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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Algorithms on Graphs II (3)

Decision Mathematics 1 · Topic 3

8 questions · 73 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

8 questions · 73 marks · 6 sub-topics



SUB-TOPIC 1.2

BIN PACKING AND SORTING

Sub-topic 1.2 · 1 question · 5 marks · lowest tariff first



1. A gardener needs the following lengths of string. All lengths are in metres.

4.3 6.1 5.1 4.7 2.5 5.9 3.4 1.7 2.1 0.4 1.3

She cuts the lengths from balls of string. Each ball contains 10m of string.

- (a) Calculate a lower bound for the number of balls of string the gardener needs.
You must make your method clear.

(2)

- (b) Use the first-fit bin packing algorithm to determine how the lengths could be cut
from the balls of string.

(3)

(Total for Question 1 is 5 marks)



SUB-TOPIC 2.1

MINIMUM SPANNING TREES

Sub-topic 2.1 · 1 question · 6 marks · lowest tariff first



1. The table below shows the lengths, in km, of the roads in a network connecting seven towns, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	24	–	22	35	–	–
B	24	–	25	27	–	–	–
C	–	25	–	33	31	36	26
D	22	27	33	–	–	42	–
E	35	–	31	–	–	37	29
F	–	–	36	42	37	–	40
G	–	–	26	–	29	40	–

- (a) By adding the arcs from vertex D along with their weights, complete the drawing of the network on Diagram 1 in the answer book. (2)
- (b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree. (3)
- (c) State the weight of the minimum spanning tree. (1)

(Total for Question 1 is 6 marks)



SUB-TOPIC 2.2

SHORTEST PATHS: DIJKSTRA AND FLOYD

Sub-topic 2.2 · 1 question · 10 marks · lowest tariff first



3. The initial distance matrix (Table 1) shows the lengths, in metres, of the corridors connecting six classrooms, A, B, C, D, E and F, in a school. For safety reasons, some of the corridors are one-way only.

	A	B	C	D	E	F
A	–	12	32	24	29	11
B	12	–	17	8	∞	∞
C	32	17	–	4	12	∞
D	24	∞	4	–	∞	13
E	∞	∞	12	18	–	12
F	11	∞	∞	13	12	–

Table 1

- (a) By adding the arcs from vertex A, along with their weights, complete the drawing of this network on Diagram 1 in the answer book.

(2)

Floyd's algorithm is to be used to find the complete network of shortest distances between the six classrooms.

The distance matrix after **two** iterations of Floyd's algorithm is shown in Table 2.

	A	B	C	D	E	F
A	–	12	29	20	29	11
B	12	–	17	8	41	23
C	29	17	–	4	12	40
D	24	36	4	–	53	13
E	∞	∞	12	18	–	12
F	11	23	40	13	12	–

Table 2

- (b) Perform the next two iterations of Floyd's algorithm that follow from Table 2. You should show the distance matrix after each iteration.

(4)



The final distance matrix after completion of Floyd's algorithm is shown in Table 3.

	A	B	C	D	E	F
A	–	12	24	20	23	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	23	29	12	16	–	12
F	11	23	17	13	12	–

Table 3

Yinka must visit each classroom. He will start and finish at E and wishes to minimise the total distance travelled.

- (c) (i) Use the nearest neighbour algorithm, starting at E, to find two Hamiltonian cycles in the completed network of shortest distances.
- (ii) Find the length of each of the two cycles.
- (iii) State, with a reason, which of the two cycles provides the better upper bound for the length of Yinka's route.

(4)

(Total for Question 3 is 10 marks)



SUB-TOPIC 3.1

ROUTE INSPECTION

Sub-topic 3.1 · 2 questions · 26 marks · lowest tariff first



5.

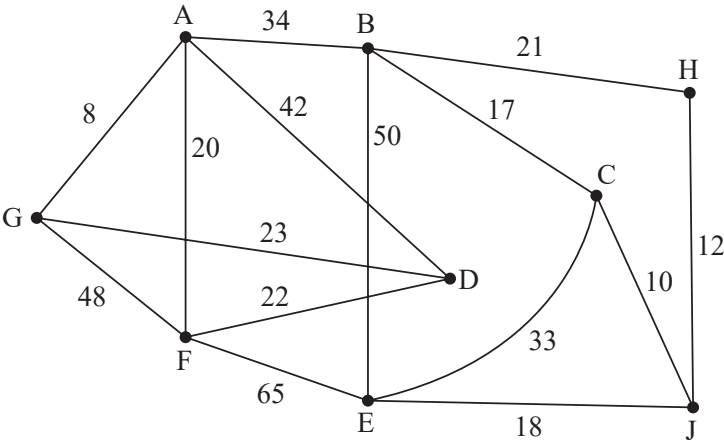


Figure 5

[The total weight of the network is 423]

Direct roads between nine towns, A, B, C, D, E, F, G, H and J, are represented in Figure 5. The number on each arc represents the length, in miles, of the corresponding road.

The table below shows the shortest distances, in miles, between the nine towns.

	A	B	C	D	E	F	G	H	J
A	–	34	51	31	79	20	8	55	61
B	34	–	17	65	45	54	42	21	27
C	51	17	–	82	28	71	59	22	10
D	31	65	82	–	87	22	23	86	92
E	79	45	28	87	–	65	87	30	18
F	20	54	71	22	65	–	28	75	81
G	8	42	59	23	87	28	–	63	69
H	55	21	22	86	30	75	63	–	12
J	61	27	10	92	18	81	69	12	–

Table of shortest distances



A route is needed that minimises the total distance required to traverse each road at least once.

The route must start at F and finish at J.

(a) (i) By considering the pairings of all relevant nodes, find the roads that would need to be traversed twice.

(ii) State the total length of this route.

(5)

(b) Starting at A, use Prim's algorithm to find the minimum spanning tree for the table of shortest distances. You must state the order in which you select the arcs of your tree.

(3)

Pete needs to visit all nine towns, starting and finishing in the same town, and wishes to minimise the total distance he travels.

(c) Starting at G, use the nearest neighbour algorithm on the table of shortest distances to find an upper bound for the length of Pete's route. Write down the route that gives this upper bound.

(2)

(d) By deleting G and all of its arcs, find a lower bound for the length of Pete's route.

(2)

Pete decides to take the route he found in (c).

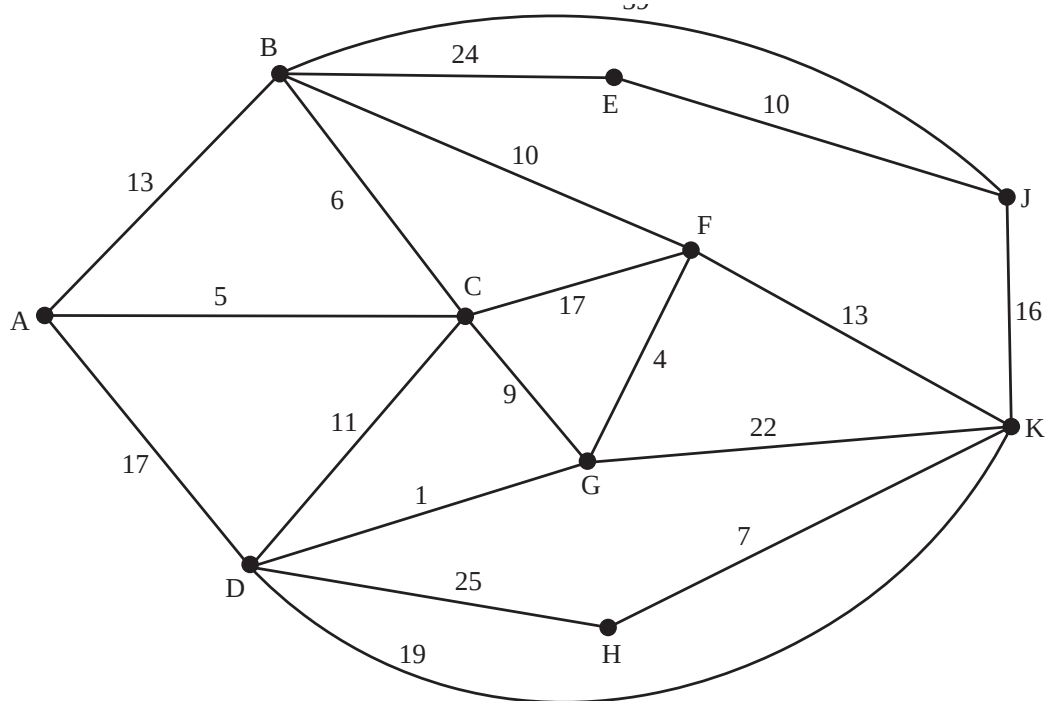
(e) Interpret the route in terms of the actual towns visited.

(1)

(Total for Question 5 is 13 marks)



4.

**Figure 2**

[The total weight of the network is 268]

Figure 2 represents a network of corridors between 10 offices, A, B, C, D, E, F, G, H, J and K, in a building. The number on each edge represents the length, in metres, of the corresponding corridor.

On Monday morning, Turvi needs to walk from J to K via A. She wishes to minimise the distance she travels along the corridors.

- (a) (i) Use Dijkstra's algorithm, starting at A, to find the shortest route from J to K via A.
 (ii) State the length of this route.

(6)

On Monday afternoon, Turvi needs a route that traverses each corridor at least once. She plans to start at J and finish at A and wants to minimise the distance travelled.

- (b) (i) By considering the pairings of all relevant nodes, find the corridors that would need to be traversed twice. You must make your method and working clear.
 (ii) State the total length of this route.

(5)

Turvi discovers that she cannot walk along corridors AB, CD and JK because they are being decorated. Turvi walks from J to A travelling along all the other corridors at least once. She does this in the minimum distance possible.

- (c) Determine the difference between the length of the route Turvi travels and the length of the route found in (b). You must make the values used in your calculation clear.

(2)**(Total for Question 4 is 13 marks)**

SUB-TOPIC 3.2

TRAVELLING SALESMAN PROBLEM

Sub-topic 3.2 · 1 question · 8 marks · lowest tariff first



3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

Table 1

Table 1 shows the shortest distances, in miles, between eight towns, A, B, C, D, E, F, G and H.

- (a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this table of distances. You must clearly state the order in which you select the edges of your tree. (3)
- (b) State the weight of the minimum spanning tree. (1)

	A	B	C	D	E	F	G	H
J	31	27	50	29	43	25	49	35

Table 2

Table 2 shows the distances, in miles, between town J and towns A, B, C, D, E, F, G and H.

Pranav needs to visit all of the towns, starting and finishing at J, and wishes to minimise the total distance he travels.

- (c) Starting at J, use the nearest neighbour algorithm to obtain an upper bound for the length of Pranav's route. You must state your route and its length. (2)
- (d) Starting by deleting J, and all of its edges, find a lower bound for the length of Pranav's route. (2)

(Total for Question 3 is 8 marks)



SUB-TOPIC 4.3

CRITICAL PATH AND EVENT TIMES

Sub-topic 4.3 · 2 questions · 18 marks · lowest tariff first



2.

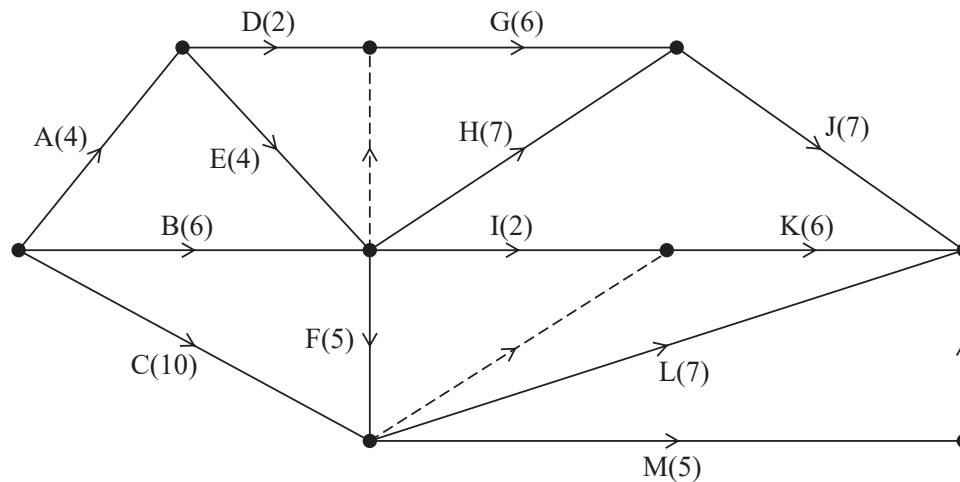


Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity.

- (a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

Each activity requires one worker and the project must be completed in the shortest possible time using as few workers as possible.

- (b) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.

(2)

- (c) Schedule the activities using Grid 1 in the answer book.

(3)

(Total for Question 2 is 9 marks)



4.

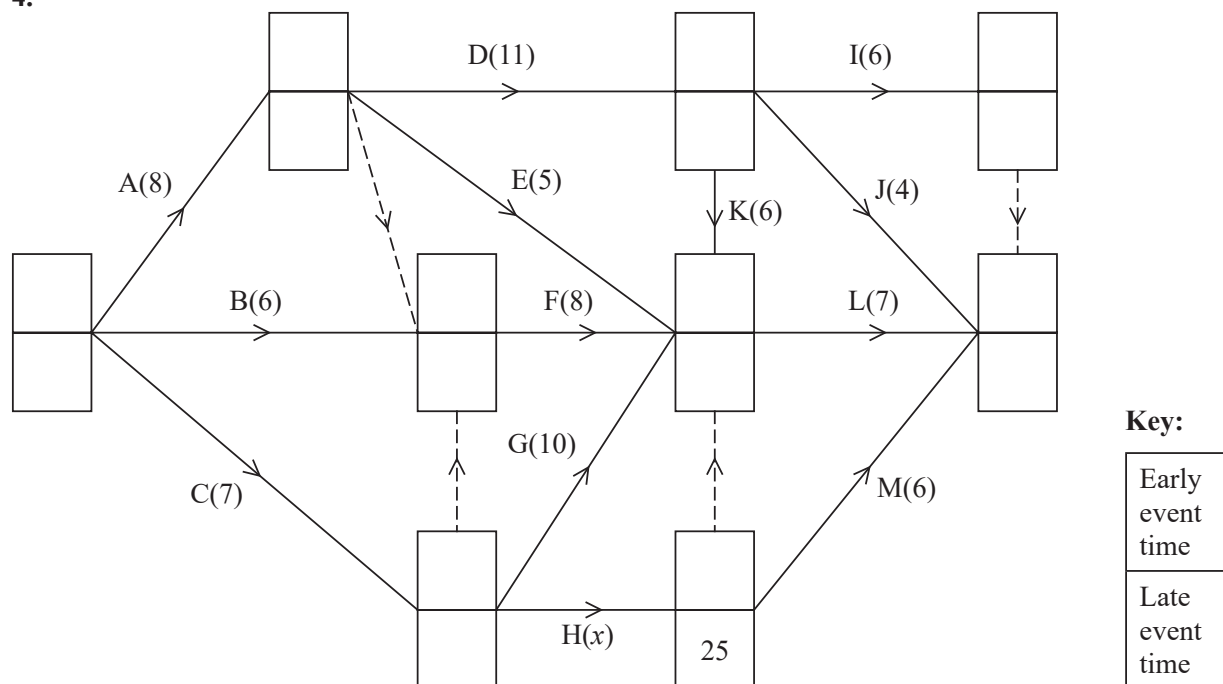


Figure 3

The network in Figure 3 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and one late event time has been completed for you.

The total float of activity H is 7 days.

- Explain, with detailed reasoning, why $x = 11$ (2)
 - Determine the missing early event times and late event times, and hence complete Diagram 1 in your answer book. (3)
- Each activity requires one worker and the project must be completed in the shortest possible time using as few workers as possible.
- Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. (1)
 - Schedule the activities using Grid 1 in the answer book. (3)

(Total for Question 4 is 9 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
1(a)	$\frac{4.3+6.1+\dots+1.3}{10} = \frac{37.5}{10} = 3.75$ so lower bound is four (balls of string)	M1 A1	1.1b 2.2a
		(2)	
(b)	Bin 1: 4.3 5.1 0.4 Bin 2: 6.1 2.5 1.3 Bin 3: 4.7 3.4 1.7 Bin 4: 5.9 2.1	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(5 marks)			
Notes for Question 1			
a1M1: Attempt to find the lower bound $(37.5 \pm 6.1)/10$ (a value of 3.75 seen with no working can imply this mark) a1A1: cso – a lower bound of 4 with either a correct calculation seen or 3.75 or ‘total is 37.5 and if each ball contains 10 this gives a lower bound of 4’. An answer of 4 with no working (or from part (b)) scores M0A0. Any incorrect working loses this mark e.g. a correct calculation followed by an incorrect value followed by 4 is A0 b1M1: First four items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the boxed values) b1A1: First eight items placed correctly (the boxed and bold values), and all eleven correct values only placed in bins (so no additional/repeated values) b2A1: cso (no additional/repeated values) Condone working in cm provided consistent No MR in this question – mark according to the scheme			

Question	Scheme	Marks	AOs
1(a)		M1 A1	1.1b 1.1b
		(2)	
(b)	Kruskal's algorithm: AD, AB, BC, CG, reject BD, EG, reject CE, reject CD, reject AE, CF (reject EF, reject FG, reject DF)	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)	Weight of MST: 162 (km)	B1	1.1b
		(1)	
(6 marks)			
Notes for Question 1			
(a) M1: Either all arcs correct (ignore weights) or two arcs correct (including correct weights) A1: CAO (b) M1: Kruskal's algorithm – first three arcs correctly chosen and at least one rejection seen at some point A1: All six arcs selected correctly AD, AB, BC, CG, EG, CF only A1: CSO – all selections and rejections correct (in correct order and at the correct time) (c) B1: CAO (condone lack of units)			

Question	Scheme	Marks	AOs																																																																																																		
3(a)		M1 A1	1.1b 1.1b																																																																																																		
		(2)																																																																																																			
(b)	Third iteration: <table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr><tr><td>A</td><td>-</td><td>12</td><td>29</td><td>20</td><td>29</td><td>11</td></tr><tr><td>B</td><td>12</td><td>-</td><td>17</td><td>8</td><td>29</td><td>23</td></tr><tr><td>C</td><td>29</td><td>17</td><td>-</td><td>4</td><td>12</td><td>40</td></tr><tr><td>D</td><td>24</td><td>21</td><td>4</td><td>-</td><td>16</td><td>13</td></tr><tr><td>E</td><td>41</td><td>29</td><td>12</td><td>16</td><td>-</td><td>12</td></tr><tr><td>F</td><td>11</td><td>23</td><td>40</td><td>13</td><td>12</td><td>-</td></tr></table> Fourth iteration: <table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr><tr><td>A</td><td>-</td><td>12</td><td>24</td><td>20</td><td>29</td><td>11</td></tr><tr><td>B</td><td>12</td><td>-</td><td>12</td><td>8</td><td>24</td><td>21</td></tr><tr><td>C</td><td>28</td><td>17</td><td>-</td><td>4</td><td>12</td><td>17</td></tr><tr><td>D</td><td>24</td><td>21</td><td>4</td><td>-</td><td>16</td><td>13</td></tr><tr><td>E</td><td>40</td><td>29</td><td>12</td><td>16</td><td>-</td><td>12</td></tr><tr><td>F</td><td>11</td><td>23</td><td>17</td><td>13</td><td>12</td><td>-</td></tr></table>		A	B	C	D	E	F	A	-	12	29	20	29	11	B	12	-	17	8	29	23	C	29	17	-	4	12	40	D	24	21	4	-	16	13	E	41	29	12	16	-	12	F	11	23	40	13	12	-		A	B	C	D	E	F	A	-	12	24	20	29	11	B	12	-	12	8	24	21	C	28	17	-	4	12	17	D	24	21	4	-	16	13	E	40	29	12	16	-	12	F	11	23	17	13	12	-	M1 A1 M1 A1	1.1b 1.1b 1.1b 1.1b
	A	B	C	D	E	F																																																																																															
A	-	12	29	20	29	11																																																																																															
B	12	-	17	8	29	23																																																																																															
C	29	17	-	4	12	40																																																																																															
D	24	21	4	-	16	13																																																																																															
E	41	29	12	16	-	12																																																																																															
F	11	23	40	13	12	-																																																																																															
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A	-	12	24	20	29	11																																																																																															
B	12	-	12	8	24	21																																																																																															
C	28	17	-	4	12	17																																																																																															
D	24	21	4	-	16	13																																																																																															
E	40	29	12	16	-	12																																																																																															
F	11	23	17	13	12	-																																																																																															
		(4)																																																																																																			
(c)	Nearest neighbour routes: E – C – D – F – A – B – E Length = 76 E – F – A – B – D – C – E Length = 59 The cycle that begins E – F – ... is the better upper bound as the value is the smaller of the two	M1 A1 A1 A1	3.4 1.1b 1.1b 2.4																																																																																																		
		(4)																																																																																																			
(10 marks)																																																																																																					

Notes for Question 3

a1M1: Correct five arcs from A

a1A1: cao no additional arcs or arrows, correct weights and arc AE directed from A to E

In the distance matrix for part (b) ignore whatever is written in the lead diagonal (top left to bottom right) and just consider the values (ignore shading etc.) – check bottom of page 9 for replaced matrices

b1M1: No change in the third row and third column with at least two values reduced correctly (no blank entries – apart from the lead diagonal)

b1A1: cao for the third iteration

b2M1: No change in their fourth row and fourth column (following their third iteration) with at least two values correctly reduced follow through from their previous iteration (no blank entries – apart from the lead diagonal)

b2A1: cso for both iterations (no follow through from an incorrect third iteration even if the fourth iteration is ‘correct’ so M1A0M1A1 is not possible in this part)

c1M1: Either cycle correct (must include returning to E) – nodes must be in the correct order and therefore not reversed but allow if stated in terms of arcs. If only one cycle correct then M1 only

c1A1: Both cycles correct – nodes must be in the correct order but allow if stated in terms of arcs

c2A1: Both cycles and corresponding lengths correct

c3A1: Correct reasoning that the smallest (oe) value is the better upper bound – dependent on all previous marks in this part (cso)



Question	Scheme	Marks	AOs
5.(a)	(i) Route must start at F and finish at J therefore need to consider pairings of the nodes F, C, D and G	M1	3.1b
	$CD + FG = 82 + 28 = 110$	A1	1.1b
	$CF + DG = 71 + 23 = 94$	A1	1.1b
	$CG + DF = 59 + 22 = 81$		
	Repeated roads: CB, BA, AG, and DF	A1	2.2a
	(ii) Length of route = $423 + 81 = 504$ (miles)	A1ft	2.2a
		(5)	
(b)	Prim's starting at A: AG, AF, DF, AB, BC, CJ, HJ, EJ	M1	1.1b
		A1	1.1b
		A1	1.1b
		(3)	
(c)	NNA starting at G: $G - A - F - D - B - C - J - H - E - G$ $8 + 20 + 22 + 65 + 17 + 10 + 12 + 30 + 87 = 271$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$(141 - 8) + 8 + 23 = 164$ (miles)	M1	3.1b
		A1	2.2a
		(2)	
(e)	GAFD GA BCJH J E JCBA G	B1	3.2a
		(1)	
(13 marks)			

Notes for Question 5

(a)**M1:** The correct three pairings of the correct four nodes (F, C, D and G)**A1:** Two rows correct including pairings **and** totals**A1:** All three rows correct including pairings **and** totals**A1:** CAO (CB, BA, AG, DF only but in any order) – must be stated as edges so **A0** for any of CG or CBAG or CG via B and A**A1ft:** 423 + weight of their shortest pairing**(b)****M1:** First three arcs correctly chosen in order {AG, AF, DF, ...} or first four nodes correctly chosen in order {A, G, F, D, ...}. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of one of the tables {1, -, -, 4, -, 3, 2, -, -} so please check the top of the tables carefully**A1:** First five arcs correctly chosen in order {AG, AF, DF, AB, BC, ...} or all nine nodes correctly chosen in order {A, G, F, D, B, C, J, H, E}. Order of nodes may be seen at the top of one of the tables so for the first two marks accept {1, 5, 6, 4, 9, 3, 2, 8, 7} (do not condone any missing numbers e.g. the number 9 must be above E)**A1:** CSO – all arcs correct stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of one of the tables unless the correct list of arcs (in the correct order) is also seen)**(c)****M1:** Nearest neighbour route starting at G – must have at least G – A – F – D – B – C – ... allow if stated in terms of arcs**A1:** CAO on length (271) **and** route (must return to G and can be stated in terms of arcs)**(d)****M1:** For one of the following

- RMST arcs: AF, DF, AB, BC, CJ, HJ, EJ (stated in any order) – condone AG added too but must be removed at some later stage (e.g. subtracting 8 at some point)
- RMST weight calculation: $20 + 22 + 34 + 17 + 10 + 12 + 18$ (not just 20, 22, etc.)
- RMST weight stated: 133 or $141 - 8$
- Lower bound stated: 164

A1: CAO (164)**(e)****B1:** CAO (GAFD **GA** BCJH **J E** JCBA G or in terms of arcs)

Question	Scheme	Marks	AOs
4(a)	(i) Route from J to K via A: JEBACAGFK (ii) Length of shortest route from J to K via A is 76 (m)	M1 A1 (ACBG D) A1 (FKE) A1ft (HJ) A1 A1ft	1.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(b)(i)	$BC + D(GF)K = 6 + 18 = 24$ $B(FG)D + C(GF)K = 15 + 26 = 41$ $B(F)K + C(G)D = 23 + 10 = 33$ Edges to traverse twice are BC, DG, FG, FK	M1 A1 A1 A1	3.1b 1.1b 1.1b 2.2a
(ii)	Length of route = $268 + 24 = 292$ (m)	A1ft	2.2a
		(5)	
(c)	E.g. $[24 -] (45 - 13 - 11 - 16)$ Or $[292 -] (268 + 45 - (13 + 11 + 16))$ = 19 (m)	M1 A1	3.1b 2.2a
		(2)	
(13 marks)			

Notes:

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 22 21 18 in that order (so 22 18 21 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4,... will be penalised once (see notes below) but 1, 2, 3, 5, 6,... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

If starting at J or K instead of A M0

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either B or D or F or H or J or K

a1A1: All values at A, C, B, G and D correct and working values in the correct order. Condone lack of 0 in As working value

a2A1: All values at F, K and E correct and the working values in the correct order. Penalise order of labelling only once per question (F, K and E must be labelled in that order and F must be labelled after A, C, B, G and D)

a3A1ft: All values at H and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working values at H follow from the candidate's final values from the feeds into H (which will come from nodes D and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from B, E and K with the order of labelling determined by the candidate's order of labelled at B, E and K)

a4A1: CAO (correct path from J to K via A – JEBCACGFK) (accept in terms of arcs)

a5A1ft: ft their final value at J + their final value at K only (if 76 stated and 76 is not the sum of these final values then A0)

If route and length are written on the wrong lines this can still score both marks

bi1M1: The correct three pairings of the correct four nodes (B, C, D and K)

bi1A1: Two rows correct including pairings and totals

bi2A1: All three rows correct including pairings and totals

bi3A1: Correct repeats (BC, DG, FG, FK)

bii4A1ft: Length of route correct on the follow through (268 + smallest repeat) but must be from a choice of 3 totals

c1M1: For using their final value at J and the blocked lengths in attempt to work out the length or change in length of the new route (sight of "45" 13 11 and 16) (follow through their final value at J for this mark) (condone use of 40 for 13 + 11 + 16)

c1A1: CAO (19)



Question	Scheme	Marks	AOs
3(a)	Prim's starting at A: AH, AB, DH; CD, CE; DG, EF	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(b)	Weight of MST is 183 (miles)	B1	2.2a
		(1)	
(c)	NNA: J – F – E – C – D – H – A – B – G – J	B1	1.1b
	Upper bound is 267 (miles)	B1	2.2a
		(2)	
(d)	$183 + 27 + 25 = \dots$	M1	3.1b
	$\dots = 235$ (miles)	A1	2.2a
		(2)	
(8 marks)			
Notes: (a) M1: First three arcs correctly chosen in order {AH, AB, DH, ...} or first four nodes correctly chosen in order {A, H, B, D, ...}. If any rejections seen at any point then M1 (max) only. Order of nodes may be seen at the top of the matrix {1, 3, -, 4, -, -, -, 2} so please check the top of the matrix carefully A1: First five arcs correctly chosen in order {AH, AB, DH, CD, CE, ...} or all eight nodes correctly chosen in order {A, H, B, D, C, E, G, F}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {1, 3, 5, 4, 6, 8, 7, 2} (do not condone any missing numbers e.g. the number 8 must be above F) A1: cso – all arcs correct stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) (b) B1: cao (183) (c) B1: cao (for route – must return to J) B1: cao (for upper bound of 267) (d) M1: Their answer to (b) + 27 + 25 (the two smallest arcs incident to J) A1: cao (235)			

Question	Scheme	Marks	AOs
2(a)		M1 A1 M1 A1	2.1 1.1b 1.1b 1.1b
		(4)	
(b)	$\frac{71}{22} = \dots$	M1	1.1b
	$\dots = 3.22\dots$ therefore 4 workers	A1	2.2a
		(2)	
(c)	e.g.	M1 A1 A1	2.1 1.1b 1.1b
		(3)	
(9 marks)			
Notes: (a) M1: All top boxes completed, number generally increasing L to R (condone one “rogue”) A1: cao - top boxes (including zero at the source node) M1: All bottom boxes completed, numbers generally decreasing R to L (condone one “rogue”) A1: cao - bottom boxes (including zero at the sink node)			

(b)

M1: Attempt to find the lower bound $(71 \pm 10) /$ their completion time (a value of 3.2... seen with no working can imply this mark)

A1: cso - correct calculation seen or 3.2 followed by 4. An answer of 4 with no working scores

M0A0

(c)

M1: Not a cascade chart, 4 'workers' used at most and at least 9 different activities placed

A1: 4 workers. All 13 activities present (just once – so if an activity appears for two different workers and is happening at the same time this is A0). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

A1: 4 workers. All 13 activities present (just once). No errors

Activity	Duration	Time Interval	IPA
A	4	0 – 4	-
B	6	0 – 8	-
C	10	0 – 15	-
D	2	4 – 9	A
E	4	4 – 8	A
F	5	8 – 15	B, E
G	6	8 – 15	B, D, E
H	7	8 – 15	B, E
I	2	8 – 16	B, E
J	7	15 – 22	G, H
K	6	13 – 22	C, F, I
L	7	13 – 22	C, F
M	5	13 – 22	C, F



Qu	Scheme	Marks	AOs
4(a)	The early event time at the end of activity C is 7 (as no other activity leads into this event). Therefore the float on activity H is $25 - 7 - x$	B1	3.1a
	The float on activity H is given as 7 and so therefore $25 - 7 - x = 7$ implies that the value of x is equal to $25 - 7 - 7 = 11$	dB1	2.4
		(2)	
(b)		M1 A1 A1	2.1 1.1b 1.1b
		(3)	
(c)	$\frac{95}{32} = 2.968... = 3$ workers	B1	2.2a
		(1)	
(d)	e.g.	M1 A1 A1	2.1 1.1b 1.1b
		(3)	
(9 marks)			

Notes for Question 4

(a) B1: correct reasoning for why the float on activity H is given by $25 - 7 - x$, must mention that the early event time at the end of activity C is 7 or the early event time at the start of H is 7 **and** that the total float for H is therefore $25 - 7 - x$ (or $25 - x - 7$ but not just $18 - x$) (no reason for why the early event time at the end of C is 7 is required)

dB1: correct explanation for why $x = 11$ (dependent on previous B mark) – as a minimum must equate $25 - 7 - x$ to 7 (allow $18 - x = 7$ as they must have shown where the 18 comes from to get the first B mark) and hence $x = 11$

SC B1B0 – for those who write or imply $25 - 7 - x = 7$ (but not just $18 - x = 7$) and state $x = 11$ without any mention of the early event time at the end of C or the total float of activity H. However, $25 - 7 - 7 = 11$ only is no marks in this part

(b) M1: All top boxes and all bottom boxes completed. Values generally increasing left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0s at the source node or the 32 in the bottom box at the sink node for M only. Condone one rogue value in top boxes and one rogue value in bottom boxes. For a rogue in the top boxes if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value (with a similar definition for bottom boxes but in reverse)

A1: CAO - Top boxes (including zero at the source node)

A1: CAO - Bottom boxes (including zero at the sink node)

(c) B1: Correct calculation seen then 3 – an answer of 3 with no working scores B0

(d) M1: Not a cascade chart. 4 ‘workers’ used at most and at least 10 different activities placed

A1: 4 workers. All 13 activities present (just once – so if an activity appears for two different workers and is happening at the same time this is A0). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

A1: 4 workers. All 13 activities present (just once). No errors

Activity	Duration	Time interval	IPA
A	8	0 - 8	-
B	6	0 - 17	-
C	7	0 - 14	-
D	11	8 - 19	A
E	5	8 - 25	A
F	8	8 - 25	A, B, C
G	10	7 - 25	C
H	11	7 - 25	C
I	6	19 - 32	D
J	4	19 - 32	D
K	6	19 - 25	D
L	7	25 - 32	E, F, G, H, K
M	6	18 - 32	H

