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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Differential Equations (9)

Core Pure Mathematics 2 · Topic 9

3 questions · 40 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.
Click any row below to jump straight to that sub-topic or question.
On every question page, click ANSWER in the top-right corner to jump to its mark scheme.
Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
9.5 SECOND ORDER NON-HOMOGENEOUS DES		2 Qs · 28 marks	3
1. Q3	Paper 2: Core Pure Mathematics 2 October2020	14	4
2. Q6	Paper 2: Core Pure Mathematics 2 June2024	14	8
9.8 DAMPED OSCILLATIONS		1 Qs · 12 marks	12
3. Q5	Paper 2: Core Pure Mathematics 2 June2019	12	13



QUESTIONS

3 questions · 40 marks · 2 sub-topics



SUB-TOPIC 9.5

SECOND ORDER NON-HOMOGENEOUS DES

Sub-topic 9.5 · 2 questions · 28 marks · lowest tariff first



Question 3 continued



Question 3 continued



Question 3 continued

(Total for Question 3 is 14 marks)





Question 6 continued



Question 6 continued

(Total for Question 6 is 14 marks)



SUB-TOPIC 9.8

DAMPED OSCILLATIONS

Sub-topic 9.8 · 1 question · 12 marks · lowest tariff first







(Total for Question 5 is 12 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
3(a)	$100m^2 + 60m + 13 = 0 \Rightarrow m = -0.3 \pm 0.2i$	M1	1.1b
	$x = e^{-0.3t} (A \cos 0.2t + B \sin 0.2t)$	A1	1.1b
	PI: $x = 2$	B1	1.1b
	$x = e^{-0.3t} (A \cos 0.2t + B \sin 0.2t) + 2$	A1ft	2.2a
	(4)		
(b)	$t = 0, x = 0 \Rightarrow A = -2$	M1	3.4
	$\frac{dx}{dt} = -0.3e^{-0.3t} (-2 \cos 0.2t + B \sin 0.2t) + e^{-0.3t} (0.4 \sin 0.2t + 0.2B \cos 0.2t)$ $t = 0, \frac{dx}{dt} = 10 \Rightarrow B = \dots$ (NB $B = 47$)	M1	3.4
	$x = e^{-0.3t} (47 \sin 0.2t - 2 \cos 0.2t) + 2$	A1	1.1b
	$-0.3e^{-0.3t} (47 \sin 0.2t - 2 \cos 0.2t) + e^{-0.3t} (9.4 \cos 0.2t + 0.4 \sin 0.2t) = 0$ $\Rightarrow t = \dots$ or $x = \sqrt{2213} e^{-0.3t} \sin(0.2t - 0.0425) + 2$ p $\frac{dx}{dt} = -0.3\sqrt{2213} e^{-0.3t} \sin(0.2t - 0.0425)$ $+ 0.2\sqrt{2213} e^{-0.3t} \cos(0.2t - 0.0425)$ p $t = \dots$	M1	3.1b
	$\tan 0.2t = \frac{100}{137} \Rightarrow 0.2t = 0.630\dots$ or $\tan(0.2t - 0.0425) = \frac{2}{3}$ p $0.2t = 0.630$	M1	2.1
	$t = 3.15\dots$ weeks	A1	1.1b
	$x = e^{-0.3 \times "3.15\dots"} (47 \sin(0.2 \times "3.15\dots") - 2 \cos(0.2 \times "3.15\dots")) + 2$	M1	3.4
	= awrt 12.1 {µg/ml}	A1	3.2a
	(8)		
	$t = 10 \Rightarrow x = e^{-3} (47 \sin(2) - 2 \cos(2)) + 2 = 4.16\dots$	M1	3.4
(c)	The model suggests that it would be safe to give the second dose	A1ft	2.2a
	(2)		
(14 marks)			
Notes			
(a) M1: Uses the model to form and solve the auxiliary equation A1: Correct CF, does not need $x =$ B1: Correct PI A1ft: Deduces the correct GS (follow through their CF + PI). Must have $x = f(t)$ and PI not 0 (b) M1: Uses the model and the initial conditions to establish the value of "A" M1: Differentiates their model using the product rule and uses the initial conditions to establish the value of "B". Must be using $x = 0$ and $\frac{dx}{dt} = 10$ A1: Correct particular solution. This can be implied by the correct constants found following a correct answer to part (a).			

M1: Uses their solution to the model with a correct strategy to obtain the required value of t e.g. differentiates, sets equal to zero and solves for t

M1: Uses a correct trigonometric approach that leads to a value for t

A1: Correct value for t

M1: Uses the model and their value for t to find the maximum concentration.

A1: Correct value

(c)

M1: Uses the model to find the concentration when $t = 10$

A1ft: Makes a suitable comment that is consistent with their calculated value

Special case: If the candidate's maximum value is less than 5 then

M1: never reaches 5 as maximum is.... or max is less than 5

A1: yes, it is safe

Question	Scheme	Marks	AOs
6(a)	$2m^2 + 5m + 2 = 0 \Rightarrow m = -\frac{1}{2}, -2$	M1	3.4
	$x = Ae^{-0.5t} + Be^{-2t}$	A1	1.1b
	PI is of the form $x = pt + q$	B1	1.1b
	$\frac{dx}{dt} = p, \frac{d^2x}{dt^2} = 0 \Rightarrow 5p + 2pt + 2q = 4t + 12 \Rightarrow p = \dots, q = \dots$	M1	3.4
	$p = 2, q = 1$	A1	1.1b
	$x = Ae^{-0.5t} + Be^{-2t} + 2t + 1$	A1ft	1.1b
	(6)		
(b)	$t = 0, x = 3 \Rightarrow 3 = A + B + 1$	M1	3.4
	$\frac{dx}{dt} = -0.5Ae^{-0.5t} - 2Be^{-2t} + 2$ $t = 0, \frac{dx}{dt} = -2 \Rightarrow -\frac{1}{2}A - 2B + 2 = -2$ $\Rightarrow A = \dots, B = \dots$	M1	3.4
	$x = 2e^{-2t} + 2t + 1$	A1	1.1b
	(3)		
(c)(i)	$\frac{dx}{dt} = -4e^{-2t} + 2$	M1	3.1b
	$e^{-2t} = \frac{1}{2} \Rightarrow -2t = \ln \frac{1}{2} \Rightarrow t = \frac{1}{2} \ln 2$	dM1	2.1
	$x = 2e^{-2t} + 2t + 1 = 1 + \ln 2 + 1 = 2 + \ln 2^*$	A1*	1.1b
(c)(ii)	$\frac{d^2x}{dt^2} = 8e^{-2t}$ is > 0 for all values of t so distance is a minimum	B1ft	2.4
	(4)		
(d)	Examples: For large values of $t, [e^{-2t} \rightarrow 0 \Rightarrow] x \rightarrow 2t + 1$ so constant speed For large values of $t, [e^{-2t} \rightarrow 0 \Rightarrow] \frac{dx}{dt} \rightarrow 2$ so constant speed For large values of $t, [e^{-2t} \rightarrow 0 \Rightarrow] \frac{d^2x}{dt^2} \rightarrow 0$ so constant speed Conclusion: so the model is suitable	B1ft	3.2b
	(1)		
(14 marks)			
Notes			
(a) M1: Attempts to solve $2m^2 + 5m + 2 = 0$, usual rules apply for solving a quadratic equation. A1: Correct CF. Do not need “$x =$” here, but must be in terms of t B1: Correct form for the PI i.e. $x = pt + q$ or $x = at^2 + bt + c$ M1: Differentiates their PI (of the forms $x = pt + q$ or $x = at^2 + bt + c$) twice and substitutes into the given differential equation finding values for their constants to obtain a PI of the form $pt + q$, $p, q \neq 0$ A1: Correct PI A1ft: Correct GS or correct ft GS, which is the sum of their CF and PI. This is dependent on achieving both previous M marks. Must have $x =$ and their GS must be in terms of t.			

(b)

M1: Substitutes $x = 3$ when $t = 0$ into their GS to establish an equation in A and B , allow minor slips if the intention is clear.

M1: Differentiates their answer to part (a) which must be in terms of t only, and sets $= -2$ with $t = 0$ to establish another equation in A and B and solves simultaneously to find A and B .

Do not be concerned about how their simultaneous equations are solved; award this mark if they then go on to write values for A and B .

Functions which require the use of product rule or trigonometric functions must be differentiated appropriately.

A1: Correct PS. **Need** “ $x =$ ” here, their answer must be in terms of t and no other variable.

(c) Mark (i) and (ii) together but only award for work done in (c)

(i)

M1: Differentiates their Particular Solution of the form $x = ae^{-kt} + bt + c$ where c may be 0 to obtain an expression of the form $Ce^{-kt} + D$

dM1: Solves an equation of the form $Ce^{-kt} + D = 0$, $C \times D < 0$, to obtain $t = -\frac{1}{k} \ln\left(\frac{-D}{C}\right)$

A1*: Substitutes $t = \frac{1}{2} \ln 2$ or to obtain the printed answer with no errors.

Condone going from $2e^{-2\left(\frac{1}{2}\ln 2\right)} + 2\left(\frac{1}{2}\ln 2\right) + 1 = 2 + \ln 2$

(c)(ii)

B1ft: Obtains a second derivative of the form $\frac{d^2x}{dt^2} = \lambda e^{-\mu t}$, $\lambda, \mu > 0$ **and** makes a conclusion e.g.

- $\frac{d^2x}{dt^2} > 0$ (for all values of t) hence minimum.
- or substitutes their value of t (even if incorrect) and states $\frac{d^2x}{dt^2} > 0$ hence minimum

(d)

B1ft: Dependent on having obtained a Particular Solution of the form $f(t) + bt + c$ where $f(t)$ only has terms in ae^{-kt} where $k > 0$ and $a, b \neq 0$

This mark is awarded for the candidate demonstrating that in the model for “large values” or “as $t \rightarrow \infty$ ”, the value of their $e^{-kt} \rightarrow 0$, so there is constant speed **and** states that the model is suitable, or equivalent statement. (See scheme for examples)



Question	Scheme	Marks	AOs
5(a)	$4m^2 + 4m + 37 = 0 \Rightarrow m = -\frac{1}{2} \pm 3i$	M1	1.1b
	$h = e^{-0.5t} (A \cos 3t + B \sin 3t)$	A1	1.1b
		(2)	
(b)	$t = 0, h = -20 \Rightarrow A = -20$	M1	3.4
	$\frac{dh}{dt} = -0.5e^{-0.5t} (A \cos 3t + B \sin 3t) + e^{-0.5t} (-3A \sin 3t + 3B \cos 3t)$ $t = 0, \frac{dh}{dt} = 55 \Rightarrow B = \dots$ (NB $B = 15$)	M1	3.4
	$(h =) e^{-0.5t} (15 \sin 3t - 20 \cos 3t)$	A1	1.1b
	$-0.5e^{-0.5t} (15 \sin 3t - 20 \cos 3t) + e^{-0.5t} (60 \sin 3t + 45 \cos 3t) = 0$ or e.g. $-0.5e^{-0.5t} (15 \sin 3t - 20 \cos 3t) + \frac{25\sqrt{37}}{2} e^{-0.5t} \sin \left(3t + \arctan \frac{22}{21} \right) = 0$ $\Rightarrow t = \dots$	M1	3.1b
	$\tan 3t = -\frac{22}{21}$ or e.g. $3t + \tan^{-1} \frac{22}{21} = 0$	A1 M1 on ePEN	2.1
	$t = 0.778 \text{ s}$	A1	1.1b
	$h = e^{-0.5 \times 0.778} (15 \sin(3 \times 0.778) - 20 \cos(3 \times 0.778))$	dM1	1.1b
	$= 16.7 \text{ cm}$	A1	3.2a
		(8)	
(c)	E.g. considers large values of t in the model for h or states that for large values of t , h becomes smaller or becomes zero	M1	3.4
	E.g. - The value of h is very small when t is large and this is likely to be correct (as the displacement of end of the board should get smaller and smaller) - This suggests the model is suitable - This is realistic - This is suitable as the board will tend towards its equilibrium position - When t is large the value of h is never zero so the model is not really appropriate for large values of t	A1 B1 on ePEN	3.2b
		(2)	
(12 marks)			
Notes			
(a) M1: Uses the model to form and solve the auxiliary equation $4m^2 + 4m + 37 = 0$ See General Guidance for awarding this mark. This can be implied by correct values for m (from calculator) A1: Correct general solution including " $h =$ " (b)			

M1: Uses the model and the initial conditions to establish the value of “A”. Need to see $t = 0$ and $h = \pm 20$ leading to a value for “A”. This may be implied by $A = -20$ or $A = 20$.

M1: Differentiates their model using the product rule and uses the initial conditions, $t = 0$ with $dh/dt = \pm 55$, to establish the value of “B”

A1: Correct particular solution or correct values for A and B

M1: Uses their solution to the model with a correct strategy to obtain a value for t e.g. differentiates or uses their derivative from earlier, sets equal to zero and solves for t

A1(M1 on ePEN): Correct equation for t

A1: Correct value for t (allow awrt 0.778 if necessary) but this value may be implied.

dM1: Uses the model and their positive value for t to find the maximum displacement - if their t is incorrect there must be some indication that they are using their h and not just a number written down. E.g. must see substitution into their h or they re-state their h and obtain a value for h .

Dependent on all the previous method marks

A1: Correct value (awrt 16.7 (units not needed))

(c)

M1: Considers the model for large values of t either by substituting values or by considering the expression and commenting on its behaviour for large values of t . E.g. as $t \rightarrow \infty$, $h \rightarrow 0$ or as

$t \rightarrow \infty$, $e^{-0.5t} \rightarrow 0$ or as $t \rightarrow \infty$ the oscillations become smaller etc.

A1: Makes a suitable comment – see scheme for examples