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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Algorithms on Graphs (2)

Decision Mathematics 1 · Topic 2

5 questions · 55 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
2.1 MINIMUM SPANNING TREES		1 Qs · 8 marks	3
1. Q3 <i>also Topic 1</i>	Option 3D: Decision Mathematics 1 June2023	8	4
2.2 SHORTEST PATHS: DIJKSTRA AND FLOYD		2 Qs · 20 marks	5
2. Q6 <i>also Topic 1 & 4 & 5</i>	Option 3D: Decision Mathematics 1 November2021	10	6
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QUESTIONS

5 questions · 55 marks · 3 sub-topics



SUB-TOPIC 2.1

MINIMUM SPANNING TREES

Sub-topic 2.1 · 1 question · 8 marks · lowest tariff first



3.

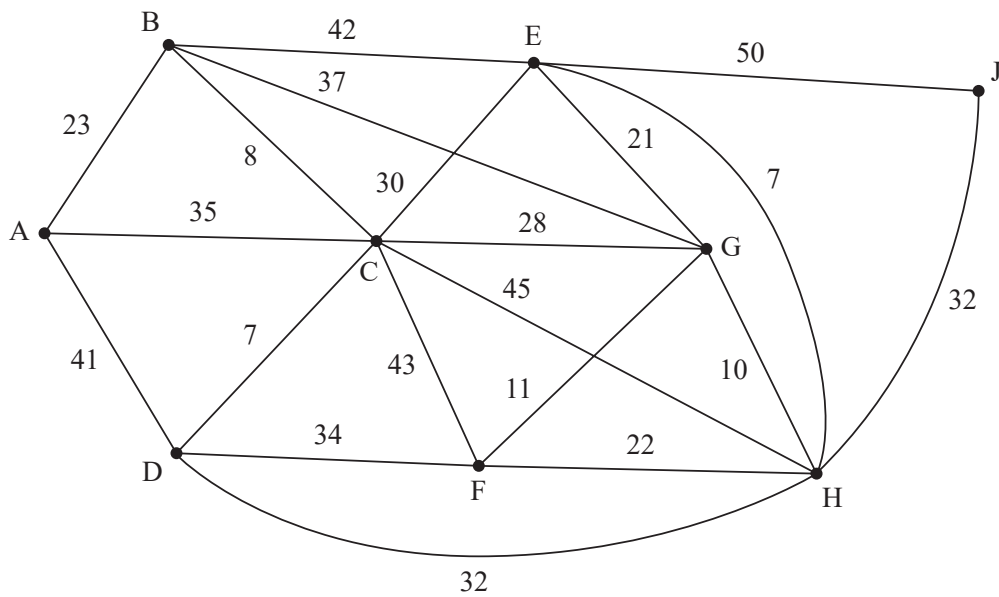
**Figure 4**

Figure 4 represents a network with nodes, A, B, C, D, E, F, G, H and J. The number on each edge gives the length of the corresponding edge.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J.

(6)

One application of Dijkstra's algorithm has order n^2 , where n is the number of nodes in the network.

It takes a computer 0.0312 seconds to find the shortest path from a given start node to a given end node in a network of 9 nodes.

(b) Calculate approximately how long it would take, in minutes, for the computer to find the shortest path from a given start node to a given end node for a network of 9000 nodes.

(2)**(Total for Question 3 is 8 marks)**

SUB-TOPIC 2.2

SHORTEST PATHS: DIJKSTRA AND FLOYD

Sub-topic 2.2 · 2 questions · 20 marks · lowest tariff first



6.

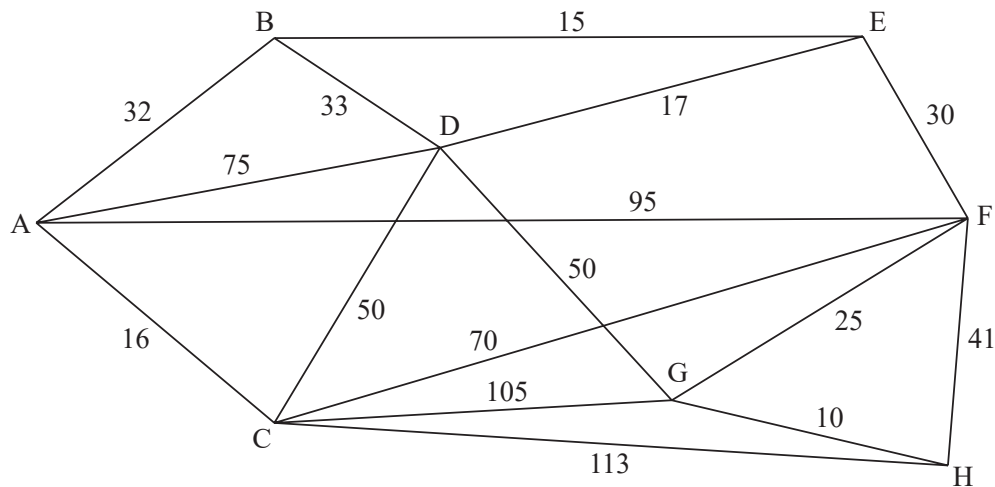


Figure 4

In Figure 4 the weights on the arcs represent distances.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to H.

(ii) State the length of the shortest path from A to H.

(6)

One application of Dijkstra's algorithm has order n^2 , where n is the number of nodes in the network. A computer produces a table of shortest distances between any two different nodes by repeatedly applying Dijkstra's algorithm from each node of the network.

It takes the computer 0.082 seconds to produce a table of shortest distances for a network of 10 nodes.

(b) Calculate approximately how long it will take, in seconds, for the computer to produce a table of shortest distances for a network with 200 nodes. You must give a reason for your answer.

(3)

(c) Explain why your answer to part (b) can only be an approximation.

(1)

(Total for Question 6 is 10 marks)



7.

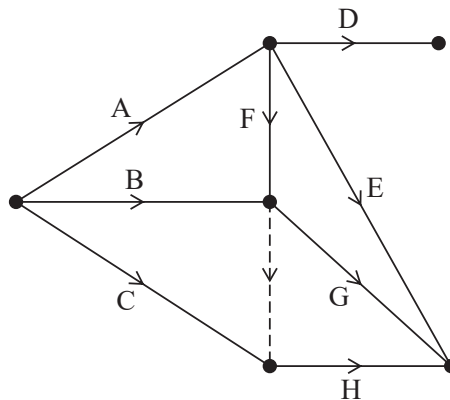


Figure 5

Figure 5 shows a partially completed activity network for a project that consists of 14 activities.

(a) Complete the precedence table in the answer book for the 8 activities in Figure 5.

(2)

The precedence table for the remaining 6 activities is given below.

Activity	Immediately preceding activities
I	D, E, G, H
J	D, E, G, H
K	E, G, H
L	I, J, K
M	J, K
N	J, K

(b) Complete the activity network in the answer book for the project. Your completed activity network must contain only the minimum number of dummies.

(4)

Given that all 14 activities have the same duration,

(c) explain why activity D cannot be critical.

(2)

(Total for Question 7 is 8 marks)



8. Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- x represent the number of hours swimming
- y represent the number of hours cycling
- z represent the number of hours running

- (a) Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(5)

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

- (b) Set up an initial tableau for solving this problem using the two-stage Simplex method.

As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6)

The following tableau T is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	-2	1	-5	62
x	1	0	1	0	0	-1	28
P	0	0	-1	1	0	1	11

- (c) Obtain a suitable pivot for a second iteration. You must give reasons for your answer.

(2)

- (d) Starting from tableau T , solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use.

(5)

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS



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Please check the examination details below before entering your candidate information

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9FM0/3D

Further Mathematics

Advanced

PAPER 3D: Decision Mathematics 1

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Answer book

Do not return the question paper with the answer book.

Total Marks

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12

1.

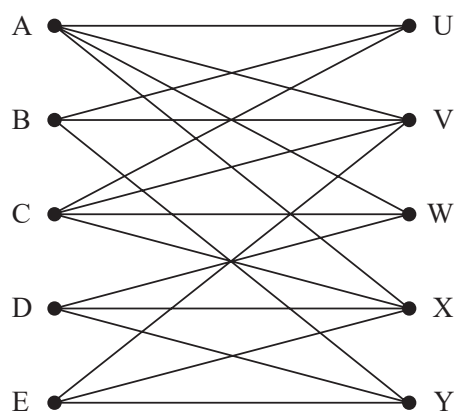


Figure 1



Question 1 continued

(Total for Question 1 is 4 marks)



Key:

Early event time
Late event time

The network diagram shows the following activities and dependencies:

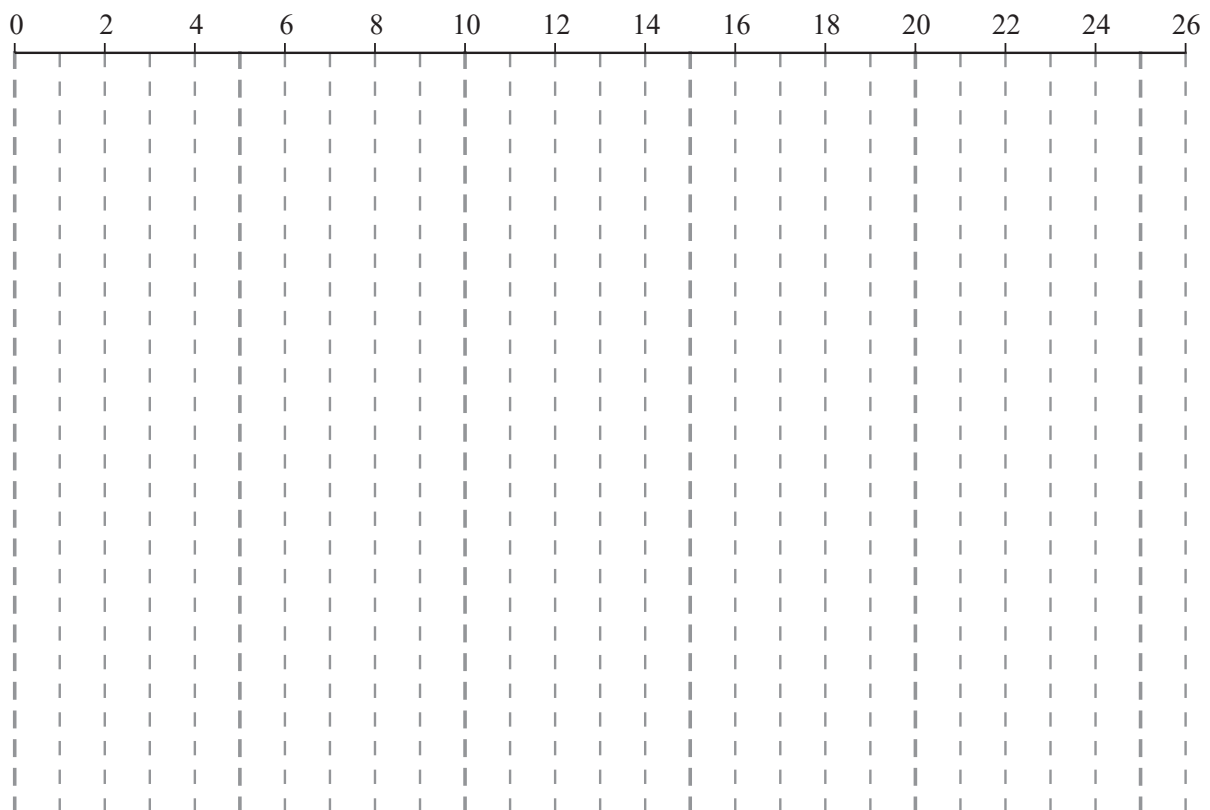
- Activity A(4) leads to B(6) and C(10).
- Activity B(6) leads to D(2) and E(4).
- Activity C(10) leads to F(5).
- Activity D(2) leads to G(6).
- Activity E(4) leads to F(5) and H(7).
- Activity F(5) leads to I(2) and L(7).
- Activity G(6) leads to H(7).
- Activity H(7) leads to J(7).
- Activity I(2) leads to K(6).
- Activity J(7) leads to K(6).
- Activity K(6) leads to L(7) and M(5).
- Activity L(7) leads to M(5).
- Activity M(5) leads to the final node.

Dashed arrows indicate dependencies from the top node of F(5) to the top node of H(7), and from the bottom node of K(6) to the bottom node of L(7).

Diagram 1

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 2 continued

[illegible]

Grid 1

(Total for Question 2 is 9 marks)

3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–



Question 3 continued

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

(Total for Question 3 is 8 marks)



4. (a)

Initial time matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Initial route matrix

	A	B	C	D	E	F
A	A	B	C	D	E	F
B	A	B	C	D	E	F
C	A	B	C	D	E	F
D	A	B	C	D	E	F
E	A	B	C	D	E	F
F	A	B	C	D	E	F

(b) 1st iteration

Time matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Route matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Question 4 continued

(Total for Question 4 is 8 marks)



5.

30

12

5

2

23

18

36

10

15

24



Question 5 continued

(Total for Question 5 is 10 marks)



Question 6 continued

(Total for Question 6 is 10 marks)



3.

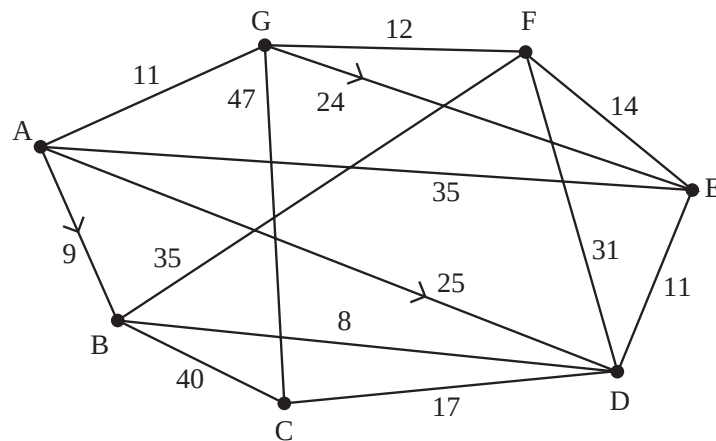


Figure 1

Direct roads between seven villages, A, B, C, D, E, F and G, are represented in Figure 1. The weight on each arc is the time, in minutes, taken to travel along the corresponding road. Three roads, AB, AD and GE, are one-way, as indicated by the arrow on the corresponding arc.

Floyd's algorithm is to be used to find the complete network of shortest times between the seven villages.

(a) Set up an initial time matrix for this network.

(2)

The time matrix after three iterations of Floyd's algorithm is shown below.

	A	B	C	D	E	F	G
A	–	9	49	17	35	44	11
B	∞	–	40	8	∞	35	87
C	∞	40	–	17	∞	75	47
D	∞	8	17	–	11	31	64
E	35	44	84	11	–	14	46
F	∞	35	75	31	14	–	12
G	11	20	47	28	24	12	–

(b) Perform the next two iterations of Floyd's algorithm that follow from the table above.

You should show only the time matrix after each iteration.

(5)



The final time matrix after completion of Floyd's algorithm is shown below.

	A	B	C	D	E	F	G
A	–	9	34	17	28	23	11
B	54	–	25	8	19	33	45
C	58	25	–	17	28	42	47
D	46	8	17	–	11	25	37
E	35	19	28	11	–	14	26
F	23	32	42	25	14	–	12
G	11	20	45	28	24	12	–

Albert must visit each village. He will start and finish at B and wishes to minimise the total time taken to visit each village.

- (c) (i) Use the nearest neighbour algorithm, starting at B, to find a Hamiltonian cycle in the complete network of shortest times shown above.
- (ii) Find the total travel time for this cycle.
- (iii) Given that he travels the Hamiltonian cycle found in (c)(i), interpret this cycle in terms of the actual villages visited by Albert.

(3)

(Total for Question 3 is 10 marks)



SUB-TOPIC 3.1

ROUTE INSPECTION

Sub-topic 3.1 · 2 questions · 27 marks · lowest tariff first



2.

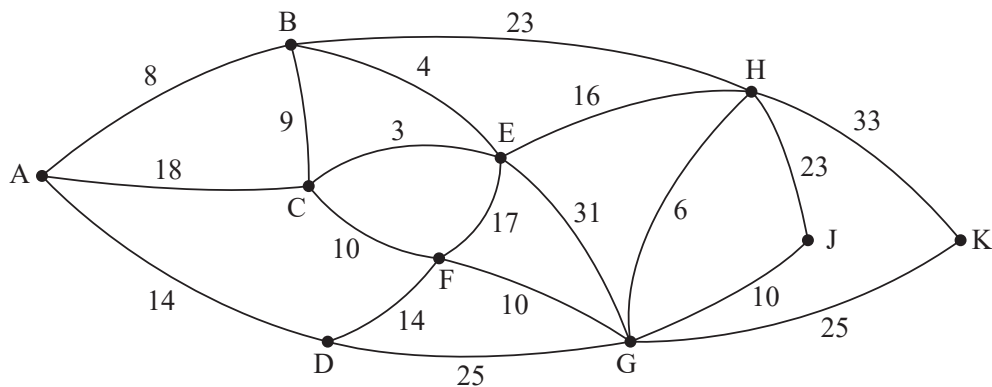


Figure 1

[The total weight of the network is 299]

Figure 1 represents a network of cycle tracks between 10 landmarks, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding track.

One day, Blanche wishes to cycle from A to K. She wishes to minimise the distance she travels.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to K.

(ii) State the length of the shortest path from A to K.

(6)

The cycle tracks between the landmarks now need to be inspected. Blanche must travel along each track at least once. She wishes to minimise the length of her inspection route. Blanche will start her inspection route at D and finish at E.

(b) (i) State the edges that will need to be traversed twice.

(ii) Find the length of Blanche's route.

(2)

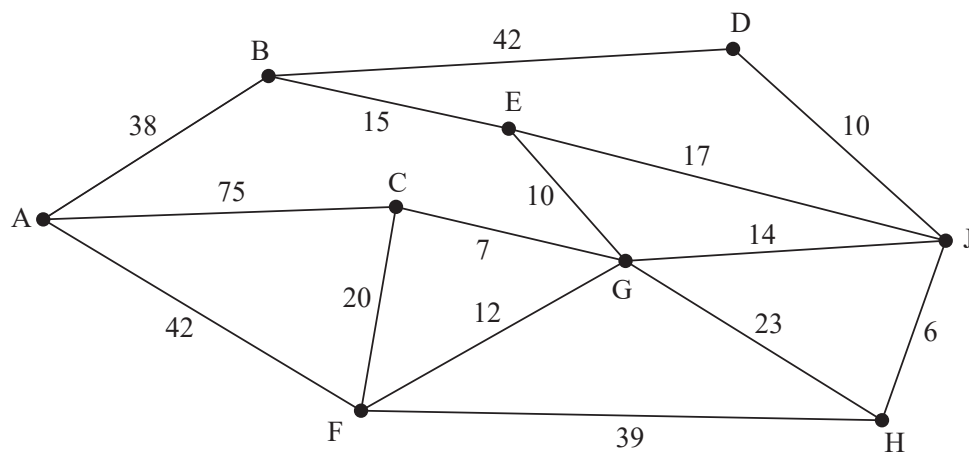
It is now decided to start the inspection route at A and finish at K. Blanche must minimise the length of her route and travel along each track at least once.

(c) By considering the pairings of all relevant nodes, find the length of Blanche's new route. You must make your method and working clear.

(5)

(Total for Question 2 is 13 marks)

2.

**Figure 1**

[The total weight of the network is 370]

Figure 1 represents a network of corridors in a building. The number on each arc represents the length, in metres, of the corresponding corridor.

- (a) Use Dijkstra's algorithm to find the shortest path from A to D, stating the path and its length.

(6)

On a particular day, Naasir needs to check the paintwork along each corridor. Naasir must find a route of minimum length. It must traverse each corridor at least once, starting at B and finishing at G.

- (b) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

(4)

- (c) Find the length of Naasir's route.

(1)

On a different day, all the corridors that start or finish at B are closed for redecorating. Naasir needs to check all the remaining corridors and may now start at any vertex and finish at any vertex. A route is required that excludes all those corridors that start or finish at B.

- (d) (i) Determine the possible starting and finishing points so that the length of Naasir's route is minimised. You must give reasons for your answer.
(ii) Find the length of Naasir's new route.

(3)**(Total for Question 2 is 14 marks)**

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
3(a)	(i)	M1 A1 (ABCD) A1 (GEH) A1ft (FJ)	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to J is ABCEHJ	A1	2.2a
	Length of shortest path is 100	A1ft	2.2a
		(6)	
(b)	$0.0312 \times \left(\frac{9000}{9} \right)^2$	M1	1.1a
	= 31 200 seconds therefore 520 minutes	A1	2.2a
		(2)	
(8 marks)			



Notes for Question 3

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at D the working values must be 41 38 in that order (so 38 41 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a) M1: A larger value replaced by a smaller value in the working values at **two** of C or D or E or F or G or H or J

A1: All values in A, B, C and D correct. Condone lack of 0 in A's working value

A1: All values G, E and H correct and the working values in the correct order. Penalise order of labelling only once per question (G, E and H must be labelled in that order and G must be labelled after A, B, C and D). Note that an additional working value of 80 at E after the 61 is not an error so 65 61 80 is fine, however, any other number or e.g. 80 65 61 in this order is incorrect and scores A0 in this part

A1ft: All values in F and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question.

To follow through F check that the working values at F follow from the candidate's final values for the nodes that are directly attached to F (which are C, D, G and H). For example, if correct then the order of labelling of nodes C, D, G and H are 3, 4, 5 and 7 respectively so the working values at F should come from C, D, G and H in that order. The first working value at F should be their 31 (the Final value at C) + 43 (the weight of the arc CF), the second working value at F should be their 38 (the Final value at D) + 34 (the weight of the arc DF), the third working value at F should be their 59 (the Final value at G) + 11 (the weight of the arc GF), the fourth working value need not be there (as $68 + 22 > 70$) but do not penalise if seen. Repeat the process for J (which will have working values from E and H with the order of these nodes determined by the candidate's order of labelling at E and H)

A1: CAO - correct path from A to J (ABCEHJ) and not the path from J to A

A1ft: ft their final value at J only (if 100 stated and 100 is not the final value at J then **A0**)

(b)

M1: Complete, correct method – allow reciprocal e.g. $0.0312 \times \left(\frac{9}{9000}\right)^2$ – allow slips in values

only e.g. 0.312 for 0.0312. If using anything other than squared e.g. n^3 then **M0**. The correct answer in seconds, minutes or hours (8 hours 40 minutes) implies this mark (and if in minutes implies the A mark too)

A1: CAO – the exact value of 520 must be stated at some point (as question specifically asked for the answer in minutes) – condone lack of units (but if present must be correct)



Question	Scheme	Marks	AOs
6(a)(i)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to H: ABEFGH	A1	2.2a
(a)(ii)	Length of shortest path from A to H is 112	A1ft	2.2a
		(6)	
(b)	Applying Dijkstra repeatedly to n nodes implies that the order is $n(n^2) = n^3$	B1	3.1b
	$t = 0.082 \left(\frac{200}{10} \right)^3$	M1	3.4
	= 656 (seconds)	A1	2.2a
		(3)	
(c)	e.g. order of n^3 does not mean that the order is proportional to n^3 (which is the assumption behind the answer in (b)) but merely means that the dominant term is of order n^3	B1	3.2b
		(1)	
(10 marks)			
Notes: In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 129 118 112 in that order (so 129 112 118 is incorrect) It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling			

(a)

M1: A larger value replaced by a smaller value in at least two of the working boxes at either D or F or G or H

A1: All values in A, B, C and E correct. Condone lack of 0 in A's working value

A1: All values D and F correct and the working values in the correct order. Penalise order of labelling only once per question (D and F must be labelled in that order and D must be labelled after A, B, C and E)

A1ft: All values in G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will most likely come from nodes C, D and/or F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from C, F and/or G with the order of these values determined by the candidate's order of labelling at C, F and G)

A1: cao - correct path from A to H (ABEFGH)

A1ft: Follow through their final value at H only (if 112 stated and 112 is not the final value at H then **A0**)

(b)

B1: Any indication that repeated application of Dijkstra has cubic order

M1: Complete method – allow 10/200 – allow slips in values e.g. 0.82 for 0.082 and accept 200/10 (or 10/200) either squared or cubed **only**

A1: cao

(c)

B1: Any indication that the run-time is not exactly proportional to n^3 e.g., may suggest that there are other terms ($n^3 + \dots$), or that n^3 is the dominant term, or that order does not imply proportionality. Do not accept only that ' n^3 is not exact'. Condone use of n^2 (oe) for n^3



Question	Scheme								Marks	AOs			
3(a)		A	B	C	D	E	F	G	B2, 1, 0	1.1b 1.1b			
	A	-	9	∞	25	35	∞	11					
	B	∞	-	40	8	∞	35	∞					
	C	∞	40	-	17	∞	∞	47					
	D	∞	8	17	-	11	31	∞					
	E	35	∞	∞	11	-	14	∞					
	F	∞	35	∞	31	14	-	12					
	G	11	∞	47	∞	24	12	-					
									(2)				
(b)	Fourth iteration:								M1 A1	2.1 1.1b			
		A	B	C	D	E	F	G					
	A	-	9	34	17	28	44	11					
	B	∞	-	25	8	19	35	72					
	C	∞	25	-	17	28	48	47					
	D	∞	8	17	-	11	31	64					
	E	35	19	28	11	-	14	46					
	F	∞	35	48	31	14	-	12					
	G	11	20	45	28	24	12	-					
	Fifth iteration:										M1 A1 A1	2.1 1.1b 2.2a	
		A	B	C	D	E	F	G					
	A	-	9	34	17	28	42	11					
	B	54	-	25	8	19	33	65					
	C	63	25	-	17	28	42	47					
	D	46	8	17	-	11	25	57					
	E	35	19	28	11	-	14	46					
	F	49	33	42	25	14	-	12					
	G	11	20	45	28	24	12	-					
									(5)				
	(c)(i)	NNA starting at B: B – D – E – F – G – A – C – B								B1			2.2a
	(ii)	Time taken: 8 + 11 + 14 + 12 + 11 + 34 + 25 = 115 (minutes)								B1			1.1b
	(iii)	Actual villages visited: BDEFGAB <u>D</u> C <u>D</u> B								B1	2.2a		
										(3)			
	(10 marks)												



Notes:

a1B1: At least three rows and three columns correct (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

a2B1: CAO

b1M1: No change in the fourth row and fourth column with at least two values reduced correctly (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

b1A1: CAO for the fourth iteration (changes in bold and blue shading)

b2M1: No change in the fifth row and fifth column with at least two values reduced correctly (follow through from their previous iteration) (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

b2A1: At least eight values reduced correctly (from the values shown in bold and orange shading)

b3A1: CSO for the fourth and fifth iterations

(if correct changes for the fifth iteration are shown in bold and orange shading)

ci1B1: CAO (Nearest neighbour cycle starting at B – must include return to B) (condone the use of arcs BD DE EF FG GA AC CB – must be written in this manner and not AG or BC)

cii2B1: Correct time

ciii3B1: Correct route – must be stated as nodes



Question	Scheme	Marks	AOs
2(a)	(i) (ii) Path from A to K: ABEHGK (iii) Length of shortest path from A to K is 59 (km)	M1 A1 A1 A1ft A1 A1ft	1.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(b)(i)	Repeated edges: AB, BE, EH	B1	2.2a
(b)(ii)	Length of route = $299 + 28 = 327$ (km)	B1ft	2.2a
		(2)	
(c)	Route must start at A and finish at K therefore need to consider pairings of the nodes D, E, H and K	M1	3.1b
	$D(AB)E + H(G)K = 26 + 31 = 57$ $D(FG)H + E(HG)K = 30 + 47 = 77$ $D(FG)K + EH = 49 + 16 = 65$	A1 A1 A1	1.1b 1.1b 1.1b
	Length of new route = $299 + 57 = 356$ (km)	A1	2.2a
		(5)	
(13 marks)			



Notes for Question 2

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded, for example at F the working values must be 29 28 25 in that order (so 29 25 28 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a 0 label at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4,... will be penalised once (see notes below) but 1, 2, 3, 5, 6,... is fine. Errors in the final values and working values are penalised before errors in the order of labelling. Condone crossed out working values if they are still legible

a1M1: A larger value replaced by a smaller value in at least **two** different Working Value boxes at either C or F or G or H or J or K

a1A1: All values at A, B, E, D and C correct. Condone lack of 0 in A's working value

a2A1: All values at F, H and G correct and the working values in the correct order. Penalise order of labelling only once per question (F, H and G must be labelled in that order and F must be labelled after A, B, E, D and C)

a3A1ft: All values in J and K correct on the follow through and the working values in the correct order. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are G and H). For example, **if** correct then the order of labelling of nodes G and H are 8 and 7 respectively so the working values at J should come from H and G in that order. The first working value at J should be their 28 (the Final value at H) + 23 (the weight of the arc HJ), the second working value at J should be their 34 (the Final value at G) + 10 (the weight of the arc GJ). Repeat the process for K (which will have working values from G and H with the order of these nodes determined by the candidate's order of labelling at G and H)

a4A1: cao (correct path from A to K – ABEHGK). Do not accept KGHEBA. Condone the length of the shortest path and the path interchanged on the answer lines in the answer book. Allow the paths written in terms of arcs, e.g. AB, BE, EH, HG, GK

a5A1ft: ft their final value at K only (if 59 stated and 59 is not the final value at K then A0)

bi1B1: cao (AB, BE, EH only but in any order) – must be stated as edges so B0 for ABEH

bii1B1ft: 327 or follow through their final value at H from **(a)** + 299

c1M1: The correct three pairings of the correct four nodes (D, E, H and K)

c1A1: One row correct including pairing **and** total

c2A1: Two rows correct including pairings **and** totals

c3A1: All three rows correct including pairings **and** totals

c4A1: cao (356) – dependent on all previous marks in this part

Condone lack of units throughout



Qu	Scheme	Marks	AOs
2(a)	Path from A to D is AFGJD Length of path from A to D is 78 metres	M1 A1 A1 A1ft A1 A1ft	1.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(b)	$A(FG)C + E(J)H = 61 + 23 = 84$ $A(B)E + C(GJ)H = 53 + 27 = 80^*$ $A(FGJ)H + C(G)E = 74 + 17 = 91$ Repeat arcs: AB, BE, CG, GJ and JH	M1 A1ft A1 A1	3.1b 1.1b 1.1b 2.2a
		(4)	
(c)	Length of the route is $370 + 80 = 450$ metres	B1ft	2.2a
		(1)	
(d)(i)	If node B is removed this makes D, C, G and H odd The shortest path between any two odd nodes is CG (so repeat CG) so the route should start at D and finish at H (or vice-versa)	M1 A1	3.1b 2.2a
(ii)	Length of new route is $370 - 38 - 42 - 15 + 7 = 282$ metres	B1	2.2a
		(3)	
(14 marks)			

Notes for Question 2

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 81 77 74 in that order (so 81 74 77 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a) M1: A larger value replaced by a smaller value in at least two of the working boxes at either C or D or E or G or J

A1: All values in A, B, F and E correct. Condone lack of 0 in A's working value

A1: All values G, C and J correct and the working values in the correct order. Penalise order of labelling only once per question (G, C and J must be labelled in that order and G must be labelled after A, B, F and E). Note that an additional working value of 63 at G after the 54 is not an error so 54 63 is fine, however, any other number or 63 54 in this order is incorrect and scores A0 in this part

A1ft: All values in H and D correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working value at H follows from the candidate's final values from their feeds into H (which will come from nodes F, G and/or J (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for D (which will possibly have working values from B and J with the order of these values determined by the candidate's order of labelling at B and J)

A1: CAO - correct path from A to D (AFGJD)

A1ft: ft their final value at D only (if 78 stated and 78 is not the final value at D then A0)

(b) M1: correct three pairings of the correct four odd nodes (A, C, E and H)

A1ft: any row correct including pairing **and** total (ft the final values from **(a)** for their shortest paths from A to the three other nodes C, E or H only (so the pairing that does not include A must be correct))

A1: all three rows correct including pairings **and** totals

A1: selecting the shortest pairing, and stating that these arcs (AB, BE, CG, GJ and JH) should be repeated. Must be these arcs and not e.g. ABE, CGJH or AE via B, etc.

(c) B1ft: For 370 + their smallest repeat out of a choice of at least **two** totals seen in **(b)** – this mark is dependent on M1 in (b)

(d) M1: Mention of the fact that these four nodes D, C, G and H only are now odd **or** clear consideration of these four nodes only

A1: CAO D and H **and** must have clearly indicated that the shortest path is from C to G (or vice-versa) – but A0 if clearly selecting the shortest pairing first before selecting the shortest path (as the shortest path is embedded in the shortest pairing)

B1: CAO (282)

