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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Work, Energy and Power (2)

Further Mechanics 1 · Topic 2

12 questions · 128 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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1.1 CONSERVATION OF MOMENTUM		1 Qs · 9 marks	3
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QUESTIONS

12 questions · 128 marks · 6 sub-topics



SUB-TOPIC 1.1

CONSERVATION OF MOMENTUM

Sub-topic 1.1 · 1 question · 9 marks · lowest tariff first



(Total for Question 1 is 9 marks)



SUB-TOPIC 2.1

WORK-ENERGY PRINCIPLE

Sub-topic 2.1 · 3 questions · 27 marks · lowest tariff first



(Total for Question 2 is 7 marks)



6.

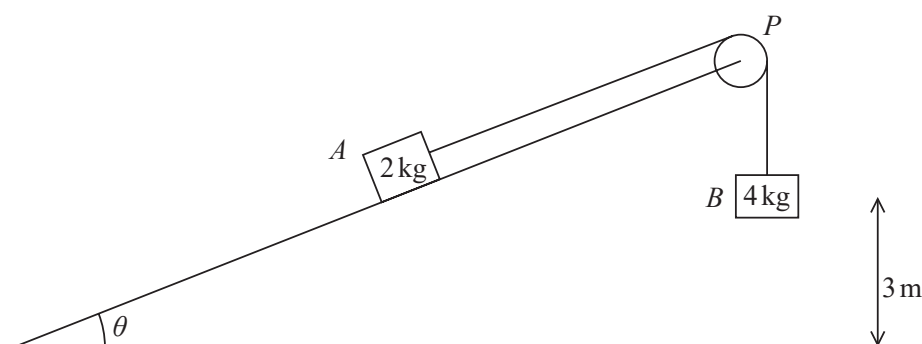


Figure 4

Two blocks, A and B , of masses 2 kg and 4 kg respectively are attached to the ends of a light inextensible string.

Initially A is held on a fixed rough plane. The plane is inclined to horizontal ground at an angle θ , where $\tan \theta = \frac{3}{4}$

The string passes over a small smooth light pulley P that is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane.

Block A is held on the plane with the distance AP greater than 3 m .

Block B hangs freely below P at a distance of 3 m above the ground, as shown in Figure 4.

The coefficient of friction between A and the plane is μ

Block A is released from rest with the string taut.

By modelling the blocks as particles,

(a) find the potential energy lost by the whole system as a result of B falling 3 m .

(3)

Given that the speed of B at the instant it hits the ground is 4.5 m s^{-1} and ignoring air resistance,

(b) use the work-energy principle to find the value of μ

(6)

After B hits the ground, A continues to move up the plane but does not reach the pulley in the subsequent motion.

Block A comes to instantaneous rest after moving a total distance of $(3 + d)\text{ m}$ from its point of release.

Ignoring air resistance,

(c) use the work-energy principle to find the value of d

(4)





(Total for Question 3 is 15 marks)



Question 4 continued

SUB-TOPIC 2.2

POWER

Sub-topic 2.2 · 2 questions · 20 marks · lowest tariff first









SUB-TOPIC 4.1

NEWTON'S LAW OF RESTITUTION

Sub-topic 4.1 · 2 questions · 25 marks · lowest tariff first



5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e .

- (a) Find the range of possible values of e , justifying your answer.

(8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

- (b) find the value of e .

(3)

Question 5 continued





Question 5 continued

(Total for Question 5 is 11 marks)



2. Two particles, A and B , are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle A has mass $5m$ and particle B has mass $3m$.

The coefficient of restitution between A and B is e , where $e > 0$

Immediately **after** the collision the speed of A is v and the speed of B is $2v$.

Given that A and B are moving in the same direction after the collision,

- (a) find the set of possible values of e .

(8)

Given also that the kinetic energy of A immediately after the collision is 16% of the kinetic energy of A immediately before the collision,

- (b) find

- the value of e ,
- the magnitude of the impulse received by A in the collision, giving your answer in terms of m and v .

(6)





Question 2 continued

(Total for Question 2 is 14 marks)



SUB-TOPIC 5.1

OBLIQUE IMPACTS BETWEEN SPHERES

Sub-topic 5.1 · 1 question · 14 marks · lowest tariff first



- (3)



Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 14 marks)



SUB-TOPIC 5.2

OBLIQUE IMPACTS WITH WALLS AND PLANES

Sub-topic 5.2 · 3 questions · 33 marks · lowest tariff first



5.

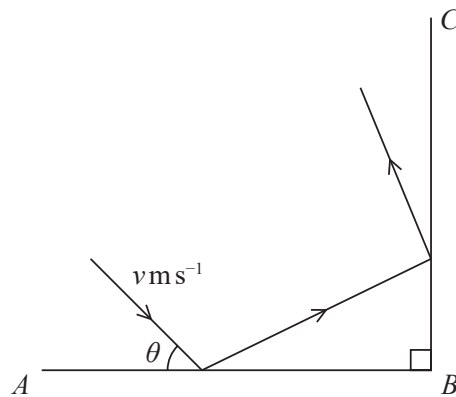


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls, with AB perpendicular to BC .

A small ball is projected along the floor towards the wall AB . Immediately before hitting the wall AB the ball is moving with speed $v \text{ ms}^{-1}$ at an angle θ to AB .

The ball hits the wall AB and then hits the wall BC .

The coefficient of restitution between the ball and the wall AB is $\frac{1}{3}$.

The coefficient of restitution between the ball and the wall BC is e .

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall AB .

(a) Find the exact value of $\cos \theta$.

(5)

The ball loses half of its remaining kinetic energy in the impact with the wall BC .

(b) Find the exact value of e .

(5)

Question 5 continued



Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)



7.

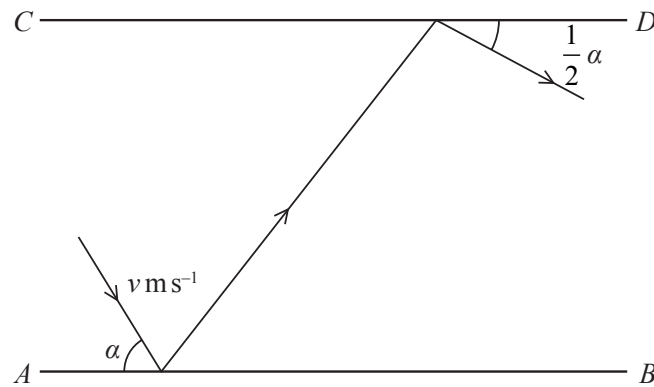


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD .

A small ball is projected along the floor towards wall AB . Immediately before hitting wall AB , the ball is moving with speed $v \text{ ms}^{-1}$ at an angle α to AB , where $0 < \alpha < \frac{\pi}{2}$.

The ball hits wall AB and then hits wall CD .

After the impact with wall CD , the ball is moving at angle $\frac{1}{2}\alpha$ to CD .

The coefficient of restitution between the ball and wall AB is $\frac{2}{3}$.

The coefficient of restitution between the ball and wall CD is also $\frac{2}{3}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that $\tan\left(\frac{1}{2}\alpha\right) = \frac{1}{3}$ (7)

(b) Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts. (4)

[illegible]

Question 7 continued

(Total for Question 7 is 11 marks)



6. A particle P of mass m is falling vertically when it strikes a fixed smooth inclined plane. The plane is inclined to the horizontal at an angle α , where $0 < \alpha \leq 45^\circ$

At the instant immediately before the impact, the speed of P is u .

At the instant immediately after the impact, P is moving horizontally with speed v .

- (a) Show that the magnitude of the impulse exerted on the plane by P is $mu \sec \alpha$ (5)

The coefficient of restitution between P and the plane is e , where $e > 0$

- (b) Show that $v^2 = u^2(\sin^2 \alpha + e^2 \cos^2 \alpha)$ (3)

- (c) Show that the kinetic energy lost by P in the impact is

$$\frac{1}{2}mu^2(1-e^2)\cos^2\alpha \quad (2)$$

- (d) Hence find, in terms of m , u and e **only**, the kinetic energy lost by P in the impact. (2)



Question 6 continued

Question 6 continued



Question 6 continued

(Total for Question 6 is 12 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
1(a)	Form an expression for total KE	M1	3.1a
	$\frac{1}{2} \times 3 \times (3^2 + (-1)^2) + \frac{1}{2} \times 2 \times ((-6)^2 + 2^2)$	A1	1.1b
	= 55 (J)	A1	1.1b
		(3)	
1(b)	Use of change in momentum	M1	3.1a
	$3 \left((-2\mathbf{i} + \frac{2}{3}\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) \right)$	A1	1.1b
	= $(-15\mathbf{i} + 5\mathbf{j})$ (N s)	A1	1.1b
		(3)	
1(c)	Either: Use impulse on B to form equation	M1	3.1a
	$(15\mathbf{i} - 5\mathbf{j}) = 2(\mathbf{v}_B - (-6\mathbf{i} + 2\mathbf{j}))$	A1	1.1b
	$\mathbf{v}_B = (1.5\mathbf{i} - 0.5\mathbf{j})$ (m s ⁻¹)	A1	1.1b
	Or: Use CLM to form equation	M1	3.1a
	$3(3\mathbf{i} - \mathbf{j}) + 2(-6\mathbf{i} + 2\mathbf{j}) = 3(-2\mathbf{i} + \frac{2}{3}\mathbf{j}) + 2\mathbf{v}_B$	A1	1.1b
	$\mathbf{v}_B = (1.5\mathbf{i} - 0.5\mathbf{j})$ (m s ⁻¹)	A1	1.1b
		(3)	
(9 marks)			
Notes: Ignore units in final answers			
(a)			
M1	Form an expression for total KE, correct number of terms and dimensionally correct. Must add 2 KE terms of the correct structure. Allow missing minus signs on components when squaring. M0 for use of $(3\mathbf{i} - \mathbf{j})^2$ unless recovered. M0 if KE terms are never added together. M0 for using $\frac{1}{2}mv$ unless the correct KE formula is also stated. (Note that $\frac{1}{2}mv$ is not dimensionally correct)		
A1	Correct unsimplified expression for the total KE. If the 2 KE terms are incorrect when the total is formed, this is A0.		
A1	Cao		
(b)	N.B. Only penalise use of column vectors in final answers once across (b) and (c). Penalise the first occurrence. Allow column vectors throughout working in (b) and (c).		

M1	Use of change in momentum, dimensionally correct (mass $\times$ velocity). Must subtract momenta but condone in wrong order. Must use mass of 3kg with velocities of A or mass of 2kg with velocities of B. M0 if speeds are used.
A1	Correct unsimplified expression for impulse
A1	Correct answer in i and j form. Penalise first occurrence if answer is given as a column vector. ISW if they continue and find the magnitude.
(c)	
M1	Either: Use of impulse-momentum equation with their Impulse. Must subtract momenta (mass $\times$ velocity) but condone in wrong order. Condone sign error on LHS.
A1	Correct unsimplified equation
A1	Correct answer in i and j form. Penalise first occurrence if answer is given as a column vector.
M1	Or: Use of CLM equation with correct number of terms, condone sign errors. Masses and velocities must be paired correctly. M0 if speeds are used.
A1	Correct unsimplified equation
A1	Correct answer in i and j form. Penalise first occurrence if answer is given as a column vector.

Question	Scheme	Marks	AOs
2(a)	Resolve perpendicular to the plane and use $F = \mu R$	M1	3.1a
	$\frac{1}{7} mg \cos \theta$	A1	1.1b
	$\frac{1}{7} mg \times \frac{4}{5} = \frac{4mg}{35} *$ or $\frac{4}{35} mg *$	A1*	1.1b
		(3)	
2(b)	Use of work-energy principle	M1	3.3
	$\frac{4mgd}{35} = \frac{1}{2} m \times 2ag - mgd \sin \theta$	A1	1.1b
		A1	1.1b
	$d = \frac{7a}{5}$ oe	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(a)			
M1	Resolve perpendicular to the plane to find an expression for R and use μR . Condone sin/cos confusion on weight component. All required terms present and no extras. Dimensionally correct.		
A1	Correct unsimplified expression for friction. Allow with $\cos \theta$ or $\frac{4}{5}$		
A1*	Given answer correctly obtained. Working out must include both $\frac{1}{7}$ and $\frac{4}{5}$ in the same line before reaching the given answer.		
(b)			
M1	Use of work-energy principle with correct number of terms: 1 work, 1 KE, 1 GPE. Condone $\pm$ sign errors on terms. All required terms present and no extras. Resolve only when required and condone cos/sin confusion for the method mark. Must use given answer from (a) in work term. M0 if Friction is not multiplied by distance (dimensionally incorrect) M0 if a term is missing. M0 for incorrect use of trig eg $d \tan \theta$, $\frac{d}{\sin \theta}$, $\frac{d}{\cos \theta}$...		
A1	Correct equation with at most one error		
A1	Correct equation		
A1	Correct answer for d . Any equivalent fraction or decimal multiple of a		



Question	Scheme	Marks	AOs
6(a)	GPE lost by B – GPE gained by A	M1	3.4
	$= 4 \times g \times 3 - 2 \times g \sin \theta \times 3$	A1	1.1b
	$= 82 \text{ (82.3) (J)}$	A1	1.1b
		(3)	
6(b)	Total KE gained $= \frac{1}{2} \times 6 \times 4.5^2 (= 60.75) \text{ (J)}$	B1	3.1b
	Max friction $\mu 2g \cos \theta (= \mu \times 2 \times 9.8 \times \cos \theta = 15.68\mu)$	B1	3.1b
	Work done against friction $= 3 \times F_{\max} (= 47.04\mu)$	B1ft	3.4
	Work-energy equation: their GPE lost = their KE gained + their WD against friction	M1	3.4
	$82.32 = 60.75 + 47.04\mu$	A1	1.1b
	$\mu = 0.459 \text{ (0.46)}$	A1	1.1b
		(6)	
6(c)	Work-energy equation for A:	M1	3.4
	$\frac{1}{2} \times 2 \times 4.5^2 = 2g \sin \theta \times d + 2g \cos \theta \times \mu d$ $\left(= 19.6 \times \frac{3}{5} \times d + 19.6 \times \frac{4}{5} \times \mu d \right)$	A1ft A1ft	1.1b 1.1b
	$d = 1.07 \text{ (1.1)}$	A1	1.1b
		(4)	
(Total 13 marks)			
Notes			
(a)M1	Expression for change in GPE. Must be dimensionally correct and resolved terms where necessary. Allow subtraction either way round		
A1	Correct unsimplified expression for the change in PE (before substitution for $\sin \theta$) Allow subtraction either way round		
A1	2 sf or 3 sf. Accept $8.4g$ or $\frac{42g}{5}$ ISW Must be positive but condone a sign change at the end without explanation		
(b) B1	Gain in KE for the system (not just for one block)		



B1	Correct unsimplified expression for $F_{\max}$ seen or implied
B1ft	Correct expression for work done: follow their $F_{\max}$ This is dependent on them having found an expression for $F_{\max}$
M1	Complete method using work-energy to form an equation in μ . Require all terms (needs to consider the KE and GPE of both blocks). Dimensionally correct. Condone sign errors.
A1	Correct unsimplified equation in μ
A1	3 sf or 2 sf only
	NB: It is possible to find the value of μ by finding the tension in the string and forming a work-energy equation for particle B, but in this case the first B1 is for KE of B and correct tension (25.7(N)) B1 for $F_{\max}$ B1ft is for work done by the tension in the string and against friction M1 for $3 \times 25.7 = 20.25 + 35.28 + 3 \times 15.68\mu$ O.E.
(c)M1	All terms required. Dimensionally correct. Condone sign errors and sin / cos confusion. If the equation uses $d + 3$ in place of d in the PE term it is correct if it also includes a term for the initial PE. If the equation uses $d + 3$ in place of d in the term for work done then it scores M0.
A1	Unsimplified equation in d and μ with at most one error
A1	Correct unsimplified equation in d and μ The ft is on their μ if they have substituted a value.
A1	3 sf or 2 sf only



Question	Scheme	Marks	AOs
4(a)	Work-energy equation:	M1	3.4
	$\frac{1}{2} \times 0.5 \times 8^2 = W + 0.5g \times 18 \sin \theta$	A1	1.1b
	$W = 16 - 9g \times \frac{1}{14} = 9.7 \quad *$	A1*	2.2a
		(3)	
(b)	Use of $F = \mu R = \mu \times 0.5g \cos \theta$	M1	3.1b
	Complete method to form a dimensionally correct equation in μ (and θ) using	M1	3.4
	• Either work done = $F \times 18$ • Or <i>suvat</i> and N2L		
	• Either $9.7 = \mu \times 0.5g \cos \theta \times 18$ • Or relevant <i>suvat</i> to find $a \left(= -\frac{16}{9} \right)$ and N2L to form	A1	1.1b
	$-\mu \times 0.5g \cos \theta \times 18 - 0.5g \sin \theta = 0.5 \left(-\frac{16}{9} \right)$		
	$\mu = 0.11 \quad (0.110)$	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(a)			
M1	Form work-energy equation in terms of W (and g and θ) only. All required terms present and no extras. All terms dimensionally correct (of correct structure). Condone $\pm$ sign errors on terms and sin/cos confusion on vertical height. M0 if a term is missing or for incorrect trig use eg $18 \tan \theta$, $\frac{18}{\sin \theta}$, $\frac{18}{\cos \theta}$. M0 for use of <i>suvat</i>		
A1	Correct unsimplified equation, no need to replace trig.		
A1*	Obtain given answer from complete and correct working. Must see a line of working between the initial equation and the given answer. Condone missing W during working but must see ' $W = \dots$ ' for the final mark.		
(b)			
M1	Correct use of $F = \mu R$ and $R = 0.5g \cos \theta$ to form an expression for Friction. Dimensionally correct. Condone sin/cos confusion. Missing g is an accuracy error not a method error.		
M1	Complete method to form a dimensionally correct equation in μ (θ and g). Trig does not need to be replaced for M mark. M0 for $W = \mu R$. May use 9.7 and work done = $F \times 18$ to form a dimensionally correct equation in μ . May see relevant <i>suvat</i> to find acceleration, followed by N2L to form dimensionally correct equation μ . N2L must contain all relevant terms and no extras. Condone sin/cos confusion on the weight components.		
A1	Correct unsimplified equation in μ (and g) with trig replaced correctly.		
A1	2 sf or 3 sf only. A0 for use of $g = 9.81 \text{ms}^{-2}$		

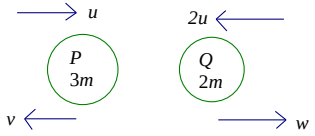


Question	Scheme	Marks	AOs	Notes
4(a)	Use of $P = Fv : F = \frac{15000}{25} (= 600)$	B1	3.3	600 or equivalent
	Equation of motion:	M1	3.4	Use the model to form the equation of motion If they start with two separate equations each one must be correct.
	$F - (200 + 200 + 25\lambda) = 0$	A1	1.1b	Correct unsimplified equation
	$\lambda = 8 *$	A1*	2.2a	Deduce given answer from correct working.
		(4)		
4(b)	Equation of motion:	M1	3.4	Use the model to form the equation of motion for the car (with $v = 10$ used). All terms required. Dimensionally correct. Condone sign error and sin/cos confusion
	$\frac{15000}{10} - 280 - 600g \sin \theta = 600a$	A1 A1	1.1b 1.1b	Unsimplified equation with at most one error. Correct unsimplified equation
	$a = 1.38 \text{ m s}^{-2} \text{ (1.4)}$	A1	1.1b	2 or 3 sf only – follows use of 9.8
		(4)		
4(c)	Work energy equation	M1	3.1b	Complete strategy to form the work-energy equation. Condone sin/cos confusion and sign errors
	$\frac{1}{2} \times 150 \times 100 = 200d + 150gd \sin \theta$	A1 A1	1.1b 1.1b	Unsimplified equation with at most one error Correct unsimplified equation for d
	$d = 25.2 \text{ (m)} \quad (25)$	A1	1.1b	Max 3 sf – follows use of 9.8
		(4)		
(Total 12 marks)				



Question	Scheme	Marks	AOs
2(a)	Equation of motion of the van	M1	3.3
	$F - 50V = 900 \times 0.2$	A1	1.1b
	Use of $P = FV$ $\left(\frac{12000}{V} - 50V = 900 \times 0.2 \right)$	M1	3.4
	$(50V^2 + 180V - 12000 = 0)$ $(V =)14$ or better (correctly rounded)	A1	1.1b
		(4)	
(b)	Equation of motion of the van	M1	3.3
	$F - 50U - 900g \sin \alpha = 0$ $\frac{15000}{U} - 50U - 75g = 0$	A1 A1	1.1b 1.1b
	$(50U^2 + 735U - 15000 = 0)$ $(U =)11$ or 11.5	A1	1.1b
		(4)	
	(8 marks)		
Notes:			
(a)			
M1	Form an equation of motion for the van to produce an equation in V only or form an equation of motion for the van to produce an equation in V and F only where F is the driving force of the van's engine. Must be dimensionally correct with required terms. Condone sign errors.		
A1	Correct unsimplified equation.		
M1	Use of $P = FV$ where F is the driving force of the van's engine. Implied by use of $\frac{12000}{V}$ with their equation of motion. Condone use of 12 or a slip with the number of zeros for this M mark.		
A1	Obtain only 14 or better (correctly rounded). Calculator display gives 13.79615337. If a negative value is seen, it must be rejected.		
(b)			
M1	Form equation of motion for the van with $a = 0$ to produce an equation in U only or in U and F only. Must be dimensionally correct with required terms. Condone sign errors and sin/cos confusion. Condone use of another letter instead of U throughout. Condone use of 15 or a slip with the number of zeros. M0 if the answer from (a) is used in (b). M0 for use of $P = 12\,000$		
A1	Unsimplified equation in F and U (or U only) with at most one error. Trig and F do not need not be replaced.		
A1	Correct unsimplified equation in U only (trig and F replaced).		
A1	Obtain only 11 or 11.5 (2 or 3sf after use of 9.8). If a negative value is seen, it must be rejected. A0 if the answer follows use of $g = 9.81\text{ms}^{-2}$		



Question	Scheme	Marks	AOs	Notes
5(a)				
	Use of CLM	M1	3.1a	Use of CLM. All terms required. Must be dimensionally correct. Condone sign errors
	$3mu - 4mu = 2mw - 3mv \quad (-u = -3v + 2w)$	A1	1.1b	Correct unsimplified equation
	Use of impact law	M1	3.4	Use of impact law. Must be dimensionally correct and used correctly. Condone sign errors
	$w + v = 3ue$	A1	1.1b	Correct unsimplified equation Signs consistent with CLM equation
	Correct strategy to form equation in w and find critical value of $e \in (0,1)$ $(5w = u(9e - 1))$	M1	3.1a	Correct overall strategy to find the critical value of e in $(0,1)$ in e eg by using CLM and impact law to form equation or inequality in w and solve for e .
	$w > 0 : e > \frac{1}{9}$	A1	1.1b	One inequality for e correct Condone $e \geq \frac{1}{9}$
	Complete strategy to justify the range of values of e $(5v = u(1 + 6e)) \quad v > 0 : \text{true for all } e$	M1	3.1a	Correct strategy to find the range of possible value of e . i.e find second speed and form second inequality
	Therefore $\frac{1}{9} < e \leq 1$	A1	2.2a	Correct final conclusion
		(8)		



Question	Scheme	Marks	AOs	Notes
5(b)	Final KE = 25% of initial KE	M1	3.1a	Use KE to form equation in e . 25% should be used correctly Condone if mass cancelled throughout
	$\frac{1}{2} \times 2m \times \frac{u^2(9e-1)^2}{25} = \frac{1}{4} \times \frac{1}{2} \times 2m \times 4u^2 \quad \left(\text{or } w = \frac{1}{2} \times 2u \right)$	A1ft	1.1b	Correct unsimplified equation – follow their w
	$\Rightarrow (9e-1)^2 = 25, \quad e = \frac{2}{3} \text{ only}$	A1	1.1b	Or equivalent. Correct conclusion ISW after correct answer.
		(3)		
(11marks)				



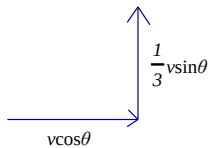
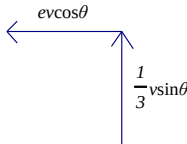
Question	Scheme	Marks	AOs
2(a)			
	Use of CLM	M1	3.1a
	$5mv + 6mv (= 11mv) = 5mx - 3my \quad (11v = 5x - 3y)$	A1	1.1b
	Use of impact law	M1	3.1a
	$v = e(x + y)$	A1	1.1b
	$\begin{cases} 11ev = 5ex - 3ey \\ 3v = 3ex - 3ey \end{cases} \Rightarrow x = \frac{v}{8e}(11e + 3)$	M1	3.1a
	$y = \frac{v}{8e}(5 - 11e)$	A1	1.1b
	$e > 0 \quad (\Rightarrow x > 0) \Rightarrow 5 - 11e > 0$	M1	3.4
	$\Rightarrow 0 < e < \frac{5}{11}$	A1	2.2a
		(8)	
(b)	Form equation for KE	M1	2.1
	$\frac{1}{2} \times 5m \times v^2 = \frac{16}{100} \times \frac{1}{2} \times 5m \times \frac{v^2}{64e^2} (11e + 3)^2$	A1ft	1.1b
	$(4(11e + 3) = (\pm)80e) \quad e = \frac{1}{3}$	A1	1.1b
	Impulse = $-5m(v - x)$	M1	3.1a
	$= -5m \left(v - \frac{11v}{8} - \frac{3v}{8e} \right)$ Or: $3m \left(2v + \frac{5v}{8e} - \frac{11v}{8} \right)$	A1ft	1.1b
	Magnitude = $\frac{15}{2}mv$	A1	2.2a
		(6)	
Alt(b)	Form equation for KE	M1	2.1
	$\frac{1}{2} \times 5m \times v^2 = \frac{16}{100} \times \frac{1}{2} \times 5m \times x^2$	A1	1.1b
	$\Rightarrow x = \frac{5v}{10} \quad v = \frac{v}{2} \quad \Rightarrow e = \frac{1}{2}$	A1	1.1b

	Impulse = $-5m(v - x)$	M1	3.1a
	$= -5m\left(v - \frac{5v}{2}\right)$ Or: $3m\left(2v + \frac{v}{2}\right)$	A1	1,16
	Magnitude = $\frac{15}{2}mv$	A1	2.2a
		(6)	
(14 marks)			
Notes:			
(a)M1	All terms required. Dimensionally correct. Condone sign errors		
A1	Correct unsimplified equation		
M1	Used correctly. Condone sign errors		
A1	Correct unsimplified equation		
M1	Use their correctly formed equations to solve for v or w or a multiple of v or w		
A1	Both velocities correct		
M1	Use their velocities (in general form – not by considering one specific value) to form inequality for both moving in the same direction.		
A1	Correct only.		
(b)M1	Dimensionally correct. Condone 16% on wrong side Allow M or $5m$		
A1ft	Or equivalent. Correct unsimplified equation. Follow their x Allow M or $5m$		
A1	Correct answer only Allow M or $5m$		
M1	Correct use of $I = mv - mu$. Must be subtracting.		
A1ft	Accept $\pm$ Follow their x, y, e		
A1	Correct only. Must be positive.		

Question	Scheme	Marks	AOs
5(a)			
	Components perpendicular to the line of centres after the collision: $\mathbf{v}_{Pj} = 2\mathbf{j}(\text{ms}^{-1})$, $\mathbf{v}_{Qj} = \mathbf{j}(\text{ms}^{-1})$	B1	3.4
	Kinetic energy:	M1	3.1a
	$\frac{1}{2} \times 0.2 \times (v^2 + 1) = \frac{1}{2} \times \frac{1}{2} \times 0.2 \times (9 + 1)$	A1	1.1b
	CLM parallel to line of centres:	M1	3.1a
	$0.3 \times 4 - 0.2 \times 3 = 0.2v - 0.3u \quad (6 = 2v - 3u)$	A1	1.1b
	Impact law parallel to line of centres	M1	3.1a
	$v + u = e(4 + 3)$	A1	1.1b
	Solve for $\mathbf{v}_P$, $\mathbf{v}_Q$ or e	M1	1.1b
	$\mathbf{v}_P = \frac{2}{3}\mathbf{i} + 2\mathbf{j}(\text{ms}^{-1})$ and $\mathbf{v}_Q = 2\mathbf{i} + \mathbf{j}(\text{ms}^{-1})$	A1	1.1b
	$e = \frac{4}{21}$	A1	1.1b
	If $v = -2 \Rightarrow u = -\frac{10}{3} \Rightarrow P$ and Q have passed through each other: impossible, so solution is unique *	A1*	2.4
		(11)	
(b)	Use trig to find angle between velocities	M1	3.1a
	$\cos \theta = \frac{\frac{8}{3} + 4}{\sqrt{20}\sqrt{4\frac{4}{9}}}$ or $\theta = \tan^{-1} \frac{2}{2/3} - \tan^{-1} \frac{1}{2}$	A1ft	1.1b
	$\theta = 45^\circ \left(\frac{\pi}{4} \text{ rads} \right)$	A1	1.1b
		(3)	
(14 marks)			
Notes:			

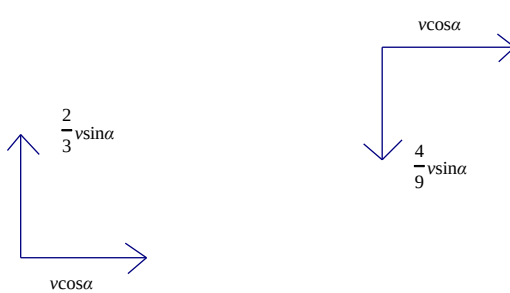
(a)B1	Seen or implied. Correct only
M1	Equation for KE of Q. Dimensionally correct. Condone $\frac{1}{2}$ on the wrong side.
A1	Correct unsimplified equation in v^2
M1	Equation for CLM. Correct terms required. Condone sign errors. Dimensionally correct.
A1	Correct unsimplified equation
M1	Correct use of impact law. Condone sign errors
A1	Correct unsimplified equation.
M1	Complete method to solve for $\mathbf{v}_P, \mathbf{v}_Q$ or e
	(Working in e gives $v = \frac{1}{5}(6 + 21e)$ and $441e^2 + 252e - 64 = 0$)
A1	Both velocities correct. Need to see answers in the form $a\mathbf{i} + b\mathbf{j}$ or equivalent
A1	Correct only. 0.19 or better (0.19047....)
A1*	Or equivalent justification of given result. e.g. a negative value for e is not possible
(b) M1	Use of trig or equivalent to find a relevant angle between two velocities e.g by scalar product or difference between angles.
A1ft	Correct unsimplified equation in θ . Follow their $\mathbf{v}_P$
A1	Correct only. (0.785... radians) Do not ISW



Question	Scheme	Marks	AOs
5(a)	Use the model to find components of velocity after first impact: 	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{2} \times \frac{1}{2} m v^2 = \frac{1}{2} m \left(v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$	M1	3.1b
	$\frac{1}{2} = \frac{1}{9} + \frac{8}{9} \cos^2 \theta$	M1	1.1b
	$\frac{7}{16} = \cos^2 \theta, \quad \cos \theta = \frac{\sqrt{7}}{4}$	A1	1.1b
	(5)		
(a) alt	Working with initial velocity $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$, after impact $\mathbf{v} = x\mathbf{i} + \frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
	KE: $\frac{1}{2} \times \frac{1}{2} m (x^2 + y^2) = \frac{1}{2} m \left(x^2 + \frac{1}{9} y^2 \right)$	M1	3.1b
	$y^2 = \frac{9}{7} x^2, \quad \frac{y}{x} = \tan \theta = \frac{3}{\sqrt{7}}$	M1	1.1b
	$\cos \theta = \frac{\sqrt{7}}{4}$	A1	1.1b
	(5)		
(b)	Use the model to find components of velocity after second impact: 	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{4} \times \frac{1}{2} m v^2 = \frac{1}{2} m \left(e^2 v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$ or $\frac{1}{2} \times \frac{1}{2} m \left(v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right) = \frac{1}{2} m \left(e^2 v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$	M1	3.1b
	$\frac{1}{4} = \frac{7}{16} e^2 + \frac{1}{9} \times \frac{9}{16}$	M1	1.1b
	$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
	(5)		
(b) alt	After second impact $\mathbf{v} = -e x \mathbf{i} + \frac{1}{3} y \mathbf{j}$	B1 B1	1.1b 3.4

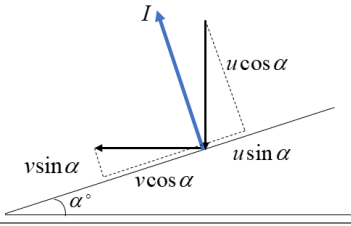
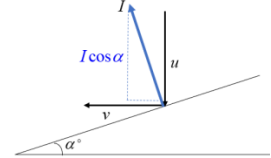
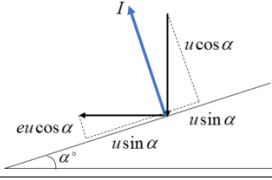


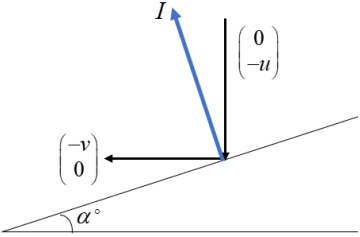
	$\text{KE: } \frac{1}{4} \times \frac{1}{2} m(x^2 + y^2) = \frac{1}{2} m\left(e^2 x^2 + \frac{1}{9} y^2\right)$	M1	3.1b
	$4e^2 x^2 + \frac{4}{9} y^2 = x^2 + y^2, \quad 4e^2 = 1 + \frac{5}{9} \left(\frac{y}{x}\right)^2$	M1	1.1b
	$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
(10 marks)			
Notes:			
(a) B1 B1	Parallel component correct Perpendicular component correct Check the diagram		
M1	Equation for KE in v, θ . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side		
M1	Form and solve equation in $\cos \theta$		
A1	Or exact equivalent		
(b) B1 B1	Parallel component correct Perpendicular component correct		
M1	Equation for KE in x, y . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side		
M1	Use their $\cos \theta$ to form and solve equation in e		
A1	Or exact equivalent		

Question	Scheme	Marks	AOs
7(a)	Use model to find components of velocity after the impacts: 	B1	3.1b
		B1	3.4
		B1	3.1b
		B1	3.4
	$\tan \frac{\alpha}{2} = \frac{\frac{4}{9} v \sin \alpha}{v \cos \alpha} \left(= \frac{4}{9} \tan \alpha \right)$	M1	3.1b
(b)	$t = \tan \frac{\alpha}{2} \Rightarrow t = \frac{4 \times 2t}{9(1-t^2)}$	M1	1.1b
	$1-t^2 = \frac{8}{9}, \quad t = \frac{1}{3} *$	A1*	2.2a
		(7)	
	$\tan \alpha = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$	B1	1.1b
	change in KE $\frac{1}{2}mv^2 - \frac{1}{2}m \left(v^2 \cos^2 \alpha + \left(\frac{4}{9}v \right)^2 \sin^2 \alpha \right)$	M1	3.1b
	$\% \text{ of KE lost} = 100 \left(1 - \frac{\frac{1}{2}mv^2 \left(\frac{16}{25} + \frac{16}{81} \times \frac{9}{25} \right)}{\frac{1}{2}mv^2} \right)$	M1	1.1b
	$= 28.888...(\%)$	A1	1.1b
		(4)	
(11 marks)			
Notes:			
(a)B1 B1 B1 B1	One mark for each component correct.		
M1	Form expression for $\tan \frac{\alpha}{2}$ in terms of $\tan \alpha$		

M1	Form and solve equation in $\tan \frac{\alpha}{2}$
A1*	Obtain given answer from correct working
	NB: This is a “Show that ..” question. A candidate who assumes, without proof, that $\tan \frac{\alpha}{2} = e^2 \tan \alpha$ can only score the last two marks.
(b)B1	Correct use of $t = \frac{1}{3}$ Must be seen / used in part (b)
M1	Dimensionally correct expression for change in KE NB note that they may not show component parallel to the wall
M1	Dimensionally correct expression for the percentage of KE lost.
A1	Accept 29(%) or better Accept $\frac{260}{9}$



Question	Scheme	Marks	AOs
6(a)			
	CLM along the plane:	M1	3.1a
	$(m)u \sin \alpha = (m)v \cos \alpha$	A1	1.1b
	Impulse-momentum perp to the plane:	M1	3.1a
	$I = m(v \sin \alpha - (-u \cos \alpha))$	A1	1.1b
	$I = m\left(\frac{u \sin^2 \alpha}{\cos \alpha} + u \cos \alpha\right) = \frac{mu}{\cos \alpha}(\sin^2 \alpha + \cos^2 \alpha) = mu \sec \alpha^*$	A1*	2.2a
	(5)		
6(a) alt1		M1	3.1a
	Impulse-momentum vertically.	M1	3.1a
	$I \cos \alpha = m(0 - -u)$	A1	1.1b
		A1	1.1b
	$I = mu \sec \alpha^*$	A1*	2.2a
	(5)		
6(a) alt 2	Introduce and use an expression for e 		
	CLM along the plane:	M1	3.1a
	$u \sin \alpha$ unchanged	A1	1.1b
	Finds an expression for e together with Impulse-momentum perpendicular to the plane $\tan \alpha = \frac{eu \cos \alpha}{u \sin \alpha} \Rightarrow e = \tan^2 \alpha$ and $I = m(eu \cos \alpha - (-u \cos \alpha))$	M1	3.1a
	$I = m(u \cos \alpha \tan^2 \alpha - (-u \cos \alpha))$	A1	1.1b
	$I = m\left(\frac{u \sin^2 \alpha}{\cos \alpha} + u \cos \alpha\right) = \frac{mu}{\cos \alpha}(\sin^2 \alpha + \cos^2 \alpha) = mu \sec \alpha^*$	A1*	2.2a
	(5)		

6(a) alt 3	Use a vector approach and magnitude of impulse			
	CLM along the plane:		M1	3.1a
	$(m)u \sin \alpha = (m)v \cos \alpha$ (this leads to $v = u \tan \alpha$)		A1	1.1b
	Impulse-momentum as a vector equation followed by Pythagoras to find the magnitude. $I = m \begin{pmatrix} -v \\ u \end{pmatrix}$ and $ I = m\sqrt{v^2 + u^2}$		M1	3.1a
	$ I = m\sqrt{u^2 \tan^2 \alpha + u^2}$		A1	1.1b
	$I = m\sqrt{u^2(1 + \tan^2 \alpha)} = m\sqrt{u^2 \sec^2 \alpha} = mu \sec \alpha^*$		A1*	2.2a
			(5)	
6(b)	NEL: $eu \cos \alpha = v \sin \alpha$		M1	3.4
	Squaring and adding their expressions for $v \sin \alpha$ and $v \cos \alpha$.		M1	1.1b
	$v^2 = u^2(\sin^2 \alpha + e^2 \cos^2 \alpha)^*$		A1*	1.1b
			(3)	
6(c)	KE loss = $\frac{1}{2}mu^2 - \frac{1}{2}mu^2(\sin^2 \alpha + e^2 \cos^2 \alpha)$.		M1	2.1
	Use $\sin^2 \alpha + \cos^2 \alpha = 1$ to give KE loss = $\frac{1}{2}mu^2(1 - e^2) \cos^2 \alpha^*$		A1*	1.1b
			(2)	
6(d)	Use $\tan^2 \alpha = e$ to eliminate α in given expression from (c)		M1	3.1a
	KE Loss = $\frac{1}{2}mu^2(1 - e)$ or $\frac{1}{2}mu^2 \frac{1}{1 + e}(1 - e^2)$		A1	1.1b
			(2)	
(12 marks)				
Notes:				
(a)				
M1	Correct no. of terms, dimensionally correct, mass $\times$ velocity, condone sin/cos confusion.			
A1	Correct equation			
M1	Dimensionally correct. Must be subtracting, but condone subtracting in the wrong order and sin/cos confusion			

A1	Correct unsimplified equation
A1*	Given answer correctly obtained. Must be EXACT factorisation.
(b)	
M1	Attempt at NEL
M1	Squaring and adding their expressions for $v \sin \alpha$ and $v \cos \alpha$ to obtain v^2 .
A1*	Given answer correctly obtained. Must be EXACT.
(c)	
M1	Expression for difference of KE in terms of m , u , α and e
A1*	Given answer correctly obtained. Factorisation must be EXACT.
(d)	
M1	Complete method to eliminate α e.g. using $\tan^2 \alpha = e$ to eliminate α Any trig identity used must be correct eg $\sec^2 \alpha = 1 + e$ or $\cos^2 \alpha = \frac{1}{1+e}$
A1	Correct answer.

Question	Scheme	Marks	AOs
	Note: The diagram below is an aide for marking. In reality, the velocity components cannot be represented by the side lengths of the snooker table. The magnitude of PC is not the magnitude of $U \cos \alpha$		