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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Algebra and Functions (4)

Core Pure Mathematics 2 · Topic 4

8 questions · 59 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

8 questions · 59 marks · 5 sub-topics



SUB-TOPIC 4.1

ROOTS OF POLYNOMIALS

Sub-topic 4.1 · 1 question · 8 marks · lowest tariff first



Question 6 continued



SUB-TOPIC 4.3

STANDARD SERIES FORMULAE

Sub-topic 4.3 · 1 question · 9 marks · lowest tariff first





(Total for Question 4 is 9 marks)



SUB-TOPIC 4.4

METHOD OF DIFFERENCES

Sub-topic 4.4 · 1 question · 6 marks · lowest tariff first







(Total for Question 4 is 6 marks)



SUB-TOPIC 4.5

MACLAURIN SERIES

Sub-topic 4.5 · 4 questions · 30 marks · lowest tariff first



(Total for Question 3 is 6 marks)

$$f(x) = \tanh^{-1}\left(\frac{3-x}{6+x}\right) \quad |x| < \frac{3}{2}$$
$$f'(x) = -\frac{1}{2x+3} \quad (4)$$

(c) Hence show that the Maclaurin series for $f(x)$, up to and including the term in x^2 , is

$$\ln p + qx + rx^2$$







SUB-TOPIC 4.6

STANDARD MACLAURIN SERIES

Sub-topic 4.6 · 1 question · 6 marks · lowest tariff first



2. (a) Write down the Maclaurin series of e^x , in ascending power of x , up to and including the term in x^3

(1)

- (b) Hence, without differentiating, determine the Maclaurin series of

$$e^{(e^x - 1)}$$

in ascending powers of x , up to and including the term in x^3 , giving each coefficient in simplest form.

(5)



(Total for Question 2 is 6 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
6(a)	$2x^4 + Ax^3 - Ax^2 - 5x + 6 = 0 \Rightarrow x^4 + \frac{A}{2}x^3 - \frac{A}{2}x^2 - \frac{5}{2}x + \frac{6}{2} = 0$ $\alpha\beta\gamma\delta = \frac{6}{2}$ $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{5}{2}$	B1	3.1a
	$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta} = \frac{3(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)}{\alpha\beta\gamma\delta} = \frac{3 \times \left(\frac{5}{2}\right)}{\left(\frac{6}{2}\right)} = \dots$	M1	1.1b
	$= \frac{5}{2}$	A1	1.1b
		(3)	
(b)	$\alpha + \beta + \gamma + \delta = -\frac{A}{2} \quad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = -\frac{A}{2}$	B1	1.1b
	$(\alpha + \beta + \gamma + \delta)^2 =$ $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$	M1 A1	3.1a 1.1b
	$\left(-\frac{A}{2}\right)^2 = -\frac{3}{4} + 2\left(-\frac{A}{2}\right) \Rightarrow A^2 + 4A + 3 = 0 \Rightarrow A = \dots$	dM1	1.1b
	$A = -1, -3$	A1	1.1b
		(5)	
(8 marks)			
Notes:			
(a) B1: Identifies the correct values for the product and triple pair sum. If seen these must be correct, but they may be implied if not explicitly extracted, e.g. $\sum \frac{3}{\alpha_i} = 3 \frac{\sum \alpha_i \alpha_j \alpha_k}{\alpha\beta\gamma\delta} = 3 \times \frac{5}{6} = \frac{5}{2}$ does not technically contain an incorrect step if one realises the 2's will cancel. A good indication will be what happens in part (b) – if there they realise the 2's are involved then allow b.o.d. for a calculation such as here given in (a). Note $\sum \alpha\beta\gamma\delta$ is a correct notation for the product of roots. M1: Uses the correct identity and their values of the product and triple sum. (The values need not be correct for this mark.) A1: Correct value following correct identity with no indication of incorrect values for triple sum and product (see comment on the B mark).			

(b)

B1: Identifies the correct values for the sum and pair sum. Allow when first seen – some will list all these in part (a), which is fine for this mark.

M1: Attempts to find the identity for $(\alpha + \beta + \gamma + \delta)^2$ in terms of the sum of squares and pair sum (seen or implied). Allow attempts where the “2” is incorrect with no other method shown.

A1: Correct identity (seen or implied).

dM1: Dependent on previous method mark. Substitutes the sum and pair sum of the roots into their identity and forms and solves a 3TQ for A (usual rules).

A1: Correct values of A

6(a) Alt	$x = \frac{3}{w} \Rightarrow 2\left(\frac{3}{w}\right)^4 + \dots - 5\left(\frac{3}{w}\right) + 6 = 0$ or $x = \frac{1}{w} \Rightarrow 2\left(\frac{1}{w}\right)^4 + \dots - 5\left(\frac{1}{w}\right) + 6 = 0$	B1	3.1a
	$\Rightarrow \dots + \dots w + \dots - 15w^3 + 6w^4 = 0$ or $\Rightarrow 2 + \dots - 5w^3 + 6w^4 = 0$ $\Rightarrow \sum \frac{3}{\alpha_i} = -\left(\frac{-15}{6}\right)$ or $\Rightarrow 3 \sum \frac{1}{\alpha_i} = 3 \times -\left(\frac{-5}{6}\right)$	M1	1.1b
	$= \frac{5}{2}$	A1	1.1b
		(3)	

Alt (a): Some may use a transformation. These can be scored as

B1: Makes a correct substitution into the equation to solve the problem. This will probably be $x = \frac{3}{w}$

but note that $x = \frac{1}{w}$ can also be used.

M1: Multiplies through by w^4 and extracts the correct sum of roots from the new equation. If using $x = \frac{1}{w}$ they must also multiply through by the 3 to gain this mark.

A1: Correct answer from correct work on the relevant coefficients (the others need not be seen).



Question	Scheme	Marks	AOs
4(a)	A complete attempt to find the sum of the cubes of the first n odd numbers using three of the standard summation formulae. Attempts to find $\sum (2r+1)^3$ or $\sum (2r-1)^3$ by expanding and using summation formulae	M1	3.1a
	$\sum_{r=1}^n (2r-1)^3 = \sum_{r=1}^n (8r^3 - 12r^2 + 6r - 1) = 8 \sum_{r=1}^n r^3 - 12 \sum_{r=1}^n r^2 + 6 \sum_{r=1}^n r - \sum_{r=1}^n 1$ or $\sum_{r=0}^{n-1} (2r+1)^3 = \sum_{r=0}^{n-1} (8r^3 + 12r^2 + 6r + 1) = 8 \sum_{r=0}^{n-1} r^3 + 12 \sum_{r=0}^{n-1} r^2 + 6 \sum_{r=0}^{n-1} r + \sum_{r=0}^{n-1} 1$	M1	1.1b
	$= 8 \frac{n^2}{4} (n+1)^2 - 12 \frac{n}{6} (n+1)(2n+1) + 6 \frac{n}{2} (n+1) - n$ or $= 8 \frac{(n-1)^2}{4} (n)^2 + 12 \frac{(n-1)}{6} (n)(2n-1) + 6 \frac{(n-1)}{2} (n) + n$	M1 A1	1.1b 1.1b
	Multiplies out to achieve a correct intermediate line for example $n \quad n+1 \quad 2n^2 - 2n + 1 \quad -n = 2n^4 - 2n^3 + n^2 + 2n^3 - 2n^2 + n - n$ $2n^4 + 4n^3 + 2n^2 - 4n^3 - 6n^2 - 2n + 3n^2 + 3n - n$ leading to $= n^2 (2n^2 - 1) \text{ cso } *$	A1 *	2.1
		(5)	
(b)	$\sum_{r=n}^{n+9} (2r-1)^3 = \sum_{r=1}^{n+9} (2r-1)^3 - \sum_{r=1}^{n-1} (2r-1)^3$ $= (n+9)^2 (2(n+9)^2 - 1) - (n-1)^2 (2(n-1)^2 - 1) = 99800$ or $\sum_{r=n+1}^{n+10} (2r-1)^3 = \sum_{r=1}^{n+10} (2r-1)^3 - \sum_{r=1}^n (2r-1)^3$ $= (n+10)^2 (2(n+10)^2 - 1) - (n)^2 (2n^2 - 1) = 99800$ or $\sum_{r=n-9}^n (2r-1)^3 = \sum_{r=1}^n (2r-1)^3 - \sum_{r=1}^{n-10} (2r-1)^3$ $= (n)^2 (2n^2 - 1) - (n-10)^2 (2(n-10)^2 - 1) = 99800$	M1	3.1a
	$80n^3 + 960n^2 + 5820n - 86760 = 0$ or $80n^3 + 1200n^2 + 7980n - 79900 = 0$ or $80n^3 - 1200n^2 + 7980n - 119700 = 0$	A1	1.1b
	Solves cubic equation	dM1	1.1b



	Achieves $n = 6$ and the smallest number as 11 or Achieves $n = 5$ and the smallest number as 11 or Achieves $n = 15$ and the smallest number as 11	A1	2.3
		(4)	
(9 marks)			
Notes:			
(a) M1: A complete attempt to find the sum of the cubes of n odd numbers using three of the standard summation formulae. M1: Expands $\sum_{r=1}^n (2r-1)^3$ or $\sum_{r=0}^{n-1} (2r+1)^3$ and splits into four appropriate sums. M1: Applies the result for at least three summations $\sum_{r=0}^{n-1} r^3$, $\sum_{r=0}^{n-1} r^2$, $\sum_{r=0}^{n-1} r$ and $\sum_{r=0}^{n-1} 1$ or $\sum_{r=1}^n r^3$, $\sum_{r=1}^n r^2$, $\sum_{r=1}^n r$ and $\sum_{r=1}^n 1$ as appropriate to their expansion provided that there is an attempt at cubing some values. A1: Correct unsimplified expression. A1 *: Multiplies out to achieve a correct intermediate expression which clearly leads to the correct expression. cso Special case: If uses $\sum_{r=1}^n (2r+1)^3$ leading to $= 8\frac{n^2}{4}(n+1)^2 + 12\frac{n}{6}(n+1)(2n+1) + 6\frac{n}{2}(n+1) + n$ max score is M1 M0 M1 A1 A0			
(b) M1: Uses the answer to part (a) to find the sum of the cubes of the first $N + 10$ odd numbers minus the sum of the first N odd numbers and sets equal to 99800 or equivalent. A1: Correct simplified cubic equation. dM1: Uses their calculator to solve their cubic equation, dependent on previous method mark. A1: cao			

Question	Scheme	Marks	AOs
4	$\frac{2}{(r+4)(r+6)} \equiv \frac{A}{r+4} + \frac{B}{r+6} \Rightarrow A = \dots, B = \dots$	M1	3.1a
	$\frac{2}{(r+4)(r+6)} \equiv \frac{1}{r+4} - \frac{1}{r+6}$	A1	1.1b
	$\sum_{r=1}^n \frac{2}{(r+4)(r+6)} = \sum_{r=1}^n \left(\frac{1}{r+4} - \frac{1}{r+6} \right)$ $\frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \frac{1}{7} - \frac{1}{9} + \dots$ $+ \frac{1}{n+2} - \frac{1}{n+4} + \frac{1}{n+3} - \frac{1}{n+5} + \frac{1}{n+4} - \frac{1}{n+6}$	M1	2.1
	$= \frac{1}{5} + \frac{1}{6} - \frac{1}{n+5} - \frac{1}{n+6}$	A1	2.2a
	$= \frac{1}{5} + \frac{1}{6} - \frac{1}{n+5} - \frac{1}{n+6} = \frac{11(n+5)(n+6) - 30(n+6) - 30(n+5)}{30(n+5)(n+6)}$	M1	1.1b
	$= \frac{n(11n+61)}{30(n+5)(n+6)}$	A1	1.1b
		(6)	
(6 marks)			
Notes			
M1: Recognises the need to find partial fractions and applies a correct method leading to finding values for A and B Allow a slip when finding the constants A1: Correct partial fractions seen at any stage. Not just values for A and B listed Note: Proof by induction will not score the next 4 marks. M1: Starts the process of finding terms at the start and at the end, in order to establish the non-cancelling terms Must have attempted a minimum of $r = 1, r = 2, \dots, r = n-1$ and $r = n$, this may be implied by their correct non-cancelling terms. Follow through on their values of A and B. Look for $r = 1 \text{ ® } \frac{A}{5} - \frac{B}{7} \qquad r = 2 \text{ ® } \frac{A}{6} - \frac{B}{8}$ $r = n-1 \text{ ® } \frac{A}{n+3} - \frac{B}{n+5} \qquad r = n \text{ ® } \frac{A}{n+4} - \frac{B}{n+6}$ A1: Correct non-cancelling terms which may be listed separately. Correct fractions from the beginning and end that do not cancel stated. M1: Combines ‘their’ fractions of the form $p + \frac{q}{n+5} + \frac{r}{n+6}$ over a correct common denominator which does not need to be the lowest common denominator and obtains a quadratic expression in the numerator. A1: Correct answer. Note: If they start with $r = 0$ the maximum they can score is M1A1M0A0M1A0			

Question	Scheme	Marks	AOs
3(a)	$f'(x) = A(1-x^2)^{-\frac{1}{2}} \quad f''(x) = Bx(1-x^2)^{-\frac{3}{2}} \text{ and}$ $f'''(x) = C(1-x^2)^{-\frac{3}{2}} + Dx^2(1-x^2)^{-\frac{5}{2}} \text{ or } \frac{C(1-x^2)^{\frac{3}{2}} + Dx^2(1-x^2)^{\frac{1}{2}}}{(1-x^2)^3}$	M1	2.1
	$f'(x) = (1-x^2)^{-\frac{1}{2}} \text{ or } \frac{1}{\sqrt{1-x^2}} \quad f''(x) = x(1-x^2)^{-\frac{3}{2}} \text{ or } \frac{x}{(1-x^2)^{\frac{3}{2}}} \text{ and}$ $f'''(x) = (1-x^2)^{-\frac{3}{2}} + 3x^2(1-x^2)^{-\frac{5}{2}} \text{ or } \frac{1}{(1-x^2)^{\frac{3}{2}}} + \frac{3x^2}{(1-x^2)^{\frac{5}{2}}}$ from quotient rule $\frac{(1-x^2)^{\frac{3}{2}} + 3x^2(1-x^2)^{\frac{1}{2}}}{(1-x^2)^3}$	A1	1.1b
	Finds $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ and applies the formula $f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6}$ $\{f(0) = 0, f'(0) = 1, f''(0) = 0, f'''(0) = 1\}$	M1	1.1b
	$f(x) = x + \frac{x^3}{6} \text{ cso}$	A1	1.1b
		(4)	
(b)	$\arcsin\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{\left(\frac{1}{2}\right)^3}{6} = \frac{\pi}{6} \Rightarrow \pi = \dots$	M1	1.1b
	$\pi = \frac{25}{8} \text{ o.e.}$	A1ft	2.2b
		(2)	
(6 marks)			
Notes:			
(a) M1: Finds the correct form of the first three derivatives, may be unsimplified – the third may come later. A1: Correct first three derivatives, may be unsimplified – the third may come later. M1: Finds $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ and applies to the correct formula, needs to go up to x^3. A1: $x + \frac{x^3}{6}$ cso ignore any higher terms whether correct or not Special case: If they think that their $f''(0) \neq 0$ then maximum score M1 A0 M1 A0 M1 for correct form of the first two derivatives M1 Correctly uses their $f(0)$, $f'(0)$, $f''(0)$ and applies to the correct formula			



Note: If candidates do not find the first three derivatives but use
 $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, $f'''(0) = 1$ and use these correctly in the formula this can score M0
 A0 M1 A0

(b)

M1: Substitutes $x = \frac{1}{2}$ into both sides and rearranges to find $\pi = \dots$

A1ft: Infers that $\pi = \frac{25}{8}$ o.e. Follow through their $6f\left(\frac{1}{2}\right)$



Question	Scheme	Marks	AOs
2(a)	$\frac{d\left(\frac{3-x}{6+x}\right)}{dx} = \frac{-(6+x)-(3-x)}{(6+x)^2}$	M1 A1	3.1a 1.1b
	$f(x) = \tanh^{-1}\left(\frac{3-x}{6+x}\right) \Rightarrow f'(x) = \frac{1}{1-\left(\frac{3-x}{6+x}\right)^2} \times \frac{-9}{(6+x)^2}$	dM1	3.1a
	$= \frac{(6+x)^2}{36+12x+x^2-9+6x-x^2} \times \frac{-9}{(6+x)^2} = \frac{-9}{18x+27} = \frac{-1}{2x+3}^*$	A1*	2.1
	(4)		
	Alternative 1 for part (a)		
	$\frac{d\left(\frac{3-x}{6+x}\right)}{dx} = \frac{-(6+x)-(3-x)}{(6+x)^2}$	M1 A1	3.1a 1.1b
	$y = \tanh^{-1}\left(\frac{3-x}{6+x}\right) \Rightarrow \tanh y = \frac{3-x}{6+x} \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = \frac{-9}{(6+x)^2}$ $\frac{dy}{dx} = \frac{1}{1-\tanh^2 y} \times \frac{-9}{(6+x)^2} = \frac{1}{1-\left(\frac{3-x}{6+x}\right)^2} \times \frac{-9}{(6+x)^2}$	dM1	3.1a
	$= \frac{(6+x)^2}{36+12x+x^2-9+6x-x^2} \times \frac{-9}{(6+x)^2} = \frac{-9}{18x+27} = \frac{-1}{2x+3}^*$	A1*	2.1
	(4)		
	Alternative 2 for part (a)		
	$f(x) = \tanh^{-1}\left(\frac{3-x}{6+x}\right) = \frac{1}{2} \ln \left(\frac{1+\frac{3-x}{6+x}}{1-\frac{3-x}{6+x}} \right) = \frac{1}{2} \ln \left(\frac{9}{3+2x} \right)$	M1 A1	3.1a 1.1b
	$f(x) = \tanh^{-1}\left(\frac{3-x}{6+x}\right) \Rightarrow f'(x) = \frac{1}{2} \times \frac{3+2x}{9} \times \frac{-18}{(3+2x)^2}$	dM1	3.1a
	$= \frac{-1}{2x+3}^*$	A1*	1.1b
	(4)		
	Alternative 3 for part (a)		
	$\tanh y = f(x) = \left(\frac{3-x}{6+x}\right) \Rightarrow \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{3-x}{6+x} \text{ or } \frac{e^{2y} - 1}{e^{2y} + 1} = \frac{3-x}{6+x}$ $e^{2y} = \frac{9}{2x+3}$	M1 A1	3.1a 1.1b
	$y = \frac{1}{2} \ln \left(\frac{9}{2x+3} \right) \Rightarrow f'(x) = \frac{1}{2} \times \frac{2x+3}{9} \times \frac{-18}{(2x+3)^2}$	dM1	3.1a
	$= \frac{-1}{2x+3}^*$	A1	1.1b
	(4)		

Notes

(a) **Do not allow misreads of $\tan x$ for $\tanh x$ in this question; this would be a maximum of M1A1 for part (a)**

M1: Attempts to use the quotient (or product) rule on $\frac{3-x}{6+x}$ to obtain an expression of the form

$$\frac{A(6+x) - B(3-x)}{(6+x)^2}, \quad B > 0 \quad \text{or} \quad C(3-x)(6+x)^{-2} + D(6+x)^{-1}$$

Alternatively, candidates may also write $\frac{3-x}{6+x}$ as $-1 + \frac{9}{6+x}$ and then differentiate to find an

expression of the form $\frac{E}{(6+x)^2}$ which may be seen embedded in their working.

They may also write $\frac{3-x}{6+x}$ as $\frac{3}{6+x} - \frac{x}{6+x}$ and then differentiate to find an expression of the

form $\frac{E}{(6+x)^2}$ or

A1: Correct expression in any form.

dM1: A complete method to find the derivative using the chain rule to obtain

$$\frac{1}{1 - \left(\frac{3-x}{6+x}\right)^2} \times \left(\text{their } \frac{-9}{(6+x)^2} \right)$$

This is dependent on the first M mark.

A1*: Reaches the printed answer with sufficient working and no errors, with at least one intermediate line.

Alternative 1:

M1: see main scheme

A1: see main scheme

dM1: A complete method to find the derivative using the chain rule:

Rearranges the equation and uses implicit differentiation. Must proceed from $\operatorname{sech}^2 y$ or equivalent to $1 - \tanh^2 y$ and then substitute for $\tanh y$

This is dependent on the first M mark.

A1*: see main scheme

Alternative 2:

M1: Uses the logarithmic form of artanh to obtain $k \ln \left(\frac{1 + \frac{3-x}{6+x}}{1 - \frac{3-x}{6+x}} \right)$

A1: Correct simplified expression

dM1: A complete method to find the derivative to obtain

$$f'(x) = k \times \left(1 \div \left(\text{their } \frac{9}{3+2x} \right) \right) \times \left(\text{their } \frac{9}{3+2x} \right)^2$$

Alternatively uses log rules to partition their expression and then differentiates each term.

This is dependent on the first M mark.



A1*: Reaches the printed answer with sufficient working and no errors.

Alternative 3:

M1: Takes $\tanh$ of both sides to obtain $\tanh y$ in terms of x and expresses $\tanh y$ correctly in terms of exponentials and makes e^{2y} the subject.

A1: A correct expression for e^{2y}

dM1: Rearranges their function to the form $k \ln \left(\frac{a}{bx+c} \right)$ **and** uses the chain rule to find a

derivative of the form $f'(x) = \frac{1}{2} \times \frac{bx+c}{a} \times \frac{k}{(bx+c)^2}$ or uses log rules to partition their expression

and then differentiates each term.

This is dependent on the first M mark.

A1*: Reaches the printed answer with sufficient working and no errors

(b)	$[f''(x) =] \frac{2}{(2x+3)^2}$	B1	1.1b
		(1)	
(c)	$f(0) = \tanh^{-1} \left(\frac{1}{2} \right) \left(= \frac{1}{2} \ln 3 \right), f'(0) = -\frac{1}{3}, f''(0) = \frac{2}{9}$	M1	1.1b
	$[f(x)] = f(0) + xf'(0) + \frac{x^2}{2} f''(0)$	M1	1.1b
	$= \ln \sqrt{3} - \frac{1}{3}x + \frac{1}{9}x^2$	A1	1.1b
		(3)	

(8 marks)

Notes

(b)

B1: Correct second derivative in any form such as e.g. $\frac{2}{(4x^2+12x+9)}$ or $2(2x+3)^{-2}$

(c)

M1: Attempts at least two of the values of $f(0)$, $f'(0)$ and $f''(0)$

M1: Correct application of the Maclaurin series for **all** of $f(0)$, $f'(0)$ and $f''(0)$ where all are non-zero values. Substitutes their values into a correct expression. This mark is not dependent.

A1: Correct expansion.

Award if written as a single expression, or p , q and r written separately.

Ignore any extra terms written as powers of x^3 and above, isw when a correct answer is seen.



Question	Scheme	Marks	AOs
9(a)(i)	$\frac{dy}{dx} = \dots \cosh^{n-1} x \sinh x$ $\frac{d^2y}{dx^2} = \dots \cosh^{n-2} x \sinh^2 x + \dots \cosh^{n-1} x \cosh x$ Alternatively $y = \left(\frac{e^x + e^{-x}}{2}\right)^n \text{ leading to } \frac{dy}{dx} = \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-1} \left(\frac{e^x - e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-2} \left(\frac{e^x - e^{-x}}{2}\right)^2 + \dots \left(\frac{e^x + e^{-x}}{2}\right)^n$	M1	1.1b
	$\frac{dy}{dx} = n \cosh^{n-1} x \sinh x$ $\frac{d^2y}{dx^2} = n(n-1) \cosh^{n-2} x \sinh^2 x + n \cosh^n x$ Alternatively $\frac{dy}{dx} = n \left(\frac{e^x + e^{-x}}{2}\right)^{n-1} \left(\frac{e^x - e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = n(n-1) \left(\frac{e^x + e^{-x}}{2}\right)^{n-2} \left(\frac{e^x - e^{-x}}{2}\right)^2 + n \left(\frac{e^x + e^{-x}}{2}\right)^n$	A1	2.1
	$\frac{d^2y}{dx^2} = n(n-1) \cosh^{n-2} x (\cosh^2 x - 1) + n \cosh^n x$	M1	2.1
	$\frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x \text{ * cso}$	A1*	1.1b
		(4)	
(a)(ii)	$\frac{d^3y}{dx^3} = \dots \cosh^{n-1} x \sinh x - \dots \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = \dots \cosh^{n-2} x \sinh^2 x + \dots \cosh^n x - \dots \cosh^{n-4} x \sinh^2 x - \dots \cosh^n x$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3 \cosh^{n-1} x \sinh x - n(n-1)(n-2) \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^3(n-1) \cosh^{n-2} x \sinh^2 x + n^3 \cosh^n x - n(n-1)(n-2)(n-3) \cosh^{n-4} x \sinh^2 x - n(n-1)(n-2) \cosh^{n-2} x$	A1	1.1b
		(2)	
	Alternative 1 using $\frac{d^2y}{dx^2} = n^2 y - n(n-1) \cosh^{n-2} x$ leading to $\frac{d^3y}{dx^3} = n^2 \frac{dy}{dx} - \dots \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^2 \frac{d^2y}{dx^2} - \dots \cosh^{n-4} x \sinh^2 x - \dots \cosh^{n-2} x$	M1	1.1b

	$\frac{d^3y}{dx^3} = n^2 \frac{dy}{dx} - n(n-1)(n-2) \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^2 \frac{d^2y}{dx^2} - n(n-1)(n-2)(n-3) \cosh^{n-4} x \sinh^2 x$ $- n(n-1)(n-2) \cosh^{n-2} x$	A1	1.1b
		(2)	
	Alternative 2 $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$ $y = \cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = \dots \cosh^{n-2} x - \dots \cosh^{n-4} x$ $\frac{d^4y}{dx^4} = n^2 [n^2 \cosh^n x - n(n-1) \cosh^{n-2} x]$ $- n(n-1) [\dots \cosh^{n-2} x - \dots \cosh^{n-4} x]$	M1	1.1b
	$y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$ $y = \cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = (n-2)^2 \cosh^{n-2} x - (n-2)(n-3) \cosh^{n-4} x$ $\frac{d^4y}{dx^4} = n^2 [n^2 \cosh^n x - n(n-1) \cosh^{n-2} x]$ $- n(n-1) [(n-2)^2 \cosh^{n-2} x - (n-2)(n-3) \cosh^{n-4} x]$	A1	1.1b
		(2)	
	Alternative 3 Using $\frac{d^2y}{dx^2} = n^2 \left(\frac{e^x + e^{-x}}{2}\right)^n - n(n-1) \left(\frac{e^x + e^{-x}}{2}\right)^{n-2}$ leading to $\frac{d^3y}{dx^3} = \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-1} \left(\frac{e^x - e^{-x}}{2}\right) - \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-3} \left(\frac{e^x - e^{-x}}{2}\right)$ $\frac{d^4y}{dx^4} = \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-2} \left(\frac{e^x - e^{-x}}{2}\right)^2 + \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-2}$ $- \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-4} \left(\frac{e^x - e^{-x}}{2}\right)^2 - \dots \left(\frac{e^x + e^{-x}}{2}\right)^{n-2}$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3 \left(\frac{e^x + e^{-x}}{2}\right)^{n-1} \left(\frac{e^x - e^{-x}}{2}\right) - n(n-1)(n-2) \left(\frac{e^x + e^{-x}}{2}\right)^{n-3} \left(\frac{e^x - e^{-x}}{2}\right)$	A1	1.1b

	$\frac{d^4 y}{dx^4} = n^3(n-1)\left(\frac{e^x + e^{-x}}{2}\right)^{n-2} \left(\frac{e^x - e^{-x}}{2}\right)^2 + n^3\left(\frac{e^x + e^{-x}}{2}\right)^{n-2}$ $-n(n-1)(n-2)(n-3)\left(\frac{e^x + e^{-x}}{2}\right)^{n-4} \left(\frac{e^x - e^{-x}}{2}\right)^2 - n(n-1)(n-2)\left(\frac{e^x + e^{-x}}{2}\right)^{n-2}$		
		(2)	
(b)	When $x = 0$ $y = 1, y' = 0, y'' = n^2 - n(n-1), y^{(3)} = 0,$ $y^{(4)} = n^3 - n(n-1)(n-2)$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \dots$	M1	1.1b
	$y = 1 + \frac{nx^2}{2} + \frac{(3n^2 - 2n)x^4}{24} + \dots$ CSO	A1	2.5
		(2)	

(8 marks)

Notes:
(a)(i)

M1: Uses the chain rule and product rule to find the first and second derivatives which must be of the required form, condone sign slips

Alternatively uses the exponential definition and uses the chain rule and product rule to find the first and second derivatives which must be of the required form.

A1: Correct unsimplified first and second derivatives, may be in exponential form.

M1: Uses the identity $\pm \cosh^2 x \pm \sinh^2 x = 1$

A1*: Achieves the printed answer with no errors or omissions e.g. missing x 's

(a)(ii)

M1: Uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 1

M1: Using $\frac{d^2 y}{dx^2} = n^2 y - n(n-1) \cosh^{n-2} x$ to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 2

M1: Using $y = \cosh^n x \Rightarrow \frac{d^2 y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$

$y = \cosh^{n-2} x \Rightarrow \frac{d^2 y}{dx^2} = \dots \cosh^{n-2} x - \dots \cosh^{n-4} x$ leading to



$$\frac{d^4 y}{dx^4} = n^2 [n^2 \cosh^n x - n(n-1) \cosh^{n-2} x] - n(n-1) \left[\text{their } \frac{d(\cosh^{n-2} x)}{dx} \right]$$

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 3

M1: Uses the exponential definition and uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form.

A1: Correct fourth derivative, does not need to be simplified ISW

(b)

M1: Attempts the evaluation of all four of their derivatives at $x = 0$ and applies the Maclaurin formula with their values. Note that $y^{(1)}(0) = 0$ and $y^{(3)}(0) = 0$ may be implied as they will have a multiple of $\sinh 0$. If their $y^{(3)}(0) \neq 0$ they allow this mark for their first 3 non-zero terms

A1: Correct simplified expansion from correct derivatives cso

Question	Scheme	Marks	AOs
8(a)	$\frac{dy}{dx} = A \sin x \sinh x + B \cos x \cosh x$	M1	1.1b
	$\frac{dy}{dx} = -\sin x \sinh x + \cos x \cosh x$ and $\frac{d^2y}{dx^2} = -2 \sin x \cosh x$	A1	1.1b
	$\frac{d^2y}{dx^2} = C \sin x \cosh x$ $\frac{d^3y}{dx^3} = D \cos x \cosh x + E \sin x \sinh x$ $\frac{d^4y}{dx^4} = F \cos x \sinh x$ (unsimplified)	M1	2.1
	$\frac{d^3y}{dx^3} = -2 \cos x \cosh x - 2 \sin x \sinh x$ and $\frac{d^4y}{dx^4} = -4 \cos x \sinh x$	A1	1.1b
	$\frac{d^4y}{dx^4} = -4y$	A1	2.1
		(5)	
(b)	$\frac{d^5y}{dx^5} = -4 \frac{dy}{dx}$ or $\frac{d^5y}{dx^5} = -4 \cos x \cosh x + 4 \sin x \sinh x$ or $y^{(5)} = -4 \times \text{their } y'$	B1ft	2.2a
	When $x = 0$ $y = 0, y' = 1, y'' = 0, y^{(3)} = -2, y^{(4)} = 0, y^{(5)} = -4$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y^{(3)}(0) + \frac{x^4}{4!} y^{(4)}(0) + \frac{x^5}{5!} y^{(5)}(0) + \dots$	M1	1.1b
	$(y =) x - \frac{x^3}{3} - \frac{x^5}{30} + \dots$	A1	2.5
		(3)	
		(8 marks)	
Notes:			
(a) M1: Uses the product rule to find the correct form for at least the first derivative. A1: Achieves correct first and second derivatives. Need not be simplified. M1: Uses the product rule to find the correct form for the second, third and fourth derivatives. A1: Correct (unsimplified) expressions for the third and fourth derivatives. A1: Correct fourth derivative in terms of y from correct work. Note do not accept just $k = -4$, we must see the $\frac{d^4y}{dx^4} = -4y$			

(b)

B1ft: Correct fifth derivative or value for the fifth derivative deduced following through on their k from part (a). If they have stated y' incorrectly allow for $y^{(5)} = -4$ × their y' provided their y' is non-zero

M1: Attempts the evaluation of all the derivatives at $x = 0$ and applies the Maclaurin formula correctly with their values to achieve at least three non-zero terms.

A1: Correct simplified expansion. Accept with $f(x) = \dots$ or with nothing before – mark the expression. Accept as a list of terms.

8(a) Alt	$u = \cos x \quad v = \sinh x$ $u' = -\sin x \quad v' = \cosh x$ $u'' = -\cos x \quad v'' = \sinh x$ $u''' = \sin x \quad v''' = \cosh x$ $u^{iv} = \cos x \quad v^{iv} = \sinh x$	M1 A1	1.1b 1.1b
	$\frac{d^4 y}{dx^4} = uv^{iv} + 4u'v''' + 6u''v'' + 4u'''v' + u^{iv}v = \dots$	M1	2.1
	$= \cos x \sinh x - 4 \sin x \cosh x - 6 \cos x \sinh x + 4 \sin x \cosh x + \cos x \sinh x$	A1	1.1b
	$\frac{d^4 y}{dx^4} = (-4 \cos x \sinh x) - 4y$	A1	2.1
		(5)	

There may be attempts via Leibnitz theorem.

M1: Attempts first four derivatives for each of $\cos x$ and $\sinh x$

A1: Correct derivatives.

M1: Applies Leibnitz theorem correctly with their derivatives.

A1: Correct unsimplified expression.

A1: Correct fourth derivative in terms of y .

There may be rare cases where exponential forms are used in part (a). For reference here are the derivatives they should get.

$$y = \frac{1}{2}(e^x - e^{-x})\cos x$$

$$\frac{dy}{dx} = \frac{1}{2}(e^x + e^{-x})\cos x - \frac{1}{2}(e^x - e^{-x})\sin x = \frac{1}{2}e^x(\cos x - \sin x) + \frac{1}{2}e^{-x}(\cos x + \sin x)$$

$$\frac{d^2 y}{dx^2} = -(e^x + e^{-x})\sin x \left[= \frac{1}{2}e^x(\cos x - \sin x) + \frac{1}{2}e^x(-\sin x - \cos x) - \frac{1}{2}e^{-x}(\cos x + \sin x) + \frac{1}{2}e^{-x}(-\sin x + \cos x) \right]$$

$$\frac{d^3 y}{dx^3} = -(e^x - e^{-x})\sin x - (e^x + e^{-x})\cos x$$

$$\frac{d^4 y}{dx^4} = -(e^x + e^{-x})\sin x - (e^x - e^{-x})\cos x - (e^x - e^{-x})\cos x + (e^x + e^{-x})\sin x = -2(e^x - e^{-x})\cos x = -4\sinh x \cos x$$

NB Some are misreading the function as $y = \cosh x \sinh x$ or $y = \cos x \sin x$. These can be score method marks as long as work is equivalent (ie via product rule), but reduction of the problem to triviality via an identity is less demand and so will score M0. Allow SC A1 for the first A if the equivalent form is reached via the misread to that shown in the scheme.





Question	Scheme	Marks	AOs
2(a)	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ or $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$	B1	1.1b
		(1)	
(b)	Version 1 $e^{(e^x-1)} = e^{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots-1}$ or Version 2 $e^{(e^x-1)} = 1 + (e^x - 1) + \frac{(e^x - 1)^2}{2!} + \frac{(e^x - 1)^3}{3!} + \dots$	M1	1.1b
	Version 1.1 $= 1 + \left(x + \frac{x^2}{2!} + \frac{x^3}{3!} \right) + \frac{1}{2} \left(x + \frac{x^2}{2!} + \dots \right)^2 + \frac{1}{6} (x + \dots)^3 + \dots$ Or Version 1.2 $e^{x+\frac{x^2}{2}+\frac{x^3}{6}} = e^x \times e^{\frac{x^2}{2}} \times e^{\frac{x^3}{6}}$ $= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \right) \left(1 + \frac{x^2}{2} + \dots \right) \left(1 + \frac{x^3}{6} + \dots \right)$ Or Version 2.1 $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \dots$ $= \frac{1}{3} + \frac{1}{6}e^{3x} + \frac{1}{2}e^x$ $= \frac{1}{3} + \frac{1}{6} \left(1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{6} \right) + \frac{1}{2} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \right)$ Or Version 2.2 $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \dots$	M1	3.1a



$= 1 + x + \left(\frac{1}{2} + \frac{1}{2}\right)x^2 + \left(\frac{1}{6} + \frac{1}{2} \times 2 \times \frac{1}{2} + \frac{1}{6}\right)x^3 + \dots$	dM1	1.1b
$= 1 + x + x^2 + \frac{5}{6}x^3 + \dots$	A1 A1	1.1b 2.1
	(5)	
(6 marks)		

Notes:**(a)****B1:** Correct series (ignore terms beyond x^3).**(b)****M1:** Correctly applies the exponential Maclaurin expansion at least once, either to the base exponent or in the index. Allow 2 for 2! and 6 for 3! in the cube term. Follow through on their series seen in (a)**M1:** A complete attempt to use the exponential Maclaurin series to produce a cubic expression in terms of x only. Allow if the 3! is incorrect for this mark, but a polynomial in x must have been achieved. Condone a slip with one term. Follow through on their series seen in (a)**dM1:** Dependent on previous method mark only. Expands the brackets and gathers terms (not necessarily fully simplified, but should have a single term for each power).**A1:** Any two correct from coefficients of x , x^2 and x^3 , need not be simplified.**A1:** Fully correct answer with simplified terms.

NB: Question instructs to use standard Maclaurin series, so use of differentiation scores no mark.

Special case: Using $e^{(e^x - 1)} = e^{e^x} \times e^{-1}$ can score M1M1M0A0A0 for using Maclaurin series e^{e^x} and then on e^x , e^{2x} , e^{3x} 