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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Linear Programming (5)

Decision Mathematics 1 · Topic 5

12 questions · 148 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

12 questions · 148 marks · 5 sub-topics



SUB-TOPIC 2.2

SHORTEST PATHS: DIJKSTRA AND FLOYD

Sub-topic 2.2 · 1 question · 10 marks · lowest tariff first



6.

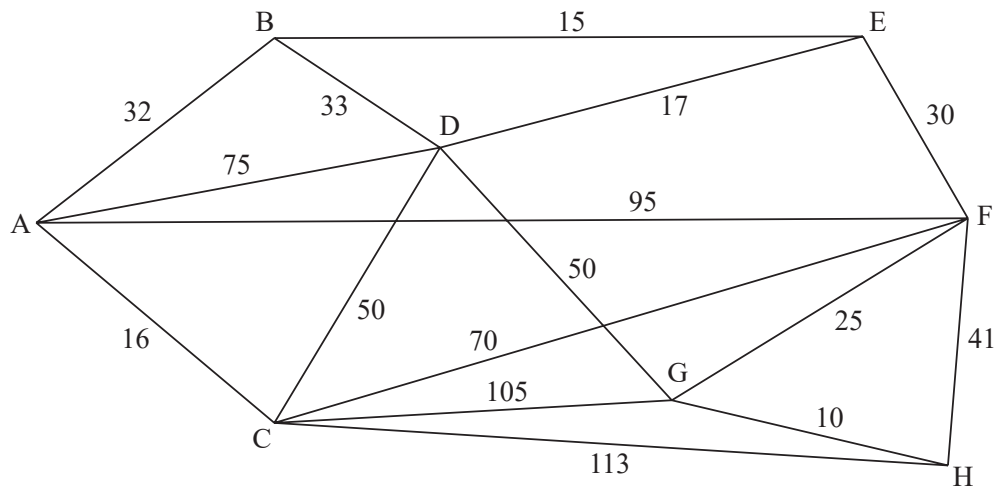


Figure 4

In Figure 4 the weights on the arcs represent distances.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to H.

(ii) State the length of the shortest path from A to H.

(6)

One application of Dijkstra's algorithm has order n^2 , where n is the number of nodes in the network. A computer produces a table of shortest distances between any two different nodes by repeatedly applying Dijkstra's algorithm from each node of the network.

It takes the computer 0.082 seconds to produce a table of shortest distances for a network of 10 nodes.

(b) Calculate approximately how long it will take, in seconds, for the computer to produce a table of shortest distances for a network with 200 nodes. You must give a reason for your answer.

(3)

(c) Explain why your answer to part (b) can only be an approximation.

(1)

(Total for Question 6 is 10 marks)



7.

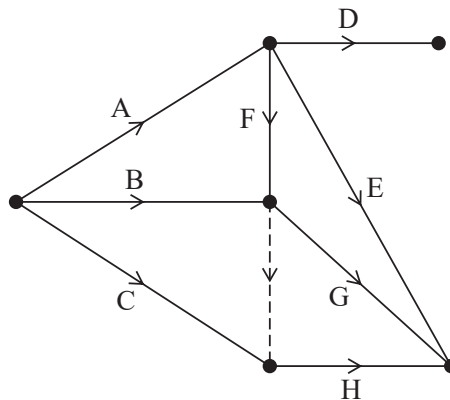


Figure 5

Figure 5 shows a partially completed activity network for a project that consists of 14 activities.

(a) Complete the precedence table in the answer book for the 8 activities in Figure 5.

(2)

The precedence table for the remaining 6 activities is given below.

Activity	Immediately preceding activities
I	D, E, G, H
J	D, E, G, H
K	E, G, H
L	I, J, K
M	J, K
N	J, K

(b) Complete the activity network in the answer book for the project. Your completed activity network must contain only the minimum number of dummies.

(4)

Given that all 14 activities have the same duration,

(c) explain why activity D cannot be critical.

(2)

(Total for Question 7 is 8 marks)



8. Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- x represent the number of hours swimming
- y represent the number of hours cycling
- z represent the number of hours running

- (a) Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(5)

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

- (b) Set up an initial tableau for solving this problem using the two-stage Simplex method.

As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6)

The following tableau T is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	-2	1	-5	62
x	1	0	1	0	0	-1	28
P	0	0	-1	1	0	1	11

- (c) Obtain a suitable pivot for a second iteration. You must give reasons for your answer.

(2)

- (d) Starting from tableau T , solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use.

(5)

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS



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Time 1 hour 30 minutes**Paper
reference****9FM0/3D****Further Mathematics****Advanced****PAPER 3D: Decision Mathematics 1****Answer book**

Do not return the question paper with the answer book.

Total Marks



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1.

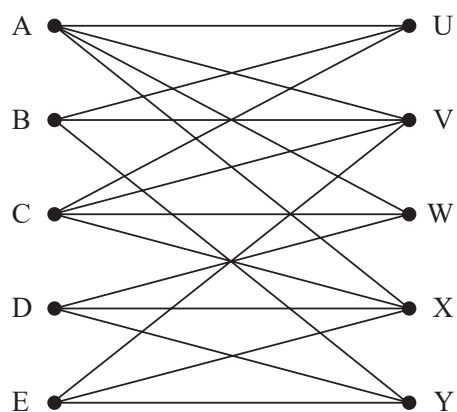


Figure 1



Question 1 continued

(Total for Question 1 is 4 marks)



2.

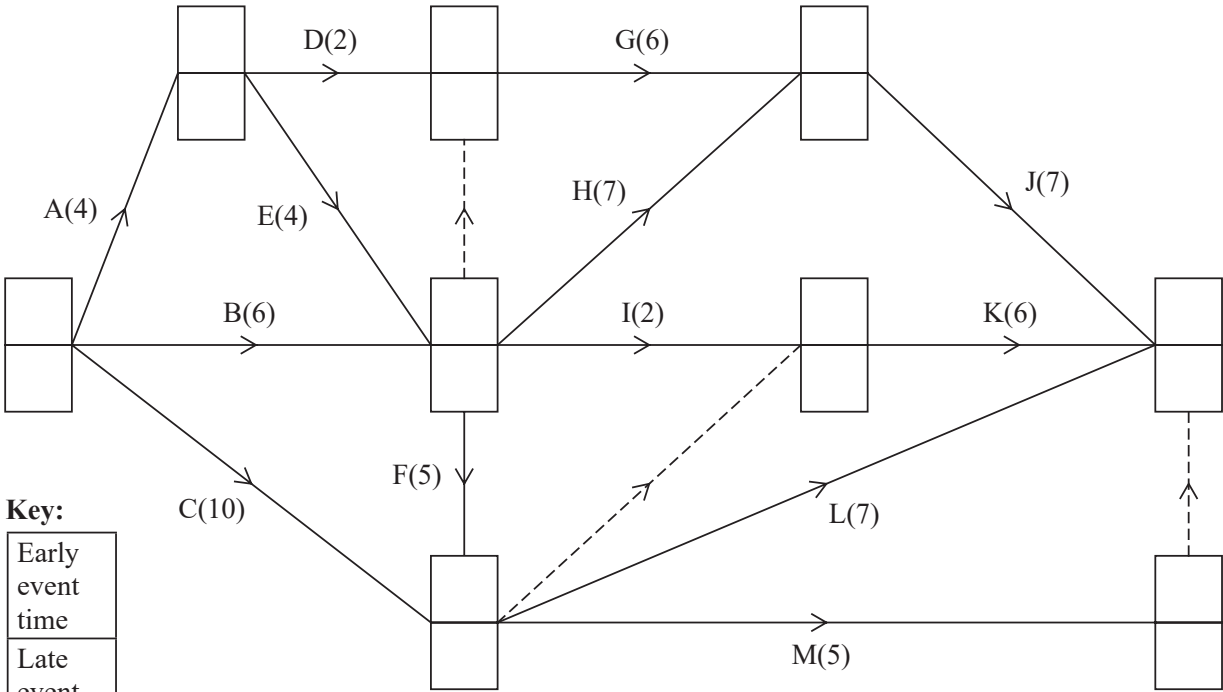
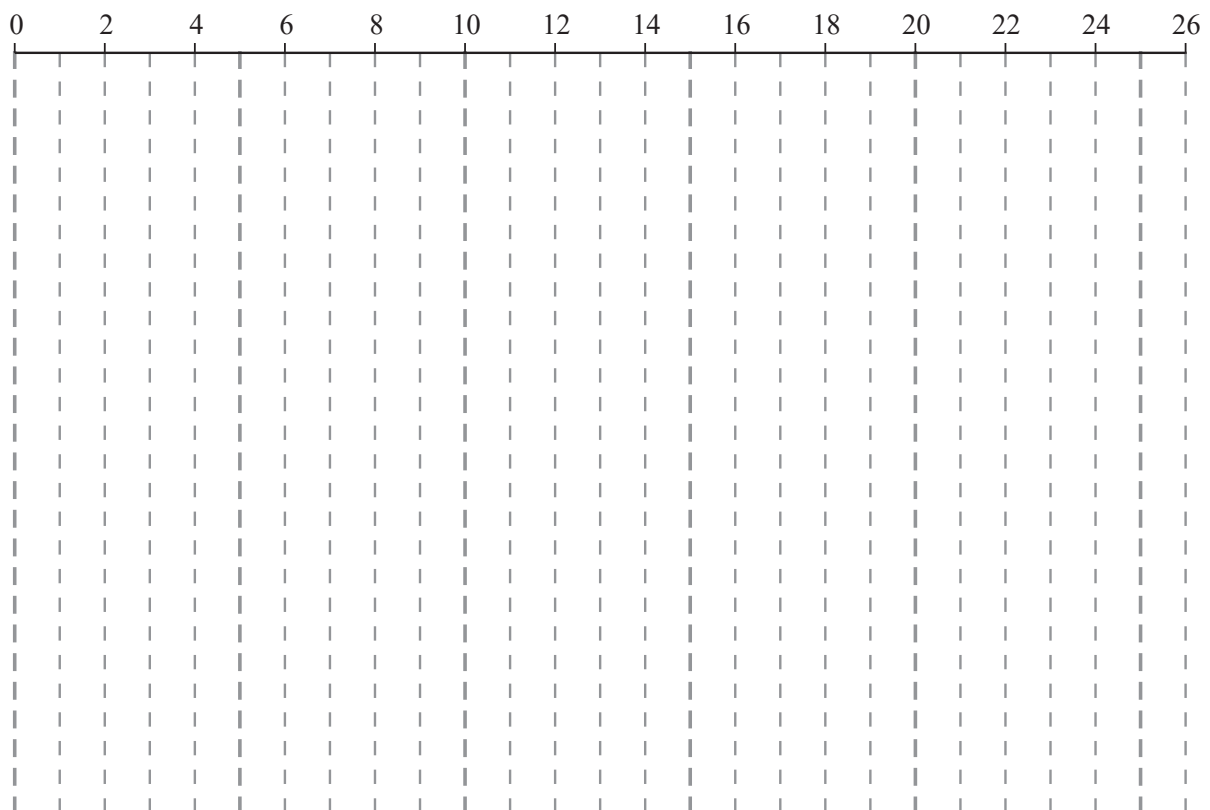


Diagram 1

Blank area for student response.



Question 2 continued

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Grid 1

(Total for Question 2 is 9 marks)

3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–



Question 3 continued

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

(Total for Question 3 is 8 marks)



4. (a)

Initial time matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Initial route matrix

	A	B	C	D	E	F
A	A	B	C	D	E	F
B	A	B	C	D	E	F
C	A	B	C	D	E	F
D	A	B	C	D	E	F
E	A	B	C	D	E	F
F	A	B	C	D	E	F

(b) 1st iteration

Time matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Route matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						



Question 4 continued

(Total for Question 4 is 8 marks)



5.

30

12

5

2

23

18

36

10

15

24



Question 5 continued

(Total for Question 5 is 10 marks)



Question 6 continued

(Total for Question 6 is 10 marks)



SUB-TOPIC 5.1

FORMULATING LP PROBLEMS

Sub-topic 5.1 · 3 questions · 40 marks · lowest tariff first



5. Two friends, Anaira and Tommi, play a game involving two positive numbers x and y

Anaira gives Tommi the following clues to see if he can correctly determine the value of x and the value of y

- x is greater than y and the difference between the two is at least 100
- x is at most 5 times as large as y
- the sum of $2x$ and $3y$ is at least 350
- the sum of x and y is as small as possible

Tommi decides to solve this problem by using the big-M method.

(a) Set up an initial tableau for solving this problem using the big-M method.

As part of your solution, you must show

- how the constraints were made into equations using one slack variable, exactly two surplus variables and exactly two artificial variables
- how the objective function was formed

(6)

The big-M method is applied until the tableau containing the optimal solution to the problem is found. One row of this final tableau is as follows.

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value
x	1	0	$-\frac{3}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	130

(b) (i) State the value of x

(ii) Hence deduce the value of y , making your reasoning clear.

(3)

(Total for Question 5 is 9 marks)

7. A shop sells two types of watch, analogue watches and digital watches.

The shop manager knows that, each month, she should order at least 60 watches in total. In addition, at most 80% of the watches she orders must be digital.

Let x be the number of analogue watches ordered and let y be the number of digital watches ordered.

- (a) Write down inequalities, in terms of x and y , to model these constraints.

(2)

Two further constraints are

$$y + 3x \geq 140$$

$$4y + x \geq 80$$

- (b) Represent all these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R .

(4)

The cost to the shop of ordering an analogue watch is five times the cost of ordering a digital watch. The shop manager wishes to minimise the total cost.

- (c) Determine the number of each type of watch the shop manager should order. You must make your method clear.

(3)

Given that the minimum total cost of ordering the watches is £4455

- (d) determine the cost of ordering one analogue watch and the cost of ordering one digital watch. You must make your method clear.

(3)

(Total for Question 7 is 12 marks)



7. A publisher plans to produce three versions of the same book: a paperback, a hardcover, and a deluxe edition.

- Each paperback takes 4 minutes to print and 1 minute to bind
- Each hardcover takes 8 minutes to print and 5 minutes to bind
- Each deluxe edition takes 15 minutes to print and 12 minutes to bind

The printing machine is available for at most 150 hours and the binding machine must be used for at least 60 hours.

The publisher decides to produce

- at least 1600 books in total
- at least three times as many paperbacks as hardcovers

The profit on each paperback sold is £8, the profit on each hardcover sold is £20 and the profit on each deluxe edition sold is £40

Let x , y and z represent the number of paperbacks, hardcovers and deluxe editions produced.

- (a) Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(5)

The publisher decides to solve this linear programming problem by using the two-stage simplex method.

- (b) Set up an initial tableau for solving this problem using the two-stage simplex method.

As part of your solution, you must show how

- the constraints have been made into equations by using slack variables, exactly two surplus variables and exactly two artificial variables
- the rows for the two objective functions are formed

(6)



The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

b.v.	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	0	0	0	1	1	3	0	-1	-3	600
z	0	$\frac{4}{11}$	1	0	$-\frac{1}{11}$	$\frac{1}{11}$	0	$\frac{1}{11}$	$-\frac{1}{11}$	$\frac{2000}{11}$
x	1	$\frac{7}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	0	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
s_4	0	$\frac{40}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	1	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
P	0	$-\frac{4}{11}$	0	0	$-\frac{32}{11}$	$-\frac{56}{11}$	0	$\frac{32}{11}$	$\frac{56}{11}$	$\frac{204800}{11}$
I	0	0	0	0	0	0	0	1	1	0

- (c) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method to obtain a new tableau. Make your method clear by stating the row operations you use.

(5)



After three iterations of the second stage of the two-stage simplex method, the following tableau is obtained.

b.v.	x	y	z	s_1	s_2	s_3	s_4	Value
s_2	0	0	0	1	1	3	0	600
z	0	0	1	$\frac{1}{10}$	0	$\frac{1}{2}$	$-\frac{1}{10}$	100
x	1	0	0	$-\frac{3}{40}$	0	$-\frac{9}{8}$	$-\frac{7}{40}$	1125
y	0	1	0	$-\frac{1}{40}$	0	$-\frac{3}{8}$	$\frac{11}{40}$	375
P	0	0	0	$\frac{29}{10}$	0	$\frac{7}{2}$	$\frac{1}{10}$	20500

Given that the publisher produces the optimal number of each version of the book,

(d) (i) state the maximum profit the publisher can earn,

(ii) find the number of hours the binding machine will be used.

(2)

(e) Give a reason why the publisher may not earn the profit stated in (d)(i).

(1)

(Total for Question 7 is 19 marks)



SUB-TOPIC 5.2

GRAPHICAL SOLUTIONS

Sub-topic 5.2 · 2 questions · 25 marks · lowest tariff first



4.

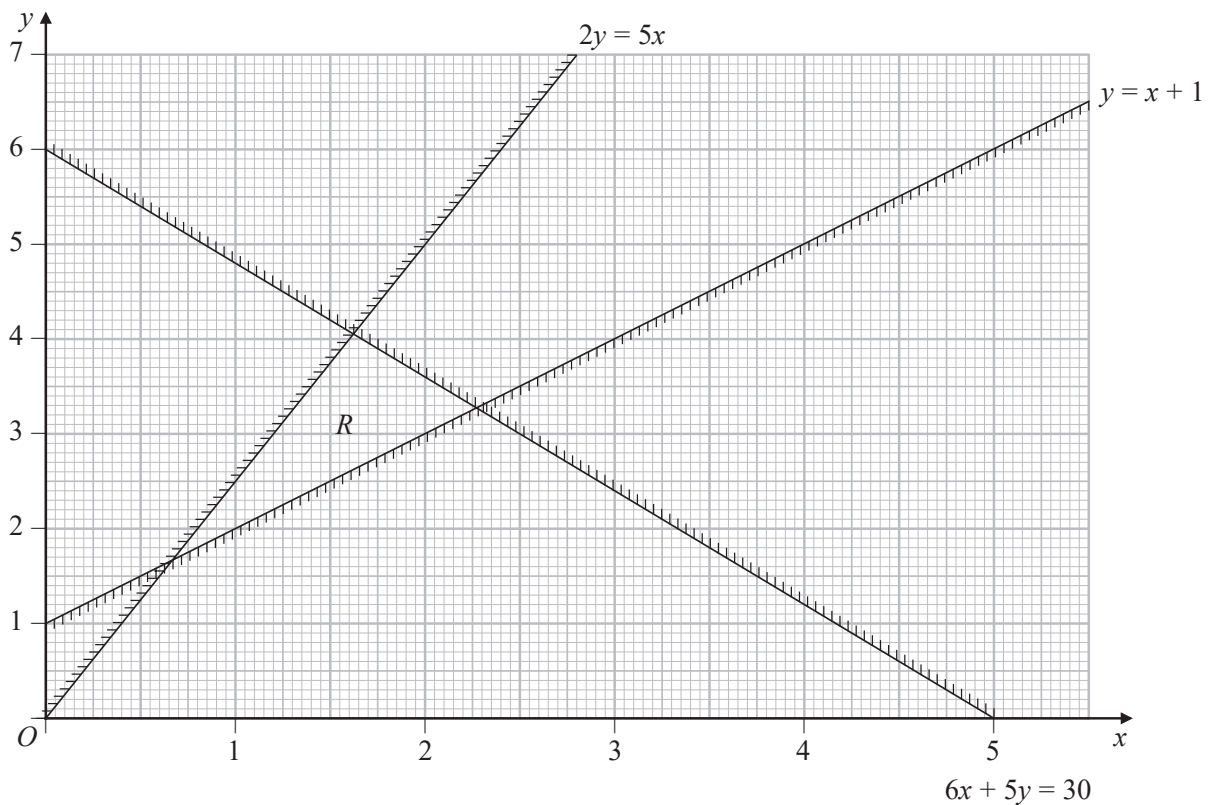


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y , where R is the feasible region.

(a) Write down the inequalities that define R .

(2)

The objective is to maximise P , where $P = 3x + y$

(b) Obtain the exact value of P at each of the three vertices of R and hence find the optimal vertex, V .

(4)

The objective is changed to maximise Q , where $Q = 3x + ay$. Given that a is a constant and the optimal vertex is still V ,

(c) find the range of possible values of a .

(4)

(Total for Question 4 is 10 marks)



7.

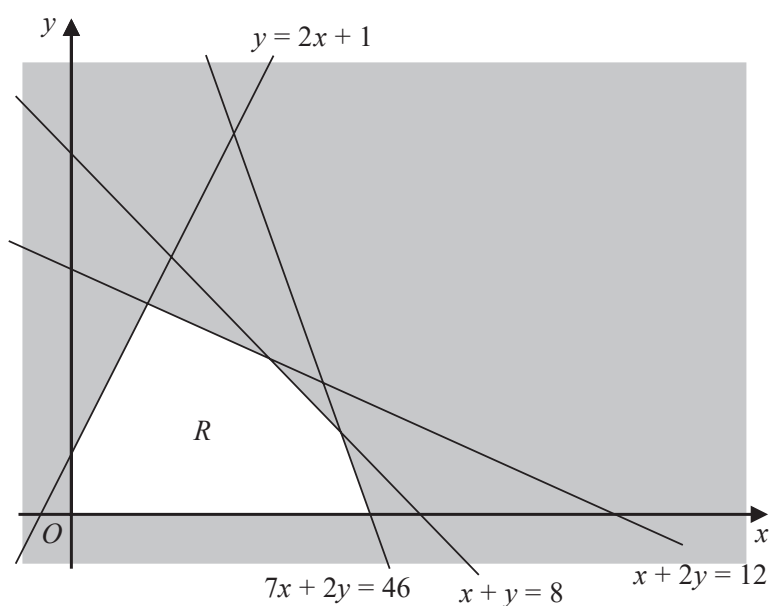
**Figure 5**

Figure 5 shows the constraints of a linear programming problem in x and y where R is the feasible region.

The objective is to maximise $P = x + ky$, where k is a positive constant.

The optimal vertex of R is to be found using the Simplex algorithm.

- (a) Set up an initial tableau for solving this linear programming problem using the Simplex algorithm.

(5)

After two iterations of the Simplex algorithm a possible tableau T is

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5
P	0	0	0	$\frac{1}{5} + \frac{2}{5}k$	0	$-\frac{2}{5} + \frac{1}{5}k$	$5k + 2$

(b) State the value of each variable after the second iteration.

(1)

It is given that T does not give an optimal solution to the linear programming problem.

After a third iteration of the Simplex algorithm the resulting tableau does give an optimal solution to the problem.

(c) Perform the third iteration of the Simplex algorithm and hence determine the range of possible values for P . You should state the row operations you use and make your method and working clear.

(9)

(Total for Question 7 is 15 marks)



SUB-TOPIC 5.3

THE SIMPLEX ALGORITHM

Sub-topic 5.3 · 3 questions · 38 marks · lowest tariff first



6. A linear programming problem in x , y and z is described as follows.

Maximise $P = 2x + 2y - z$

subject to $3x + y + 2z \leq 30$

$$x - y + z \geq 8$$

$$4y + 2z \geq 15$$

$$x, y, z \geq 0$$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

(7)

After a first iteration of the big-M method, the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	0	1.5	1	0	0.25	0	-0.25	26.25
a_1	1	0	1.5	0	-1	-0.25	1	0.25	11.75
y	0	1	0.5	0	0	-0.25	0	0.25	3.75
P	$-(2 + M)$	0	$2 - 1.5M$	0	M	$-0.5 + 0.25M$	0	$0.5 + 0.75M$	$7.5 - 11.75M$

(c) State the value of each variable after the first iteration.

(1)

(d) Explain why the solution given by the first iteration is not feasible.

(1)

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

(2)

(Total for Question 6 is 12 marks)



7. A maximisation linear programming problem in x , y and z is to be solved using the Simplex method.

The tableau after the 1st iteration is shown below.

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	0	$-\frac{1}{2}$	$\frac{3}{2}$	1	$-\frac{1}{2}$	0	30
x	1	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	10
s_3	0	1	1	0	0	1	26
P	0	$-\frac{1}{4}$	$-\frac{11}{4}$	0	$\frac{3}{4}$	0	30

- (a) State the column that contains the pivot value for the 1st iteration. You must give a reason for your answer. (2)
- (b) By considering the equations represented in the above tableau, formulate the linear programming problem in x , y and z only. State the objective and list the constraints as inequalities with integer coefficients. (5)
- (c) Taking the most negative number in the profit row to indicate the pivot column, perform the 2nd iteration of the Simplex algorithm, to obtain a new tableau, T. Make your method clear by stating the row operations you use. (4)
- (d) (i) Explain, using T, how you know that an optimal solution to the original linear programming problem has **not** been found after the 2nd iteration. (2)
- (ii) State the values of the basic variables after the 2nd iteration.



A student attempts the 3rd iteration of the Simplex algorithm and obtains the tableau below.

b.v.	x	y	z	s_1	s_2	s_3	Value
z	0	0	1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{43}{2}$
x	1	0	0	$\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{57}{4}$
y	0	1	0	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{2}$
P	0	1	0	$\frac{5}{4}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{361}{4}$

(e) Explain how you know that the student's attempt at the 3rd iteration is **not** correct.

(1)

(Total for Question 7 is 14 marks)



5. Consider the following linear programming problem in x , y and z .

$$\text{Maximise } P = 3x + 4y + 2z$$

$$\text{subject to } 2x + 2y + 2z \leq 21$$

$$2x - y - z \leq 18$$

$$-3x + y - 2z \leq 1$$

$$x \geq 4 \quad y \geq 1 \quad z \leq -3$$

- (a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem in its current form.

(1)

- (b) Use the substitutions $x = X + 4$, $y = Y + 1$, $z = -Z - 3$ and $P = Q + 10$ to reformulate the linear programming problem into a form in which the Simplex algorithm can be used to solve the problem. You should write the constraints as simplified inequalities with integer coefficients.

(3)

After a second iteration of the Simplex algorithm, a possible tableau T is

b.v.	X	Y	Z	s_1	s_2	s_3	Value
X	1	0	$-\frac{3}{4}$	$\frac{1}{8}$	0	$-\frac{1}{4}$	$\frac{5}{8}$
s_2	0	0	$\frac{9}{4}$	$\frac{1}{8}$	1	$\frac{3}{4}$	$\frac{117}{8}$
Y	0	1	$-\frac{1}{4}$	$\frac{3}{8}$	0	$\frac{1}{4}$	$\frac{63}{8}$
Q	0	0	$-\frac{5}{4}$	$\frac{15}{8}$	0	$\frac{1}{4}$	$\frac{267}{8}$

- (c) Write down the profit equation given by T .

(1)

- (d) Using your answer to (c), explain why T is not optimal.

(1)

- (e) (i) Perform the third iteration of the Simplex algorithm. You should state the row operations you use.

- (ii) Hence determine the maximum value of P , and the corresponding values of x , y and z .

(6)

(Total for Question 5 is 12 marks)



SUB-TOPIC 5.4

TWO-STAGE SIMPLEX AND BIG-M

Sub-topic 5.4 · 3 questions · 35 marks · lowest tariff first



4. A linear programming problem in x , y and z is to be solved using the big-M method. The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	-2	2	0	-1	0	1	0	8
a_2	3	0	-4	0	0	-1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

- (a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

- (b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total for Question 4 is 6 marks)



7. A maximisation linear programming problem in x , y and z is to be solved using the two-stage simplex method.

The partially completed initial tableau is shown below.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	-1	0	1	0	9
a_2	-1	0	4	0	0	-1	0	1	4
P	-2	-1	-3	0	0	0	0	0	0
A									

- (a) Using the information in the above tableau, formulate the linear programming problem. State the objective and list the constraints as inequalities.

(4)

- (b) Complete the bottom row of Table 1 in the answer book. You should make your method and working clear.

(2)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$-\frac{7}{12}$	$-\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{45}{4}$
A	0	0	0	0	0	0	1	1	0

- (c) (i) Explain how the above tableau shows that a basic feasible solution has been found for the original linear programming problem.

- (ii) Write down the basic feasible solution for the second stage.

(3)

- (d) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method, to obtain a new tableau, T . Make your method clear by stating the row operations you use.

(5)



- (e) (i) Explain, using T , whether or not an optimal solution to the original linear programming problem has been found.
- (ii) Write down the value of the objective function.
- (iii) State the values of the basic variables.

(3)**(Total for Question 7 is 17 marks)**

7.

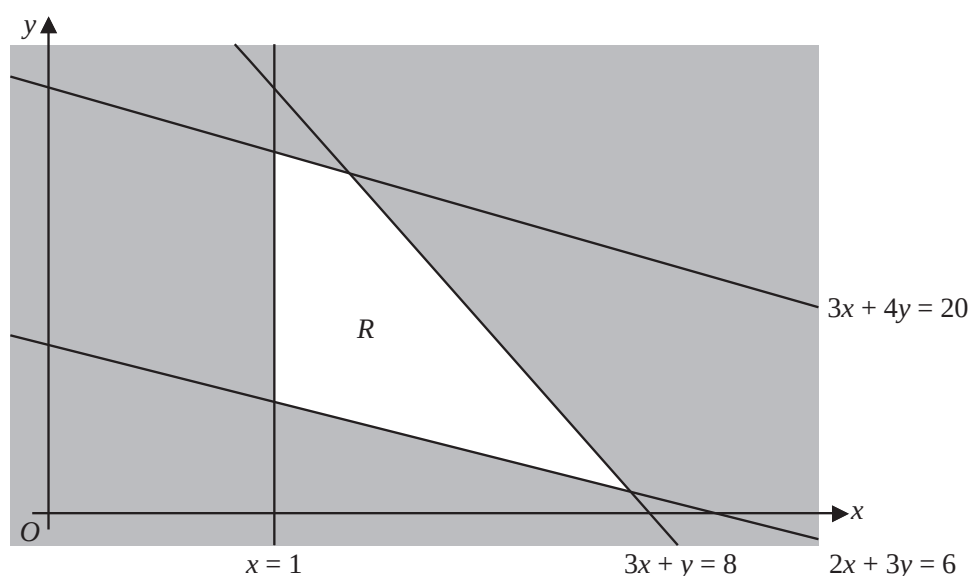


Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y , where R is the feasible region.

The objective is to maximise $P = 11x + ky$, where k is a positive constant.

The optimal value of P is to be found using the big-M method.

- (a) Set up an initial tableau for solving this linear programming problem using the big-M method.

You should use exactly 2 slack variables, 2 surplus variables and 2 artificial variables.

(7)

After a third iteration of the big-M method, a possible tableau is

b.v.	x	y	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	0	0	1	0	$\frac{9}{7}$	$-\frac{1}{7}$	0	$-\frac{9}{7}$	$\frac{78}{7}$
x	1	0	0	0	$\frac{1}{7}$	$\frac{3}{7}$	0	$-\frac{1}{7}$	$\frac{18}{7}$
y	0	1	0	0	$-\frac{3}{7}$	$-\frac{2}{7}$	0	$\frac{3}{7}$	$\frac{2}{7}$
s_2	0	0	0	1	$\frac{1}{7}$	$\frac{3}{7}$	-1	$-\frac{1}{7}$	$\frac{11}{7}$
P	0	0	0	0	$\frac{11}{7} - \frac{3}{7}k$	$\frac{33}{7} - \frac{2}{7}k$	M	$M - \frac{11}{7} + \frac{3}{7}k$	$\frac{198}{7} + \frac{2}{7}k$

(b) Given that the third iteration gives the optimal value for P , determine this value.

(5)

(Total for Question 7 is 12 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
6(a)(i)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to H: ABEFGH	A1	2.2a
(a)(ii)	Length of shortest path from A to H is 112	A1ft	2.2a
		(6)	
(b)	Applying Dijkstra repeatedly to n nodes implies that the order is $n(n^2) = n^3$	B1	3.1b
	$t = 0.082 \left(\frac{200}{10} \right)^3$	M1	3.4
	= 656 (seconds)	A1	2.2a
		(3)	
(c)	e.g. order of n^3 does not mean that the order is proportional to n^3 (which is the assumption behind the answer in (b)) but merely means that the dominant term is of order n^3	B1	3.2b
		(1)	
(10 marks)			
Notes: In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 129 118 112 in that order (so 129 112 118 is incorrect) It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling			

(a)

M1: A larger value replaced by a smaller value in at least two of the working boxes at either D or F or G or H

A1: All values in A, B, C and E correct. Condone lack of 0 in A's working value

A1: All values D and F correct and the working values in the correct order. Penalise order of labelling only once per question (D and F must be labelled in that order and D must be labelled after A, B, C and E)

A1ft: All values in G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will most likely come from nodes C, D and/or F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from C, F and/or G with the order of these values determined by the candidate's order of labelling at C, F and G)

A1: cao - correct path from A to H (ABEFGH)

A1ft: Follow through their final value at H only (if 112 stated and 112 is not the final value at H then **A0**)

(b)

B1: Any indication that repeated application of Dijkstra has cubic order

M1: Complete method – allow 10/200 – allow slips in values e.g. 0.82 for 0.082 and accept 200/10 (or 10/200) either squared or cubed **only**

A1: cao

(c)

B1: Any indication that the run-time is not exactly proportional to n^3 e.g., may suggest that there are other terms ($n^3 + \dots$), or that n^3 is the dominant term, or that order does not imply proportionality. Do not accept only that ' n^3 is not exact'. Condone use of n^2 (oe) for n^3



Qn	Scheme	Marks	AOs																																													
5.(a)	$x - y \geq 100 \Rightarrow x - y - s_1 + a_1 = 100$	B3,2,1	3.3																																													
	$x - 5y \leq 0 \Rightarrow x - 5y + s_2 = 0$		3.3																																													
	$2x + 3y \geq 350 \Rightarrow 2x + 3y - s_3 + a_2 = 350$		3.3																																													
	$P = -x - y - M(a_1 + a_2)$ and substitute expressions for a_1 and a_2 $(P + (1 - 3M)x + (1 - 2M)y + Ms_1 + Ms_3 = -450M)$	M1	2.1																																													
	e.g. <table><tr><td>b.v</td><td>x</td><td>y</td><td>s_1</td><td>s_2</td><td>s_3</td><td>a_1</td><td>a_2</td><td>Value</td></tr><tr><td>a_1</td><td>1</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>100</td></tr><tr><td>s_2</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>a_2</td><td>2</td><td>3</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>350</td></tr><tr><td>P</td><td>$1 - 3M$</td><td>$1 - 2M$</td><td>M</td><td>0</td><td>M</td><td>0</td><td>0</td><td>$-450M$</td></tr></table>	b.v	x	y	s_1	s_2	s_3	a_1	a_2	Value	a_1	1	-1	-1	0	0	1	0	100	s_2	1	-5	0	1	0	0	0	0	a_2	2	3	0	0	-1	0	1	350	P	$1 - 3M$	$1 - 2M$	M	0	M	0	0	$-450M$	M1 A1	3.3 2.2a
b.v	x	y	s_1	s_2	s_3	a_1	a_2	Value																																								
a_1	1	-1	-1	0	0	1	0	100																																								
s_2	1	-5	0	1	0	0	0	0																																								
a_2	2	3	0	0	-1	0	1	350																																								
P	$1 - 3M$	$1 - 2M$	M	0	M	0	0	$-450M$																																								
		(6)																																														
(b) (i)	$x = 130$	B1	3.4																																													
(ii)	When $x = 130 \Rightarrow y \leq 30, y \geq 26, y < 130$ and $y \geq 30$	M1	3.1a																																													
	$y = 30$	A1	2.2a																																													
		(3)																																														
(9 marks)																																																

Notes for Question 5

(a) NOTE: if correct they must use one slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable

B1: one correct equation or two correct inequalities (do not accept strict inequalities)

B1: two correct equations or three correct inequalities

B1: all three equations correct (please check suffices on s and a terms carefully – they may be in a different order)

M1: setting up the new objective which must be $P = -x - y - M(a_1 + a_2)$ and an attempt to substitute for their a_1 and a_2 (accept any equivalent form, which may not be fully simplified) (accept the use of Q instead of P throughout)

M1: setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark) **(Note the order of rows may be different from above) Check that the slack, surplus and artificial variables correspond to their equations**

A1: CAO (any equivalent correct form, but the terms in the objective row must be simplified)

(b)

(i) B1: CAO ($x = 130$)

(ii) M1: Substitute $x = 130$ into candidate's inequalities from (a) (at least 3 inequalities seen or both $y \geq 30$ and $y \leq 30$) (condone $y \leq 130$)

A1: CAO ($y = 30$) we must see both $y \geq 30$ and $y \leq 30$ explicitly stated for this mark (must not follow from any incorrect working)



Qu	Scheme	Marks	AOs
7(a)	$x + y \geq 60$	B1	3.3
	$y \leq \frac{4}{5}(x + y)$	B1	3.3
		(2)	
(b)		B1 B1 B1 B1	1.1b 1.1b 1.1b 2.2a
		(4)	
(c)	objective line drawn or point-testing	M1 A1	3.1a 1.1b
	(20, 80) so 20 analogue watches and 80 digital watches	A1	3.2a
		(3)	
(d)	$20a + 80d = 4455$	B1ft	3.1b
	$a = 5d$	B1	2.1
	Leading to $a = 123.75$ and $d = 24.75$ so an analogue watch costs £123.75 and a digital watch costs £24.75	dB1	2.2a
		(3)	
(12 marks)			

Notes for Question 7

(a) B1: CAO – allow any equivalent form of $x + y \geq 60$ - do not condone strict inequality

B1: CAO – allow any equivalent form of $y \leq \frac{4}{5}(x + y)$ (but not $y \leq 80\%(x + y)$ only) and need not be simplified - do not condone strict inequality – isw if correct answer is incorrectly simplified

In **(b)**, lines must be long enough to define the correct feasible region and would pass if extended through one small square of the points stated:

$x + y = 60$ must pass within one small square of its intersection with the axes – (0, 60) and (60, 0)

$y + 3x = 140$ must pass within one small square of its intersection with the axes – (0, 140) and

$(\frac{140}{3}, 0)$ (so at 46.666..., 0)

$4y + x = 80$ must pass within one small square of its intersection with the axes – (0, 20) and (80, 0)

$y = 4x$ must pass within one small square of (0, 0) and (25, 100)

In (b) condone for full marks lines which are drawn as dashed rather than solid

(b) B1: 2 lines drawn correctly

B1: 3 lines drawn correctly

B1: 4 lines drawn correctly

B1: Region, R , correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part

(c) M1: Drawing the correct objective line (with gradient -5) or its reciprocal (with gradient $-\frac{1}{5}$).

Line must be correct to within one small square if extended from axis to axis. If lines shorter than (5, 0) to (0, 25) or (0, 5) to (25, 0) then M0. Or point testing at least two exact coordinates of their R using their objective function which must be of the form $k(5x + y)$ or $k(x + 5y)$ for some positive real value k

A1: Correct objective line – condone lack of labelling of the objective line. Or point testing at least two of the correct exact coordinates which are (20, 80), (40, 20), (80, 0) and $(\frac{160}{3}, \frac{20}{3})$ using a correct objective function of the form $k(5x + y)$

A1: Correct number of watches – **must be in context** (and not just in terms of x and y) – dependent on a correct feasible region in **(b)** (so must have scored the first three marks in **(b)** but may not have labelled the FR as R)

Condone use of x for a and y for d in part (d)

(d) B1ft: A ‘correct’ equation (e.g. $20a + 80d = 4455$) involving their optimal point from **(c)** (accept any values even if non-integer) and 4455 – note that for those who have done point testing in **(c)** the calculation $4455 / (\text{their value for } P)$ where $P = 5x + y$ or $x + 5y$ using their optimal point implies this mark

B1: CAO on the relationship between the costs of the two types of watches ($a = 5d$) – this mark may be implied e.g. $20(5d) + 80d = 4455$ would score the first two marks in this part – note that for those who have done point testing in **(c)** the calculation $4455 / (\text{their value for } P)$ where $P = 5x + y$ using their optimal point implies this mark e.g. just seeing $4455 / 180$ is the first two marks in this part

dB1: CAO (dependent on first two B marks) – this mark is dependent on having the correct optimal point (20, 80) and is dependent on a correct feasible region in **(b)** (so must have scored at least the first three marks in **(b)**) – allow for $a = 123.75$ and $d = 24.75$ (so does not need to be in context or

units) – the correct answers with no working scores no marks in this part (however, note that $4455 / 180$ is the minimum amount of working that is acceptable)



Ques tion	Scheme	Marks	AOs																																																																													
7.(a)	Maximise $P = 8x + 20y + 40z$	B1	3.3																																																																													
	$4x + 8y + 15z \leq 9000$ $x + 5y + 12z \geq 3600$	M1 A1	3.3 1.1b																																																																													
	$x + y + z \geq 1600$	B1	3.3																																																																													
	$-x + 3y \leq 0$ $(x, y, z \geq 0)$	B1	3.3																																																																													
		(5)																																																																														
(b)	$4x + 8y + 15z + s_1 = 9000$ $x + 5y + 12z - s_2 + a_1 = 3600$ $x + y + z - s_3 + a_2 = 1600$ $-x + 3y + s_4 = 0$	B1 B1	1.1b 2.5																																																																													
	$I = -(a_1 + a_2)$ where $a_1 = 3600 - x - 5y - 12z + s_2$ and $a_2 = 1600 - x - y - z + s_3$	M1	2.1																																																																													
	$I - 2x - 6y - 13z + s_2 + s_3 = -5200$ and $P - 8x - 20y - 40z = 0$	A1	2.2a																																																																													
	e.g. <table><tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>a_1</th><th>a_2</th><th>Value</th></tr><tr><td>s_1</td><td>4</td><td>8</td><td>15</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>9000</td></tr><tr><td>a_1</td><td>1</td><td>5</td><td>12</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>3600</td></tr><tr><td>a_2</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>1600</td></tr><tr><td>s_4</td><td>-1</td><td>3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>P</td><td>-8</td><td>-20</td><td>-40</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>I</td><td>-2</td><td>-6</td><td>-13</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>-5200</td></tr></table>	b.v.	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	Value	s_1	4	8	15	1	0	0	0	0	0	9000	a_1	1	5	12	0	-1	0	0	1	0	3600	a_2	1	1	1	0	0	-1	0	0	1	1600	s_4	-1	3	0	0	0	0	1	0	0	0	P	-8	-20	-40	0	0	0	0	0	0	0	I	-2	-6	-13	0	1	1	0	0	0	-5200	M1 A1	3.4 2.2a
b.v.	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	Value																																																																						
s_1	4	8	15	1	0	0	0	0	0	9000																																																																						
a_1	1	5	12	0	-1	0	0	1	0	3600																																																																						
a_2	1	1	1	0	0	-1	0	0	1	1600																																																																						
s_4	-1	3	0	0	0	0	1	0	0	0																																																																						
P	-8	-20	-40	0	0	0	0	0	0	0																																																																						
I	-2	-6	-13	0	1	1	0	0	0	-5200																																																																						
		(6)																																																																														

(c)	b.v.	x	y	z	s_1	s_2	s_3	s_4	Value	Row Ops	B1	1.1b
	s_3	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	1	0	200	$\frac{1}{3}r_1$	M1	2.1
	z	0	$\frac{4}{11}$	1	$-\frac{1}{33}$	$-\frac{4}{33}$	0	0	$\frac{1800}{11}$	$r_2 - \frac{1}{11}R_1$	A1	1.1b
	x	1	$\frac{7}{11}$	0	$\frac{4}{11}$	$\frac{5}{11}$	0	0	$\frac{18000}{11}$	$r_3 + \frac{12}{11}R_1$	A1	1.1b
	s_4	0	$\frac{40}{11}$	0	$\frac{4}{11}$	$\frac{5}{11}$	0	1	$\frac{18000}{11}$	$r_4 + \frac{12}{11}R_1$	B1	2.4
	P	0	$-\frac{4}{11}$	0	$\frac{56}{33}$	$-\frac{40}{33}$	0	0	$\frac{216000}{11}$	$r_5 + \frac{56}{11}R_1$		
											(5)	
(d)	(i) £20 500										B1	3.4
	(ii) $1125 + 5(375) + 12(100) = (4200 \text{ minutes so}) 70 \text{ (hours)}$										B1	2.2a
											(2)	
(e)	e.g. there is no guarantee that all the books will be sold										B1	3.5b
											(1)	
(19 marks)												
Notes for Question 7												
(a)												
B1: Correct objective function ($8x + 20y + 40z$) plus ‘maximise’ or ‘max’ but not ‘maximum’ (and ‘ $P =$ ’ is not required)												
M1: Either one correct inequality (need not be simplified) or both $4x + 8y + 15z \leq k_1$ and $x + 5y + 12z \geq k_2$ where $k_1, k_2 > 0$												
A1: Both correct ($4x + 8y + 15z \leq 9000$, $x + 5y + 12z \geq 3600$) – allow equivalent answers (provided 4 terms only and integer coefficients e.g. $2x + 10y + 24z - 7200 \geq 0$)												
B1: CAO ($x + y + z \geq 1600$ oe provided 4 terms only and integer coefficients)												
B1: CAO ($3y \leq x$ oe provided 2 terms only and integer coefficients)												
(b) Note in this part that the numbering of the suffices for the slack, surplus and artificial variables seen in both the candidate’s equations and Simplex tableau will most likely be different from what is seen in the MS (e.g. $4x + 8y + 15z + s_3 = 9000$ is correct). The correct values in the rows of the tableau do NOT imply the first four marks (the question explicitly asked for the constraints as equations and the rows for the two objectives to be explicitly stated)												

B1: Any two correct inequalities converted into equations correctly (condone the same letter, say s_3 , being used twice) – any equivalent forms of the correct equations are acceptable (e.g. variables do not need to be on the same side)

B1: All four correct equations (using distinct slack, surplus and artificial variables) – any equivalent forms of the correct equations are acceptable

M1: Using $I = -(a_1 + a_2)$ with their expressions for a_1 and a_2 (allow slips in forming I from their two expressions for the two artificial variables) but must be a clear intention to calculate $I = -(a_1 + a_2)$ – must be **exactly two** artificial variables in their expression for I for this mark

A1: CAO for I and P (must be stated as $P - 8x - 20y - 40z = 0$ (**A0** if $= 0$ missing) and $I - 2x - 6y - 13z + s_2 + s_3 = -5200$ - so variables on one side and constant on the other)

M1: setting up initial tableau – **all** six rows complete (with no blanks) and two correct rows (but ignore b.v. column for this mark)

A1: CAO (any equivalent correct form) - **note that the candidate's order in which the rows appear in the tableau (and choice of letter to represent the slack, surplus and artificial variables) may be different – check to ensure that the basic variable column is consistent with their choice of lettering for the slack and artificial variables**

(c)

B1: Pivot (top) row completely correct including change of b.v. (but not 'Row Ops' column)

M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are y, s_1, s_2 or Value) correct (must have pivoted on the correct value)

A1: Row operations used correctly at least twice, i.e. two of the 'non zero and one' columns (s_1, s_2, y or Value) correct

A1: CAO **all** values including b.v. column – ignore 'Row Ops' column for this mark

B1: Correct row operations stated – alternatives are $\frac{1}{3}r_1, r_2 - \frac{1}{33}r_1, r_3 + \frac{12}{33}r_1, r_4 + \frac{12}{33}r_1, r_5 + \frac{56}{33}r_1$

(d)(i)

B1: CAO (20 500)

(ii)

B1: CAO (70 only)

(e)

B1: CAO – must **explicitly** mention the fact that it is possible that not all the books will be sold. Ignore reasons for why, provided they do not relate to the publisher producing less than the optimal number of books



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Question	Scheme	Marks	AOs
4(a)	$2y \leq 5x, y \geq x+1, 6x+5y \leq 30$	B2,1,0	1.1b 2.5
		(2)	
(b)	$\left(\frac{2}{3}, \frac{5}{3}\right), \left(\frac{60}{37}, \frac{150}{37}\right), \left(\frac{25}{11}, \frac{36}{11}\right)$ $\left(\frac{2}{3}, \frac{5}{3}\right) \rightarrow P = \frac{11}{3}$ $\left(\frac{60}{37}, \frac{150}{37}\right) \rightarrow P = \frac{330}{37}$ $\left(\frac{25}{11}, \frac{36}{11}\right) \rightarrow P = \frac{111}{11}$ so optimal vertex is $\left(\frac{25}{11}, \frac{36}{11}\right)$	B1 B1 M1 A1	1.1b 1.1b 2.1 2.2a
		(4)	
(c)	$Q = 3x + ay$ $3\left(\frac{25}{11}\right) + \frac{36a}{11} > 3\left(\frac{60}{37}\right) + \frac{150a}{37}$ $\Rightarrow a < \frac{5}{2}$ $3\left(\frac{25}{11}\right) + \frac{36a}{11} > 3\left(\frac{2}{3}\right) + \frac{5a}{3}$ $\Rightarrow a > -3$	M1 A1 M1 A1	3.1a 2.2a 1.1b 2.2a
		(4)	
(10 marks)			
Notes for Question 4			
(a) B1: Any two correct (accept strict inequalities) – accept equivalent inequalities B1: CAO (accept equivalent inequalities but inequalities must not be strict) (b) B1: One correct vertex (must be exact) B1: All three correct vertices (must be exact) M1: Testing all three of their vertices in the correct objective function A1: Correct three values of P and correct optimal vertex either stated or clearly indicated on the graph			

(c)

M1: Their optimal point from (b) evaluated in Q compared to their $\left(\frac{60}{37}, \frac{150}{37}\right)$ evaluated in Q (with correct inequality)

A1: $a < \frac{5}{2}$

M1: Their optimal point from (b) evaluated in Q compared to their $\left(\frac{2}{3}, \frac{5}{3}\right)$ evaluated in Q (with correct inequality)

A1: $a > -3$



Qu	Scheme	Marks	AOs																																																						
7 (a)	$x + y, 8 \Rightarrow x + y + s_1 = 8$ $x + 2y, 12 \Rightarrow x + 2y + s_2 = 12$ $7x + 2y, 46 \Rightarrow 7x + 2y + s_3 = 46$ $y, 2x + 1 \Rightarrow -2x + y + s_4 = 1$ $P = x + ky \Rightarrow P - x - ky = 0$	M1 A1 B1	3.4 1.1b 1.1b																																																						
	e.g. <table><tr><th>b.v.</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>Value</th></tr><tr><td>s_1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>8</td></tr><tr><td>s_2</td><td>1</td><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>12</td></tr><tr><td>s_3</td><td>7</td><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>46</td></tr><tr><td>s_4</td><td>-2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>P</td><td>-1</td><td>-k</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	b.v.	x	y	s_1	s_2	s_3	s_4	Value	s_1	1	1	1	0	0	0	8	s_2	1	2	0	1	0	0	12	s_3	7	2	0	0	1	0	46	s_4	-2	1	0	0	0	1	1	P	-1	-k	0	0	0	0	0	M1 A1	3.3 2.2a						
b.v.	x	y	s_1	s_2	s_3	s_4	Value																																																		
s_1	1	1	1	0	0	0	8																																																		
s_2	1	2	0	1	0	0	12																																																		
s_3	7	2	0	0	1	0	46																																																		
s_4	-2	1	0	0	0	1	1																																																		
P	-1	-k	0	0	0	0	0																																																		
		(5)																																																							
(b)	$x = 2, y = 5, s_1 = 1, s_2 = 0, s_3 = 22, s_4 = 0$ ($P = 5k + 2$)	B1	3.4																																																						
		(1)																																																							
(c)	<table><tr><th>b.v.</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>Value</th><th>Row Ops</th></tr><tr><td>s_4</td><td>0</td><td>0</td><td>5</td><td>-3</td><td>0</td><td>1</td><td>5</td><td>5r1</td></tr><tr><td>x</td><td>1</td><td>0</td><td>2</td><td>-1</td><td>0</td><td>0</td><td>4</td><td>r2 + 0.4R1</td></tr><tr><td>s_3</td><td>0</td><td>0</td><td>-12</td><td>5</td><td>1</td><td>0</td><td>10</td><td>r3 - 2.4R1</td></tr><tr><td>y</td><td>0</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>4</td><td>r4 - 0.2R1</td></tr><tr><td>P</td><td>0</td><td>0</td><td>2 - k</td><td>k - 1</td><td>0</td><td>0</td><td>4k + 4</td><td>r5 - (0.2k - 0.4)R1</td></tr></table>	b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops	s_4	0	0	5	-3	0	1	5	5r1	x	1	0	2	-1	0	0	4	r2 + 0.4R1	s_3	0	0	-12	5	1	0	10	r3 - 2.4R1	y	0	1	-1	1	0	0	4	r4 - 0.2R1	P	0	0	2 - k	k - 1	0	0	4k + 4	r5 - (0.2k - 0.4)R1	B1 M1 A1 A1 B1	1.1b 2.1 1.1b 1.1b 2.4
b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops																																																	
s_4	0	0	5	-3	0	1	5	5r1																																																	
x	1	0	2	-1	0	0	4	r2 + 0.4R1																																																	
s_3	0	0	-12	5	1	0	10	r3 - 2.4R1																																																	
y	0	1	-1	1	0	0	4	r4 - 0.2R1																																																	
P	0	0	2 - k	k - 1	0	0	4k + 4	r5 - (0.2k - 0.4)R1																																																	
	Optimal value of P is $4k + 4$ (at $x = y = 4$)	M1	3.4																																																						
	Second iteration not optimal $\Rightarrow -\frac{2}{5} + \frac{1}{5}k < 0 \therefore k < 2$	B1	3.1a																																																						
	Third iteration optimal $\Rightarrow 2 - k \dots 0$ and $k - 1 \dots 0$ ($\therefore k \dots 1$)	dM1	3.4																																																						
	1,, $k < 2 \Rightarrow 8$,, $P < 12$	A1	2.2a																																																						
		(9)																																																							
(15 marks)																																																									

Notes for Question 7

a1M1: Correctly re-writing any two inequalities as equations with slack variables (can be implied by two correct rows in the Simplex tableau ignoring b.v. column). Or correctly stating all four constraints as inequalities

a1A1: Correctly re-writing all inequalities as equations with slack variables (can be implied by the four correct constraint rows in the Simplex tableau ignoring b.v. column)

a1B1: Correctly re-writes objective function (can be implied by correct row in tableau)

a2M1: **Any** two rows correct including consistent b.v. column entries **or** any three rows correct (ignoring b.v. column)

a2A1: **Any** two rows correct including consistent b.v. column – **note that the candidate's order in which the rows appear in the tableau (and choice of letter to represent the slack variable) may be different. A correct tableau implies full marks in this part**

b1B1: **Any** two rows correct including consistent b.v. column entries **or** any three rows correct (ignoring b.v. column)

c1B1: Pivot row completely correct including change of b.v.

c1M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are s_1, s_2 or Value) correct (must have pivoted on the correct value)

c1A1: Row operations used correctly at least twice, i.e. two of the 'non zero and one' columns (s_1, s_2 or Value) correct

c2A1: **Any** two rows correct including consistent b.v. column entries **or** any three rows correct (ignoring b.v. column)

c2B1: Correct row operations stated

(alternatives row operations are $5r_1; r_2 + 2r_1; r_3 - 12r_1; r_4 - r_1; r_5 - (k - 2)r_1$)

c2M1: Their optimal value (as a linear expression in k) stated correctly following their third iteration (must have pivoted on a positive value from the s_4 column and completed the bottom row of the tableau). Condone this expression ($4k + 4$ **if** correct) being stated as part of an equation/inequality (or as part of their final answer) – sight of this expression (but must be seen **outside** of the tableau) scores this mark

c3B1: Correctly inferring that $k < 2$ either from $-\frac{2}{5} + \frac{1}{5}k < 0$ or from $4k + 4 > 5k + 2$ – just stating $k < 2$ without it being clear where this comes from is B0

c3dM1: Considering (at least) two of their linear expressions in k from their objective row (not including the Value column) after the third iteration ...0 (**dependent on the previous M mark**) – note that working may be minimal here so please follow through their expressions in k from their objective row (so **if** correct, stating $k \geq 2$ and $k \leq 1$ implies this mark)

c3A1: **Any** two rows correct including consistent b.v. column entries **or** any three rows correct (ignoring b.v. column)

Qu	Scheme	Marks	AOs																																																		
6(a)	Simplex can only be applied when the non-negativity constraints are $\leq$	B1	3.5b																																																		
		(1)																																																			
(b)	$3x + y + 2z \leq 30 \Rightarrow 3x + y + 2z + s_1 = 30$	B1	1.1b																																																		
	$x - y + z \geq 8 \Rightarrow x - y + z - s_2 + a_1 = 8$	B1	2.5																																																		
	$4y + 2z \geq 15 \Rightarrow 4y + 2z - s_3 + a_2 = 15$	B1	1.1b																																																		
	$P = 2x + 2y - z \Rightarrow P = 2x + 2y - z - M(a_1 + a_2)$ together with $a_1 + a_2 = 23 - x - 3y - 3z + s_2 + s_3$	M1	2.1																																																		
	$P - (2 + M)x - (2 + 3M)y - (-1 + 3M)z + Ms_2 + Ms_3 = -23M$	A1	1.1b																																																		
	<table><tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>s₁</td><td>s₂</td><td>s₃</td><td>a₁</td><td>a₂</td><td>Value</td></tr><tr><td>s₁</td><td>3</td><td>1</td><td>2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr><tr><td>a₁</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>8</td></tr><tr><td>a₂</td><td>0</td><td>4</td><td>2</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>15</td></tr><tr><td>P</td><td>-(2 + M)</td><td>-(2 + 3M)</td><td>-(3M - 1)</td><td>0</td><td>M</td><td>M</td><td>0</td><td>0</td><td>-23M</td></tr></table>	b.v	x	y	z	s ₁	s ₂	s ₃	a ₁	a ₂	Value	s ₁	3	1	2	1	0	0	0	0	30	a ₁	1	-1	1	0	-1	0	1	0	8	a ₂	0	4	2	0	0	-1	0	1	15	P	-(2 + M)	-(2 + 3M)	-(3M - 1)	0	M	M	0	0	-23M	M1 A1	3.3 2.2a
b.v	x	y	z	s ₁	s ₂	s ₃	a ₁	a ₂	Value																																												
s ₁	3	1	2	1	0	0	0	0	30																																												
a ₁	1	-1	1	0	-1	0	1	0	8																																												
a ₂	0	4	2	0	0	-1	0	1	15																																												
P	-(2 + M)	-(2 + 3M)	-(3M - 1)	0	M	M	0	0	-23M																																												
		(7)																																																			
(c)	$s_1 = 26.25, a_1 = 11.75, y = 3.75, x = z = s_2 = s_3 = a_2 = 0$	B1	3.4																																																		
		(1)																																																			
(d)	The solution after the 1 st iteration is not feasible because $a_1 = 11.75$ is an artificial variable which must be zero in a feasible solution	B1	2.4																																																		
		(1)																																																			
(e)	The most negative value in the objective row is $2 - 1.5M$ so the pivot is a value from the z-column	B1	2.4																																																		
	The 0.5 in the y row is the pivot because $\frac{3.75}{0.5}$ is less than both $\frac{26.25}{1.5}$ and $\frac{11.75}{1.5}$	dB1	2.2a																																																		
		(2)																																																			
(12 marks)																																																					

Notes for Question 6

(a)

B1: CAO – e.g. not all of the constraints are $\leq$, the origin is not a (basic feasible) solution of the LP

(b)

B1: CAO $3x + y + 2z + s_1 = 30$ (may be seen in the simplex tableau – allow any s_i (or s) for s_1)**B1:** CAO $x - y + z - s_2 + a_1 = 8$ (may be seen in the simplex tableau – allow any consistent s_i for s_2 (or t say) but not the same s_i as in the previous mark and allow any a_i for a_1)**B1:** CAO $4y + 2z - s_3 + a_2 = 15$ (may be seen in the simplex tableau – same conditions as above)**M1:** setting up the new objective which must be $P = 2x + 2y - z - M(a_1 + a_2)$ and substituting for their a_1 and a_2 (if no working then the **correct** objective line in the tableau implies this mark)**A1:** CAO $P - (2 + M)x - (2 + 3M)y - (-1 + 3M)z + Ms_2 + Ms_3 = -23M$ (any equivalent form – need not be factorised and does not need to be re-arranged into this form - if no working then the **correct** objective line in the tableau implies this mark)**M1:** setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark)**A1:** CAO (any equivalent correct form)

(c)

B1: CAO $s_1 = 26.25, a_1 = 11.75, y = 3.75, x = z = s_2 = s_3 = a_2 = 0$ (ignore expression for P if given)

(d)

B1: correct reasoning of why the solution is not feasible e.g. a_1 is not zero but B0 for just stating that the artificial variable is non-zero (so must see either a_1 or 11.75 being stated as non-zero)

(e)

B1: correct reasoning of why the pivot comes from a value from the z -column so must say that the most negative value (in the objective row) is $2 - 1.5M$ (or this expression clearly implied)**dB1:** correct justification of why the 0.5 in the third row is the next pivot (dependent on previous B mark) – so must compare or state that $\frac{3.75}{0.5}$ or 7.5 is less than both $\frac{26.25}{1.5}$ or 17.5 and $\frac{11.75}{1.5}$ or 7.8(3333....) – just stating that the 0.5 in the third row is the next pivot without reasoning is no marks in this part

Ques tion	Scheme	Marks	AOs																																								
7.(a)	The pivot for this first iteration came from the x -column	B1	1.1b																																								
	as it is now a basic variable	dB1	2.5																																								
		(2)																																									
(b)	x -row: $4x + y - z \leq 40$ or s_3 - row: $y + z \leq 26$	B1	3.4																																								
	Eliminating s_2 from the s_1 row using the x row: $-\frac{1}{2}y + \frac{3}{2}z + s_1 - 2\left(10 - x - \frac{1}{4}y + \frac{1}{4}z\right) = 30$	M1	2.1																																								
	Eliminating s_2 from the P row using the x row: $P - \frac{1}{4}y - \frac{11}{4}z + 3\left(10 - x - \frac{1}{4}y + \frac{1}{4}z\right) = 30$	M1	1.1b																																								
	For either $P - 3x - y - 2z = 0$ or $2x + z + s_1 = 50$	A1	1.1b																																								
	Alternatively <table border="1"><thead><tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>Value</th></tr></thead><tbody><tr><td>s_1</td><td>2</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>50</td></tr><tr><td>s_2</td><td>4</td><td>1</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>40</td></tr><tr><td>s_3</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>26</td></tr><tr><td>P</td><td>-3</td><td>-1</td><td>-2</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></tbody></table>	b.v.	x	y	z	s_1	s_2	s_3	Value	s_1	2	0	1	1	0	0	50	s_2	4	1	-1	0	1	0	40	s_3	0	1	1	0	0	1	26	P	-3	-1	-2	0	0	0	0		
b.v.	x	y	z	s_1	s_2	s_3	Value																																				
s_1	2	0	1	1	0	0	50																																				
s_2	4	1	-1	0	1	0	40																																				
s_3	0	1	1	0	0	1	26																																				
P	-3	-1	-2	0	0	0	0																																				
	LP: (Maximise) $P = 3x + y + 2z$ Subject to: $2x + z \leq 50$ $4x + y - z \leq 40$ $y + z \leq 26$ ($x, y, z \geq 0$)	A1	2.2a																																								
		(5)																																									



(c)	<table><tr><td>b.v.</td><td>x</td><td>y</td><td>z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>Value</td><td>Row Ops</td></tr><tr><td>z</td><td>0</td><td>$-\frac{1}{3}$</td><td>1</td><td>$\frac{2}{3}$</td><td>$-\frac{1}{3}$</td><td>0</td><td>20</td><td>$\frac{2}{3}r_1$</td></tr><tr><td>x</td><td>1</td><td>$\frac{1}{6}$</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>0</td><td>15</td><td>$r_2 + 0.25R_1$</td></tr><tr><td>s_3</td><td>0</td><td>$\frac{4}{3}$</td><td>0</td><td>$-\frac{2}{3}$</td><td>$\frac{1}{3}$</td><td>1</td><td>6</td><td>$r_3 - R_1$</td></tr><tr><td>P</td><td>0</td><td>$-\frac{7}{6}$</td><td>0</td><td>$\frac{11}{6}$</td><td>$-\frac{1}{6}$</td><td>0</td><td>85</td><td>$r_4 + 2.75R_1$</td></tr></table>	b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops	z	0	$-\frac{1}{3}$	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	20	$\frac{2}{3}r_1$	x	1	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	15	$r_2 + 0.25R_1$	s_3	0	$\frac{4}{3}$	0	$-\frac{2}{3}$	$\frac{1}{3}$	1	6	$r_3 - R_1$	P	0	$-\frac{7}{6}$	0	$\frac{11}{6}$	$-\frac{1}{6}$	0	85	$r_4 + 2.75R_1$	B1	1.1b
	b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops																																							
	z	0	$-\frac{1}{3}$	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	20	$\frac{2}{3}r_1$																																							
	x	1	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	15	$r_2 + 0.25R_1$																																							
	s_3	0	$\frac{4}{3}$	0	$-\frac{2}{3}$	$\frac{1}{3}$	1	6	$r_3 - R_1$																																							
P	0	$-\frac{7}{6}$	0	$\frac{11}{6}$	$-\frac{1}{6}$	0	85	$r_4 + 2.75R_1$																																								
	M1	2.1																																														
	A1	1.1b																																														
	B1	2.4																																														
		(4)																																														
(d)(i)	After the second iteration an optimal solution has not been found as the profit row still contains negative values	dB1	2.4																																													
(ii)	$z=20, x=15, s_3=6$	dB1ft	2.2a																																													
		(2)																																														
(e)	e.g. Their attempt is not correct as, if y is now a basic variable, then the y column should contain only one value of 1 (in the third row) and therefore the entry of 1 in the profit row is incorrect (and should be 0)	B1	2.3																																													
		(1)																																														
(14 marks)																																																

Notes for Question 7

(a)

B1: CAO (correct column of x stated)**dB1:** **Dependent on first B1** Correct statement that x is now a basic variable (oe) Accept that x now appears in the first (bv) column or states x column has one 1 and rest 0 or s_2 row has been replaced with x

(b)

B1: Either the constraint for the x -row **or** s_3 - row correct (allow any equivalent form including non-integer coefficients but must be inequalities) Do not accept strict inequalities**M1:** Eliminating s_2 from the equation from the s_1 row using the equation from the x row Correct constraint implies this mark**M1:** Eliminating s_2 from the equation from the P row using the equation from the x row Correct objective implies this mark**A1:** $P - 3x - y - 2z = 0$ **or** $2x + z + s_1 = 50$ (allow any equivalent form including non-integer coefficients but must have been simplified to a single term in each variable) Correct constraint or objective implies the corresponding M mark**A1:** Correct LP formulation (condone lack of 'maximise' and condone lack of the non-negative trivial constraints)**Note: It is possible to score M1 M0 A1 A0 or M0 M1 A1 A0**

Alternatively – reproduces original tableau

B1: s_2 row correct**M1:** s_1 row correct - Correct constraint implies this mark**M1:** P row correct - Correct objective implies this mark**A1:** fully correct tableau**A1:** Correct LP formulation (condone lack of 'maximise' and condone lack of the non-negative trivial constraints)(c) **Note – accept correct recurring decimals in place of fractions****B1:** Pivot row completely correct including change of b.v.**M1:** All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are y , s_1 , s_2 or Value) correct (must have pivoted on the correct value)**A1:** **cao all** values including b.v. column – ignore 'Row Ops' column for this mark**B1:** Correct row operations stated (allow alternative numbering of rows as long as this is clear. Condone use of R_1 throughout)

(d)(i)

dB1: CAO (profit row still contains negative values (but not negative variables) – dependent on the M mark in (c) and a completed profit row in (c) (accept P row or objective row but not operations row or bottom row) Do not accept that not all values are positive**dB1ft:** Follow through their values of z , x , and s_3 – dependent on the M mark in (c) and a completed tableau in (c) ignore mention of any other variables including P **All values must be positive**

(e)

B1: CAO (e.g. correct indication that the (basic variable) column for y is not correct – see bold statement for the minimum acceptable) Accept there should not be a 1 in the objective row in the y column or that there should not be two 1s in the y column. If s_2 column also mentioned then B0

Question	Scheme	Marks	AOs																																													
5(a)	The Simplex algorithm cannot be used as the problem has no obvious basic feasible solution since the origin is not in the feasible region Alternatively Simplex can only be used with non-negative values of variables	B1	3.5b																																													
		(1)																																														
(b)	$2(X + 4) + 2(Y + 1) + 2(-Z - 3) \leq 21 \Rightarrow 2X + 2Y - 2Z \leq 17$ $2(X + 4) - (Y + 1) - (-Z - 3) \leq 18 \Rightarrow 2X - Y + Z \leq 8$ $-3(X + 4) + (Y + 1) - 2(-Z - 3) \leq 1 \Rightarrow -3X + Y + 2Z \leq 6$ $Q + 10 = 3(X + 4) + 4(Y + 1) + 2(-Z - 3) \Rightarrow Q = 3X + 4Y - 2Z$ $(X \geq 0, Y \geq 0, Z \geq 0)$	M1 A1 A1	3.3 1.1b 1.1b																																													
		(3)																																														
(c)	$Q - \frac{5}{4}Z + \frac{15}{8}s_1 + \frac{1}{4}s_3 = \frac{267}{8}$	B1	3.4																																													
		(1)																																														
(d)	$Q = \frac{5}{4}Z - \frac{15}{8}s_1 - \frac{1}{4}s_3 + \frac{267}{8}$ so therefore, we can increase the profit by increasing Z	B1	2.4																																													
		(1)																																														
(e)(i)	<table><tr><th>b.v.</th><th>X</th><th>Y</th><th>Z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>Value</th><th>Row Ops</th></tr><tr><td>X</td><td>1</td><td>0</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>0</td><td>$\frac{11}{2}$</td><td>$r1 + \frac{3}{4} \times R2$</td></tr><tr><td>Z</td><td>0</td><td>0</td><td>1</td><td>$\frac{1}{18}$</td><td>$\frac{4}{9}$</td><td>$\frac{1}{3}$</td><td>$\frac{13}{2}$</td><td>$\frac{4}{9} \times r2$</td></tr><tr><td>Y</td><td>0</td><td>1</td><td>0</td><td>$\frac{7}{18}$</td><td>$\frac{1}{9}$</td><td>$\frac{1}{3}$</td><td>$\frac{19}{2}$</td><td>$r3 + \frac{1}{4} \times R2$</td></tr><tr><td>Q</td><td>0</td><td>0</td><td>0</td><td>$\frac{35}{18}$</td><td>$\frac{5}{9}$</td><td>$\frac{2}{3}$</td><td>$\frac{83}{2}$</td><td>$r4 + \frac{5}{4} \times R2$</td></tr></table>	b.v.	X	Y	Z	s_1	s_2	s_3	Value	Row Ops	X	1	0	0	$\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{11}{2}$	$r1 + \frac{3}{4} \times R2$	Z	0	0	1	$\frac{1}{18}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{13}{2}$	$\frac{4}{9} \times r2$	Y	0	1	0	$\frac{7}{18}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{19}{2}$	$r3 + \frac{1}{4} \times R2$	Q	0	0	0	$\frac{35}{18}$	$\frac{5}{9}$	$\frac{2}{3}$	$\frac{83}{2}$	$r4 + \frac{5}{4} \times R2$	B1 M1 A1 A1	1.1b 2.1 1.1b 1.1b
b.v.	X	Y	Z	s_1	s_2	s_3	Value	Row Ops																																								
X	1	0	0	$\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{11}{2}$	$r1 + \frac{3}{4} \times R2$																																								
Z	0	0	1	$\frac{1}{18}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{13}{2}$	$\frac{4}{9} \times r2$																																								
Y	0	1	0	$\frac{7}{18}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{19}{2}$	$r3 + \frac{1}{4} \times R2$																																								
Q	0	0	0	$\frac{35}{18}$	$\frac{5}{9}$	$\frac{2}{3}$	$\frac{83}{2}$	$r4 + \frac{5}{4} \times R2$																																								
(ii)	$X = \frac{11}{2}, Y = \frac{19}{2}, Z = \frac{13}{2}, Q = \frac{83}{2}$ ($X = 5.5$ $Y = 9.5$ $Z = 6.5$ $Q = 41.5$) $x = \frac{19}{2}, y = \frac{21}{2}, z = -\frac{19}{2}, P = \frac{103}{2}$ ($x = 9.5$ $y = 10.5$ $z = -9.5$ $P = 51.5$)	M1 A1ft	3.4 2.2a																																													
		(6)																																														
(12 marks)																																																



Notes:

a1B1: Correct reason why Simplex cannot be used to solve the LP problem. Please mark positively and award if a correct statement is seen.

SC accept Because $z \leq -3$ (and variables must be ≥ 0)

b1M1: Substituting given equations into all three given inequalities to form inequalities or equations with slack variables in terms of X , Y and Z (condone at most two slips but see Special Case below)

b1A1: At least two of the four expressions simplified correctly (three inequalities and the new objective) (accept any equivalent rearrangement but the objective must be in terms of Q not P) (accept equations with slack variables)

b2A1: CAO (must be inequalities)

c1B1: CAO (must be in terms of Q)

d1B1: CAO (Must include mention that Z can be increased but do not award if any incorrect statement seen)

Do not accept just a statement that the objective row contains a negative value and therefore it is not optimal

Note – accept correct recurring decimals in place of fractions or any equivalent fractions

ei1B1: Pivot row (Z row) correct including change of b.v. (ignore row ops)

ei1M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (s_1, s_2, s_3 or value) correct

ei1A1: Row operations used correctly at least twice, i.e. **two** of the non-pivot rows or **two** of the non zero and one columns (s_1, s_2, s_3 or value) – ignore row operations for this mark

ei2A1: CAO all values and row operations correctly stated including b.v column (allow alternative numbering of rows as long as this is clear. Condone use of r_2 throughout. Do not accept in terms of b.v.) (Accept in terms of original r_2 so $r_1 + 1/3r_2$, $4/9r_2$, $r_3 + 1/9r_2$, $r_4 + 5/9r_2$) (Row ops for pivot row may be written as $r_2 \div 9/4$)

eii2M1: Stating (or implying) optimal values of X , Y , Z and Q (see special case)

eii3A1ft: CAO for x , y , z and P (the correct 4 values implies both marks) (follow through their values for X , Y , Z and Q from the tableau)

Special Case

(b) If they make a consistent error when substituting by using an incorrect expression for X , Y or Z (e.g. substituting $x = X + 1$ instead of $x = X + 4$ in all terms) they may score M1 A1 (for at least two of their four expressions correct) A0

(e) If they use the same incorrect substitution and do not state the values of X , Y , Z and Q they may score M1 A0 for the implied values of X , Y , Z and Q from their x , y , z and P



Question	Scheme	Marks	AOs																														
6(a)	<table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>-</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>C</td></tr><tr><td>F</td><td>C</td></tr></tbody></table> <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>G</td><td>A, B, E</td></tr><tr><td>H</td><td>D, G</td></tr><tr><td>I</td><td>A, B, E</td></tr><tr><td>J</td><td>H, I</td></tr><tr><td>K</td><td>A, B, E</td></tr><tr><td>L</td><td>A, B, E, F</td></tr><tr><td>M</td><td>A, B, E, F</td></tr></tbody></table>	Activity	IPA	A	-	B	-	C	-	D	A	E	C	F	C	Activity	IPA	G	A, B, E	H	D, G	I	A, B, E	J	H, I	K	A, B, E	L	A, B, E, F	M	A, B, E, F	B1 B1	1.1b 1.1b
Activity	IPA																																
A	-																																
B	-																																
C	-																																
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L	A, B, E, F																																
M	A, B, E, F																																
		(2)																															
(b)		M1 A1 M1 A1	1.1b 1.1b 1.1b 1.1b																														
		(4)																															
(c)		M1 A1 A1 A1	2.1 1.1b 1.1b 1.1b																														
		(4)																															

(d)	Activity	Workers	Activity	Workers	B3, 2, 1	3.4 1.1b 1.1b
	A	1	G	2		
	B	2	H	1		
	C	1	I	2		
	D	2	J	2		
	E	3	K	1		
	F	1	L	1		
			M	1		
(13 marks)						
Notes:						
a1B1: Five correct rows (not including A, B and C) a2B1: All rows correct (accept blanks for A, B and C) b1M1: All top boxes complete, values generally increasing in the direction of the arrows (‘left to right’), condone one rogue value which is a number in a top box greater than the subsequent value b1A1: CAO – top boxes b2M1: All bottom boxes complete, values generally decreasing in the opposite direction of the arrows (‘right to left’), condone one rogue value which is a number in a bottom box greater than the previous value. Condone missing 0 and/or their 20 (at the end event) for the M mark only b2A1: CAO – bottom boxes c1M1: Gantt (cascade) chart with at least 9 activities labelled and at least four activities having non-zero float. A scheduling diagram scores M0 c1A1: Critical activities (C, E, G, H and J) correct (note this may be seen as separate activities listed alphabetically instead of at the top of the chart) c2A1: At least 4 non-critical activities correct c3A1: CAO All 13 activities present (just once). No errors. d1B1: At least four correct d2B1: At least eight correct d3B1: CAO For reference these are the early start, late end and length of float for the non-critical activities						
Activity	Start	Late End	Float	Critical	Start	End
A	0	5	3	C	0	3
B	0	7	3	E	3	7
D	2	12	3	G	7	12
F	3	10	2	H	12	18
I	7	18	1	J	18	20
K	7	20	7			
L	8	20	2			
M	8	20	5			

Question	Scheme	Marks	AOs
4(a)	Constraints: $2x + 3y + 4z \leq 13$ $x - 2y + 2z \leq 8$ $3x - 4z \leq 12$ $(x, y, z \geq 0)$	B1 B1	3.4 2.5
	Objective functions: Maximise $-2x + 3y + z$ Minimise $2x - 3y - z$	M1 A1	3.1a 2.2a
		(4)	
(b)	(Because M is big) the only negative in the objective row is the $2 - 4M$ so the pivot is from the x -column	B1	2.4
	The 3 in the a_2 row is the pivot as $\frac{12}{3}$ is less than both $\frac{8}{1}$ and $\frac{13}{2}$	B1	2.2a
		(2)	
(6 marks)			
Notes for Question 4			
a1B1: One correct non-trivial inequality (allow strict inequality provided direction of inequality sign is correct) – equations with slack variables etc. scores no marks unless replaced with correct inequalities a2B1: All three non-trivial inequalities correct a1M1: Either expression stated correctly (allow equal to (or an inequality with) any letter e.g. $P = -2x + 3y + z$ but not equal to a value e.g. $= 0$) – ignore any mention of maximum/minimum for this mark a1A1: Both expressions correct including max/min correctly matched with each expression (allow equal to any letter only) – do not isw if they continue and place their expression(s) equal to a value(s) b1B1: Correct reasoning that the pivot is a value from the x-column – as a minimum must state that the $2 - 4M$ is the <u>only negative</u> (condone <u>most negative</u>) in the <u>objective row</u> (allow <u>profit row</u> or <u>P row</u>, condone ‘<u>bottom row</u>’) b2B1: Correct justification of why the 3 in the a_2 row or the 3 in the x column is the pivot – so must state the correct pivot in a clear unambiguous way (so just saying the pivot is ‘the 3’ is B0) and comparing or stating that $\frac{12}{3}$ or 4 is less than/least positive for both $\frac{8}{1}$ or 8 and $\frac{13}{2}$ or 6.5 – must see all three values so do check the table for possibly stating the θ values there. However, just stating that the 3 is the pivot because it is the smallest θ value (without seeing anywhere these θ values) is B0			



Question	Scheme				Marks	AOs



Question	Scheme	Marks	AOs																																													
7(a)	Maximise ($P =$) $2x + y + 3z$	B1	3.4																																													
	$x + 2y + 3z \leq 45$	B1	3.4																																													
	$3x + 2y \geq 9$	B1	3.4																																													
	$-x + 4z \geq 4$	B1	1.1b																																													
		(4)																																														
(b)	$A = -(a_1 + a_2) \Rightarrow -(9 - 3x - 2y + s_2 + 4 + x - 4z + s_3)$	M1	2.1																																													
	$A - 2x - 2y - 4z + s_2 + s_3 = -13$ therefore bottom row of the table is <table><tr><td>A</td><td>-2</td><td>-2</td><td>-4</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>-13</td></tr></table>	A	-2	-2	-4	0	1	1	0	0	-13	A1	2.2a																																			
A	-2	-2	-4	0	1	1	0	0	-13																																							
		(2)																																														
(c)(i)	In the given tableau the value of the objective A is equal to zero indicating that a basic feasible solution has been found	B1	2.4																																													
(c)(ii)	$x = 3, y = 0, z = \frac{7}{4}, s_1 = \frac{147}{4}, s_2 = s_3 = 0$	B1 B1	3.4 1.1b																																													
		(3)																																														
(d)	<table><tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>s₁</td><td>s₂</td><td>s₃</td><td>Value</td><td>Row Ops</td></tr><tr><td>s₂</td><td>0</td><td>$\frac{10}{7}$</td><td>0</td><td>$\frac{12}{7}$</td><td>1</td><td>$\frac{9}{7}$</td><td>63</td><td>$R1 \div \frac{7}{12}$</td></tr><tr><td>x</td><td>1</td><td>$\frac{8}{7}$</td><td>0</td><td>$\frac{4}{7}$</td><td>0</td><td>$\frac{3}{7}$</td><td>24</td><td>$R2 + \frac{1}{3}R1$</td></tr><tr><td>z</td><td>0</td><td>$\frac{2}{7}$</td><td>1</td><td>$\frac{1}{7}$</td><td>0</td><td>$-\frac{1}{7}$</td><td>7</td><td>$R3 + \frac{1}{12}R1$</td></tr><tr><td>P</td><td>0</td><td>$\frac{15}{7}$</td><td>0</td><td>$\frac{11}{7}$</td><td>0</td><td>$\frac{3}{7}$</td><td>69</td><td>$R4 + \frac{11}{12}R1$</td></tr></table>	b.v	x	y	z	s ₁	s ₂	s ₃	Value	Row Ops	s ₂	0	$\frac{10}{7}$	0	$\frac{12}{7}$	1	$\frac{9}{7}$	63	$R1 \div \frac{7}{12}$	x	1	$\frac{8}{7}$	0	$\frac{4}{7}$	0	$\frac{3}{7}$	24	$R2 + \frac{1}{3}R1$	z	0	$\frac{2}{7}$	1	$\frac{1}{7}$	0	$-\frac{1}{7}$	7	$R3 + \frac{1}{12}R1$	P	0	$\frac{15}{7}$	0	$\frac{11}{7}$	0	$\frac{3}{7}$	69	$R4 + \frac{11}{12}R1$	M1 A1 M1 A1 A1	2.1 1.1b 1.1b 1.1b 1.1b
b.v	x	y	z	s ₁	s ₂	s ₃	Value	Row Ops																																								
s ₂	0	$\frac{10}{7}$	0	$\frac{12}{7}$	1	$\frac{9}{7}$	63	$R1 \div \frac{7}{12}$																																								
x	1	$\frac{8}{7}$	0	$\frac{4}{7}$	0	$\frac{3}{7}$	24	$R2 + \frac{1}{3}R1$																																								
z	0	$\frac{2}{7}$	1	$\frac{1}{7}$	0	$-\frac{1}{7}$	7	$R3 + \frac{1}{12}R1$																																								
P	0	$\frac{15}{7}$	0	$\frac{11}{7}$	0	$\frac{3}{7}$	69	$R4 + \frac{11}{12}R1$																																								
		(5)																																														
(e)(i)	Yes, an optimal solution has been found as there are no negative values in the objective (P) row	B1	2.4																																													
(e)(ii)	$P = 69$	B1ft	3.4																																													
(e)(iii)	$s_2 = 63, x = 24, z = 7$	B1ft	3.4																																													
		(3)																																														
(17 marks)																																																

Notes for Question 7

(a)**B1:** CAO – including maximise (or max)**B1:** CAO (oe)**B1:** CAO (oe)**B1:** CAO (oe)**(b)****M1:** Setting up the new objective and substituting for a_1 and a_2 **A1:** Correct values substituted into Table 1**(c)****B1:** CAO – mention that $A = 0$ **B1:** At least three values stated correctly**B1:** All six values correct (ignore values stated for a_1, a_2 and P)**(d)****M1:** Correct pivot located, attempt to divide row**A1:** Pivot row correct including change of b.v.**M1:** **All** values in one of the non-pivot rows correct **or** one of the non zero and one columns (y , s_1, s_3 or value) correct following through their choice of pivot from column s_2 or s_3 **A1:** Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (y , s_1, s_3 or value) correct**A1:** CAO all values and row operations correctly stated – allow if row operations given in terms of old row 1 – **ignore b.v. column for this mark****(e)(i)****B1:** Correct reasoning of why solution is optimal or using $P = 69 - \frac{15}{7}y - \frac{11}{7}s_1 - \frac{3}{7}s_3$ andmentioning increasing y, s_1, s_3 would decrease P (oe)**(e)(ii)****B1ft:** their value of P – dependent on both M marks in (d)**(e)(iii)****B1ft:** their values of the basic variables **only** – dependent on both M marks in (d)

Qu	Scheme										Marks	AOs	
7(a)	$3x + 4y \leq 20 \Rightarrow 3x + 4y + s_1 = 20$										B1	2.5	
	$3x + y \leq 8 \Rightarrow 3x + y + s_4 = 8$												
	$x \geq 1 \Rightarrow x - s_2 + a_1 = 1$										B1	2.5	
	$2x + 3y \geq 6 \Rightarrow 2x + 3y - s_3 + a_2 = 6$										B1	1.1b	
	$P = 11x + ky - M(a_1 + a_2)$ $a_1 + a_2 = 7 - 3x - 3y + s_2 + s_3$ $P = 11x + ky - M(7 - 3x - 3y + s_2 + s_3)$										M1	2.1	
	$P - (11 + 3M)x - (k + 3M)y + Ms_2 + Ms_3 = -7M$										A1	2.2a	
	e.g.										M1	3.3	
	b.v.	x	y	s ₁	s ₂	s ₃	s ₄	a ₁	a ₂	Value			
	s ₁	3	4	1	0	0	0	0	0	20			
	a ₁	1	0	0	-1	0	0	1	0	1			
	a ₂	2	3	0	0	-1	0	0	1	6		A1	2.2a
	s ₄	3	1	0	0	0	1	0	0	8			
	P	-(11 + 3M)	-(k + 3M)	0	M	M	0	0	0	-7M			
											(7)		
(b)	Using $x = \frac{18}{7}, y = \frac{2}{7}$ or stating $P = \frac{198}{7} + \frac{2}{7}k$										B1	3.4	
	If optimal after the third iteration, then $\frac{11}{7} - \frac{3}{7}k \geq 0$ and $\frac{33}{7} - \frac{2}{7}k \geq 0$										M1	3.1a	
	$(0 <) k \leq \frac{11}{3}$										A1	2.2a	
	Maximum P when $k = \frac{11}{3}$ and $P = 11\left(\frac{18}{7}\right) + \frac{11}{3}\left(\frac{2}{7}\right)$										dM1	3.4	
	Optimal value of P is $\frac{88}{3}$ accept answers from a stated value of k from $(0 <) k \leq \frac{11}{3}$ so $\frac{198}{7} < P \leq \frac{88}{3}$										A1	2.2a	
											(5)		
(12 marks)													

Notes:

If correct they must use two slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable

Please check suffices on s and a terms carefully – they may be in a different order – check that these are consistent

a1B1: Correctly re-writing the two $\leq$ inequalities as equations with slack variables (can be implied by the corresponding two correct rows in Simplex tableau – our row 1 and 4)

a2B1: Correctly re-writing one of the $\geq$ inequalities as an equation with surplus and artificial variables (can be implied by a correct corresponding row in Simplex tableau – our row 2 or 3)

a3B1: Correctly re-writing both $\geq$ inequalities

a1M1: Forming an objective of the form $P = 11x + ky - M(a_1 + a_2)$ and substituting for a_1 and a_2 (we must see the substitution but this does not need to be a correct expression for this mark)

a1A1: CAO for new objective (accept equivalent equation with terms in x and y collected) (M1 A1 may be implied by a correct objective row in the tableau)

a2M1: Any two rows correct on the ft from the candidate's stated equations (ignore b.v. for this mark)

a2A1: CAO (including consistent b.v. column) – note that the candidate's order in which the rows appear in the tableau (and choice of slack variable) may be different

A fully correct tableau implies all marks in (a) provided that there are no errors seen in the formation of the objective function

b1B1: Either using the correct values of x and y in the objective function or stating $P = \frac{198}{7} + \frac{2}{7}k$

b1M1: Considering at least one of the expressions (s_3 or s_4 columns) in the P row that involve k and compare with 0 (accept any correct inequality or equals)

b1A1: Correct range of values for k (condone missing $0 <$) but must have considered both possibilities and chosen $11/3$ (may be implied by subsequent working) (allow $k = 11/3$ stated as the maximum value)

b2dM1: Dependent on previous M mark – using their k in given P which must come from a correct inequality or equation (they may choose a value of k from the correct range e.g. $k = 3$)

b2A1: CAO - Correct value of P for their choice of k

(accept answers in the range $\frac{198}{7} < P \leq \frac{88}{3}$ if $k = 3$ $P = 204/7$)

