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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Transportation Problems (1)

Decision Mathematics 2 · Topic 1

5 questions · 54 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
1.1 NORTH-WEST CORNER METHOD		1 Qs · 9 marks	3
1. Q3	Option 4D: Decision Mathematics 2 June2023	9	4
1.2 STEPPING-STONE METHOD		3 Qs · 36 marks	5
2. Q1	Option 4D: Decision Mathematics 2 June2019	10	6
3. Q3	Option 4D: Decision Mathematics 2 October2020	16	7
4. Q3	Option 4D: Decision Mathematics 2 June2025	10	8
1.3 TRANSPORTATION AS LINEAR PROGRAMMING		1 Qs · 9 marks	9
5. Q5	Option 4D: Decision Mathematics 2 June2022	9	10



QUESTIONS

5 questions · 54 marks · 3 sub-topics



SUB-TOPIC 1.1

NORTH-WEST CORNER METHOD

Sub-topic 1.1 · 1 question · 9 marks · lowest tariff first



3. The table below shows the stock held at each supply point and the stock required at each demand point in a standard transportation problem. The table also shows the cost, in pounds, of transporting the stock from each supply point to each demand point.

	Q	R	S	Supply
A	23	18	12	45
B	8	10	14	27
C	11	14	21	34
D	19	15	11	50
Demand	75	37	44	

The problem is partially described by the linear programming formulation below.

Let x_{ij} be the number of units transported from i to j

where $i \in \{A, B, C, D\}$

$j \in \{Q, R, S\}$ and $x_{ij} \geq 0$

$$\text{Minimise } P = 23x_{AQ} + 18x_{AR} + 12x_{AS} + 8x_{BQ} + 10x_{BR} + 14x_{BS} \\ + 11x_{CQ} + 14x_{CR} + 21x_{CS} + 19x_{DQ} + 15x_{DR} + 11x_{DS}$$

- Write down, as inequalities, the constraints of the linear program. (2)
- Use the north-west corner method to obtain an initial solution to this transportation problem. (1)
- Taking AS as the entering cell, use the stepping-stone method to find an improved solution. Make your route clear. (2)
- Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and the
 - shadow costs
 - improvement indices
 - entering cell and exiting cell(4)

(Total for Question 3 is 9 marks)



SUB-TOPIC 1.2

STEPPING-STONE METHOD

Sub-topic 1.2 · 3 questions · 36 marks · lowest tariff first



1. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of four demand points, P, Q, R and S. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

	P	Q	R	S	Supply
A	15	14	17	11	23
B	10	9	16	12	42
C	11	13	8	10	18
D	15	13	16	17	19
Demand	25	45	12	20	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	P	Q	R	S
A	23			
B	2	40		
C		5	12	1
D				19

Table 2

- (a) Taking DQ as the entering cell, use the stepping-stone method to find an improved solution. Make your method clear. (2)
- (b) Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by stating the
- shadow costs
 - improvement indices
 - route
 - entering cell and exiting cell.
- (4)
- (c) Determine whether the solution obtained from this second iteration is optimal, giving a reason for your answer. (3)
- (d) State the cost of the solution found in (b). (1)

(Total for Question 1 is 10 marks)



3. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to three sales points, P, Q and R. It also shows the number of units held at each supply point and the number of units required at each sales point. A minimum cost solution is required.

	P	Q	R	Supply
A	25	24	17	42
B	7	12	14	68
C	13	11	20	25
D	16	15	13	40
Demand	59	72	44	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	P	Q	R
A	42		
B	17	51	
C		21	4
D			40

Table 2

- (a) Taking AR as the entering cell, use the stepping-stone method to find an improved solution. Make your method clear. (2)
- (b) Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by stating
- shadow costs
 - improvement indices
 - route
 - entering cell and exiting cell.
- (4)
- (c) Determine whether the solution obtained from this second iteration is optimal, giving the reason for your answer. (3)
- (d) Formulate this situation as a linear programming problem. You must define your decision variables and make the objective function and constraints clear. (6)
- (e) Explain why the Simplex algorithm cannot be used to solve transportation linear programming problems such as that formulated in (d). (1)

(Total for Question 3 is 16 marks)



3. The table below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to three demand points, P, Q and R. It also shows the stock held at each supply point and the number of units required at each demand point.

A minimum cost solution is required.

	P	Q	R	Supply
A	27	23	25	35
B	29	30	28	41
C	29	33	26	29
D	32	34	36	45
Demand	57	31	62	

- (a) Use the north-west corner method to obtain an initial solution to this transportation problem. (1)
- (b) Perform one iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and stating the
- shadow costs
 - improvement indices
 - entering cell and exiting cell
- (4)
- (c) Formulate the transportation problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear. (5)

(Total for Question 3 is 10 marks)



SUB-TOPIC 1.3

TRANSPORTATION AS LINEAR PROGRAMMING

Sub-topic 1.3 · 1 question · 9 marks · lowest tariff first



5. A standard transportation problem is described in the linear programming formulation below.

Let x_{ij} be the number of units transported from i to j

where $i \in \{A, B, C, D\}$

$j \in \{R, S, T\}$ and $x_{ij} \geq 0$

Minimise $P = 23x_{AR} + 17x_{AS} + 24x_{AT} + 15x_{BR} + 29x_{BS} + 32x_{BT}$
 $+ 25x_{CR} + 25x_{CS} + 27x_{CT} + 19x_{DR} + 20x_{DS} + 25x_{DT}$

subject to

$$\sum x_{Aj} \leq 34$$

$$\sum x_{Bj} \leq 27$$

$$\sum x_{Cj} \leq 41$$

$$\sum x_{Dj} \leq 18$$

$$\sum x_{iR} \geq 44$$

$$\sum x_{iS} \geq 37$$

$$\sum x_{iT} \geq k$$

Given that the problem is balanced,

- (a) state the value of k . (1)

- (b) Explain precisely what the constraint $\sum x_{iR} \geq 44$ means in the transportation problem. (2)

- (c) Use the north-west corner method to obtain the cost of an initial solution to this transportation problem. (2)

- (d) Perform one iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and the

- shadow costs
- improvement indices
- entering cell and exiting cell.

(4)

(Total for Question 5 is 9 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs																																																												
3(a)	$\sum x_{Aj} \leq 45$ $\sum x_{Bj} \leq 27$ $\sum x_{Cj} \leq 34$ $\sum x_{Dj} \leq 50$ $\sum x_{iQ} \geq 75$ $\sum x_{iR} \geq 37$ $\sum x_{iS} \geq 44$	M1 A1	3.3 2.5																																																												
		(2)																																																													
(b)	<table border="1"> <tr><td></td><td>Q</td><td>R</td><td>S</td></tr> <tr><td>A</td><td>45</td><td></td><td></td></tr> <tr><td>B</td><td>27</td><td></td><td></td></tr> <tr><td>C</td><td>3</td><td>31</td><td></td></tr> <tr><td>D</td><td></td><td>6</td><td>44</td></tr> </table>		Q	R	S	A	45			B	27			C	3	31		D		6	44	B1	1.1b																																								
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D		6	44																																																												
		(1)																																																													
(c)	<table border="1"> <tr><td></td><td>Q</td><td>R</td><td>S</td></tr> <tr><td>A</td><td>$45 - \theta$</td><td></td><td>θ</td></tr> <tr><td>B</td><td>(27)</td><td></td><td></td></tr> <tr><td>C</td><td>$3 + \theta$</td><td>$31 - \theta$</td><td></td></tr> <tr><td>D</td><td></td><td>$6 + \theta$</td><td>$44 - \theta$</td></tr> </table> giving <table border="1"> <tr><td></td><td>Q</td><td>R</td><td>S</td></tr> <tr><td>A</td><td>14</td><td></td><td>31</td></tr> <tr><td>B</td><td>27</td><td></td><td></td></tr> <tr><td>C</td><td>34</td><td></td><td></td></tr> <tr><td>D</td><td></td><td>37</td><td>13</td></tr> </table>		Q	R	S	A	$45 - \theta$		θ	B	(27)			C	$3 + \theta$	$31 - \theta$		D		$6 + \theta$	$44 - \theta$		Q	R	S	A	14		31	B	27			C	34			D		37	13	M1 A1	2.1 2.2a																				
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C	$3 + \theta$	$31 - \theta$																																																													
D		$6 + \theta$	$44 - \theta$																																																												
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		(2)																																																													
(d)	0 -15 -12 -1 <table border="1"> <tr><td></td><td>Q</td><td>R</td><td>S</td></tr> <tr><td>A</td><td>X</td><td>2</td><td>X</td></tr> <tr><td>B</td><td>X</td><td>9</td><td>17</td></tr> <tr><td>C</td><td>X</td><td>10</td><td>21</td></tr> <tr><td>D</td><td>-3</td><td>X</td><td>X</td></tr> </table> <table border="1"> <tr><td></td><td>Q</td><td>R</td><td>S</td></tr> <tr><td>A</td><td>$14 - \theta$</td><td></td><td>$31 + \theta$</td></tr> <tr><td>B</td><td></td><td></td><td></td></tr> <tr><td>C</td><td></td><td></td><td></td></tr> <tr><td>D</td><td>θ</td><td></td><td>$13 - \theta$</td></tr> </table> giving <table border="1"> <tr><td></td><td>Q</td><td>R</td><td>S</td></tr> <tr><td>A</td><td>1</td><td></td><td>44</td></tr> <tr><td>B</td><td>27</td><td></td><td></td></tr> <tr><td>C</td><td>34</td><td></td><td></td></tr> <tr><td>D</td><td>13</td><td>37</td><td></td></tr> </table> Entering cell is DQ and exiting cell is DS		Q	R	S	A	X	2	X	B	X	9	17	C	X	10	21	D	-3	X	X		Q	R	S	A	$14 - \theta$		$31 + \theta$	B				C				D	θ		$13 - \theta$		Q	R	S	A	1		44	B	27			C	34			D	13	37		M1 A1 M1 A1	2.1 1.1b 1.1b 2.2a
	Q	R	S																																																												
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A	1		44																																																												
B	27																																																														
C	34																																																														
D	13	37																																																													
		(4)																																																													
(9 marks)																																																															



Notes:

(a) M1: At least five equations or inequalities with unit coefficients (with at least 3 correct)

A1: CAO must be inequalities – check signs and notation carefully. If individual terms written out in full check suffices.

(b) B1: CAO for north-west corner method

(c) M1: A valid route shown, AS chosen as entering cell, only one empty square used, θ s balance

A1: CAO (with no zero in cell CR)

(d) M1: Finding 7 shadow costs and 6 improvement indices (no additional zeroes but condone if clearly differentiated)

A1: CAO (Alternative shadow costs: columns 0, -7, -11 rows 23, 8, 11, 22)

M1: A valid route shown, their most negative II chosen, only one empty square used, θ s balance

A1: cao – including the deduction of both entering and exiting cells



Question	Scheme										Marks	AOs																													
1 (a)	B					B	2	40			1A1	1.1b																													
	C		$5 - \theta$		$1 + \theta$	C			12	6																															
	D		θ		$19 - \theta$	D		5		14																															
											(2)																														
(b)	15141618 <table><tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr><tr><td>0</td><td>A</td><td>X</td><td>0</td><td>1</td><td>-7</td></tr><tr><td>-5</td><td>B</td><td>X</td><td>X</td><td>5</td><td>-1</td></tr><tr><td>-8</td><td>C</td><td>4</td><td>7</td><td>X</td><td>X</td></tr><tr><td>-1</td><td>D</td><td>1</td><td>X</td><td>1</td><td>X</td></tr></table>											P	Q	R	S	0	A	X	0	1	-7	-5	B	X	X	5	-1	-8	C	4	7	X	X	-1	D	1	X	1	X	1M1	1.1b
		P	Q	R	S																																				
	0	A	X	0	1	-7																																			
	-5	B	X	X	5	-1																																			
	-8	C	4	7	X	X																																			
	-1	D	1	X	1	X																																			
										1A1	1.1b																														
	<table><tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr><tr><td>A</td><td>$23 - \theta$</td><td></td><td></td><td>θ</td></tr><tr><td>B</td><td>$2 + \theta$</td><td>$40 - \theta$</td><td></td><td></td></tr><tr><td>C</td><td></td><td></td><td></td><td></td></tr><tr><td>D</td><td></td><td>$5 + \theta$</td><td></td><td>$14 - \theta$</td></tr></table> Entering cell is AS and exiting cell is DS											P	Q	R	S	A	$23 - \theta$			θ	B	$2 + \theta$	$40 - \theta$			C					D		$5 + \theta$		$14 - \theta$	2M1	1.1b				
		P	Q	R	S																																				
	A	$23 - \theta$			θ																																				
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	C																																								
	D		$5 + \theta$		$14 - \theta$																																				
										2A1	2.2a																														
											(4)																														
(c)	1514911 <table><tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr><tr><td>0</td><td>A</td><td>X</td><td>0</td><td>8</td><td>X</td></tr><tr><td>-5</td><td>B</td><td>X</td><td>X</td><td>12</td><td>6</td></tr><tr><td>-1</td><td>C</td><td>-3</td><td>0</td><td>X</td><td>X</td></tr><tr><td>-1</td><td>D</td><td>1</td><td>X</td><td>8</td><td>7</td></tr></table> A negative II so solution is not optimal											P	Q	R	S	0	A	X	0	8	X	-5	B	X	X	12	6	-1	C	-3	0	X	X	-1	D	1	X	8	7	1M1	2.1
		P	Q	R	S																																				
	0	A	X	0	8	X																																			
	-5	B	X	X	12	6																																			
	-1	C	-3	0	X	X																																			
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										2A1	2.4																														
											(3)																														
(d)	(£)1086										1B1	1.1b																													
											(1)																														
(10 marks)																																									

Notes:

(a) 1M1: A valid route, only one empty square used, θ 's balance.

Note: If not entering in DQ, allow this mark for a valid route and entry in CP or DP only.

1A1: CAO

(b) 1M1: Finding (exactly) 8 shadow costs and (exactly) 9 improvement indices for their improved solution

1A1: Shadow costs [Alt: A(15), B(10), C(7), D(14), P(0), Q(-1), R(1), S(3)] and II CAO. Please check top table for Shadow Costs.

2M1: A valid route, their most negative II chosen, only one empty square used, θ 's balance

2A1: CAO – including the deduction of all entering and exiting cells

(c) 1M1: finding all 8 shadow costs **and** all 9 negative improvement indices **or** sufficient number of shadow costs for at least 1 negative II found (May just see SC: A, C, P and S and II: PC). This mark is dependent on the previous M mark in (b) which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution

1A1: CAO negative II from correct working

2A1: CSO for (a), (b), (c) including the correct reasoning that the solution is not optimal because there is a negative II. Do not allow for 'some IIs are not positive' o.e. [Alt shadow costs: A(15), B(10), C(14), D(14), P(0), Q(-1), R(-6), S(-4)].

(d) 1B1: CAO ignore lack of or incorrect units.

Question	Scheme	Marks	AOs
	Subtracting each entry from a value ≥ 203 e.g. $\begin{bmatrix} 100 & 106 & 129 & 123 \\ 2 & 48 & 58 & 48 \\ 92 & 123 & 126 & 111 \\ 0 & 15 & 66 & 19 \end{bmatrix}$ $\begin{bmatrix} 0 & 6 & 29 & 23 \\ 0 & 46 & 56 & 46 \end{bmatrix}$	1B1	1.1b



Question	Scheme								Marks	AOs																								
3 (a)	B	$17 + \theta$	$51 - \theta$		B	21	47		M1	2.1																								
	C		$21 + \theta$	$4 - \theta$	C		25		A1	1.1b																								
	D			(40)	D			40																										
									(2)																									
(b)	25 30 17 <table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>0</td><td>A</td><td>X</td><td>-6</td><td>X</td></tr><tr><td>-18</td><td>B</td><td>X</td><td>X</td><td>15</td></tr><tr><td>-19</td><td>C</td><td>7</td><td>X</td><td>22</td></tr><tr><td>-4</td><td>D</td><td>-5</td><td>-11</td><td>X</td></tr></table>									P	Q	R	0	A	X	-6	X	-18	B	X	X	15	-19	C	7	X	22	-4	D	-5	-11	X	M1	1.1b
		P	Q	R																														
	0	A	X	-6	X																													
	-18	B	X	X	15																													
-19	C	7	X	22																														
-4	D	-5	-11	X																														
									A1	1.1b																								
	<table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td>$38 - \theta$</td><td></td><td>$4 + \theta$</td></tr><tr><td>B</td><td>$21 + \theta$</td><td>$47 - \theta$</td><td></td></tr><tr><td>C</td><td></td><td>(25)</td><td></td></tr><tr><td>D</td><td></td><td>θ</td><td>$40 - \theta$</td></tr></table>									P	Q	R	A	$38 - \theta$		$4 + \theta$	B	$21 + \theta$	$47 - \theta$		C		(25)		D		θ	$40 - \theta$	M1	1.1b				
		P	Q	R																														
	A	$38 - \theta$		$4 + \theta$																														
	B	$21 + \theta$	$47 - \theta$																															
C		(25)																																
D		θ	$40 - \theta$																															
	Entering cell is DQ and exiting cell is AP								A1	2.2a																								
									(4)																									
(c)	14 19 17 <table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>0</td><td>A</td><td>11</td><td>5</td><td>X</td></tr><tr><td>-7</td><td>B</td><td>X</td><td>X</td><td>4</td></tr><tr><td>-8</td><td>C</td><td>7</td><td>X</td><td>11</td></tr><tr><td>-4</td><td>D</td><td>6</td><td>X</td><td>X</td></tr></table>									P	Q	R	0	A	11	5	X	-7	B	X	X	4	-8	C	7	X	11	-4	D	6	X	X	M1	2.1
		P	Q	R																														
	0	A	11	5	X																													
	-7	B	X	X	4																													
-8	C	7	X	11																														
-4	D	6	X	X																														
									A1	1.1b																								
	No negative IIs so solution is optimal								A1	2.4																								
									(3)																									
(d)	Let x_{ij} be the number of units (of stock) transported from (supply point) i to (sales point) j								B1	3.3																								
	where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R\}$ ($x_{ij} \geq 0$)								B1	3.3																								
	Minimise $25x_{AP} + 24x_{AQ} + 17x_{AR} + 7x_{BP} + 12x_{BQ} + 14x_{BR} + 13x_{CP} + 11x_{CQ} + 20x_{CR} + 16x_{DP} + 15x_{DQ} + 13x_{DR}$								B1	2.5																								
									B1	3.3																								

	$\sum x_{Aj} \leq 42, \sum x_{Bj} \leq 68, \sum x_{Cj} \leq 25, \sum x_{Dj} \leq 40$	accept =	B1	3.3
	$\sum x_{iP} \geq 59, \sum x_{iQ} \geq 72, \sum x_{iR} \geq 44$	accept =	B1	3.3
			(6)	
(e)	The Simplex algorithm cannot be used as not all the constraints are in the form $\sum x \leq k$ where k is a positive constant		B1	3.5b
			(1)	
(16 marks)				

Notes:

- (a) M1:** a valid route, only one empty square used, θ 's balance
A1: cao
- (b) M1:** Finding all 7 shadow costs and the 6 improvement indices for the correct 9 entries
A1: Shadow costs and II CAO
M1: A valid route, their most negative II chosen, only one empty square used, θ 's balance
A1: CAO – including the deduction of all entering and exiting cells
- (c) M1:** finding all 7 shadow costs **and** the 6 improvement indices – this mark is dependent on the previous M mark in (b) which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution
A1: CAO (shadow costs and IIs)
A1: CSO including the correct reasoning that the solution is optimal because there are no negative II
- (d) B1:** Correct definition of x_{ij}
B1: Correctly defining the set of values that i and j can take
B1: 'Minimise' + correct number of terms in objective
B1: Correct objective function
B1: Correct supply constraints (allow 'equals' or written out in full e.g. $x_{AP} + x_{AQ} + x_{AR} \leq 42$)
B1: Correct demand constraints (allow 'equals' or written out in full)
- (e) B1:** Correct justification of why the Simplex algorithm cannot be used to solve transportation LP

Question	Scheme	Marks	AOs
4(a)	CF of $u_n = A(2)_n + B(-1)_n \Rightarrow$ auxiliary equation is		
	$(m-2)(m+1) = 0$	M1	3.1a
	$m^2 - m - 2 = 0 \Rightarrow \alpha = -1, \beta = -2$	A1	2.2a



		(2)	
(b)	particular solution try $u_n = \lambda (-3)_n, u_{n+1} = \lambda (-3)_{n+1}, u_{n+2} = \lambda (-3)_{n+2}$ $9\lambda + 3\lambda - 2\lambda = 20 \Rightarrow \lambda = 2$ $u_n = A(2)_n + B(-1)_n + 2(-3)_n$ $2u_0 = u_1 \Rightarrow 2A + 2B + 4 = 2A - B - 6$ $u_4 = 164 \Rightarrow 16A + B + 162 = 164$ $A = \frac{1}{3}, B = -\frac{10}{3} \Rightarrow u_n = \frac{1}{3}(2)^n - \frac{10}{3}(-1)^n + 2(-3)^n$	M1 M1 A1 M1 M1 A1	2.1 1.1b 1.1b 1.1b 1.1b 2.2a
		(6)	
(8 marks)			
Notes:			
(a) M1: Uses given complementary function to find auxiliary equation corresponding to second-order recurrence relation A1: cao for both α and β (b) M1: substitute $u_n = \lambda (-3)_n$ into their second-order recurrence relation M1: forms linear equation in λ only A1: correct general solution M1: use $2u_0 = u_1$ to form an equation in B (and possibly A) M1: use $u_4 = 164$ to set up a second equation in A and B A1: cao			



Ques tion	Scheme	Marks	AOs																																																																
3 (a)	<table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td>35</td><td></td><td></td></tr><tr><td>B</td><td>22</td><td>19</td><td></td></tr><tr><td>C</td><td></td><td>12</td><td>17</td></tr><tr><td>D</td><td></td><td></td><td>45</td></tr></table>		P	Q	R	A	35			B	22	19		C		12	17	D			45	B1	1.1b																																												
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(b)	0 2 5 15 <table><tr><td></td><td>27</td><td>28</td><td>21</td></tr><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td>X</td><td>− 5</td><td>4</td></tr><tr><td>B</td><td>X</td><td>X</td><td>5</td></tr><tr><td>C</td><td>− 3</td><td>X</td><td>X</td></tr><tr><td>D</td><td>− 10</td><td>− 9</td><td>X</td></tr></table> <table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td></td><td></td><td></td></tr><tr><td>B</td><td>22 − θ</td><td>19 + θ</td><td></td></tr><tr><td>C</td><td></td><td>12 − θ</td><td>17 + θ</td></tr><tr><td>D</td><td>θ</td><td></td><td>45 − θ</td></tr></table><table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td>35</td><td></td><td></td></tr><tr><td>B</td><td>10</td><td>31</td><td></td></tr><tr><td>C</td><td></td><td></td><td>29</td></tr><tr><td>D</td><td>12</td><td></td><td>33</td></tr></table> Entering cell is DP and exiting cell is CQ		27	28	21		P	Q	R	A	X	− 5	4	B	X	X	5	C	− 3	X	X	D	− 10	− 9	X		P	Q	R	A				B	22 − θ	19 + θ		C		12 − θ	17 + θ	D	θ		45 − θ		P	Q	R	A	35			B	10	31		C			29	D	12		33	M1 A1 M1 A1	2.1 1.1b 1.1b 2.2a
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(c)	Let x_{ij} be the number of units (of stock) transported from (supply point) i to (demand point) j	B1	3.3																																																																
	where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R\} \Big(x_{ij} \geq 0 \Big)$	B1	2.5																																																																
	Minimise $27x_{AP} + 23x_{AQ} + 25x_{AR} + 29x_{BP} + 30x_{BQ} + 28x_{BR} + 29x_{CP} + 33x_{CQ} + 26x_{CR} + 32x_{DP} + 34x_{DQ} + 36x_{DR}$	B1	3.3																																																																
	$\sum x_{Aj} \leq 35, \sum x_{Bj} \leq 41, \sum x_{Cj} \leq 29, \sum x_{Dj} \leq 45$ accept =	B1	3.3																																																																
	$\sum x_{iP} \geq 57, \sum x_{iQ} \geq 31, \sum x_{iR} \geq 62$ accept =	B1	3.3																																																																
		(5)																																																																	
(10 marks)																																																																			



Notes for Question 3

(a)**B1:** CAO for north-west corner method**(b)****M1:** Finding 7 shadow costs and 6 improvement indices**A1:** Shadow costs and II correct (Alternative SC rows 27, 29, 32, 42 columns 0, 1, -6)**M1:** A valid route, their most negative II chosen, only one empty square used, θ 's balance**A1:** CSO (for (b)) so all previous marks in this part must have been awarded – including exiting and entering cells stated correctly. Improved solution must be 6 numbers only with no additional 0 in CQ**(c)** Check all suffixes carefully for accuracy and consistency**B1:** Correct definition of x_{ij} must clearly state that this is the **number** of units transported**B1:** Correctly defining the set of values that i and j can take**B1:** 'Minimise' + correct objective function**B1:** Correct supply constraints with unit coefficients(allow 'equals' or written out in full e.g. $x_{AP} + x_{AQ} + x_{AR} \leq 35$)**B1:** Correct demand constraints with unit coefficients(allow 'equals' or written out in full e.g. $x_{AP} + x_{BP} + x_{CP} + x_{DP} \geq 57$)

Question	Scheme	Marks	AOs																																																												
5(a)	$k = 39$	B1	2.2a																																																												
		(1)																																																													
(b)	To ensure that the total amount transported to destination R from the four supply points cannot be less than the demand of 44	B2, 1, 0	2.4 2.4																																																												
		(2)																																																													
(c)	<table border="1"> <tr><td></td><td>R</td><td>S</td><td>T</td></tr> <tr><td>A</td><td>34</td><td></td><td></td></tr> <tr><td>B</td><td>10</td><td>17</td><td></td></tr> <tr><td>C</td><td></td><td>20</td><td>21</td></tr> <tr><td>D</td><td></td><td></td><td>18</td></tr> </table>		R	S	T	A	34			B	10	17		C		20	21	D			18	B1	1.1b																																								
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B	10	17																																																													
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	2942	B1	2.2a																																																												
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(d)	0 -8 -12 -14 <table border="1"> <tr><td></td><td>R</td><td>S</td><td>T</td></tr> <tr><td>A</td><td>X</td><td>-20</td><td>-15</td></tr> <tr><td>B</td><td>X</td><td>X</td><td>1</td></tr> <tr><td>C</td><td>14</td><td>X</td><td>X</td></tr> <tr><td>D</td><td>10</td><td>-3</td><td>X</td></tr> </table> <table border="1"> <tr><td></td><td>R</td><td>S</td><td>T</td></tr> <tr><td>A</td><td>$34 - \theta$</td><td>θ</td><td></td></tr> <tr><td>B</td><td>$10 + \theta$</td><td>$17 - \theta$</td><td></td></tr> <tr><td>C</td><td></td><td></td><td></td></tr> <tr><td>D</td><td></td><td></td><td></td></tr> </table> <table border="1"> <tr><td></td><td>R</td><td>S</td><td>T</td></tr> <tr><td>A</td><td>17</td><td>17</td><td></td></tr> <tr><td>B</td><td>27</td><td></td><td></td></tr> <tr><td>C</td><td></td><td>20</td><td>21</td></tr> <tr><td>D</td><td></td><td></td><td>18</td></tr> </table> Entering cell is AS and exiting cell is BS		R	S	T	A	X	-20	-15	B	X	X	1	C	14	X	X	D	10	-3	X		R	S	T	A	$34 - \theta$	θ		B	$10 + \theta$	$17 - \theta$		C				D					R	S	T	A	17	17		B	27			C		20	21	D			18	M1 A1 M1 A1	2.1 1.1b 1.1b 2.2a
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		(4)																																																													
(9 marks)																																																															
Notes:																																																															
(a) B1: CAO (b) B1: Partial correct reasoning – must include at least two of ‘destination R’, ‘supply points’, ‘cannot be less’/‘must be at least’, ‘demand of 44’ oe (do not accept ‘greater than or equal to’) B1: Fully correct reasoning – all points covered as stated above. No incorrect statement. (c) B1: CAO for north-west corner method (six correct figures in correct cells only, no zeros) B1: CAO for initial solution (2942) (d) M1: Finding 7 shadow costs and 6 improvement indices A1: CAO																																																															

M1: A valid route shown, their most negative II chosen, only one empty square used, θ 's balance

A1:cao – (no zeros) including deducing entering and exiting cells

For reference:

	R	S	T	Supply
A	23	17	24	34
B	15	29	32	27
C	25	25	27	41
D	19	20	25	18
Demand	44	37	k	

