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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Flows in Networks (3)

Decision Mathematics 2 · Topic 3

4 questions · 47 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
3.1 CUTS AND CAPACITIES		1 Qs · 10 marks	3
1. Q1	Option 4D: Decision Mathematics 2 June2024	10	4
3.4 MULTIPLE SOURCES, SINKS AND RESTRICTED NODES		3 Qs · 37 marks	5
2. Q5	Option 4D: Decision Mathematics 2 October2020	10	6
3. Q5	Option 4D: Decision Mathematics 2 November2021	16	7
4. Q4	Option 4D: Decision Mathematics 2 June2025	11	9

QUESTIONS

4 questions · 47 marks · 2 sub-topics



SUB-TOPIC 3.1

CUTS AND CAPACITIES

Sub-topic 3.1 · 1 question · 10 marks · lowest tariff first



1.

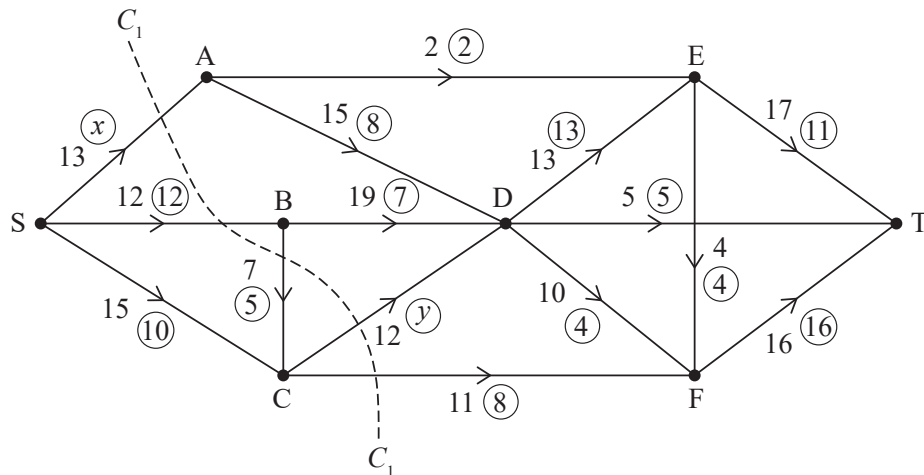


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The numbers in circles represent an initial flow from S to T. The other number on each arc represents the capacity, in litres per second, of the corresponding pipe.

(a) (i) State the value of x

(ii) State the value of y

(2)

(b) State the value of the initial flow.

(1)

(c) State the capacity of cut C_1

(1)

(d) Find, by inspection, a flow-augmenting route to increase the flow by four units. You must state your route.

(1)

The flow-augmenting route from (d) is used to increase the flow from S to T.

(e) Prove that the flow is now maximal.

(3)

A vertex restriction is now applied so that no more than 12 litres per second can flow through E.

(f) (i) Complete Diagram 1 in the answer book to show this restriction.

(ii) State the **value** of the maximum flow through the network with this restriction.

(2)

(Total for Question 1 is 10 marks)



SUB-TOPIC 3.4

MULTIPLE SOURCES, SINKS AND RESTRICTED NODES

Sub-topic 3.4 · 3 questions · 37 marks · lowest tariff first



5.

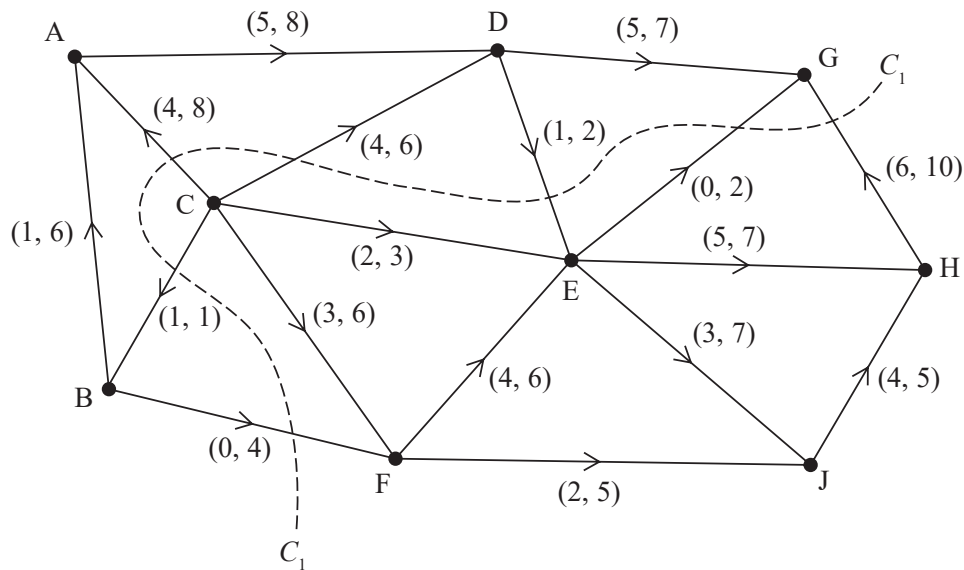


Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower capacities and upper capacities for the corresponding pipes, in litres per second.

- State the source node. (1)
- Explain why the sink node must be G. (1)
- Calculate the capacity of the cut C_1 . (1)
- Assuming that a feasible flow exists,
 - explain why arc JH must be at its upper capacity,
 - explain why arcs AD and CD must be at their lower capacities. (2)
- Use Diagram 1 in the answer book to show a flow of 18 litres per second through the system. (2)
- Prove that the answer to (e) is the maximum flow through the system. (3)

(Total for Question 5 is 10 marks)



5.

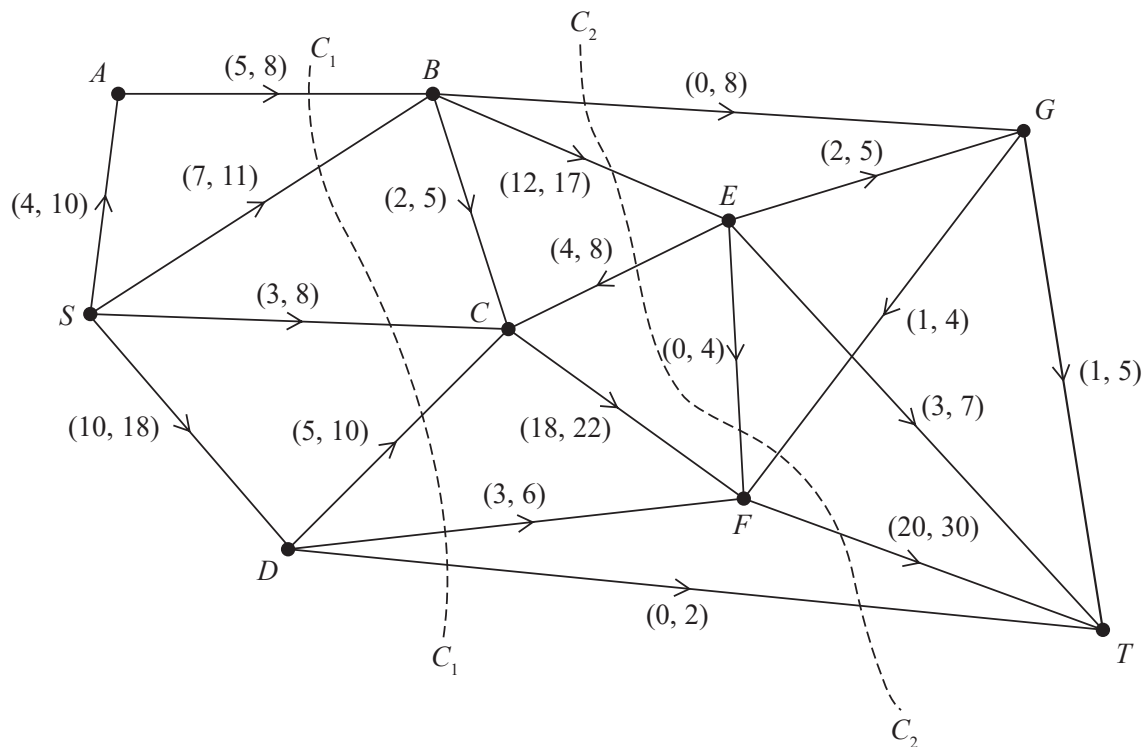


Figure 1

Figure 1 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

(ii) cut C_2

(2)

(b) Using only the capacities of cuts C_1 and C_2 , state what can be deduced about the maximum flow through the system.

(1)

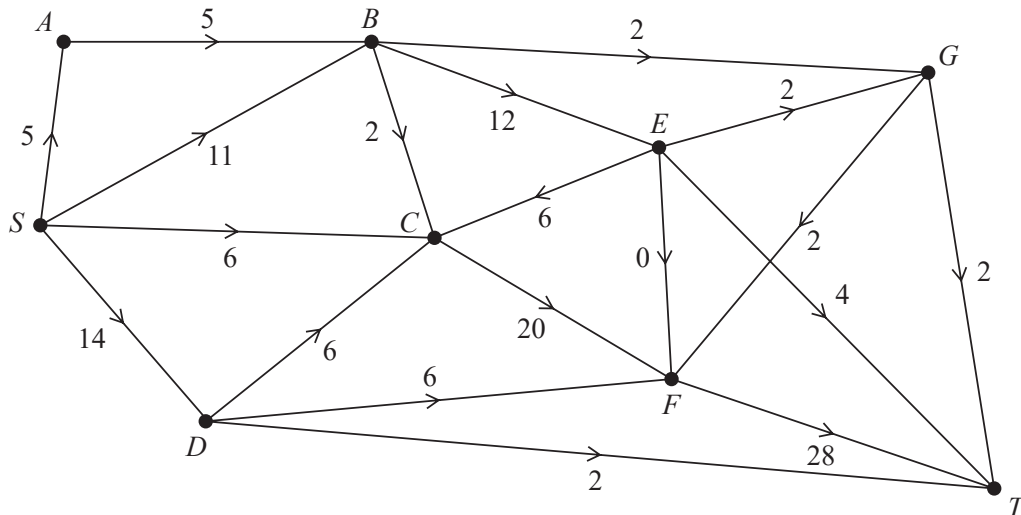
**Figure 2**

Figure 2 shows an initial flow through the same network.

- (c) State the value of the initial flow. (1)
- (d) By entering values along BC , CF and DT , complete the labelling procedure on Diagram 1 in the answer book. (2)
- (e) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)
- (f) Use your answer to (e) to find a maximum flow pattern for this system of pipes and draw it on Diagram 2 in the answer book. (1)
- (g) Prove that the answer to (f) is optimal. (3)
- A vertex restriction is now applied to B so that no more than 16 litres per second can flow through it.
- (h) (i) Complete Diagram 3 in the answer book to show this restriction. (3)
- (ii) State the value of the maximum flow with this restriction.

(Total for Question 5 is 16 marks)

4.

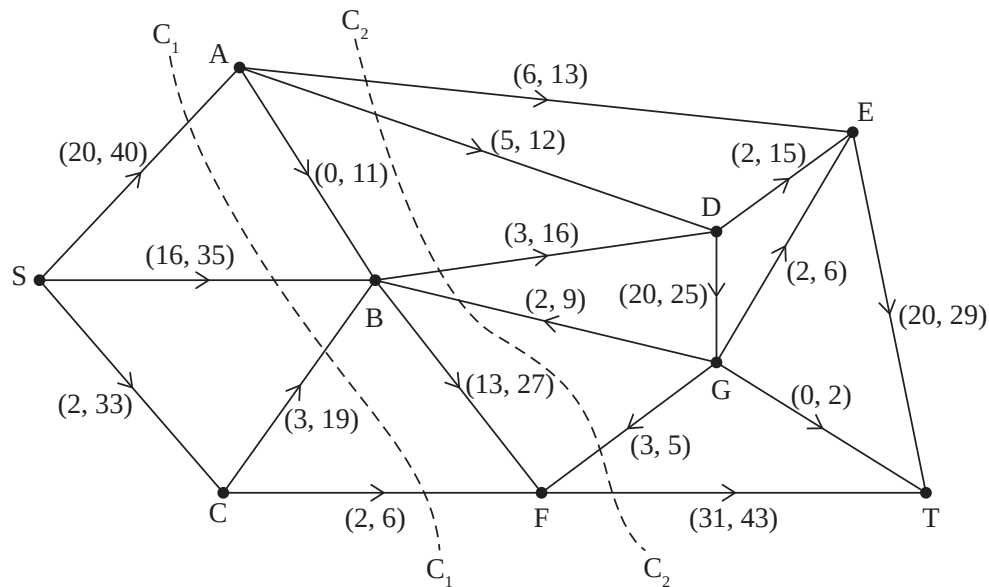


Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

(ii) cut C_2

(2)

(b) Using only the capacities of cuts C_1 and C_2 , state what can be deduced about the maximum flow through the system.

(1)



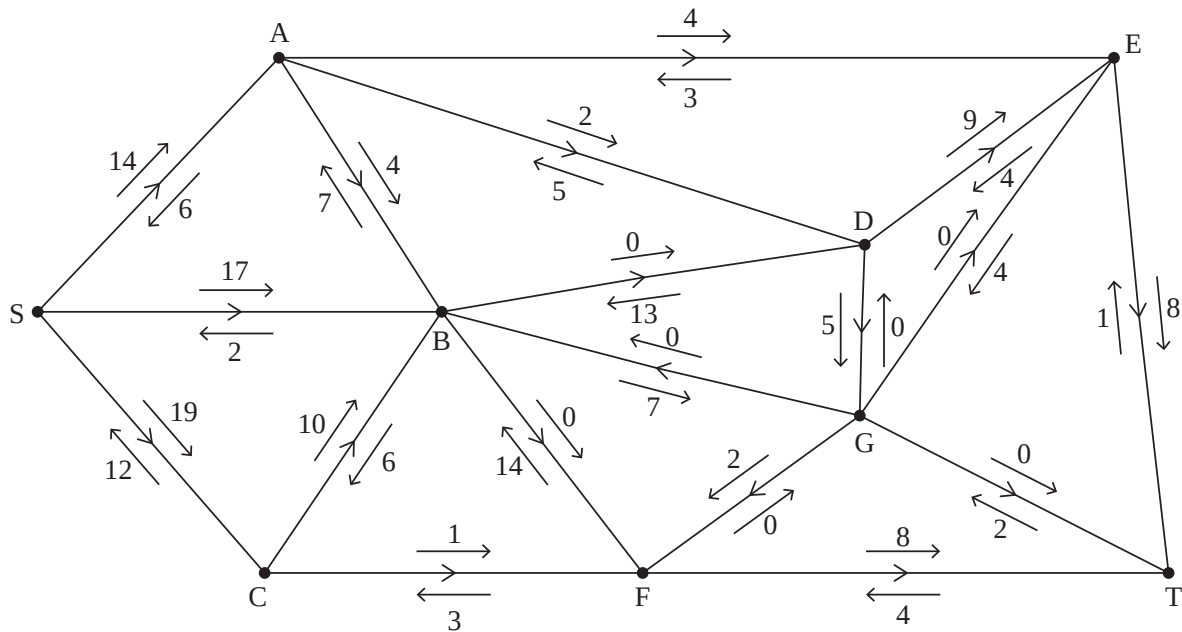


Figure 3

A scientist monitors the system of pipes in which a fluid now flows from the source, S, to the sink, T.

The scientist initialises the labelling procedure for this system. The excess and potential backflows for this are shown on the arrows either side of each arc in Figure 3.

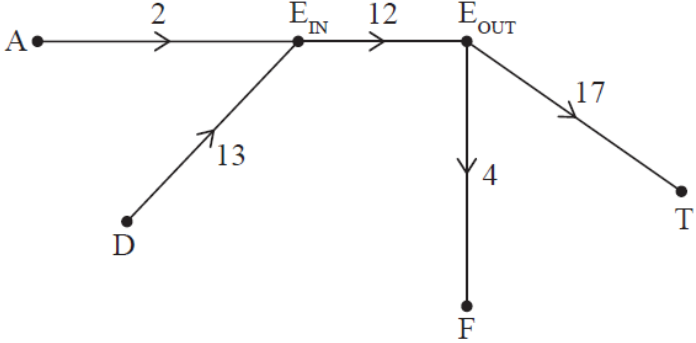
- (c) State the value of the initial flow. (1)
- (d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)
- (e) Use your answer to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book. (1)
- (f) Prove that the answer to part (e) is optimal. (3)

(Total for Question 4 is 11 marks)

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
1(a)	(i) $x = 10$	B1	1.1b
	(ii) $y = 7$	B1	1.1b
		(2)	
(b)	32	B1	1.1b
		(1)	
(c)	Cut $C_1 (= 13 + 12 + 12 + 11) = 48$	B1	1.1b
		(1)	
(d)	SCDFET	B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem Identification of cut through AE, DE, DT, EF and FT Value of flow = 36 It follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
(f)		B1	3.3
	(ii) maximum flow = 33	B1ft	2.2a
		(2)	
(10 marks)			

Notes for Question 1

(a) **B1:** CAO for x

B1: CAO for y

(b) **B1:** CAO

(c) **B1:** CAO

(d) **B1:** CAO

(e) **M1:** Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through **saturated** arcs). The cut may be drawn or stated in terms of arcs but not as nodes. (Note the only saturated arc not in the cut is SB)

A1: Use appropriate process of finding a minimum cut: cut + value correct

dA1: Must have stated the value of the flow and correct deduction that the flow is maximal.

Must use max flow = min cut all 4 words

dependent on previous A mark so M1 A0 A1 is not possible

(f) **B1:** Flows into E go to E_{IN} and flows out of E go from E_{OUT} and arc of capacity 12 from E_{IN} to E_{OUT} . All arcs must have the correct arrow and capacity shown. Split node must be labelled as E_{IN} and E_{OUT} or E_1 and E_2

B1ft: Value of their maximum flow – 3

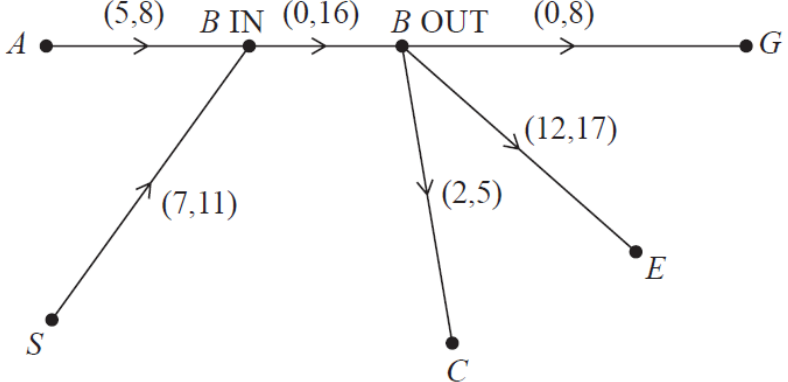


Question	Scheme	Marks	AOs
5 (a)	Source node is C	B1	1.1b
		(1)	
(b)	G is the sink node as all the arcs incident to G flow into G	B1	2.4
		(1)	
(c)	Capacity of cut $C_1 = 10 + 2 - 1 + 6 + 8 + 1 - 0 = 26$	B1	1.1b
		(1)	
(d)(i)	Arc JH must be at its upper capacity of 5 as the two arcs that flow into J (EJ and FJ) have a lower capacity of $2 + 3 = 5$	B1	2.4
(ii)	Arcs AD and CD must be at the lower capacities (which in total is 9) as the only two arcs (DG and DE) that flow out of D have a total upper capacity of $7 + 2 = 9$	B1	2.4
		(2)	
(e)		M1 A1	2.2a 1.1b
		(2)	
(f)	Use of max-flow min-cut theorem Identification of cut through DG, DE, CE, CF, CB, BA with a capacity of 18 and value of flow = 18 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
(10 marks)			
Notes:			

- (a) **B1:** cao (node C)
- (b) **B1:** correct explanation of why G is the sink node
- (c) **B1:** cao
- (d)(i) **B1:** correct explanation that JH must be at its upper capacity (must refer to arcs EJ and FJ)
- (d)(ii) **B1:** correct explanation that AD and CD must be at their lower capacities (must refer to arcs DG and DE)
- (e) **M1:** 'flow in = flow out' at all but one vertex – one number only required on each arc (condone blank for arc BF)
- A1:** a correct valid flow through the network (check that flow in must equal flow out at each vertex)
- (f) **M1:** Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs)
- A1:** Use appropriate process of finding a minimum cut (cut + value correct)
- A1:** Correct deduction that the flow is maximal



Question	Scheme	Marks	AOs
5 (a)(i)	$C_1 = 8 + 11 + 8 + 10 + 6 + 2 = 45$	B1	1.1b
(ii)	$C_2 = 8 + 17 - 4 - 0 - 1 + 30 + 2 = 52$	B1	1.1b
		(2)	
(b)	Deduces the maximum possible flow is ≤ 45 litres per second	B1ft	2.2a
		(1)	
(c)	Initial flow = 36	B1	1.1b
		(1)	
(d)		M1 A1	1.1b 1.1b
		(2)	
(e)	e.g., SABGT – 2, SCFT – 2, SDCET – 2, SABET – 1 e.g., SABGT – 3, SCFT – 2, SDCET – 2	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(f)	e.g.	B1	2.2a
		(1)	

(g)	Use of max-flow min-cut theorem Identification of cut through AB, SB, BC, EC, CF, DF and DT Value of flow = 43 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
(h)	(i) 	B1 B1	3.3 3.3
	(ii) maximum flow = 40	B1ft	2.2a
		(3)	

(16 marks)

Notes:**(a)(i) B1:** cao**(ii) B1:** cao**(b) B1ft:** Deduced from their least value given in **(a)** - must include 'less than or equal to' (oe)**(c) B1:** cao**(d) M1:** Two numbers on each arc and at least two arcs or four numbers correct (so correct numbers with the correct arrows)**A1:** cao do give bod since they might well cross these numbers out**(e) M1:** One flow augmenting route found from S to T**A1:** Two correct routes + flow values**A1:** cso – increasing the flow by 7**(f) B1:** cao**(g) M1:** Construct argument based on max-flow min-cut theorem (e.g., attempt to find a cut through saturated arcs)**A1:** Use appropriate process of finding a minimum cut: cut + value correct**A1:** Correct deduction that the flow is maximal**(h) B1:** Flows into B go to B_{IN} and flows out of B go from B_{OUT}**B1:** Arc of capacity 16 from B_{IN} to B_{OUT}**B1ft:** value of their maximum flow – 3

Question	Scheme	Marks	AOs
4(a)	$C_1 (= 40 + 35 + 19 + 6) = 100$	B1	1.1b
	$C_2 (= 13 + 12 + 16 - 2 - 3 + 43) = 79$	B1	1.1b
		(2)	
(b)	Deduces the maximum possible flow is ≤ 79 litres per second	B1ft	2.2a
		(1)	
(c)	Initial flow = 58	B1	1.1b
		(1)	
(d)	e.g. SAET – 4, SADET – 2, SBGFT – 2, SCFT – 1	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(e)	e.g. 	B1	2.2a
		(1)	
(f)	Use of max-flow min-cut theorem Identification of cut through AE, AD, BD, DG, GE, GT, GF, BF and CF Value of flow = 67 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
(11 marks)			

Notes for Question 4

(a)**B1:** CAO**B1:** CAO**(b)****B1ft:** deduced from their least value given in (a) - must include 'less than or equal to' (oe)**(c)****B1:** CAO**(d)****M1:** One flow augmenting route found from S to T**A1:** Two correct routes + flow values**A1:** CSO – increasing the flow by 9

Note possible flow augmenting routes are only $_ADET + 2_AET + 4_BGFT + 2_CFT + 1$ where $_$ represents either S, SB, SC or SCB

(SADGFT + 2 is also valid but this prevents DET being increased)

(e)

B1: CAO – one number only per arc (only SA, SB, SC, AB and CB can vary depending on their flow augmenting routes)

(f)

M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through **saturated** arcs) (A valid cut through saturated arcs must either be listed or drawn)

May be drawn on either diagram or listed as arcs or set notation $\{S, A, B, C, G\}\{D, E, F, T\}$

A1: Use appropriate process of finding a minimum cut: cut + value correct and value of flow stated

A1: Correct deduction that the flow is maximal – must see all 4 words max flow min cut and conclusion - dependent on previous A mark

