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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Calculus (5)

Core Pure Mathematics 2 · Topic 5

7 questions · 50 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
4.6 STANDARD MACLAURIN SERIES		1 Qs · 6 marks	3
1. Q2 <i>also Topic 4</i>	Paper 2: Core Pure Mathematics 2 June2023	6	4
5.1 VOLUMES OF REVOLUTION		2 Qs · 18 marks	6
2. Q7	Paper 2: Core Pure Mathematics 2 June2023	8	7
3. Q9	Paper 2: Core Pure Mathematics 2 June2024	10	11
5.2 IMPROPER INTEGRALS		1 Qs · 5 marks	16
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QUESTIONS

7 questions · 50 marks · 5 sub-topics



SUB-TOPIC 4.6

STANDARD MACLAURIN SERIES

Sub-topic 4.6 · 1 question · 6 marks · lowest tariff first



- (1)

- (5)



(Total for Question 2 is 6 marks)



SUB-TOPIC 5.1

VOLUMES OF REVOLUTION

Sub-topic 5.1 · 2 questions · 18 marks · lowest tariff first



7.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

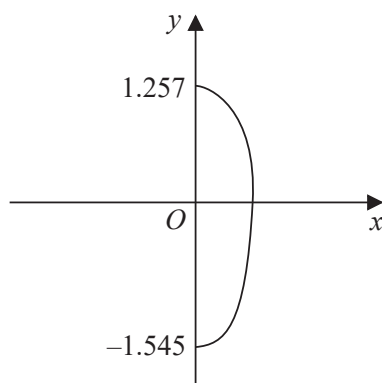


Figure 2

John picked 100 berries from a plant.

The largest berry picked was approximately 2.8 cm long.

The shape of this berry is modelled by rotating the curve with equation

$$16x^2 + 3y^2 - y \cos\left(\frac{5}{2}y\right) = 6 \quad x \geq 0$$

shown in Figure 2, about the y -axis through 2π radians, where the units are cm.Given that the y intercepts of the curve are -1.545 and 1.257 to four significant figures,

- (a) use algebraic integration to determine, according to the model, the volume of this berry.

(6)

Given that the 100 berries John picked were then squeezed for juice,

- (b) use your answer to part (a) to decide whether, in reality, there is likely to be enough juice to fill a 200 cm^3 cup, giving a reason for your answer.

(2)





9.

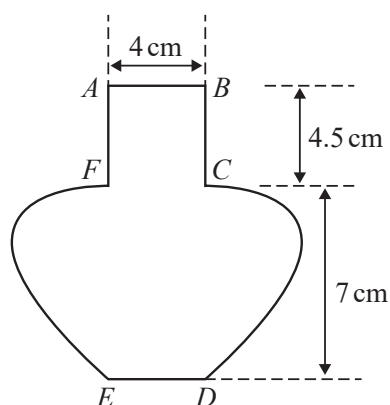


Figure 1

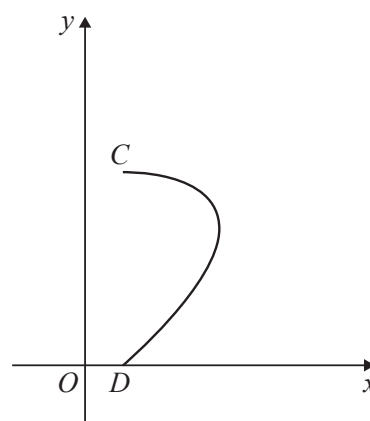


Figure 2

Figure 1 shows the central vertical cross-section $ABCDEFA$ of a vase together with measurements that have been taken from the vase.

The horizontal cross-section between AB and FC is a circle with diameter 4 cm.

The base of the vase ED is horizontal and the point E is vertically below F and the point D is vertically below C .

Using these measurements, the curve CD is modelled by the parametric equations

$$x = a + 3 \sin 2t \quad y = b \cos t \quad 0 \leq t \leq \frac{\pi}{2}$$

where a and b are constants and O is the fixed origin, as shown in Figure 2.

- (a) Determine the value of a and the value of b according to the model. (2)
- (b) Using algebraic integration and showing all your working, determine, according to the model, the volume of the vase, giving your answer to the nearest cm^3 . (7)
- (c) State a limitation of the model. (1)









SUB-TOPIC 5.2

IMPROPER INTEGRALS

Sub-topic 5.2 · 1 question · 5 marks · lowest tariff first





SUB-TOPIC 5.5

DIFFERENTIATING INVERSE TRIG FUNCTIONS

Sub-topic 5.5 · 1 question · 7 marks · lowest tariff first



Question 9 continued

SUB-TOPIC 7.3

AREAS AND TANGENTS OF POLAR CURVES

Sub-topic 7.3 · 2 questions · 14 marks · lowest tariff first



(Total for Question 1 is 4 marks)



Question 7 continued



Question 7 continued



Question 7 continued

(Total for Question 7 is 10 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
2(a)	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ or $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$	B1	1.1b
		(1)	
(b)	Version 1 $e^{(e^x-1)} = e^{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots-1}$ or Version 2 $e^{(e^x-1)} = 1 + (e^x - 1) + \frac{(e^x - 1)^2}{2!} + \frac{(e^x - 1)^3}{3!} + \dots$	M1	1.1b
	Version 1.1 $= 1 + \left(x + \frac{x^2}{2!} + \frac{x^3}{3!} \right) + \frac{1}{2} \left(x + \frac{x^2}{2!} + \dots \right)^2 + \frac{1}{6} (x + \dots)^3 + \dots$ Or Version 1.2 $e^{x+\frac{x^2}{2}+\frac{x^3}{6}} = e^x \times e^{\frac{x^2}{2}} \times e^{\frac{x^3}{6}}$ $= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \right) \left(1 + \frac{x^2}{2} + \dots \right) \left(1 + \frac{x^3}{6} + \dots \right)$ Or Version 2.1 $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \dots$ $= \frac{1}{3} + \frac{1}{6}e^{3x} + \frac{1}{2}e^x$ $= \frac{1}{3} + \frac{1}{6} \left(1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{6} \right) + \frac{1}{2} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \right)$ Or Version 2.2 $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \dots$	M1	3.1a

	$= 1 + x + \left(\frac{1}{2} + \frac{1}{2}\right)x^2 + \left(\frac{1}{6} + \frac{1}{2} \times 2 \times \frac{1}{2} + \frac{1}{6}\right)x^3 + \dots$	dM1	1.1b
	$= 1 + x + x^2 + \frac{5}{6}x^3 + \dots$	A1 A1	1.1b 2.1
		(5)	
(6 marks)			
Notes:			
(a) B1: Correct series (ignore terms beyond x^3). (b) M1: Correctly applies the exponential Maclaurin expansion at least once, either to the base exponent or in the index. Allow 2 for 2! and 6 for 3! in the cube term. Follow through on their series seen in (a) M1: A complete attempt to use the exponential Maclaurin series to produce a cubic expression in terms of x only. Allow if the 3! is incorrect for this mark, but a polynomial in x must have been achieved. Condone a slip with one term. Follow through on their series seen in (a) dM1: Dependent on previous method mark only. Expands the brackets and gathers terms (not necessarily fully simplified, but should have a single term for each power). A1: Any two correct from coefficients of x, x^2 and x^3, need not be simplified. A1: Fully correct answer with simplified terms. NB: Question instructs to use standard Maclaurin series, so use of differentiation scores no mark. Special case: Using $e^{(e^x - 1)} = e^{e^x} \times e^{-1}$ can score M1M1M0A0A0 for using Maclaurin series e^{e^x} and then on e^x, e^{2x}, e^{3x}			



Question	Scheme	Marks	AOs
7(a)	$\frac{\pi}{16} \int_{-1.545}^{1.257} \left(6 - 3y^2 + y \cos\left(\frac{5}{2}y\right) \right) \{dy\}$ $\pi \int_{-1.545}^{1.257} \left(\frac{3}{8} - \frac{3}{16}y^2 + \frac{1}{16}y \cos\left(\frac{5}{2}y\right) \right) \{dy\}$	B1	1.1a
	$\int x^2 dy = \frac{1}{16} \int 6 - 3y^2 + y \cos\left(\frac{5}{2}y\right) dy \rightarrow Ky - Ly^3 + \dots$	M1	1.1b
	$\int y \cos\left(\frac{5}{2}y\right) dy = Ay \sin\left(\frac{5}{2}y\right) + B \cos\left(\frac{5}{2}y\right) (+c)$ $\left\{ \begin{aligned} \int y \cos\left(\frac{5}{2}y\right) dy &= y \cdot \frac{2}{5} \sin\left(\frac{5}{2}y\right) - \int 1 \cdot \frac{2}{5} \sin\left(\frac{5}{2}y\right) dy \\ &= \frac{2}{5} y \sin\left(\frac{5}{2}y\right) + \frac{4}{25} \cos\left(\frac{5}{2}y\right) (+c) \end{aligned} \right\}$	M1	3.1a
	$\int x^2 dy = \frac{1}{16} \left(6y - y^3 + \frac{2}{5} y \sin\left(\frac{5}{2}y\right) + \frac{4}{25} \cos\left(\frac{5}{2}y\right) \right) (+c)$ $\int x^2 dy = \frac{3}{8}y - \frac{1}{16}y^3 + \frac{1}{40}y \sin\left(\frac{5}{2}y\right) + \frac{1}{100} \cos\left(\frac{5}{2}y\right) (+c)$	A1	1.1b
	$\int_{-1.545}^{1.257} x^2 dy = \frac{1}{16} \left[6y - y^3 + \frac{2}{5} y \sin\left(\frac{5}{2}y\right) + \frac{4}{25} \cos\left(\frac{5}{2}y\right) \right]_{-1.545}^{1.257}$ $= \frac{1}{16} (5.3954... - (-6.1101...)) = \dots$ $= (0.3372...) - (-0.3818...) = \dots$	M1	3.4
	$\text{Volume} = \pi \times \frac{11.505...}{16} = 2.26 \text{ cm}^3 \quad (2.2591159...) \text{ cso}$	A1	3.2a
		(6)	
	Max volume for 100 berries (as we know volume of the largest) is $100 \times 2.26 \approx 226$ Reason e.g. not all the berries will become juice (e.g. skin, flesh, seeds may not pulp) or not all will be as big as the largest, $150 < 200$ or $300 > 200$ If their value • is less than 220 – so not likely to produce 200 cm^3 of juice. • is between 220 and 250 – they can conclude either way • is greater than 250 – so likely to produce 200 cm^3 of juice.	B1ft	1.1b
(b)		B1ft	2.2b

		(1)	
(8 marks)			
Notes:			
(a) B1: Selects the correct volume of revolution formula to use, with correct limits in evidence, could appear later in their working M1: Attempts to integrate with the correct form for the constant and term in y^2 M1: Applies integration by parts fully on the $y \cos\left(\frac{5}{2}y\right)$ term in the correct direction. Allow slips in the coefficients, but the form must be correct. A1: Correct integration of the x^2 equation. M1: Applies the correct limits to their integral provided there was some attempt at integration. No need for the π for this mark. Condone working in degrees (5.74 ...) - (-5.38...), award this mark following correct integration. If their integration is incorrect you will need to check the use of limits in radians only if they do not show the substitution. A1: Correct volume including units. Accept awrt 2.26 cm³. Allow 0.719 π cm³. Correct solution only Note: All previous marks must have been scored to award this final accuracy mark Special case: Use of calculator for all the integration can score a maximum of B1M0M0A0M1A0 (b) B1ft: Attempts to estimate the volume of juice produced by 100 berries - look for their (a) multiplied by 100. B1ft: Draws a suitable conclusion with reason given, see scheme			



Question	Scheme	Marks	AOs
9(a)	$a = 2$ or $b = 7$	B1	3.3
	$a = 2$ and $b = 7$	B1	3.3
		(2)	
(b)	$V = (\pi) \int x^2 \frac{dy}{dt} dt = \int (2 + 3 \sin 2t)^2 (-7 \sin t) dt$	M1	3.4
	$= -7(\pi) \int (4 \sin t + 12 \sin 2t \sin t + 9 \sin^2 2t \sin t) dt$ $= -7(\pi) \int (4 \sin t + 24 \sin^2 t \cos t + 36 \sin^3 t \cos^2 t) dt$	M1	3.1a
	$= -7(\pi) \left[-4 \cos t + 8 \sin^3 t - 12 \cos^3 t + \frac{36}{5} \cos^5 t \right]$	A1ft A1	1.1b 1.1b
	Cylinder volume is $\pi \times 2^2 \times 4.5 = (18\pi)$	B1	3.4
	Total Volume = V + cylinder volume $= -7\pi \left[-4 \cos t + 8 \sin^3 t - 12 \cos^3 t + \frac{36}{5} \cos^5 t \right]_{\frac{\pi}{2}}^0 + \pi \times 2^2 \times 4.5$ or $= 7\pi \left[-4 \cos t + 8 \sin^3 t - 12 \cos^3 t + \frac{36}{5} \cos^5 t \right]_0^{\frac{\pi}{2}} + \pi \times 2^2 \times 4.5$ (NB this is $\frac{588}{5} \pi + 18\pi$)	ddM1	3.4
	$426 \text{ (cm}^3\text{)}$	A1	2.2b
		(7)	
(c)	Any one of e.g. The vase may not be completely smooth The vase may not be symmetrical The measurements may not be accurate The equation of the curve may not be a suitable model The thickness of the sides has not been considered Accept the base may have a dimple in it (the base may not be completely flat)	B1	3.5b
		(1)	
(10 marks)			
Notes			
(a) B1: Uses the model to obtain a correct value for a or b B1: Uses the model to obtain correct values for a and b			

(b)

M1: Uses the parametric curve for the model and applies $(\pi) \int x^2 \frac{dy}{dt} dt$

The π symbol may be missing here. Also do not be concerned about a missing dt at the end of their integral.

Must see an attempt at squaring x and finding $\frac{dy}{dt}$ where $x = (a + 3\sin 2t)$ and $\frac{dy}{dt} = k \sin t$ with their a and b or the letters a and b in their integral.

M1: Expands and makes progress to an integrable form by applying $\sin 2t = 2\sin t \cos t$ at least once

Integrals in terms of a and b will be equivalent to:

$$\begin{aligned} & \pm(\pi) \int (a^2 b \sin t + 6ab \sin 2t \sin t + 9b \sin^2 2t \sin t) dt \\ & = \pm(\pi) \int a^2 b \sin t + 12ab \sin^2 t \cos t + 36b \sin^3 t \cos^2 t dt \end{aligned}$$

Integrals with correct values of a and b will be equivalent to:

$$\begin{aligned} & \pm(\pi) \int (28 \sin t + 84 \sin 2t \sin t + 63 \sin^2 2t \sin t) dt \\ & = \pm(\pi) \int (28 \sin t + 168 \sin^2 t \cos t + 252 \sin^3 t \cos^2 t) dt \end{aligned}$$

A1ft: (dependent on **both** M marks) At least 2 terms integrated correctly; follow through their a and b , however their a and b must now be numerical.

$$\begin{aligned} & \pm(\pi) \left[-a^2 b \cos t + 4ab \sin^3 t - \frac{36}{3} b \cos^3 t + \frac{36b}{5} \cos^5 t \right] \\ & = \pm(\pi) \left[28 \cos t - 56 \sin^3 t + 84 \cos^3 t - \frac{252}{5} \cos^5 t \right] \end{aligned}$$

A1: All correct from correct values for a and b

Their integral will be equivalent to:

$$= \pm(\pi) \left[28 \cos t - 56 \sin^3 t + 84 \cos^3 t - \frac{252}{5} \cos^5 t \right]$$

B1: Uses the model to deduce the correct volume of the cylinder; need not be simplified.

ddM1: Dependent on **both** previous M marks. Fully correct strategy for the volume.

Applies the correct limits (either way round) and adds this to the volume of the cylinder which must have been obtained from $\pi \times 2^2 \times 4.5$ but condone $\pi \times 4^2 \times 4.5$

Both volumes must be positive when combined.

A1: Correct volume, allow awrt 426 (cm^3), units are not required but if any are given, they should be correct.

(c)

B1: See scheme. Award for a correct statement. If there is more than one statement ignore any incorrect statements as long as there are no statements that contradict their correct statement.

Comments relating to facts about the shape that are **not** worthy of marks include:

- the cross section is not a circle
- The vase is not solid
- the cylinder might not be vertical
- it depends on the material used to make the vase
- the vase may not have the same density throughout



Question	Scheme	Marks	AOs
3(a)	Because the upper limit is infinite	B1	2.4
		(1)	
(b)	$\int \frac{1}{9x^2 + 16} dx = \frac{1}{12} \arctan\left(\frac{3x}{4}\right)$	M1 A1	3.1a 1.1b
	$\int_{\frac{4}{3}}^{\infty} \frac{1}{9x^2 + 16} dx = \frac{1}{12} \lim_{t \rightarrow \infty} \left[\arctan\left(\frac{3x}{4}\right) \right]_{\frac{4}{3}}^t$ $= \frac{1}{12} \left(\lim_{t \rightarrow \infty} \arctan\left(\frac{3t}{4}\right) - \arctan(1) \right)$	dM1	1.1b
	$= \frac{1}{12} \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{\pi}{48}$	A1	2.1
		(4)	
	(5 marks)		
Notes			
(a) B1: Suitable explanation stating that one of the bounds/limits is infinite or is w e.g. “one of the limits is unbounded” or “the integral is unbounded” Do not allow this mark if they only say the limit or integral is undefined, unless they go on to say it is undefined at infinity. Commenting that the function is undefined at infinity is not enough to award this mark on its own, this question concerns the limits. If the candidates state any extra incorrect comments about the limits withhold this mark e.g. not defined at $\frac{4}{3}$ (b) M1: Integrates to obtain $\alpha \arctan(\beta x)$ where $\beta \neq 1$ A1: Correct integration, unsimplified or simplified. dM1: Applies correct limits, "t" and $\frac{4}{3}$ with evidence of applying the infinite limit to obtain a non-zero value. Allow with ∞ used as the limit (which may be implied by $\frac{\pi}{2}$) A1: Correct value obtained with evidence of use of limiting process on the upper bound. Withhold this mark if there is no evidence of using the limiting process. We must see as a minimum $\lim_{t \rightarrow \infty}$ or at some stage in their work. e.g. $\left[\frac{1}{12} \arctan\left(\frac{3x}{4}\right) \right]_{\frac{4}{3}}^{\infty} = \frac{1}{12} \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{\pi}{48}$ would score dM1A0			

Question	Scheme	Marks	AOs
9(a)	$\frac{dy}{dx} = \frac{\lambda}{\sqrt{1-9x^2}}$	M1	1.1b
	$\frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}$	A1	1.1b
		(2)	
Alt	$\frac{1}{3} \sin y = x \Rightarrow \frac{dx}{dy} = \lambda \cos y = \lambda \sqrt{1-9x^2}$	M1	1.1b
	$\frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}$	A1	1.1b
		(2)	
(b)	$\frac{dy}{dx} = \dots \sin(\arcsin 3x) \times \text{their} \left(\frac{3}{\sqrt{1-9x^2}} \right)$	M1	1.1b
	$\frac{dy}{dx} = -\frac{9x}{\sqrt{1-9x^2}}$	A1	2.1
		(2)	
Alt	$\arccos y = \arcsin 3x \left[\Rightarrow -\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \text{their} \left(\frac{3}{\sqrt{1-9x^2}} \right) \right]$ $\Rightarrow \frac{dy}{dx} = \dots \sqrt{1-y^2} \times \text{their} \left(\frac{3}{\sqrt{1-9x^2}} \right)$	M1	1.1b
	$\frac{dy}{dx} = -\frac{9x}{\sqrt{1-9x^2}}$	A1	2.1
		(2)	
(c)	$\frac{dy}{dx} = -\frac{9x}{\sqrt{1-9x^2}} = 4 \Rightarrow -9x = 4\sqrt{1-9x^2} \Rightarrow 81x^2 = 16(1-9x^2)$ Leading to a value for x or x ²	M1	3.1a
	$x^2 = \frac{16}{225}$	A1	1.1b
	$x = -\frac{4}{15}$	A1	2.3
		(3)	
(7 marks)			

Notes:**(a)**

M1: Uses the chain rule to differentiate to the correct form. (2 can be 1). Condone a missing bracket on $3x^2$ for this mark.

A1: Correct answer. Accept $\frac{dy}{dx} = \frac{3}{\sqrt{1-(3x)^2}}$ oe

Alternatively

M1: Uses implicit differentiation and trig identities to achieve the correct form for $\frac{dx}{dy}$ in terms of x

A1: Correct answer. Accept $\frac{dy}{dx} = \frac{3}{\sqrt{1-(3x)^2}}$ oe Accept $\frac{dy}{dx} = \frac{3}{\cos(\arcsin 3x)}$

(b)

M1: Uses the chain rule and their answer to part (a) to differentiate to the correct form. May use

$$\arccos y = \arcsin 3x - \text{see scheme for form but must get to } \frac{dy}{dx} = \dots$$

A1: Correct simplified answer. If using the Alt they must replace the y and simplify to achieve the same answer. Must be their answer to (b). Do not accept forms in trig functions such as

$$\frac{dy}{dx} = -3 \tan(\arcsin 3x) \text{ as these are not simplest form.}$$

Do not allow recovery if they simplify the $\sin(\arcsin \dots)$ as part of their working in part (c).

(c)

M1: A complete method to find a value for at least x^2 . Sets their answer to part (b) equal to 4, rearranges, squares both sides to reach a quadratic in x before achieving a value for x or x^2 .

Note that if they never simplify the $\sin(\arcsin 3x)$ in the derivative they will not be able to solve for x^2 and so will not gain this mark.

A1: Correct value for x^2 from a correct derivative in (b) (may be implied).

A1: Selects the correct value of x . Must have come from a correct derivative in (b).

Caution: Watch out for those who get $\frac{dy}{dx} = \frac{9x}{\sqrt{1-9x^2}}$ in (b) which will give the correct values for part (c) but should score (b) M1A0 (c) M1A0A0.

Alt

$$\textbf{M1: } \frac{dy}{dx} = -3 \tan(\arcsin 3x) = 4 \Rightarrow \tan(\arcsin 3x) = -\frac{4}{3} \Rightarrow \sin(\arcsin 3x) = -\frac{4}{5} \Rightarrow 3x = \dots$$

Uses the alt form in trig functions, set equal to 4 and proceeds to use right angle triangle or trig identities to proceed to a value for $3x$

A1: Correct value for $3x$

A1: Correct value for x



Question	Scheme	Marks	AOs
1	$\text{Area} = \frac{1}{2} \int_0^{\pi} r^2 d\theta = \frac{1}{2} \int_0^{\pi} \left[2\sqrt{\sinh \theta + \cosh \theta} \right]^2 \{d\theta\}$ or $\frac{1}{2} \int_0^{\pi} 4(\sinh \theta + \cosh \theta) \{d\theta\}$	B1	1.1b
	$= 2 \left[\cosh \theta + \sinh \theta \right]_0^{\pi}$ or $2 \int_0^{\pi} \left(\frac{e^{\theta} - e^{-\theta}}{2} + \frac{e^{\theta} + e^{-\theta}}{2} \right) d\theta = 2 \int_0^{\pi} e^{\theta} d\theta = 2 \left[e^{\theta} \right]_0^{\pi}$	M1	1.1b
	$= 2 \left((\cosh \pi + \sinh \pi) - (\cosh 0 + \sinh 0) \right)$ $= 2 \left(\left(\frac{e^{\pi} + e^{-\pi}}{2} + \frac{e^{\pi} - e^{-\pi}}{2} \right) - (1 + 0) \right)$ or $= 2(e^{\pi} - e^0)$	M1	3.1a
	$= 2e^{\pi} - 2 \text{ or } 2(e^{\pi} - 1)$	A1	2.1
		(4)	
(4 marks)			
Notes:			
B1: Correct area formula applied, including the $\frac{1}{2}$ and correct limits, may be seen later, $d\theta$ may be implied. M1: Attempts the integration $\sinh \theta \rightarrow \pm \cosh \theta$ and $\cosh \theta \rightarrow \pm \sinh \theta$ or in terms of exponentials $\int e^{\lambda \theta} d\theta = \frac{1}{\lambda} e^{\lambda \theta}$ M1: Applies their limits to the integral, subtracts (there must be an attempt to integrate) and uses exponential definitions to achieve answer in suitable form. Condone the inclusion of i or a missing $\frac{1}{2}$ from the definitions. This can be implied. A1: Correct exact answer, no i			

Question	Scheme	Marks	AOs
7(a)	$x = r \cos \theta = (1 + \tan \theta) \cos \theta = \cos \theta + \sin \theta$ $= \cos \theta + \tan \theta \cos \theta$ $\frac{dx}{d\theta} = \alpha(1 + \tan \theta) \sin \theta + \beta \sec^2 \theta \cos \theta \quad \text{or} \quad \frac{dx}{d\theta} = \alpha \sin \theta + \beta \cos \theta$ $\frac{dx}{d\theta} = \alpha \sin \theta + \beta \sec^2 \theta \cos \theta + \delta \tan \theta \sin \theta$	M1	3.1a
	$\frac{dx}{d\theta} = -(1 + \tan \theta) \sin \theta + \sec^2 \theta \cos \theta \quad \text{or} \quad \frac{dx}{d\theta} = -\sin \theta + \cos \theta$ $\frac{dx}{d\theta} = -\sin \theta + \sec^2 \theta \cos \theta - \tan \theta \sin \theta \quad \text{or} \quad \frac{dx}{d\theta} = -\sin \theta + \sec \theta - \tan \theta \sin \theta$	A1	1.1b
	For example $\left\{ \frac{dx}{d\theta} = \right\} -\sin \theta + \cos \theta = 0 \Rightarrow \tan \theta = 1 \Rightarrow \theta = \dots$ $\left\{ \frac{dx}{d\theta} = \right\} -\sin \theta + \cos \theta = 0 \Rightarrow \sin \theta = \cos \theta \Rightarrow \theta = \dots$ $\left\{ \frac{dx}{d\theta} = \right\} -\sin \theta + \cos \theta = \sqrt{2} \cos \left(\theta + \frac{\pi}{4} \right) = \theta = \dots$ or $\left\{ \frac{dx}{d\theta} = \right\} - (1 + \tan \theta) \sin \theta + \sec^2 \theta \cos \theta = 0$ $\Rightarrow -\sin \theta - \frac{\sin^2 \theta}{\cos \theta} + \frac{1}{\cos \theta} = 0 \Rightarrow -\sin \theta + \frac{1 - \sin^2 \theta}{\cos \theta} = 0$ $\Rightarrow -\sin \theta + \cos \theta = 0 \Rightarrow \tan \theta = 1 \Rightarrow \theta = \dots$ or $\left\{ \frac{dx}{d\theta} = \right\} -\sin \theta - \tan \theta \sin \theta + \sec \theta = 0$ $\Rightarrow -\frac{1}{2} \sin 2\theta - \sin^2 \theta + 1 = 0 \Rightarrow \sin 2\theta + 2 \sin^2 \theta - 1 = 1$ $\Rightarrow \sin 2\theta - \cos 2\theta = 1 \Rightarrow \sqrt{2} \sin \left(2\theta - \frac{\pi}{4} \right) = 1 \Rightarrow \theta = \dots$ or $\left\{ \frac{dx}{d\theta} = \right\} -\sin \left(\frac{\pi}{4} \right) + \cos \left(\frac{\pi}{4} \right) = 0$ $\left\{ \frac{dx}{d\theta} = \right\} - \left(1 + \tan \left(\frac{\pi}{4} \right) \right) \sin \left(\frac{\pi}{4} \right) + \sec^2 \left(\frac{\pi}{4} \right) \cos \left(\frac{\pi}{4} \right) = 0$ $\left\{ \frac{dx}{d\theta} = \right\} -\sin \left(\frac{\pi}{4} \right) + \sec^2 \left(\frac{\pi}{4} \right) \cos \left(\frac{\pi}{4} \right) - \tan \left(\frac{\pi}{4} \right) \sin \left(\frac{\pi}{4} \right) = 0$	dM1	3.1a
	$r = 1 + \tan \left(\frac{\pi}{4} \right) = 2 \quad \text{therefore } A \left(2, \frac{\pi}{4} \right)^*$	A1*	2.1
		(4)	
	Area bounded by the curve $= \frac{1}{2} \int (1 + \tan \theta)^2 \{d\theta\}$	M1	3.1a

(b)	$= \frac{1}{2} \int (1 + 2 \tan \theta + \tan^2 \theta) \{d\theta\}$ $= \frac{1}{2} \int (1 + 2 \tan \theta + [\sec^2 \theta - 1]) \{d\theta\} = \dots$		
	$= \frac{1}{2} [2 \ln \sec \theta + \tan \theta] \text{ or } \ln \sec \theta + \frac{1}{2} \tan \theta \text{ or } -\ln \cos \theta + \frac{1}{2} \tan \theta \text{ or } = \frac{1}{2} [-2 \ln \cos \theta + \tan \theta]$	A1	1.1b
	$= \frac{1}{2} \left[2 \ln \left \sec \left(\frac{\pi}{4} \right) \right + \tan \left(\frac{\pi}{4} \right) \right] - \frac{1}{2} [2 \ln \sec(0) + \tan(0)]$ $= \left(\ln \left \sec \left(\frac{\pi}{4} \right) \right + \frac{1}{2} \tan \left(\frac{\pi}{4} \right) \right) - \left(\ln \sec 0 + \frac{1}{2} \tan 0 \right)$ $\left\{ = \ln \sqrt{2} + \frac{1}{2} \right\}$	dM1	1.1b
	Area of triangle $= \frac{1}{2} xy = \frac{1}{2} \left(2 \cos \frac{\pi}{4} \right) \left(2 \sin \frac{\pi}{4} \right) = \dots \left\{ \frac{1}{2} \times \sqrt{2} \times \sqrt{2} = 1 \right\}$ The equation of the tangent is $r = \sqrt{2} \sec \theta$ then applies Area bounded of triangle $= \frac{1}{2} \int_0^{\frac{\pi}{4}} (\sqrt{2} \sec \theta)^2 \{d\theta\}$	M1	1.1b
	Finds the required area = area of triangle – area bounded by the curve $= 1 - \left[\ln \sqrt{2} + \frac{1}{2} \right]$ May be seen within an integral $= \frac{1}{2} \int (\sqrt{2} \sec \theta)^2 \{d\theta\} - \frac{1}{2} \int (1 + \tan \theta)^2 \{d\theta\}$	M1	3.1a
	$= \frac{1}{2} (1 - \ln 2) \text{ * cso}$	A1*	2.1
		(6)	
	Alternative Area bounded by the curve $= \frac{1}{2} \int (1 + \tan \theta)^2 \{d\theta\}$ $= \frac{1}{2} \int (1 + 2 \tan \theta + \tan^2 \theta) \{d\theta\} \text{ let } u = \tan \theta \Rightarrow \frac{du}{d\theta} = \sec^2 \theta$ Leading to $= \frac{1}{2} \int \frac{(1 + 2u + u^2)}{1 + u^2} \{du\} = \frac{1}{2} \int 1 + \frac{2u}{1 + u^2} \{du\} = \dots$	M1	3.1a
	$\frac{1}{2} [u + \ln(1 + u^2)]$	A1	1.1b
	$\frac{1}{2} [(1 + \ln(1 + (1)^2)) - (0 + \ln 1)] \text{ or } \frac{1}{2} \left[\left(\tan \left(\frac{\pi}{4} \right) + \ln \left(1 + \tan^2 \left(\frac{\pi}{4} \right) \right) \right) - (\tan(0) + \ln(1 + \tan^2(0))) \right]$ $\left\{ = \frac{1}{2} \ln 2 + \frac{1}{2} \right\}$	dM1	1.1b
	Area of triangle $= \frac{1}{2} xy = \frac{1}{2} \left(2 \cos \frac{\pi}{4} \right) \left(2 \sin \frac{\pi}{4} \right) = \dots \left\{ \frac{1}{2} \times \sqrt{2} \times \sqrt{2} = 1 \right\}$	M1	1.1b

	Finds the required area = area of triangle – area bounded by the curve $= 1 - \left[\ln \sqrt{2} + \frac{1}{2} \right]$	M1	3.1a
	$= \frac{1}{2}(1 - \ln 2) *$	A1*	2.1
		(6)	
(10 marks)			
Notes:			
(a) M1: Substitutes the equation of C into $x = r \cos \theta$ and differentiates to the required form A1: Fully correct differentiation dm1: Dependent on previous method mark. Sets their $\frac{dx}{d\theta} = 0$ and uses correct trig identities to find a value for θ. Alternatively substitutes $\theta = \frac{\pi}{4}$ into their $\frac{dx}{d\theta}$ and shows equals 0. A1*: Shows that $r = 2$ and hence the polar coordinates $\left(2, \frac{\pi}{4}\right)$ from correct working			
(b) M1: Applies area $= \frac{1}{2} \int r^2 \theta \, d\theta$, multiplies out, uses the identity $\pm 1 \pm \tan^2 \theta = \sec^2 \theta$ to get into an integrable form and integrates. Condone missing $d\theta$, limits are not required for this mark A1: Correct integration. Note may include $\theta - \theta$ if the one's were not cancelled earlier. dm1: Dependent on the first method mark. Applies the limits of $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied M1: Correct method to find the area of triangle seen. This may be minimal but area = 1 only is M0, they need to show some method. M1: Finds the required area = area of triangle – area bounded by the curve A1*: Correct answer, with no errors or omissions. cso Alternative M1: Applies area $= \frac{1}{2} \int r^2 \theta \, d\theta$, multiplies out, uses the substitution $u = \tan \theta$ to get into an integrable form and integrates. Limits are not required for this mark A1: Correct integration dm1: Dependent on the first method mark. Applies the limits of $u = 0$ and $u = 1$ or substitutes back using $u = \tan \theta$ and uses the limits $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied M1: Correct method to find the area of triangle M1: Finds the required area = area of triangle – area bounded by the curve A1*: Correct answer, with no errors or omissions. cso			