) \\ 0 & 2^k \end{pmatrix}$	M1	2.4
	$= \begin{pmatrix} 1 & 5 + 2 \times 5(2^k-1) \\ 0 & 2(2^k) \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 5(2^k-1) + 5(2^k) \\ 0 & 2(2^k) \end{pmatrix}$	M1 A1	1.1b 1.1b
	$\begin{pmatrix} 1 & 10(2^k) - 5 \\ 0 & 2^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 5[2(2^k) - 1] \\ 0 & 2^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 5(2^{k+1} - 1) \\ 0 & 2^{k+1} \end{pmatrix}$	A1	2.1
	“If true for $n = k$ then true for $n = k + 1$” and as it is “true for $n = 1$” the statement is “true for all (positive integers) n”	A1	2.4
		(6)	
(b)	Reflection	B1	1.1b
	Reflection in the y-axis or line $x = 0$	B1	1.1b
		(2)	
(c)	$\begin{pmatrix} 1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix}$	B1	1.1b
		(1)	
(d)	$\begin{pmatrix} -1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} 27 \\ 1 \end{pmatrix} = \begin{pmatrix} -27 + 5(2^n-1) \\ 2^n \end{pmatrix}$ Sets $-27 + 5(2^n-1) = 2^n \Rightarrow 2^n = \dots$	M1	3.1a
	$(A^n) = \begin{pmatrix} 1 & 5('8'-1) \\ 0 & '8' \end{pmatrix} = \begin{pmatrix} 1 & 35 \\ 0 & 8 \end{pmatrix}$	M1 A1	1.1b 1.1b
		(3)	
(12 marks)			

Notes:**(a)****B1:** Shows that the result holds for $n = 1$. Must see substitution in the RHS minimum required is
$$\begin{pmatrix} 1 & 5(2-1) \\ 0 & 2 \end{pmatrix} \text{ and reaches } \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$$
 No need to state “true for $n = 1$ ” for this mark.

M1: Assumes the result is true for some value of $n = k$, and sets up a matrix multiplication of the assumed result multiplied by the original matrix, either way round (no need to carry out for this mark, it is for essentially explaining the procedure). “Assume (true for) $n = k$ ” (oe) is sufficient for the assumption and may even be part of the conclusion and accept any alternative wording that indicates the assumption has been made. Allow for setting up the multiplication in reverse (ie working from $n = k + 1$ towards $n = k$, $\mathbf{A}^k = \mathbf{A}^{k+1} \mathbf{A}^{-1}$ oe)

M1: Carries out the multiplication. (Allow the reverse case.)**A1:** Achieves a correct un-simplified matrix. Accept in working in reverse.

A1: Reaches a correct simplified matrix with **no errors, the correct un-simplified matrix seen previously and at least one intermediate line which must be correct**. If working from both sides all steps in showing the two sides are equal must be seen. If working in reverse they must return to $\mathbf{A}^{k+1} = \dots$ for this mark.

A1: Correct formal conclusion. This mark is dependent on the M and previous A marks having been scored and an attempt at the check for $n = 1$ (if e.g. they didn’t show sufficient detail). It is gained by conveying the ideas of **all three bold points at the end of their solution**.

Note: Some cases may use $n = 0$ as the base case. These can score full marks if dealt with correctly, but the conclusion must be consistent with their initial check to score the final A.

For the B mark minimum $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (or **I**) and $\begin{pmatrix} 1 & 5(1-1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ should be seen. (No need to state true for $n = 0$ for this mark – but must have consistent conclusion for the final A as noted above). If unsure send to review.

(b)**B1:** Identifies the transformation as a reflection. Accept stretch with scale factor -1 .

B1: Identifies reflection and the correct line of reflection. Condone phrasing such as “across the y-axis”. Must be a single transformation – B0 if they state something else also happens.

NB Allow B1B0 for answers such as “flip in the y-axis” that convey the correct transformation in imprecise language.

(c)**B1:** Correct matrix**(d)**

M1: A complete method to find a value for 2^n (or n). Multiplies the coordinates $(27, 1)$ by their matrix **C** and sets the x and y coordinates equal to reach a value for 2^n or n (condone negative values for 2^n). Note you may allow this for reach a value of a if $a = 2^n$ is stated or clearly implied by their working.

M1: Uses their value of 2^n or n , to find the matrix $\mathbf{A}^n$. Note substituting into **C** is M0 without further work to find $\mathbf{A}^n$.

A1: Correct matrix.

Question	Scheme	Marks	AOs
2	$\begin{pmatrix} 4 & -2 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix}$ leading to an equation in x , m , c and X	M1	3.1a
	$4x - 2(mx + c) = X$ and $5x + 3(mx + c) = mX + c$	A1	1.1b
	$5x + 3(mx + c) = m(4x - 2(mx + c)) + c$ leading to $5 + 3m = 4m - 2m^2$ $(3c = -2mc + c)$	M1	2.1
	$2m^2 - m + 5 = 0 \Rightarrow b^2 - 4ac =$ $(-1)^2 - 4(2)(5) = \dots$	Solves $3c = -2mc + c \Rightarrow m = \dots$ dM1	1.1b
	Correct expression for the discriminant = $\{-39\} < 0$ therefore there are no invariant lines.	$m = -1$ and shows a contradiction in $5 + 3m = 4m - 2m^2$ therefore there are no invariant lines. A1	2.4
Alternative			
	$\begin{pmatrix} 4 & -2 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} X \\ mX \end{pmatrix}$ leading to an equation in x , m and X	M1	3.1a
	$4x - 2(mx) = X$ and $5x + 3(mx) = mX$	A1	1.1b
	$5x + 3(mx) = m(4x - 2(mx))$ leading to $5 + 3m = 4m - 2m^2$	M1	2.1
	$2m^2 - m + 5 = 0 \Rightarrow b^2 - 4ac = (-1)^2 - 4(2)(5) = \dots$	dM1	1.1b
	Correct expression for the discriminant = $\{-39\} < 0$ therefore there are no invariant lines that pass through the origin no invariant lines.	A1	2.4
		(5)	
(5 marks)			
Notes: M1: Sets up a matrix equation in an attempt to find a fixed line and extract at least one equation. A1: Correct equations. M1: Eliminates X from the simultaneous equations and equates the coefficients of x leading to a quadratic equation in terms of m . dM1: Dependent on the previous method, finds the value of the discriminant, this can be seen in an attempt to solve the quadratic using the formula. Alternatively solves $3c = -2mc + c$ and finds a value for m Note: If the quadratic equation in m is solved on a calculator and complex roots given this is M0 as they are not showing why there are no real roots. A1: Correct expression for the discriminant, states < 0 and draws the required conclusion. Alternatively, correct value for m , shows a contradiction in $5 + 3m = 4m - 2m^2$ and draws the required conclusion.			
Alternative M1: Sets up a matrix equation in an attempt to find a fixed line and extract at least one equation. A1: Correct equations.			

M1: Eliminates X from the simultaneous equations and equates the coefficients of x leading to a quadratic equation in terms of m .

dM1: Dependent on the previous method, finds the value of the discriminant.

A1: Correct expression for the discriminant, states < 0 and draws the required conclusion.



Question	Scheme	Marks	AOs
7(a)	$ \mathbf{M} = 2(-k-8) + 1(-3-12) + 1(6-3k) = 0 \Rightarrow k = \dots$	M1	1.1b
	$k \neq -5$	A1	2.4
		(2)	
(b) Way 1	$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & -6 & 4 \\ 3 & 2 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$	M1	3.1a
	$\mathbf{M}^{-1} = \frac{1}{5} \begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix}$	B1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix} \begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots$	M1	2.1
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -2p+1 \\ 15p-5 \\ 24p-7 \end{pmatrix}$	A1	1.1b
	$\left(\frac{-2p+1}{5}, 3p-1, \frac{24p-7}{5} \right)$	A1ft	2.5
		(5)	
(b) Way 2	$2x - y + z = p$ $3x - 6y + 4z = 1 \Rightarrow \text{e.g. } \begin{matrix} 8y - 5z = -1 \\ 9y - 5z = 3p - 2 \end{matrix} \Rightarrow y = \dots$ $3x + 2y - z = 0$ $\Rightarrow x = \dots, z = \dots$	M1	3.1a
	$y = 3p - 1 \text{ (or } x = \frac{-2p+1}{5} \text{ or } z = \frac{24p-7}{5} \text{)}$	B1	1.1b
	$8(3p-1) - 5z = -1 \Rightarrow z = \dots \Rightarrow x = \dots$	M1	2.1
	$z = \frac{24p-7}{5}, x = \frac{-2p+1}{5}$	A1	1.1b
	$\left(\frac{-2p+1}{5}, 3p-1, \frac{24p-7}{5} \right)$	A1ft	2.5

(c)(i)	For consistency: E.g. $5x + y = 4 - q$ and $15x + 3y = q$	M1	3.1a
	$4 - q = \frac{q}{3} \Rightarrow q = \dots$	M1	2.1
	$q = 3$	A1	1.1b
	Alternative for (c)(i): $x = 1 \Rightarrow 2 - y + z = 1, 3 + 2y - z = 0 \Rightarrow y = \dots, z = \dots$ M1 for allocating a number to one variable and solves for the other 2 $x = 1, y = -4, z = -5 \Rightarrow 3 + 20 - 20 = q$ M1 substitutes into the second equation and solves for q A1: $q = 3$		
(ii)	Three planes that intersect in a line Or Three planes that form a sheaf allow sheath !	B1	2.4
		(4)	
(11 marks)			
Notes			
(a) M1: Attempts determinant, equates to zero and attempts to solve for k in order to establish the restriction for k . For the determinant, at least 2 of the 3 “elements” should be correct. May see rule of Sarrus used for determinant e.g. $ M = (2)(k)(-1) + (4)(3)(-1) + (3)(2)(1) - (3)(k)(-1) - (2)(4)(2) - (-1)(3)(-1) = 0 \Rightarrow k = \dots$ A1: Describes the correct condition for k with no contradictions. Allow e.g. $k < -5, k > -5$ (b) Way 1 M1: A complete strategy for solving the given equations. Need to see an attempt at the inverse followed by a correct method for finding x, y and z B1: Correct inverse matrix M1: Uses their inverse and attempts the multiplication with the correct vector A1: Correct values for x, y and z in any form A1ft: Correct values given in coordinate form only. Follow through their x, y and z. Way 2 M1: A complete strategy for solving the given equations. Need to see an attempt at eliminating one variable followed by a correct method for finding x, y and z B1: One correct value M1: Uses the equations to find values for the other 2 variables A1: Correct values for x, y and z in any form A1ft: Correct values given in coordinate form only. Follow through their x, y and z. (c)(i) M1: Uses a correct strategy that will lead to establishing a value for q . E.g. eliminating one of x, y or z M1: Solves a suitable equation to obtain a value for q A1: Correct value (ii) B1: Describes the correct geometrical configuration. Must include the two ideas of planes and meeting in a line or forming a sheaf with no contradictory statements.			

Question	Scheme	Marks	AOs
2(a)(i)	$x / C = \text{number of Construction students}$ $y / D = \text{number of Design students}$ $z / H = \text{number of Hospitality students}$	B1	3.3
(ii)	The increase in number of students in 2020 $1110 \times 0.0027 \{= 2.997 \approx 3\}$ Or The number of students in 2020 $1110 \times 1.0027 = \{1112.997 \approx 1113\}$	M1	1.1b
	$x + y + z = 1110$ $C + D + H = 1110$ $x - z = 370 \text{ o.e.}$ $C - H = 370 \text{ o.e.}$ $0.0125C + 0.025D - 0.02H = 3 \text{ or } 2.997 \text{ o.e.}$ $1.0125C + 1.025D + 0.98H = 1113 \text{ or } 1112.997 \text{ o.e.}$ $0.0125x + 0.025y - 0.02z = 3 \text{ or } 2.997 \text{ o.e.}$ $1.0125x + 1.025y + 0.98z = 1113 \text{ or } 1112.997 \text{ o.e.}$	M1 A1	3.3 1.1b
(b)		(4)	
	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix} \begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1110 \\ 370 \\ 1113 \end{pmatrix}$ or $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix} \begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1110 \\ 370 \\ 3 \end{pmatrix}$	M1 A1ft	1.1b 1.1b
	$\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}^{-1} \begin{pmatrix} 1110 \\ 370 \\ 1113 \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$ or $\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}^{-1} \begin{pmatrix} 1110 \\ 370 \\ 3 \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$	dM1	1.1b
	So in 2019, 720 students studied Construction, 40 students studied Design and 350 students studied Hospitality	A1	3.2a
		(4)	
(8 marks)			
Notes:			
Mark (i) and (ii) together (a)(i) B1: Defines 3 variables, minimum e.g. construction = C , Design = D , Hospitality = H . This may be seen in text of the question, abbreviations may be used (ii)			

M1: Finds either the increase or the number of students in 2020. This may be implied by any equation which equals 1113 or 1112.997. If students use 1100 instead of 1110 this is slip and we can award this mark.

M1: Attempts to use the model to set up at least 2 equations

A1: All 3 simplified equations correct (decimals or fractions), one for each different piece of information. Award with mark even if B0 is scored and it is clear what the variables used stand for. Ignore any additional equations even if incorrect. As soon as 3 correct equations are seen you may award this mark.

Alternative approach

(i) B1: Construction = $H + 370$, Design = D , Hospitality = H

(ii) M1M1A1: $H + 370 + D + H = 1110$ o.e. $C = H + 370$ $1.0125(H + 370) + 1.025D + 0.98H = 1113$ or 1112.997 o.e. they do not need to be simplified

(b) This is M1 M1 A1 A1 on ePen but is marked M1A1M1A1

M1: Uses their equation in part(a) to set up a matrix equation of the form $\begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$, where “...” are numerical values.

A1ft: Correct matrix equation for their equations

dm1: Dependent on previous method mark. Writes $\left(\text{their } A \right)^{-1} \begin{pmatrix} 1110 \\ \text{their "370"} \\ \text{their "3"} \end{pmatrix}$ and obtains at

least one value of C , D or H . The inverse matrix need not be found, writing $A^{-1} \begin{pmatrix} 1110 \\ 370 \\ \text{their "3"} \end{pmatrix} = \dots$ is sufficient. A correct matrix equation followed by correct values implies this mark.

Condone $\begin{pmatrix} 1110 \\ \text{their "370"} \\ \text{their "3"} \end{pmatrix} A^{-1} = \dots$ as long as they reach some values. The values imply the correct method

$$\text{Note: } \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{10}{23} = 0.43\dots & \frac{18}{23} = 0.78\dots & -\frac{400}{23} = -17.39\dots \\ \frac{3}{23} = 0.13\dots & -\frac{13}{23} = -0.56\dots & \frac{800}{23} = 34.78\dots \\ \frac{10}{23} = 0.43\dots & -\frac{5}{23} = -0.21\dots & -\frac{400}{23} = -17.39\dots \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{410}{23} = 17.82\dots & \frac{18}{23} = 0.78\dots & -\frac{400}{23} = -17.39\dots \\ -\frac{797}{23} = -34.65\dots & -\frac{13}{23} = -0.56\dots & \frac{800}{23} = 34.78\dots \\ \frac{410}{23} = 17.82\dots & -\frac{5}{23} = -0.21\dots & -\frac{400}{23} = -17.39\dots \end{pmatrix}$$

A1: Interprets the answer in the context of the question, minimum is $C = 720$, $D = 40$, $H = 350$ with their variables. Condone the variable not been defined for this mark if it is clear which variable belong to what course.



Note: they must be using a matrix equation to solve the equation to score any marks.

Alternative approach

For example

Equations simplifies to $C - H = 370$, $D + 2H = 740$ and $1.025D + 1.9925H = 738.375$

$$\text{which leads to } \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 1.025 & 1.9925 \end{pmatrix} \begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 740 \\ 370 \\ 738.375 \end{pmatrix} \text{ then } \begin{pmatrix} C \\ D \\ H \end{pmatrix} =$$

$$\begin{pmatrix} 17.826 & 1 & -17.3913 \\ -34.6521 & 0 & 34.7826 \\ 17.826 & 0 & -17.3913 \end{pmatrix} \begin{pmatrix} 740 \\ 370 \\ 738.375 \end{pmatrix} = \begin{pmatrix} 720 \\ 40 \\ 350 \end{pmatrix}$$

Note: A 2 x 2 matrix is fine if it is appropriate for their equation.

Special Case: Forming an equation in one variable

(a)(i) B1: Hospitality = x , Construction = $x + 370$, Design = $740 - 2x$

(ii) M1M1A1: $1.0125(x + 370) + 1.025(740 - 2x) + 0.98x = 1113$ or 1112.997

(a)(i) B1: Hospitality = $x - 370$, Construction = x , Design = $1480 - 2x$

(ii) M1M1A1: $1.0125(x) + 1.025(1480 - 2x) + 0.98(x - 370) = 1113$ or 1112.997

(b) M0A0M0A0: They have an equation and are not forming and solving a matrix equation



Question	Scheme			Marks	AOs
6(a)	$ \mathbf{M} = k(-1-1) - 5(-1-2) + 7(1-2)$ $\{= 8 - 2k\}$			M1	1.1b
	Minors: $\begin{pmatrix} -2 & -3 & -1 \\ -12 & -k-14 & k-10 \\ -2 & k-7 & k-5 \end{pmatrix}$ Cofactors: $\begin{pmatrix} -2 & 3 & -1 \\ 12 & -k-14 & 10-k \\ -2 & 7-k & k-5 \end{pmatrix}$			M1	1.1b
	$\mathbf{M}^{-1} = \frac{1}{8-2k} \begin{pmatrix} -2 & 12 & -2 \\ 3 & -k-14 & 7-k \\ -1 & 10-k & k-5 \end{pmatrix}$			M1 A1	2.1 1.1b
				(4)	
(b)	$\mathbf{M}^{-1} = \frac{1}{4} \begin{pmatrix} -2 & 12 & -2 \\ 3 & -16 & 5 \\ -1 & 8 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix}$ Solve the equations simultaneously to achieve values for x, y and z $y + 3z = 2p - 2$ and $4y + 8z = -1p$ $x = \dots$, $y = \dots$, $z = \dots$			M1	3.1a
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} + 3p - 1 \\ \frac{3}{4} - 4p + \frac{5}{2} \\ -\frac{1}{4} + 2p - \frac{3}{2} \end{pmatrix}$			A1ft	1.1b
	$\left(\frac{12p-6}{4}, \frac{13-16p}{4}, \frac{8p-7}{4} \right)$ $\left(3p - \frac{3}{2}, \frac{13}{4} - 4p, 2p - \frac{7}{4} \right)$			A1	2.2a
				(3)	
(c)(i)					
	For consistency: E.g. eliminates z to find two equations from $3x + 2y = q + 2$ $3x + 2y = 7q - 1$ $18x + 12y = 15$	For consistency: E.g. eliminates x to find two equations from $y + 3z = 1 - 4q$ $3y + 9z = -3$ $y + 3z = 2q - 2$	For consistency: E.g. eliminates y to find two equations from $x - 2z = 2 - q$ $-x + 2z = 1 - 5q$ $-6x + 12z = -9$	M1	3.1a
	e.g. $q + 2 = 7q - 1$ $\Rightarrow q = \dots$	e.g. $-3 = 3(1 - 4q)$ $\Rightarrow q = \dots$	e.g. $-9 = 6(1 - 5q)$ $\Rightarrow q = \dots$	M1	1.1b
	$q = \frac{1}{2}$			A1	1.1b

	Alternative Equating coefficients leading to two out of three equations and solves to find values for a and b $4a + b = 2, 5a + b = 1, 7a + b = -1$ $\{a = -1, b = 6\}$	M1	3.1a
	Forms the fourth equation involving q $a + bq = 2$ and substitutes in the values of a and b to find a value for q	M1	1.1b
	$q = \frac{1}{2}$	A1	1.1b
	Finds a coordinate of intersection of the planes $4x + 5y + 7z = 1$ and $2x + y - z = 2$ e.g let $z = 0$ $4x + 5y = 1$ and $2x + y = 2$ $y = -1, x = 1.5$	M1	3.1a
	Substitutes the values for x, y and z into $x + y + z = q$ to reach a value for q	M1	1.1b
	$q = \frac{1}{2}$	A1	1.1b
(ii)	For example: $x = \lambda \Rightarrow 3\lambda + 2y = \frac{5}{2}, \lambda - 2z = \frac{3}{2} \Rightarrow y = f(\lambda), z = f(\lambda)$ $y = \lambda \Rightarrow 3x + 2\lambda = \frac{5}{2}, \lambda + 3z = 1 \Rightarrow x = f(\lambda), z = f(\lambda)$ $z = \lambda \Rightarrow 3y + 9\lambda = -3, -6x + 12\lambda = -9 \Rightarrow x = f(\lambda), y = f(\lambda)$	M1	3.1a
	Let $x = \lambda, \lambda = \frac{y - \frac{5}{4}}{-\frac{3}{2}} = \frac{z + \frac{3}{4}}{\frac{1}{2}}$ or $y = \frac{5}{4} - \frac{3}{2}\lambda, z = -\frac{3}{4} + \frac{1}{2}\lambda$ Let $y = \lambda, \lambda = \frac{x - \frac{5}{6}}{-\frac{2}{3}} = \frac{z + \frac{1}{3}}{-\frac{1}{3}}$ or $x = \frac{5}{5} - \frac{2}{3}\lambda, z = -\frac{1}{3} - \frac{1}{3}\lambda$ Let $z = \lambda, \lambda = \frac{x - \frac{3}{2}}{2} = \frac{y + 1}{-3}$ or $x = \frac{3}{2} + 2\lambda, y = -1 - 3\lambda$	A1	1.1b
	$\mathbf{r} = \frac{5}{4}\mathbf{j} - \frac{3}{4}\mathbf{k} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ o.e. $\mathbf{r} = \frac{5}{6}\mathbf{i} - \frac{1}{3}\mathbf{k} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ o.e. $\mathbf{r} = \frac{3}{2}\mathbf{i} - \mathbf{j} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ o.e.	M1 A1	1.1b 2.5
	Alternative (ii) Finds two different coordinates that lie on the line of intersection For example: setting $x = 0$ $\frac{5}{4}, -\frac{3}{4}$ setting $y = 0$ $\frac{5}{6}, 0, -\frac{1}{3}$ setting $z = 0$ $\frac{3}{2}, -1, 0$	M1 A1	3.1a 1.1b
	Finds the vector equation of the line passing through their two points	M1	1.1b
	$\mathbf{r} = \frac{5}{4}\mathbf{j} - \frac{3}{4}\mathbf{k} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ o.e.	A1	2.5

	$\mathbf{r} = \frac{5}{6}\mathbf{i} - \frac{1}{3}\mathbf{k} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \text{ o.e.}$ $\mathbf{r} = \frac{3}{2}\mathbf{i} - \mathbf{j} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \text{ o.e.}$		
	Alternative (ii): Outside the spec Finds the cross product of two normal vectors and a coordinate that lies on all three planes	M1	3.1a
	Correct cross product $\begin{vmatrix} 4 & 5 & 7 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 12 & 18 & -3 \end{vmatrix}$	A1	1.1b
	Uses the point and the direction vector to find the equation of the line	M1	1.1b
	$\mathbf{r} = \frac{5}{4}\mathbf{j} - \frac{3}{4}\mathbf{k} + t(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \text{ o.e.}$	A1	1.1b
		(7)	
(14 marks)			
Notes			
(a) M1: Correct method to find the determinant. Condone one sign slip M1: A correct first step in obtaining the inverse. Could be the matrix of minors or cofactors. Condone sign slips as long as the intention is clear. M1: Fully correct method to obtain the inverse. Attempts matrix of minors, cofactors, transposes and 1/determinant A1: Correct matrix (b) M1: A complete strategy for solving the given equations e.g. multiplies the given coordinates by their inverse or solves simultaneously to achieve values for x, y and z A1ft: Correct calculation on their inverse matrix (unsimplified) or at least on correct value if solving simultaneously A1: Correct coordinates (c)(i) M1: Uses a correct strategy that will lead to establishing a value for q. E.g. eliminating one of x, y or z M1: Solves a suitable equation to obtain a value for q A1: Correct value (c)(i) Alternative 1 M1: Equating coefficients leading to two out of three equations and solves to find values for a and b M1: Solves a suitable equation to obtain a value for q using their values for a and b (c)(i) Alternative 2 M1: Finds a coordinate of intersection of the planes $4x + 5y + 7z = 1$ and $2x + y - z = 2$ M1: Substitutes the values for x, y and z into $x + y + z = q$ to reach a value for q A1: Correct value (ii) M1: Uses a correct strategy to obtain the Cartesian equation of the line or the general coordinates A1: Correct Cartesian equation or coordinates in terms of a parameter.			



M1: Uses their Cartesian equation to correctly extract the position and direction to form a vector equation for the required line

A1: Correct equation (o.e.) look out for multiples of the direction vector

Alternative (ii)

M1: Finds two different coordinates that lie on the line of intersection

A1: Correct coordinates

M1: Uses their coordinates to find the vector equation of the line that passes through them.

A1: Correct equation (o.e.), look out for multiples of the direction vector. Must have $\mathbf{r} = \dots$

Alternative (ii) outside spec

M1: Finds the cross product between the normal vectors of two of the planes and a coordinate that lies on all three planes. If a coordinate is found in (i) it must be used in this part to award this mark.

A1: Correct cross product

M1: Uses the coordinate and the cross product to find the equation of the line

A1: Correct equation (o.e.), look out for multiples of the direction vector. Must have $\mathbf{r} = \dots$

Question	Scheme	Marks	AOs
5(i)	$\begin{vmatrix} 2 & -1 & 1 \\ 1 & p & -3 \\ 3 & 1 & -2 \end{vmatrix} = 2(p \times -2 - 1 \times -3) + (1 \times -2 - 3 \times -3) + (1 \times 1 - 3 \times p) = 0$ $= 2(-2p + 3) + 7 + 1 - 3p = 0 \Rightarrow p = \dots$	M1	2.1
	$p = 2$	A1	1.1b
		(2)	
(ii)	$\begin{cases} 2x - y + z = 3 \\ x + 2y - 3z = q \end{cases} \Rightarrow 3x + y - 2z = 3 + q$ compares with $3x + y - 2z = 4$ leading to $3 + q = 4 \Rightarrow q = \dots$ Alternatively $\begin{cases} 2x - y + z = 3 \\ 3x + y - 2z = 4 \end{cases} \Rightarrow 5x - z = 7$ $\begin{cases} 4x - 2y + 2z = 6 \\ x + 2y - 3z = q \end{cases} \Rightarrow 5x - z = 6 + q \Rightarrow 6 + q = 7 \Rightarrow q = \dots$	M1	3.1a
	$q = 1$	A1	1.1b
		(2)	
(4 marks)			
Notes:			
(i) M1: Finds the determinant of the matrix, sets = 0 and solves to find a value of p. Allow for any recognisable attempt at finding the determinant (may be slips in coefficients or signs). Accept if $\det = 14 - 7p$ appears with no working shown. A1: $p = 2$ from correct work. Alt I Using normal M1: Attempts the cross product between sets of pairs of planes and scales (if appropriate) and equates directions to solve for p. A1: Correct p FYI $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 1 & p & -3 \end{vmatrix} = (3 - p)\mathbf{i} + 7\mathbf{j} + (2p + 1)\mathbf{k}, \quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix} = \mathbf{i} + 7\mathbf{j} + 5\mathbf{k}, \quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & p & -3 \\ 3 & 1 & -2 \end{vmatrix} = (3 - 2p)\mathbf{i} - 7\mathbf{j} + (1 - 3p)\mathbf{k}$			
(ii) M1: A complete method to find the value of q. May be implied by correct value if no incorrect working is shown. E.g. Adds together equations 1 and 2 and compares with equation 3 to find a value for q.			

Alternatively: Uses equations $2x - y + z = 3$ and $3x + y - 2z = 4$ to eliminate one variable. Uses equation $x + 2y - 3z = q$ and one of the other equations to eliminate the same variable and compare to find a value for q . Another possible method is to identify a point on both planes 1 and 3 (e.g. let $x = 1$ and solve for y and z to get $(1, -3, -2)$ then substitute into middle equation to find q)

A1: $q = 1$

Alternatively II the question may be done as a whole. Eg.

$$\begin{cases} 2x - y + z = 3 \\ 3x + y - 2z = 4 \end{cases} \Rightarrow x + 2y - 3z = 1 \Rightarrow p = 2, q = 1$$
 is the most direct method. Variations are possible, e.g.

$$\begin{cases} 2x - y + z = 3 \\ x + py - 3z = q \\ 3x + y - 2z = 4 \end{cases} \Rightarrow \begin{matrix} 3(1) + (2): 7x + (p-3)y = 9 + q \\ 2(1) + (3): 7x - y = 10 \end{matrix} \Rightarrow \begin{matrix} p - 3 = -1 \Rightarrow p = ... \\ 9 + q = 10 \Rightarrow q = ... \end{matrix}$$

Score as follows:

M1: Attempts to solve at least two of the equations simultaneously to eliminate one variable or match coefficients of the third equation and uses the equation to identify one of the unknowns. Condone slips as long as the method is clear.

A1: Correct p or correct q .

M1: Full method to use the linear dependence of the equations to find both variables.

A1: Correct p and q .

Alt III: Using points on the common line.

M1: Finds two separate points on each of the first and third plane.

A1: Two correct points.

M1: Forms and solves two equations in p and q using their points.

A1: Correct p and q .

E.g.

$$\begin{matrix} x = 0 \Rightarrow \begin{cases} -y + z = 3 \\ y - 2z = 4 \end{cases} \Rightarrow y = -10, z = -7 \\ x = 1 \Rightarrow \begin{cases} -y + z = 1 \\ y - 2z = 1 \end{cases} \Rightarrow y = -3, z = -2 \end{matrix} \Rightarrow \begin{cases} -10p + 21 = q \\ 1 - 3p + 6 = q \end{cases} \Rightarrow p = ..., q = ...$$

Some useful equations :

$$\begin{aligned} \text{Eliminating } x: & \begin{cases} (2p+1)y - 7z = 2q - 3 \\ 5y - 7z = -1 \\ (3p-1)y - 7z = 3q - 4 \end{cases} \\ \text{Eliminating } y: & \begin{cases} (2p+1)x + (p-3)z = 3p + q \\ 5x - z = 7 \\ (3p-1)x + (3-2p)z = 4p - q \end{cases} \\ \text{Eliminating } z: & \begin{cases} 7x + (p-3)y = 9 + q \\ 7x - y = 10 \\ 7x + (3-2p)y = 12 - 2q \end{cases} \end{aligned}$$