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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Recurrence Relations (6)

Decision Mathematics 2 · Topic 6

9 questions · 76 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

9 questions · 76 marks · 2 sub-topics



SUB-TOPIC 6.1

RECURRENCE RELATIONS AS MODELS

Sub-topic 6.1 · 3 questions · 28 marks · lowest tariff first



7. Martina decides to open a bank account to help her to save for a holiday. Each month she puts $\pounds k$ into the account and allows herself to spend one quarter of what was in the account at the end of the previous month.

Let u_n (where $n \geq 1$) represent the amount in the account at the end of month n .

Martina has $\pounds k$ in the account at the end of the first month.

- (a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n , determine an expression for u_n in terms of n and k .

(6)

At the end of the 8th month, Martina needs to have at least $\pounds 1750$ in the account to pay for her holiday.

- (b) Determine, to the nearest penny, the minimum amount of money that Martina should put into the account each month.

(2)

(Total for Question 7 is 8 marks)



8. The owner of a new company models the number of customers that the company will have at the end of each month. The owner assumes that

- a constant proportion, p (where $0 < p < 1$), of the previous month's customers will be retained for the next month
- a constant number of new customers, k , will be added each month.

Let u_n (where $n \geq 1$) represent the number of customers that the company will have at the end of n months.

The company has 5000 customers at the end of the first month.

(a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n , determine an expression for u_n in terms of n , p and k .

(6)

The owner believes that 95% of the previous month's customers will be retained each month and that there will be 10 000 new customers each month.

According to the model, the company will first have at least 135 000 customers by the end of the m th month.

(b) Using logarithms, determine the value of m .

(3)

(Total for Question 8 is 9 marks)



5. An increasing sequence $\{u_n\}$ for $n \in \mathbb{N}$ is such that the difference between the n th term of $\{u_n\}$ and the mean of the previous two terms of $\{u_n\}$ is always 6

(a) Show that, for $n \geq 3$

$$2u_n - u_{n-1} - u_{n-2} = 12 \quad (2)$$

Given that $u_1 = 2$ and $u_2 = 8$

(b) find the solution of this second order recurrence relation to obtain an expression for u_n in terms of n .

(7)

(c) Show that as $n \rightarrow \infty$, $u_n \rightarrow kn$ where k is a constant to be determined. You must give reasons for your answer.

(2)

(Total for Question 5 is 11 marks)



SUB-TOPIC 6.2

SOLVING RECURRENCE RELATIONS

Sub-topic 6.2 · 6 questions · 48 marks · lowest tariff first



2. The general solution of the first order recurrence relation

$$u_{n+1} + au_n = bn^2 + cn + d \quad n \geq 0$$

is given by

$$u_n = A(3)^n + 5n^2 + 1$$

where A is an arbitrary non-zero constant.

By considering expressions for u_{n+1} and u_n , find the values of the constants a , b , c and d .

(Total for Question 2 is 3 marks)



2. The general solution of the second order recurrence relation

$$u_{n+2} + k_1 u_{n+1} + k_2 u_n = 0 \quad n \geq 0$$

is given by

$$u_n = (A + Bn)(-3)^n$$

where A and B are arbitrary non-zero constants.

(a) Find the value of k_1 and the value of k_2

(2)

Given that $u_0 = u_1 = 1$

(b) find the value of A and the value of B .

(2)

(Total for Question 2 is 4 marks)



5. A sequence $\{u_n\}$, where $n \geq 0$, satisfies the second order recurrence relation

$$u_{n+2} = \frac{1}{2}(u_{n+1} + u_n) + 3 \quad \text{where } u_0 = 15 \quad u_1 = 20$$

(a) By considering the sequence $\{v_n\}$, where $u_n = v_n + 2n$ for $n \geq 0$, determine an expression for u_n as a function of n .

(7)

(b) Describe the long-term behaviour of u_n

(1)

(Total for Question 5 is 8 marks)



8. A sequence $\{u_n\}$, where $n \geq 0$, satisfies the recurrence relation

$$2u_{n+2} + 5u_{n+1} = 3u_n + 8n + 2$$

- (a) Find the general solution of this recurrence relation.

(5)

A particular solution of this recurrence relation has $u_0 = 1$ and $u_1 = k$, where k is a positive constant. All terms of the sequence are positive.

- (b) Determine the value of k .

(3)

(Total for Question 8 is 8 marks)



4. Sequences $\{x_n\}$ and $\{y_n\}$ for $n \in \mathbb{N}$, are defined by

$$x_{n+1} = 2y_n + 3 \quad \text{and} \quad y_{n+1} = 3x_{n+1} - 4x_n$$

$$x_1 = 1 \quad \text{and} \quad y_1 = a$$

where a is a constant.

(a) Show that $x_{n+2} - 6x_{n+1} + 8x_n = 3$ (1)

(b) Solve the second-order recurrence relation given in (a) to obtain an expression for x_n in terms of a and n . (8)

Given that $x_7 = 28\,225$

(c) find the value of a . (2)

(Total for Question 4 is 11 marks)



7. A sequence $\{u_n\}$, where $n \geq 0$, satisfies the first order recurrence relation (*)

$$u_{n+1} = \alpha u_n - \beta(k^n) \quad (*)$$

where α, β, k are non-zero constants and $\alpha \neq k$

- (a) Determine, in terms of α, β, k and n , the general solution of this recurrence relation.

(4)

Given that $u_0 = L$ and $u_T = 0$, where T is a constant,

- (b) show that a particular solution of the recurrence relation is

$$u_n = \frac{Lk^n \left(1 - \left(\frac{k}{\alpha} \right)^{T-n} \right)}{1 - \left(\frac{k}{\alpha} \right)^T}$$

(5)

Taylor borrows a sum of money, L , to buy a house.

Taylor arranges a loan which enables them to increase their repayment as their income grows.

The terms of the loan require Taylor's repayments to be made once a year.

On each anniversary of the loan being taken out, 6% interest is charged on the outstanding balance and Taylor makes an annual repayment of the total loan.

Taylor's total annual repayment of the loan will increase by 4% each year and the whole debt will be cleared in exactly 40 years.

The recurrence relation (*) can be used to model this situation.

- (c) (i) State the value of α

- (ii) State the value of k

- (iii) State the relevance of β in (*)

(3)

- (d) Show that after 30 years, according to the model, the outstanding debt is larger than the original loan.

(2)

(Total for Question 7 is 14 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
7(a)	$u_{n+1} = 0.75u_n + k$	B1	3.3
	(aux equation $m - 0.75 = 0 \Rightarrow$) complementary function is $A(0.75)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = \lambda$ so $\lambda - 0.75\lambda = k$	M1	1.1b
	General solution is $u_n = A(0.75)^n + 4k$	A1	1.1b
	$u_1 = k \therefore 0.75A + 4k = k \Rightarrow A = \dots$	M1	3.4
	$u_n = 4k(1 - (0.75)^n)$	A1	2.2a
		(6)	
(b)	$4k(1 - 0.75^8) = 1750 \Rightarrow k = \dots$	dM1	3.4
	$k = 486.1721068 \dots \Rightarrow k = 486.18$	A1	2.2a
		(2)	
(8 marks)			
Notes:			
(a) B1: CAO B1: CAO M1: substituting their trial solution into the recurrence relation (which must be of the form $u_{n+1} = au_n + k$ where $a = 0.75$ or ± 0.25) in an attempt to find their λ A1: CAO for the general solution (may be implied by subsequent working) M1: using the conditions in the model and their expression of the form $A(\alpha)^n + Bk$ where α is 0.75 or ± 0.25 to calculate A (which if correct is $-4k$) A1: CAO for the particular solution (may be an expression without $u_n =$ accept any equivalent form e.g. $u_n = 4k - 3k(0.75)^{n-1}$) (b) dM1: Applying $n = 8$ to their general solution of the form $A(\alpha)^n + Bk$ where α is 0.75 or ± 0.25 equal to 1750 (dependent on both M marks in (a)) A1: CAO – must be 486.18			

Qu	Scheme	Marks	AOs
8(a)	$u_{n+1} = pu_n + k$	B1	3.3
	(aux equation $m - p = 0 \Rightarrow$) complementary function is $A(p)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = \lambda$ so $\lambda - p\lambda = k$	M1	1.1b
	General solution is $u_n = A(p)^n + \frac{k}{1-p}$	A1	1.1b
	$u_1 = 5000 \Rightarrow 5000 = A(p) + \frac{k}{1-p}$ and solve for A	M1	3.4
	$u_n = \left(5000 - \frac{k}{1-p}\right)p^{n-1} + \frac{k}{1-p}$	A1	2.2a
		(6)	
(b)	Set $k = 10\,000$, $p = 0.95$ and $u_m \dots 135\,000$ $\left(5000 - \frac{10000}{1-0.95}\right)(0.95)^{m-1} + \frac{10000}{1-0.95} \dots 135000$ $(0.95)^{m-1} \cdot \frac{1}{3} \Rightarrow (m-1)\log(0.95) \cdot \frac{1}{3} \Rightarrow m \dots$ $m \dots 22.418 \dots$ so 23 months after the company was first up	B1 M1 A1	3.1b 1.1b 3.2a
		(3)	
(9 marks)			
Notes:			
(a) B1: CAO B1: CAO M1: substituting their trial solution into the recurrence relation in an attempt to find their λ (which if correct is $\frac{k}{1-p}$) A1: CAO for the general solution M1: using the conditions in the model to calculate A (which if correct is $p^{-1}\left(5000 - \frac{k}{1-p}\right)$) A1: CAO for the particular solution (oe) (b) B1: Applying $u_m \dots 135000$ (or equality or strict inequality) to their general solution together with correct values for k and p (dependent on both M marks in (a)) M1: dependent on previous B mark – solving their equation using logarithms A1: CAO – must be rounded to 23 Special case: A common misread is 500 for 5000. Mark as a misread so final A marks in (a) and (b) deducted.			

Question	Scheme	Marks	AOs
5(a)	$u_n - \frac{1}{2}(u_{n-1} + u_{n-2}) = 6 \Rightarrow 2u_n - (u_{n-1} + u_{n-2}) = 12$	1M1	2.1
	$2u_n - u_{n-1} - u_{n-2} = 12$	1A1	2.2a
		(2)	
(b)	aux equation $2m^2 - m - 1 = 0 \Rightarrow m = 1, m = -\frac{1}{2}$	1B1	2.1
	$u_n = A + B\left(-\frac{1}{2}\right)^n$	2B1	1.1b
	particular solution try $u_n = \lambda n$	1M1	1.1b
	$\therefore 2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = 12 \Rightarrow \lambda [= 4]$	1A1	1.1b
	$u_n = A + B\left(-\frac{1}{2}\right)^n + 4n$	2M1	1.1b
	$u_1 = 2 \Rightarrow 2A - B = -4$	3M1	1.1b
	$u_2 = 8 \Rightarrow 4A + B = 0$	2A1	2.2a
		(7)	
(c)	$u_n \rightarrow 4n \quad (k=4)$	1M1	2.1
	As $n \rightarrow \infty, \left(-\frac{1}{2}\right)^n \rightarrow 0$ and $4n$ is considerably greater than $-\frac{2}{3}$	1A1ft	1.1b
		(2)	
(11 marks)			
Notes:			
(a) 1M1: attempt to write given information in terms of a recurrence equation (allow sign errors and errors in notation) 1A1*: CAO (NB equation is provided in the question so must see a correct unsimplified recurrence relation which is simplified with no errors to the required form for this mark). (b) 1B1: CAO for auxiliary equation and corresponding solutions (this mark can be implied by correct complementary function) 2B1: complementary function CAO. Condone lack of '$u_n =$'. 1M1: substitutes $u_n = \lambda n$ into their 2nd-order recurrence relation and solves to obtain $\lambda = \dots$ Note: $2\lambda n - \lambda(n-1) - \lambda(n-2) = 12$ o.e. $\Rightarrow \lambda = \dots$ can earn this mark Note: Do not condone sign errors or errors in coefficients e.g. $2\lambda n - \lambda(n+1) - \lambda(n-2) = 12$ is M0 $2\lambda n - 2\lambda(n-1) - \lambda(n-2) = 12$ is M0 1A1: Correct general solution. Condone lack of '$u_n =$'. 2M1: Forms one equation in A and B. General Solution must be of the form. $u_n = A + B\left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$ 3dM1: Forms a second equation in A and B. Dependent on the previous method mark. 2A1: Particular solution CAO. Do not condone lack of '$u_n = \dots$'			



(c) M1: Obtains correct limit (ft their particular solution which must be of the correct form i.e.

$$u_n = A + B \left(-\frac{1}{2}\right)^n + \mu n \text{ where } \mu \neq 0)$$

A1ft: Provides correct reasoning including comments relating to both of the following:

- $n \rightarrow \infty, \left(-\frac{1}{2}\right)^n \rightarrow 0$
- $-\frac{2}{3}$ becomes negligible (compared to $4n$ as $n \rightarrow \infty$)

Note: Condone $-\frac{2}{3}$ becomes insignificant'



Question	Scheme	Marks	AOs
2	$u_{n+1} = A(3)^{n+1} + 5(n+1)^2 + 1$	B1	1.1b
	$u_{n+1} = A(3)^n(3) + 5(n+1)^2 + 1$ $\Rightarrow u_{n+1} = 3(u_n - 5n^2 - 1) + 5(n+1)^2 + 1$	M1	1.1b
	$u_{n+1} - 3u_n = -10n^2 + 10n + 3$	A1	1.1b
		(3)	
	Alternative 1 $u_{n+1} + au_n = 0$ $u_{n+1} = -au_n$ C.F. $u_n = A(-a)^n$ $-a = 3 \Rightarrow a = -3$	B1	
	Particular solution Try $\lambda n^2 + \mu n + \nu \Rightarrow \lambda = 5 \quad \mu = 0 \quad \nu = 1$ $5(n+1)^2 + 1 - 3(5n^2 + 1) = bn^2 + cn + d$ $\Rightarrow b = -10 \quad c = 10 \quad d = 3$	M1 A1	
		(3)	
	Alternative 2 $u_{n+1} + au_n = 3A(3)^n + 5n^2 + 10n + 6$ $+ aA(3)^n + 5an^2 + a$	B1	
	Compares coefficients $3 + a = 0 \Rightarrow a = -3$ $5 + 5a = b \Rightarrow b = -10$ $c = 10$ $6 + a = d \Rightarrow d = 3$	M1 A1	
		(3)	
(3 marks)			

Notes for Question 2

B1: any correct expression for u_{n+1}

M1: eliminating A to form a first order recurrence relation containing u_{n+1} and u_n

A1: CAO for $u_{n+1} - 3u_n = -10n^2 + 10n + 3$ (need not explicitly state $a = -3, b = -10, c = 10, d = 3$)

Alternative 1

B1: Considers the C.F. and deduces $a = -3$ with no other values stated

M1: Obtains P.S. and forms equation using $u_{n+1} \pm 3u_n$

A1: CAO for $u_{n+1} - 3u_n = -10n^2 + 10n + 3$ (need not explicitly state $a = -3, b = -10, c = 10, d = 3$)

Alternative 2

B1: Forms the correct equation for $u_{n+1} + au_n$

M1: Attempts to compare coefficients – at least three terms seen

A1: CAO



Question	Scheme	Marks	AOs
2(a)	$(m+3)^2 = 0$	M1	3.1a
	$k_1 = 6$ and $k_2 = 9$	A1	1.1b
		(2)	
(b)	$u_0 = 1 \Rightarrow A(-3)^0 = 1$ $u_1 = 1 \Rightarrow (A+B)(-3) = 1$	M1	1.1b
	$A = 1$ and $B = -\frac{4}{3}$	A1	1.1b
		(2)	
(4 marks)			
Notes:			
(a) M1: Correct auxiliary equation (may be implied by 6, 9 correct) from $u_n = (A+Bn)(-3)^n$ A1: CAO – allow values stated implicitly i.e., $u_{n+2} + 6u_{n+1} + 9u_n = 0$ (b) M1: Uses $u_0 = u_1 = 1$ and attempts to find A and B A1: CAO – allow values stated implicitly e.g. $u_n = \left(1 - \frac{4}{3}n\right)(-3)^n$			

Question	Scheme	Marks	AOs
5(a)	$u_{n+2} = \frac{1}{2}(u_{n+1} + u_n) + 3$		
	$v_{n+2} + 2(n+2) = \frac{1}{2}(v_{n+1} + 2(n+1) + v_n + 2n) + 3$	M1	2.1
	$2v_{n+2} - v_{n+1} - v_n = 0$	A1	1.1b
	$2m^2 - m - 1 = 0 \Rightarrow (2m+1)(m-1) = 0 \therefore m = -0.5 \text{ and } 1$	dM1	1.1b
	$v_n = A + B\left(-\frac{1}{2}\right)^n$	A1ft	1.1b
	$u_0 = 15 \Rightarrow v_0 = 15$ $u_1 = 20 \Rightarrow v_1 = 18$	B1	1.1b
	$A + B = 15$ $A - \frac{1}{2}B = 18$ and solve for A and B	M1	1.1b
	$u_n = 17 - 2\left(-\frac{1}{2}\right)^n + 2n$	A1	2.2a
		(7)	
(b)	As $n \rightarrow \infty$, the terms of u_n are (approximately) given by the linear expression $17 + 2n$	B1	2.4
		(1)	

(8 marks)**Notes:****(a) M1:** Substituting $u_n = v_n + 2n$ to obtain a second-order recurrence relation in v_n only**A1:** CAO Correct homogeneous second-order recurrence relation**dM1:** Solving their three-term auxiliary equation (dependent on previous M mark) (if no auxiliary equation seen, correct expression implies this mark)**A1ft:** Correct general solution for v_n following their roots or complimentary function if they do not obtain homogeneous relationship.**B1:** Either converting the first two given values for the sequence for u to v or for stating the corresponding general solution for u_n **M1:** Setting up two equations using their solution and solving for their A and B (if correct $A = 17$ and $B = -2$) (dependent on general solution of the form $A + B(\alpha)^n$ where α is $-\frac{1}{2}$ or -2)**A1:** CAO (may be an expression without $u_n =$ accept any equivalent form e.g.

$$u_n = 17 + \left(-\frac{1}{2}\right)^{n-1} + 2n$$

(b) B1: Explanation that as n becomes large the terms of the sequence are (approximately) in arithmetic progression (or equivalent) dependent on correct expression for u_n 

Question	Scheme	Marks	AOs
5(a)	Special Case – does not use the transformation Max 6/8		
	$2u_{n+2} - u_{n+1} - u_n = 6$	M0	
		A0	
	$2u_{n+2} - u_{n+1} - u_n = 0$		
	$2m^2 - m - 1 = 0 \Rightarrow (2m+1)(m-1) = 0 \therefore m = -0.5 \text{ and } 1$	M1	
	C.F. $u_n = A + B\left(-\frac{1}{2}\right)^n$	A1ft	
	Trial solution λn $\lambda(n+2) - \frac{1}{2}\lambda(n+1) - \frac{1}{2}\lambda n = 3$ $\Rightarrow \lambda = 2$ P.S. $2n$ General solution $u_n = A + B\left(-\frac{1}{2}\right)^n + 2n$	B1	
	$A + B = 15$ $A - \frac{1}{2}B = 18$ and solve for A and B	dM1	
	$u_n = 17 - 2\left(-\frac{1}{2}\right)^n + 2n$	A1	
		(7)	
(b)	As $n \rightarrow \infty$, the terms of u_n are (approximately) given by the linear expression $17 + 2n$	B1	
		(1)	
(8 marks)			
Notes:			
(a) M0: Transformation not used A0: M1: Setting up and solving their three-term auxiliary equation (if no auxiliary equation seen, correct expression implies this mark) A1ft: Correct complementary function for u_n following their roots B1: Finding the correct particular solution and stating the corresponding general solution for u_n			

dM1: Setting up two equations using their solution and solving for their A and B (if correct $A = 17$ and $B = -2$) dependent on previous M mark (dependent on general solution of the form

$A + B(\alpha)^n + Cn$ where α is $-\frac{1}{2}$ or -2)

A1: CAO (may be an expression without $u_n =$ accept any equivalent form e.g.

$$u_n = 17 + \left(-\frac{1}{2}\right)^{n-1} + 2n$$

(b) B1: Explanation that as n becomes large the terms of the sequence are (approximately) in arithmetic progression (or equivalent) dependent on correct expression for u_n



Question	Scheme	Marks	AOs
8 (a)	aux. equation is $2m^2 + 5m - 3 = 0 \Rightarrow m = \dots$	M1	2.1
	complementary function is $A(0.5)^n + B(-3)^n$	A1	1.1b
	Particular solution try $u_n = an + b$ and substitute into recurrence relation	M1	1.1b
	$2(a(n+2) + b) + 5(a(n+1) + b) = 3(an + b) + 8n + 2$ and by comparing linear and constant terms gives $4a = 8$ $9a + 4b = 2$	dM1	1.1b
	$(u_n =) A(0.5)^n + B(-3)^n + 2n - 4$	A1ft	1.1b
		(5)	
(b)	$A + B - 4 = 1$ $0.5A - 3B - 2 = k$ leading to $A = \dots$ and $B = \dots$	ddM1	3.4
	$(u_n =) \left(\frac{34+2k}{7}\right)(0.5)^n + \left(\frac{1-2k}{7}\right)(-3)^n + 2n - 4$ and setting their $\frac{1-2k}{7} = 0$	dddM1	3.1a
	$k = 0.5$	A1	2.2a
		(3)	
(8 marks)			

Notes for Question 8

Note mark (a) and (b) together

(a) M1: correct auxiliary equation and attempt to solve (leading to two distinct values of m)

A1: CAO for complementary function (accept if seen in subsequent working)

M1: correct form for particular solution and substituted $n + 2$, $n + 1$, n into recurrence relation

dM1: compares coefficients and setting up both equations in a, b (so one equation in a only and one equation in a and b (although a may have already been found from the first equation)) – dependent on previous M mark ($a = 2$ and $b = -4$)

A1ft: correct general solution following through their complementary function (must be their C.F. + $2n - 4$) Please award for a fully correct solution seen either here or being used in (b)

(b) ddM1: use correct initial conditions correctly to form simultaneous equations in their A, B and k and attempt to solve for A and B – dependent on the previous two M marks. Award this mark for **Either**

- eliminating either A **or** B from the correct two equations leading to one of
 $7A = 2k + 34$ **or** $7B = 1 - 2k$ (accept any equivalent form)

Or

- eliminating both A and B from their equations using the initial conditions and obtaining expressions in terms of k

If correct $A = \frac{34 + 2k}{7}, B = \frac{1 - 2k}{7}$

dddM1: setting coefficient (which must be a linear expression in k) of their $(-3)^n$ equal to zero – dependent on the previous three M marks (and at least one root of the auxiliary equation being negative)

A1: CAO (must be from correct working)

Question	Scheme	Marks	AOs
4(a)	$x_{n+2} = 2y_{n+1} + 3 \Rightarrow x_{n+2} = 2(-4x_n + 3x_{n+1}) + 3$ Leading to $x_{n+2} - 6x_{n+1} + 8x_n = 3^*$	B1	2.2a
		(1)	
(b)	aux equation $m^2 - 6m + 8 = 0 \Rightarrow m = 2, m = 4$ $x_n = A(2)^n + B(4)^n$ particular solution try $x_n = \lambda$ $\therefore \lambda - 6\lambda + 8\lambda = 3 \Rightarrow \lambda [=1]$ $x_n = A(2)^n + B(4)^n + 1$ $x_1 = 1 \Rightarrow 2A + 4B = 0$ $y_1 = a \Rightarrow x_2 = 2a + 3$ $4A + 16B + 1 = 2a + 3$ $A = -\frac{(a+1)}{2}, B = \frac{(a+1)}{4} \Rightarrow x_n = (a+1)(4)^{n-1} - (a+1)(2)^{n-1} + 1$ (oe)	B1 B1 M1 A1 M1 B1 M1 A1	2.1 1.1b 1.1b 2.2a 1.1b 3.1a 1.1b 2.2a
		(8)	
(c)	As $x_7 = 28225 \Rightarrow (a+1)(4)^6 - (a+1)(2)^6 + 1 = 28225$ leading to $a = \dots$ $a = 6$	M1 A1	3.4 2.2a
		(2)	
(11 marks)			
Notes: (a) B1: Correct reasoning to derive given result – sufficient working must be shown as recurrence equation given in question (b) B1: cao for auxiliary equation and corresponding solutions (this mark can be implied by the correct complementary function) B1: cao for the complementary function M1: Substitute $x_n = \lambda$ into their second order recurrence relation and solve for λ A1: Correct general solution M1: Forms one equation in A and B using $x_1 = 1$ B1: Uses original recurrence relation for x_{n+1} to derive the expression $2a + 3$ M1: Setting up a second equation in A and B A1: cao (oe e.g., $x_n = 0.25(a+1)(4)^n - 0.5(a+1)(2)^n + 1$) (c) M1: Using $x_7 = 28225$ to form a linear equation in a and attempt to solve for a A1: cao for a			

Question	Scheme	Marks	AOs
7(a)	aux. equation is $m - \alpha = 0 \Rightarrow$ complementary function is $A\alpha^n$	B1	2.1
	Particular solution try $u_n = Bk^n$ and substitute into recurrence relation	M1	1.1b
	$Bk^{n+1} - \alpha Bk^n = -\beta k^n \Rightarrow Bk - \alpha B = -\beta$ and solve for B	dM1	1.1b
	$u_n = A\alpha^n + \frac{\beta k^n}{\alpha - k}$	A1	2.2a
		(4)	
(b)	From given conditions: $u_0 = L \Rightarrow A + \frac{\beta}{\alpha - k} = L$ and $u_T = 0 \Rightarrow A\alpha^T + \frac{\beta k^T}{\alpha - k} = 0$	M1	3.4
	Eliminating β which if correct is $A\alpha^T + (L - A)k^T = 0$	dM1	3.1a
	Re-arrange to get an expression for A in terms of L, k, T and α (which for reference if correct is $A = \frac{Lk^T}{k^T - \alpha^T}$)	ddM1	3.1a
	Eliminate A and β from their general solution to obtain an expression for u_n in terms of L, k, T, n and α only $u_n = \left(\frac{Lk^T}{k^T - \alpha^T} \right) \alpha^n + \left(L - \frac{Lk^T}{k^T - \alpha^T} \right) k^n$	dddM1	1.1b
	$u_n = \frac{Lk^n}{\alpha^T - k^T} (\alpha^T - k^{T-n}\alpha^n) = \frac{Lk^n}{\frac{\alpha^T}{\alpha^T} - \frac{k^T}{\alpha^T}} \left(\frac{\alpha^T}{\alpha^T} - \frac{k^{T-n}\alpha^n}{\alpha^T} \right)$ $u_n = \frac{Lk^n}{1 - \left(\frac{k}{\alpha} \right)^T} \left(1 - \left(\frac{k}{\alpha} \right)^{T-n} \right)^*$	A1	2.2a
		(5)	
(c)	(i) $\alpha = 1.06$ (ii) $k = 1.04$ (iii) β is the amount that Taylor repaid on the first anniversary of the loan being taken out	B1 B1 B1	2.2a 2.2a 3.1b
		(3)	
(d)	$u_{30} = \frac{L(1.04)^{30}}{1 - \left(\frac{1.04}{1.06} \right)^{40}} \left[1 - \left(\frac{1.04}{1.06} \right)^{40-30} \right]$	M1	3.4
	$u_{30} = (1.05\dots)L > L$ so, after 30 years the outstanding debt (which is $(1.05\dots)L$) is larger than the original loan (L)	A1	3.2a
		(2)	
(14 marks)			

Notes for Question 7

(a)

B1: CAO for complementary function (condone if stated as $u_n = A\alpha^n$)**M1:** correct form for particular solution and substituted into recurrence relation**dM1:** simplifying (by eliminating n) and solving for their constant (B)**A1:** correct general solution (must be $u_n =$) (must be fully simplified)

(b)

M1: Form two equations using their general solution, and both given conditions**dM1:** Eliminate either A or β from their two equations (formed from the given conditions)**ddM1:** Re-arrange to obtain an expression for either β or A in terms of L, k, T, n and α They may eliminate A and then obtain an expression for $\frac{\beta}{\alpha - k}$ and then substitute for this**dddM1:** Eliminate both A and β from their general solution to obtain an expression for u_n in terms of L, k, T, n and α only**A1:** CSO – correctly derive given result (so sufficient working must be shown)

(c)(i)

B1: CAO for α

(ii)

B1: CAO for k

(iii)

B1: CAO for the relevance of β in (*) must be clear that this is the amount repaid one year after the loan is taken out. Accept e.g original amount repaid or initial repayment and condone reference to 0th year if clear that this is the first repayment (do not accept annual repayment)

(d)

M1: Substitute $T = 40, n = 30$ and their values of α, k into correct particular solution (accept sight of 1.05L for this mark even if full calculation not shown)**A1:** CAO that the outstanding debt after 30 years is larger than the original debt (so must see some reference to 1.05L (or better) and L) (accept $1.05L > L$) (must follow from a correct calculation)In part (c) for those that attempt to verify (rather than show the given result) then the first 3 M marks are as in the main scheme (M1 for using the given conditions to form two equations, dM1 for eliminating A , ddM1 for obtaining an expression for β (which for reference if correct is $\beta = \frac{L\alpha^T(\alpha - k)}{\alpha^T - k^T}$), then the fourth M mark is for obtaining an expression for $u_{n+1} - \alpha u_n$ in terms of L, k, T, n and α only – which if correct should be equivalent to $u_{n+1} - \alpha u_n = \frac{Lk^n}{1 - \left(\frac{k}{\alpha}\right)^T} \left\{ k - k\left(\frac{k}{\alpha}\right)^{T-n} \left(\frac{\alpha}{k}\right) - \alpha + \alpha\left(\frac{k}{\alpha}\right)^{T-n} \right\} = -k^n \left\{ \frac{L(\alpha - k)}{1 - \left(\frac{k}{\alpha}\right)^T} \right\}$. The final A mark is for correctly showingthat $\frac{L(\alpha - k)}{1 - \left(\frac{k}{\alpha}\right)^T}$ is equivalent to $\frac{L\alpha^T(\alpha - k)}{\alpha^T - k^T}$ (so explicitly showing that $u_{n+1} - \alpha u_n$ is equal to $-\beta(k^n)$ with a correct expression for β explicitly seen (in terms of L, k, T and α)).

Alternative for (b)

If they eliminate A first

$$\beta = \frac{L\alpha^T(\alpha - k)}{(\alpha^T - k^T)} \text{ and } A = L - \frac{L\alpha^T}{(\alpha^T - k^T)} = \frac{-Lk^T}{\alpha^T - k^T}$$

