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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Game Theory (5)

Decision Mathematics 2 · Topic 5

4 questions · 60 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
5.4 MIXED STRATEGIES (GRAPHICAL)		1 Qs · 15 marks	3
1. Q6	Option 4D: Decision Mathematics 2 June2025	15	4
5.5 MIXED STRATEGIES VIA LINEAR PROGRAMMING		3 Qs · 45 marks	5
2. Q4	Option 4D: Decision Mathematics 2 June2019	14	6
3. Q6	Option 4D: Decision Mathematics 2 October2020	14	7
4. Q8	Option 4D: Decision Mathematics 2 June2023	17	8



QUESTIONS

4 questions · 60 marks · 2 sub-topics



SUB-TOPIC 5.4

MIXED STRATEGIES (GRAPHICAL)

Sub-topic 5.4 · 1 question · 15 marks · lowest tariff first



6.

		Player B		
		Option P	Option Q	Option R
Player A	Option X	4	-2	-5
	Option Y	-1	1	3

A two person zero-sum game is represented by the pay-off matrix for player A shown above.

(a) Verify that there is no stable solution to this game.

(2)

Let player A play option X with probability p

(b) Use a graphical method to find the optimal value of p and hence find the best strategy for player A in this game.

(6)

A third option, Z, is added to player A's options. Option Z has the pay-offs shown in the matrix below.

		Player B		
		Option P	Option Q	Option R
Player A	Option X	4	-2	-5
	Option Y	-1	1	3
	Option Z	4	-1	1

Player A now intends to make a random choice between options X, Y and Z, choosing option X with probability x , option Y with probability y and option Z with probability z

Player A decides to use the Simplex algorithm to find the optimal values of x , y and z

(c) Determine an initial Simplex tableau to solve this 3×3 game, making your variables clear.

(4)

In the optimal Simplex tableau for this 3×3 game, $x = 0$ and $y = \frac{5}{7}$

(d) By considering the values of the two games, determine how much better the 3×3 game (with option Z) is for player A compared with the 2×3 game (without option Z).

You must make your reasoning clear.

(3)

(Total for Question 6 is 15 marks)



SUB-TOPIC 5.5

MIXED STRATEGIES VIA LINEAR PROGRAMMING

Sub-topic 5.5 · 3 questions · 45 marks · lowest tariff first



4.

		Player B		
		Option X	Option Y	Option Z
Player A	Option P	3	-2	0
	Option Q	-4	4	-2
	Option R	1	2	-1

A two person zero-sum game is represented by the pay-off matrix for player A shown above.

(a) Verify that there is no stable solution to this game.

(2)

Player A intends to make a random choice between options P, Q and R, choosing option P with probability p_1 , option Q with probability p_2 and option R with probability p_3 .

Player A wants to find the optimal values of p_1 , p_2 and p_3 using the Simplex algorithm. Player A formulates the following linear programming problem for the game, writing the constraints as inequalities.

Maximise $P = V$

subject to $V \geq 3p_1 - 4p_2 + p_3$
 $V \geq -2p_1 + 4p_2 + 2p_3$
 $V \geq -2p_2 - p_3$
 $p_1 + p_2 + p_3 \leq 1$
 $p_1 \geq 0, p_2 \geq 0, p_3 \geq 0, V \geq 0$

(b) Correct the errors made by player A in the linear programming formulation of the game, giving reasons for your answer.

(3)

(c) Write down an initial Simplex tableau for the corrected linear programming problem.

(3)

The Simplex algorithm is used to solve the corrected linear programming problem.

The optimal values are $p_1 = 0.6$, $p_2 = 0$ and $p_3 = 0.4$

(d) Calculate the value of the game to player A.

(2)

(e) Determine the optimal strategy for player B, making your reasoning clear.

(4)

(Total for Question 4 is 14 marks)



6.

		Player B		
		Option X	Option Y	Option Z
Player A	Option Q	1	5	3
	Option R	4	-3	1
	Option S	2	-4	-2
	Option T	3	-2	0

A two person zero-sum game is represented by the pay-off matrix for player A, shown above.

- (a) Explain, with justification, why this matrix may be reduced to a 3×3 matrix by removing option S from player A's choices.

(2)

- (b) Verify that there is no stable solution to the reduced game.

(3)

Player A intends to make a random choice between options Q, R and T, choosing option Q with probability p_1 , option R with probability p_2 and option T with probability p_3 .

Player A wants to find the optimal values of p_1 , p_2 and p_3 using the Simplex algorithm. Player A formulates the following linear programme, writing the constraints as inequalities.

Maximise $P = V$, where V = the value of original game + 3

subject to $V \leq 4p_1 + 7p_2 + 6p_3$

$$V \leq 8p_1 + p_3$$

$$V \leq 6p_1 + 4p_2 + 3p_3$$

$$p_1 + p_2 + p_3 \leq 1$$

$$p_1 \geq 0, p_2 \geq 0, p_3 \geq 0, V \geq 0$$

- (c) Explain why V cannot exceed any of the following expressions

$$4p_1 + 7p_2 + 6p_3 \qquad 8p_1 + p_3 \qquad 6p_1 + 4p_2 + 3p_3$$

(1)

- (d) Explain why it is necessary to use the constraint $p_1 + p_2 + p_3 \leq 1$

(1)

The Simplex algorithm is used to solve the linear programming problem.

Given that the optimal value of $p_1 = \frac{7}{11}$ and the optimal value of $p_3 = 0$

- (e) calculate the value of the game to player A.

(3)

Player B intends to make a random choice between options X, Y and Z, choosing option X with probability q_1 , option Y with probability q_2 and option Z with probability q_3 .

- (f) Determine the optimal strategy for player B, making your working clear.

(4)

(Total for Question 6 is 14 marks)



8. A two-person zero-sum game is represented by the pay-off matrix for player A shown below.

		Player B		
		Option X	Option Y	Option Z
Player A	Option Q	−3	2	5
	Option R	2	−1	0
	Option S	4	−2	−1
	Option T	−4	0	2

- (a) Verify that there is no stable solution to this game. (2)
- (b) Explain why player A should never play option T. You must make your reasoning clear. (2)

Player A intends to make a random choice between options Q, R and S, choosing option Q with probability p_1 , option R with probability p_2 and option S with probability p_3 .

Player A wants to calculate the optimal values of p_1 , p_2 and p_3 using the Simplex algorithm.

- (c) (i) Formulate the game as a linear programming problem for player A. You should write the constraints as equations. (7)
- (ii) Write down an initial Simplex tableau for this linear programming problem, making your variables clear.

The linear programming problem is solved using the Simplex algorithm. The optimal value of p_1 is $\frac{6}{11}$ and the optimal value of p_2 is 0.

- (d) Find the best strategy for player B, defining any variables you use. (6)

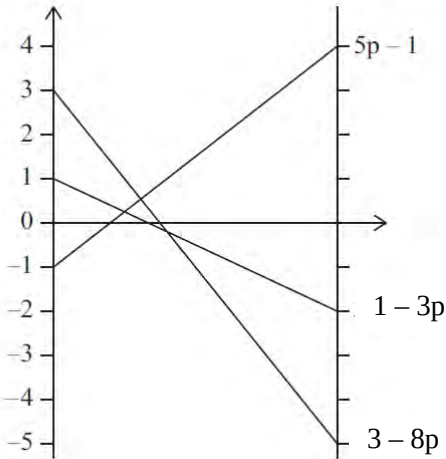
(Total for Question 8 is 17 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
6(a)	Row minima: $-5, -1$ max is -1 Column maxima: $4, 1, 3$ min is 1	M1	1.1b
	Row maximin $(-1) \neq$ Column minimax (1) so not stable	A1	2.4
		(2)	
(b)	If B plays option P, A's gains are $4p + (-1)(1-p) = -1 + 5p$ If B plays option Q, A's gains are $-2p + 1(1-p) = 1 - 3p$ If B plays option R, A's gains are $-5p + 3(1-p) = 3 - 8p$	M1 A1	1.1b 1.1b
		M1 A1	1.1b 1.1b
	$1 - 3p = -1 + 5p \Rightarrow p = 1/4$	A1	1.1b
	A should play option X with probability $1/4$ and option Y with probability $3/4$	A1ft	3.2a
		(6)	

(c)	e.g. $\begin{pmatrix} 4 & -2 & -5 \\ -1 & 1 & 3 \\ 4 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 9 & 3 & 0 \\ 4 & 6 & 8 \\ 9 & 4 & 6 \end{pmatrix}$	B1	1.1b																																																												
	e.g. $V - 9x - 4y - 9z + r = 0$ $V - 3x - 6y - 4z + s = 0$ $V - 8y - 6z + t = 0$ $x + y + z + u = 1$ $P - V = 0$																																																														
	e.g. <table border="1"><thead><tr><th>b.v.</th><th>V</th><th>x</th><th>y</th><th>z</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr></thead><tbody><tr><td>r</td><td>1</td><td>-9</td><td>-4</td><td>-9</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>s</td><td>1</td><td>-3</td><td>-6</td><td>-4</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>t</td><td>1</td><td>0</td><td>-8</td><td>-6</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>P</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></tbody></table>	b.v.	V	x	y	z	r	s	t	u	Value	r	1	-9	-4	-9	1	0	0	0	0	s	1	-3	-6	-4	0	1	0	0	0	t	1	0	-8	-6	0	0	1	0	0	u	0	1	1	1	0	0	0	1	1	P	-1	0	0	0	0	0	0	0	0	B1 B1 B1	3.3 1.1b 1.1b
b.v.	V	x	y	z	r	s	t	u	Value																																																						
r	1	-9	-4	-9	1	0	0	0	0																																																						
s	1	-3	-6	-4	0	1	0	0	0																																																						
t	1	0	-8	-6	0	0	1	0	0																																																						
u	0	1	1	1	0	0	0	1	1																																																						
P	-1	0	0	0	0	0	0	0	0																																																						
		(4)																																																													
(d)	$x=0, y=\frac{5}{7} \Rightarrow z=\frac{2}{7}$	B1	1.1b																																																												
	e.g. $V \leq \frac{38}{7}, \frac{38}{7}, \frac{52}{7} \Rightarrow V = \frac{38}{7}$ so the value of the 3×3 game is $\frac{3}{7}$	M1	3.4																																																												
	The value of the 3×3 game is $\frac{3}{7}$ which is $\frac{5}{28}$ better than the value of the 2×3 game which was $\frac{1}{4}$	A1	2.4																																																												
		(3)																																																													
(15 marks)																																																															

Notes for Question 6

(a)

M1: Attempt to calculate row minima and column maxima (all 5 values) – condone one error**A1:** Correct reasoning that the game is not stable (accept $-1 \neq 1 +$ statement) – dependent on correct row maximin and column minimax which must be clearly identified either around the table or in their statement (must have all 5 max and min values correct)

(b)

M1: setting up three expressions in terms of p **A1:** all three expressions correct**M1:** axes correct, at least two lines correctly drawn for their expressions, horizontal scale from 0 to 1**A1:** correct graph**A1:** using the graph to obtain the correct probability expressions leading to the correct value of p **A1ft:** interpret their value of p in the context of the question – must refer to play and name options(c) Condone use of $p_1 p_2 p_3$ instead of $x y z$ **B1:** Correct augmentation (by at least 5) – possibly implied by later working in tableau**B1:** Any one (numerical in nature) constraint row (r, s, t or u) correct (ignore labelling of b.v. column) or one correct constraint equation stated (so must be using columns)**B1:** Any two (numerical in nature) constraint row (r, s, t or u) correct (ignore labelling of b.v. column) or two correct constraint equations stated**B1:** CAO (including b.v. column)

(d)

B1: correct value for z (may be implied by subsequent working)**M1:** Attempts to calculate the value of the 3×3 game – must substitute in to at least two of their equations which may either be augmented or unaugmented(if using the original game then expect to see (if correct) $V \leq \frac{3}{7}, \frac{3}{7}, \frac{17}{7} \Rightarrow V = \frac{3}{7}$ so the value of the 3×3 game is $\frac{3}{7}$)**A1:** Correct values for both games seen together with some indication of how much better the new game is (could be given as a % increase, for example '71.4% better', or equivalent or as a multiplier e.g. $\frac{12}{7}$ or 1.714)

Question	Scheme	Marks	AOs																																																												
4(a)	Row minima: $-2, -4, -1$ max is -1	1M1	1.2																																																												
	Column maxima: $3, 4, 0$ min is 0																																																														
	Row maximin $(-1) \neq$ Column minimax (0) so not stable	1A1	2.4																																																												
		(2)																																																													
(b)	As the value of V must be non-negative the coefficients of the three inequalities involving V must be non-negative so add at least 4 to each value $\left[\begin{array}{ccc} 8 & 3 & 5 \\ 1 & 9 & 3 \\ 6 & 7 & 4 \end{array} \right]$	1B1	2.3																																																												
	Furthermore, as V is the minimum that A can expect to win, the constraints should be $V \leq \dots$	2B1	2.3																																																												
	e.g. ‘adding 5’ $V \leq 8p_1 + p_2 + 6p_3$ $V \leq 3p_1 + 9p_2 + 7p_3$ $V \leq 5p_1 + 3p_2 + 4p_3$	e.g. ‘adding 4’ $V \leq 7p_1 + 5p_3$ $V \leq 2p_1 + 8p_2 + 6p_3$ $V \leq 4p_1 + 2p_2 + 3p_3$	3B1	1.1b																																																											
		(3)																																																													
(c)	‘adding 5’ <table><tr><th>b.v.</th><th>V</th><th>p_1</th><th>p_2</th><th>p_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr><tr><td>r</td><td>1</td><td>-8</td><td>-1</td><td>-6</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>s</td><td>1</td><td>-3</td><td>-9</td><td>-7</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>t</td><td>1</td><td>-5</td><td>-3</td><td>-4</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>P</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	b.v.	V	p_1	p_2	p_3	r	s	t	u	Value	r	1	-8	-1	-6	1	0	0	0	0	s	1	-3	-9	-7	0	1	0	0	0	t	1	-5	-3	-4	0	0	1	0	0	u	0	1	1	1	0	0	0	1	1	P	-1	0	0	0	0	0	0	0	0	1B1	1.2
b.v.	V	p_1	p_2	p_3	r	s	t	u	Value																																																						
r	1	-8	-1	-6	1	0	0	0	0																																																						
s	1	-3	-9	-7	0	1	0	0	0																																																						
t	1	-5	-3	-4	0	0	1	0	0																																																						
u	0	1	1	1	0	0	0	1	1																																																						
P	-1	0	0	0	0	0	0	0	0																																																						
		1M1	3.3																																																												
		1A1	1.1b																																																												
		(3)																																																													
(d)	(ii) Substitute p values to obtain $V \leq 4.6$	1M1	3.4																																																												
	Value of the game to player A = $4.6 - 5 = -0.4$	1A1	2.2a																																																												
		(2)																																																													
(e)	Player B’s choices are options Y and Z only	1B1	3.4																																																												
	Either $2q = 0.4$ or $-2q + 1(1 - q) = 0.4$ (where q is the probability Player B plays their option Y and $(1 - q)$ is the corresponding probability for option Z)	1M1	3.1a																																																												
	$q = 0.2$	1A1	1.1b																																																												
	Player B should never play their option X, they should play their option Y with probability 0.2 and option Z with probability 0.8	2A1ft	3.2a																																																												
		(4)																																																													
(14 marks)																																																															

Notes:

(a) **1M1:** attempt at row minima and column maxima – condone one error

1A1: correct reasoning that the game is not stable (accept “ $-1 \neq 0$ ” + statement) – dependent on correct row minima and column maxima

(b) **1B1:** Indicates that coefficients are incorrect because V must be non-negative. Must convey both underlined aspects. Condone ‘positive’ for ‘non-negative’

2B1: Indicates that inequality signs are the wrong way because V is the minimum (so expected winnings are $\geq V$). Must convey both aspects but give bod.

3B1: CAO (all three inequalities). Give this mark for correct equations with slack variables provided 2B1 has been awarded.

(c) **1B1:** All row and column labels correct for Simplex tableau

1M1: Setting up the Simplex model - any two of my ‘r’, ‘s’ or ‘t’ rows correct. Or a completely correct answer with either one column or one row missing – condone lack of basic variable column. Should follow from **changed** constraints of the correct form from b). So, constraints must have been of the form $V \leq ap_1 + bp_2 + cp_3$ ($a, b, c \geq 0$) o.e.

1A1: CAO on numerical values.

Note: The B mark is for labelling the simplex tableau correctly; the M and A marks are for values only.

(d) **1M1:** substitutes their p values into all three expressions for the upper bound of V . Condone use of an equals sign here. But not ‘ $V \geq \dots$ ’

May see one of:

- $V \leq 3(0.6) - 4(0) + 0.4 = 2.2 \{= \frac{11}{5}\}; V \leq -2(0.6) - 4(0) + 2(0.4) = -0.4 \{= -\frac{2}{5}\}; V \leq -2(0) - (0.4) = -0.4 \{= -\frac{2}{5}\}$
- $V \leq 8(0.6) + (0) + 6(0.4) = 7.2 \{= \frac{36}{5}\}; V \leq 3(0.6) + 9(0) + 7(0.4) = 4.6 \{= \frac{23}{5}\}; V \leq 5(0.6) + 3(0) + 4(0.4) = 4.6 \{= \frac{23}{5}\}$
- $V \leq 7(0.6) + 5(0.4) = 6.2 \{= \frac{31}{5}\}; V \leq 2(0.6) + 8(0) + 6(0.4) = 3.6 \{= \frac{18}{5}\}; V \leq 4(0.6) + 2(0) + 3(0.4) = 3.6 \{= \frac{18}{5}\}$

1A1: CAO for the value of the game to player A

(e) **1B1:** CAO – uses the model to determine that Player B only plays Y and Z

1M1: A *correct* equation for B (where value of game to B = $-1 \times$ **their** value of game to A)

1A1: CAO for q (the probability that B plays Y)

2A1ft: Correct optimal strategy in context (not just in terms of q) following through their q

Alternative for (e)

1B1: CAO – uses the model to determine that Player B only plays Y and Z

1M1: Formulates two correct expressions for the expected value of the game to B and finds the intersection: $2q = -2q + 1(1 - q)$ ($= 1 - 3q$)

1A1: As above

2A1ft: As above



Question	Scheme	Marks	AOs
6 (a)	Option R (or option T) <u>dominates</u> option S	B1	1.2
	Because e.g. $4 > 2$ and $-3 > -4$ and $1 > -2$	B1	2.4
		(2)	
(b)	Row minima: 1, -3, -2 max is 1	M1	1.1b
	Column maxima: 4, 5, 3 min is 3	A1	1.1b
	Row maximin (1) $\neq$ Column minimax (3) so not stable	A1	2.4
		(3)	
(c)	V is less than or equal to each of these three expressions since we need to find the maximum value of the worst possible augmented expected pay-off for each value of p	B1	2.3
		(1)	
(d)	It is necessary to use an inequality because it enables the Simplex algorithm to pivot on a row that will increase the value of P	B1	3.5a
		(1)	
(e)	$p_2 = \frac{4}{11}$	B1	1.1b
	Substitute p values to obtain $V \leq \frac{56}{11}, \frac{56}{11}, \frac{58}{11}$	M1	3.4
	Value of the game to player A = $\frac{56}{11} - 3 = \frac{23}{11}$	A1	2.2a
		(3)	
(f)	$q_1 + 5q_2 + 3q_3 = \frac{23}{11}$	M1	3.1a
	$4q_1 - 3q_2 + q_3 = \frac{23}{11}$	A1ft	1.1b
	$q_1 + q_2 + q_3 = 1$	A1	1.1b
	Player B should play option X with probability $\frac{8}{11}$, option Y with probability $\frac{3}{11}$ and never play option Z	A1	3.2a
		(4)	
(14 marks)			
Notes:			
(a) B1: correct statement – must include the word ‘dominate’ (note that T dominates S too)			
B1: correct inequalities – must be clear that all inequalities must hold			



- (b) **M1:** attempt at row minima and column maxima – condone one error
A1: correct max(row min) and min(col max)
A1: correct reasoning that the game is not stable (accept $1 \neq 3$ + statement)
- (c) **B1:** an understanding that for each value of p we are seeking the minimum possible output
- (d) **B1:** as a minimum accept an answer that implies that an inequality is required so that we can apply the Simplex algorithm
- (e) **B1:** cao
M1: substitute their p values into all three expressions for the upper bound of V
A1: cao for the value of the game to player A
- (f) **M1:** Attempt to set up at least three equations in q_1, q_2, q_3 using the value of the game from (e)
A1ft: Two correct fit “their” V
A1: cao (for exactly three equations correct)
A1: cao in context



Question	Scheme	Marks	AOs																																																												
8(a)	Row minima are $-3, -1, -2, -4$ Column maxima are $4, 2, 5$	M1	1.1b																																																												
	As Row maximin is not equal to Column minimax ($-1 \neq 2$) the game is not stable	A1	2.4																																																												
		(2)																																																													
(b)	Option Q dominates option T	B1	1.2																																																												
	Because e.g., $-3 > -4, 2 > 0$ and $5 > 2$	B1	2.4																																																												
		(2)																																																													
(c)(i)	e.g., augment by $+3 \begin{pmatrix} -3 & 2 & 5 \\ 2 & -1 & 0 \\ 4 & -2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 5 & 8 \\ 5 & 2 & 3 \\ 7 & 1 & 2 \end{pmatrix}$	B1	1.1b																																																												
	Maximise $P - V = 0$ where V is the value of the augmented game to A	B1	3.3																																																												
	$V - 5p_2 - 7p_3 + r = 0$ $V - 5p_1 - 2p_2 - p_3 + s = 0$ $V - 8p_1 - 3p_2 - 2p_3 + t = 0$	M1 A1	3.3 1.1b																																																												
	$p_1 + p_2 + p_3 + u = 1$	B1	3.3																																																												
(ii)	<table><tr><th>b.v.</th><th>V</th><th>p_1</th><th>p_2</th><th>p_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr><tr><td>r</td><td>1</td><td>0</td><td>-5</td><td>-7</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>s</td><td>1</td><td>-5</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>t</td><td>1</td><td>-8</td><td>-3</td><td>-2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>P</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	b.v.	V	p_1	p_2	p_3	r	s	t	u	Value	r	1	0	-5	-7	1	0	0	0	0	s	1	-5	-2	-1	0	1	0	0	0	t	1	-8	-3	-2	0	0	1	0	0	u	0	1	1	1	0	0	0	1	1	P	-1	0	0	0	0	0	0	0	0	M1 A1	3.3 1.1b
b.v.	V	p_1	p_2	p_3	r	s	t	u	Value																																																						
r	1	0	-5	-7	1	0	0	0	0																																																						
s	1	-5	-2	-1	0	1	0	0	0																																																						
t	1	-8	-3	-2	0	0	1	0	0																																																						
u	0	1	1	1	0	0	0	1	1																																																						
P	-1	0	0	0	0	0	0	0	0																																																						
		(7)																																																													



(d)	$p_3 = \frac{5}{11}$	B1	1.1b
	$V \leq \frac{35}{11}, V \leq \frac{35}{11}, V \leq \frac{58}{11} \Rightarrow V = \frac{35}{11}$	M1	3.1a
	Let Player B play option X with probability q_1 , option Y with probability q_2 and option Z with probability q_3	B1	1.1b
	$5q_2 + 8q_3 = \frac{35}{11}$ $7q_1 + q_2 + 2q_3 = \frac{35}{11}$ $q_1 + q_2 + q_3 = 1$	dM1 A1ft	3.1a 1.1b
	Player B should play option X with probability $\frac{4}{11}$, option Y with probability $\frac{7}{11}$ and never play option Z	A1	3.2a
		(6)	
(17 marks)			
Notes:			
(a) M1: Clear attempt to find the row maximin and column minimax (either the row minimums or column maximums correct or at least five (of the seven) values stated correctly) A1: CAO (dependent on all rowmins and colmaxs correct) states – 1 ≠ 2 (or row (maximin) ≠ col (minimax) as long as – 1 is clearly identified as the row maximin and 2 as the column minimax) (b) B1: Correct statement – must include the word ‘dominate’ (or exact equivalent) B1: Correct inequalities – must be clear that all inequalities must hold (c)(i) B1: Correct augmentation – possibly implied by later working B1: Correct objective defined M1: At least three equations in V, p_1, p_2, p_3 and at least one dummy variable seen A1: CAO B1: Correct probability equation (ii) M1: Any two (numerical in nature) row correct A1: CAO including correct row and column labels for Simplex tableau (d) B1: p_3 correctly stated			



M1: Attempt to find V by substituting probabilities into all three equations or corresponding inequalities involving V, p_1, p_2, p_3 (or using the original value of the game) (Note candidates may reject B plays Z here as V is worse than playing X or Y)

B1: Defining probabilities for Player B

dM1: Dependent on previous M mark - setting up at least three equations in their q_1, q_2, q_3 using their V (or original value) (may be just q and $(1 - q)$ if Option Z rejected)

A1ft: Correct three equations (two if Option Z rejected) following through their augmented matrix in (c) – A0 if option R for player A is used (and not rejected later)

A1: Correct options for Player B in context (must say that Player B should never play option Z but this may appear earlier in their working) dependent on previous A mark

Alternative if augmenting by +4

(c)(i)	e.g., augment by +4 $\begin{pmatrix} -3 & 2 & 5 \\ 2 & -1 & 0 \\ 4 & -2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 6 & 9 \\ 6 & 3 & 4 \\ 8 & 2 & 3 \end{pmatrix}$	B1	1.1b																																																												
	Maximise $P - V = 0$ where V is the value of the augmented game to A	B1	3.3																																																												
	$V - p_1 - 6p_2 - 8p_3 + r = 0$ $V - 6p_1 - 3p_2 - 2p_3 + s = 0$ $V - 9p_1 - 4p_2 - 3p_3 + t = 0$	M1 A1	3.3 1.1b																																																												
	$p_1 + p_2 + p_3 + u = 1$	B1	3.3																																																												
(ii)	<table border="1"> <tr> <th>b.v.</th> <th>V</th> <th>p_1</th> <th>p_2</th> <th>p_3</th> <th>r</th> <th>s</th> <th>t</th> <th>u</th> <th>Value</th> </tr> <tr> <td>r</td> <td>1</td> <td>-1</td> <td>-6</td> <td>-8</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>s</td> <td>1</td> <td>-6</td> <td>-3</td> <td>-2</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>t</td> <td>1</td> <td>-9</td> <td>-4</td> <td>-3</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>u</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>P</td> <td>-1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </table>	b.v.	V	p_1	p_2	p_3	r	s	t	u	Value	r	1	-1	-6	-8	1	0	0	0	0	s	1	-6	-3	-2	0	1	0	0	0	t	1	-9	-4	-3	0	0	1	0	0	u	0	1	1	1	0	0	0	1	1	P	-1	0	0	0	0	0	0	0	0	M1 A1	3.3 1.1b
b.v.	V	p_1	p_2	p_3	r	s	t	u	Value																																																						
r	1	-1	-6	-8	1	0	0	0	0																																																						
s	1	-6	-3	-2	0	1	0	0	0																																																						
t	1	-9	-4	-3	0	0	1	0	0																																																						
u	0	1	1	1	0	0	0	1	1																																																						
P	-1	0	0	0	0	0	0	0	0																																																						
		(7)																																																													

(d)	$p_3 = \frac{5}{11}$	B1	1.1b
	$V \leq \frac{46}{11}, V \leq \frac{46}{11}, V \leq \frac{69}{11} \Rightarrow V = \frac{46}{11}$	M1	3.1a
	Let Player B play option X with probability q_1 , option Y with probability q_2 and option Z with probability q_3	B1	1.1b
	$q_1 + 6q_2 + 9q_3 = \frac{46}{11}$ $8q_1 + 2q_2 + 3q_3 = \frac{46}{11}$ $q_1 + q_2 + q_3 = 1$	dM1 A1ft	3.1a 1.1b
	Player B should play option X with probability $\frac{4}{11}$, option Y with probability $\frac{7}{11}$ and never play option Z	A1	3.2a
	For notes see main scheme above	(6)	

	Alternative for (d)		
(d)	$p_3 = \frac{5}{11}$	B1	
	Recognises that Option Y dominates Option Z and reduces matrix to either 3 x 2 or 2 x 2 (if A plays R removed) X Y Q -3 2 R 2 -1 S 4 -2 or X Y Q 0 5 R 5 2 S 7 1 or X Y Q 1 6 R 6 3 S 8 2	M1	
	Define probabilities for B e.g. play Option X with probability q and Option Y with probability $(1 - q)$	B1	
	e.g. $-3q + 2(1 - q) \Rightarrow 2 - 5q$ $4q - 2(1 - q) \Rightarrow 6q - 2$	dM1	
	$2 - 5q = 6q - 2$ e.g. $q = \frac{4}{11}$	A1ft	
	Player B should play option X with probability $\frac{4}{11}$, option Y with probability $\frac{7}{11}$ and never play option Z	A1	
		(6)	
Notes			
(d) B1: p_3 correctly stated M1: Uses dominance argument (may be implied) to reduce either the original Matrix or their augmented Matrix to either 3 x 2 or 2 x 2 (if A plays R deleted). The Matrix may be transposed and signs changed. B1: Defining probabilities for Player B (condone use of p instead of q) dM1: Dependent on previous M mark - setting up at least two equations (using A plays Q and S) or expressions in terms of their q A1ft: Solves their expressions or equations to obtain q (may use graphical approach). If solving multiple pairs of expressions, they must clearly choose their solution – A0 if option R for player A is used (and not rejected later) A1: Correct options for Player B in context (must say that Player B should never play option Z but this may appear earlier in their working) dependent on previous A mark			

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