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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Elastic Strings and Springs, and Elastic Energy (3)

Further Mechanics 1 · Topic 3

7 questions · 78 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
3.1 HOOKE'S LAW		1 Qs · 7 marks	3
1. Q5	Option 3C: Further Mechanics 1 June2024	7	4
3.2 ELASTIC ENERGY		6 Qs · 71 marks	6
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QUESTIONS

7 questions · 78 marks · 2 sub-topics



SUB-TOPIC 3.1

HOOKE'S LAW

Sub-topic 3.1 · 1 question · 7 marks · lowest tariff first



(Total for Question 5 is 7 marks)



SUB-TOPIC 3.2

ELASTIC ENERGY

Sub-topic 3.2 · 6 questions · 71 marks · lowest tariff first



Question 6 continued

Question 6 continued

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Question 6 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)



Question 7 continued

Question 7 continued



Question 7 continued

(Total for Question 7 is 12 marks)



4. A light elastic string has natural length $2a$ and modulus of elasticity $4mg$. One end of the elastic string is attached to a fixed point O . A particle P of mass m is attached to the other end of the elastic string. The particle P hangs freely in equilibrium at the point E , which is vertically below O .

(a) Find the length OE .

(4)

Particle P is now pulled vertically downwards to the point A , where $OA = 4a$, and released from rest. The resistance to the motion of P is a constant force of

magnitude $\frac{1}{4}mg$.

(b) Find, in terms of a and g , the speed of P after it has moved a distance a .

(7)

Particle P is now held at O

Particle P is released from rest and reaches its maximum speed at the point B .

The resistance to the motion of P is again a constant force of magnitude $\frac{1}{4}mg$.

(c) Find the distance OB .

(4)

Question 4 continued



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
5.	GPE from E to instantaneous rest eg mgx , $mg(d - a)$	M1	1.1b
	Use of conservation of energy principle	M1	3.1a
	For example, $\frac{1}{2}m\frac{9ag}{4} + mgx = \frac{2mg(a+x)^2}{4a} - \frac{2mga^2}{4a}$ oe $\frac{1}{2}m\frac{9ag}{4} + mg(d - a) = \frac{2mgd^2}{2(2a)} - \frac{2mga^2}{2(2a)}$ oe	A1	1.1b
	Accept equivalent rearrangements in one unknown length	A1	1.1b
	$x = \frac{3a}{2}$ or $d = \frac{5a}{2}$	A1	1.1b
	Use of EPE formula	M1	2.1
	$\frac{25mga}{8}$	A1	2.2a
		(7)	
(7 marks)			
Notes:			
M1	Use of GPE for unknown distance from E to instantaneous rest. May be implied by a difference of 2 GPE terms.		
M1	Use of conservation of energy principle to form an equation with one KE term, one GPE term, two EPE terms. For the method mark only we will, condone only one EPE term. However, all terms are required for A marks. Dimensionally correct energy equation (energy terms must have the correct structure). Note there are common rearrangements. Eg Initial Energy = Final Energy, Energy Loss = Energy Gain oe Condone $\pm$ sign errors. M0 for use of <i>suvat</i> . M0 if 'E' or similar is used to represent unknown EPE unless recovered.		
A1	All 4 terms present in an energy equation with one unknown length. At most one error. A0 if an energy term is missing.		
A1	Fully correct equation in one unknown length.		
A1	cao		
M1	Use of EPE formula at least once. May be seen here or in earlier working. EPE must have the form $\frac{\lambda x^2}{k a}$ where λ is modulus of elasticity, k is a constant and x is extension.		
A1	Correct answer		



Question	Scheme	Marks	AOs
6(a)	Thrust in the spring $= \frac{3mg2l}{3l} (= 2mg)$	B1	2.1
	Equation of motion:	M1	3.3
	$2mg - mg \sin \alpha - \frac{1}{3} mg \cos \alpha = ma$ $\left(2mg - \frac{3mg}{5} - \frac{4mg}{15} = ma \right)$	A1ft A1ft	1.1b 1.1b
	$a = \frac{17g}{15} \quad *$	A1*	2.2a
		(5)	
(b)	Initial EPE $= \frac{3mg4l^2}{2 \times 3l} (= 2mgl)$	B1	3.4
	Gain in GPE $= mg2l \sin \alpha \left(= \frac{6}{5} mgl \right)$	B1	3.4
	Work done against friction $= \frac{1}{3} mg \cos \alpha \times 2l \left(= \frac{8}{15} mgl \right)$	B1	3.4
	Work-energy equation:	M1	3.1a
	$\frac{1}{2} mv^2 + \frac{2}{3} mgl \cos \alpha + 2mgl \sin \alpha = 2mgl$	A1	1.1b
	$v = \sqrt{\frac{8gl}{15}}$	A1	1.1b
		(6)	
(11 marks)			
Notes:			
(a) B1	Correct unsimplified expression for the thrust		
M1	Equation of motion. All required terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion		
A1ft	Unsimplified equation with at most one error (in T or their T)		
A1ft	Correct unsimplified equation (in T or their T)		
A1*	Obtain given result from correct working		
(b) B1	Use model to obtain one correct term		
B1	Use model to obtain two correct terms		
B1	Use model to obtain three correct terms		



M1	All required terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion.
A1	Correct unsimplified equation
A1	Accept $0.73\sqrt{gl}$



Question	Scheme	Marks	AOs
6(a)	Work done against friction = $3l \times \mu mg \cos \theta \quad \left(= \frac{9mgl}{13} \right)$	B1	3.4
	Gain in EPE = $\frac{kmg \times 4l^2}{2l} \quad (= 2kmg l)$	B1	3.4
	Gain in GPE = $mg \times 3l \sin \theta \quad \left(= \frac{15mgl}{13} \right)$	B1	3.4
	Work energy equation:	M1	2.1
	$\frac{1}{2} m \times 6gl = \frac{9mgl}{13} + 2kmg l + \frac{15mgl}{13}$	A1	1.1b
	$2k = 3 - \frac{24}{13} = \frac{15}{13}, \quad k = \frac{15}{26} \quad *$	A1*	2.2a
		(6)	
(b)	Tension in the string at B: $\frac{\frac{15}{26}mg \times 2l}{l} \quad \left(= \frac{15mg}{13} \right)$	B1	3.1a
	Equation of motion: tension + component of weight – friction = ma	M1	3.3
	$\frac{15mg}{13} + mg \sin \theta - \frac{1}{4}mg \cos \theta = ma$ $\left(mg \left(\frac{15}{13} + \frac{5}{13} - \frac{3}{13} \right) = ma \right)$	A1 A1	1.1b 1.1b
	$a = \frac{17g}{13}$	A1	1.1b
		(5)	
(11 marks)			
Notes:			
(a)B1	Use model to obtain one correct term		
B1	Use model to obtain two correct terms		
B1	Use model to obtain three correct terms		
M1	Work-energy equation. Need all terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion.		
A1	Correct unsimplified equation		
A1*	Obtain given result from correct working		
	NB: The use of <i>suvat</i> equations is not a valid alternative method because the acceleration is not constant		
(b) B1	Correct unsimplified expression for the tension in the string		



M1	Equation of motion. Need all terms and no extras. Condone sign errors and sin/cos confusion Allow with T or their T
A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
A1	Exact answer or accept 12.8 or 13 (ms^{-2})



Question	Scheme	Marks	AOs	Notes
7(a)	From A to B EPE lost = GPE gained	M1	2.1	Use conservation of energy with $EPE = \frac{\lambda x^2}{2a}$. (Condone $EPE = \frac{\lambda x^2}{a}$ here). All three terms required. Must be dimensionally correct. Condone sign errors.
	$\frac{kmg \times 4a^2}{2a} - \frac{kmg \times \frac{a^2}{4}}{2a} = mg \times \frac{5a}{2}$	A1	1.1b	Correct unsimplified equation in k
	$k = \frac{4}{3} *$	A1*	2.2a	Derive given result from correct working.
		(3)		
7(b)	At A, equation of motion:	M1	3.1a	Use $T = \frac{\lambda x}{a}$ and N2L to form equation of motion. All terms required. Dimensionally correct. Condone sign errors
	$(T - mg) = \frac{4mg \times 2a}{3a} - mg = m \times \text{acceleration}$	A1	1.1b	Correct unsimplified equation
	$\Rightarrow \text{acceleration} = \frac{5g}{3}$	A1	1.1b	Correct only ISW. Condone 1.7g or better. Accept + / -
		(3)		



Question	Scheme	Marks	AOs	Notes
7(c)	Max speed at equilibrium position	M1	3.1a	Maximum speed at equilibrium seen or implied, and correct method to find e
	$\frac{4mge}{3a} = mg, \quad e = \frac{3a}{4}$	A1	1.1b	Correct e
				Alternative: form energy equation for movement through a height of h and differentiate v^2 wrt h to find h for max v M1 $h = \frac{5a}{4}$ A1
	Forms equation using conservation of energy	M1	3.1a	Form energy equation for movement from A to equilibrium position. Need all 4 terms. Correct form for EPE. Dimensionally correct. Condone sign errors. Allow in a, g and e (with e defined)
	$\frac{4mg \times 4a^2}{3 \times 2a} = \frac{4mg \times \frac{9a^2}{16}}{3 \times 2a} + \frac{1}{2}mv^2 + mg \times \frac{5a}{4}$	A1ft A1ft	1.1b 1.1b	Unsimplified equation in their e with at most one error Correct unsimplified equation (using their e) for v
	$v = \frac{5}{2}\sqrt{\frac{ga}{3}}$	A1	1.1b	Any equivalent form. Accept $1.44\sqrt{ag}$ or $1.4\sqrt{ag}$
		(6)		
				SHM is not on this specification, but you might see some candidates using it. See over for SHM alternative for parts (b) and (c)



	At equilibrium, $\frac{4mge}{3a} = mg, e = \frac{3a}{4}$			They need to start by showing that they have SHM in order to justify using the standard results. No marks scored for this at this stage.
	Equation of motion: $mg - \frac{4mg}{3a}(e + x) = m\ddot{x}$, so $\ddot{x} = -\frac{4g}{3a}x$ Hence SHM			
	(b) Use of $x = \frac{5a}{4}$ and their ω^2	M1		Substitute to find acceleration
	$\ddot{x} = -\frac{4g}{3a} \times \frac{5a}{4} = -\frac{5g}{3}, \ddot{x} = \frac{5g}{3}$	A1		Correct only ISW. Condone 1.7g or better
		(2)		
	(c) $\frac{4mge}{3a} = mg,$	M1		This work now scores the two marks provided it is used in part (c)
	$e = \frac{3a}{4}$	A1		
	Use of $v_{\max} = \omega a$	M1		Correct method to find max v
	$v_{\max} = \sqrt{\frac{4g}{3a}} \times \frac{5a}{4}$	A2ft		Follow their e and ω
	$v_{\max} = \frac{5}{2} \sqrt{\frac{ga}{3}}$	A1		Any equivalent form. Accept $1.44\sqrt{ag}$ or $1.4\sqrt{ag}$
		(6)		
(Total 12 marks)				



Question	Scheme	Marks	AOs
7(a)	EPE at $A = \frac{\lambda a^2}{2a}$ or EPE at $B = \frac{\lambda(2a)^2}{2a}$	M1	2.1
	Form work-energy equation:	M1	3.3
	$\frac{\lambda a^2}{2a} + mg \times 3a = \frac{\lambda(2a)^2}{2a} \quad \left(\frac{\lambda a}{2} + 3mga = 2\lambda a \right)$	A1 A1	1.1b 1.1b
	$3mg = \frac{3\lambda}{2} \Rightarrow \lambda = 2mg \quad *$	A1*	2.2a
		(5)	
7(b)	Extension at equilibrium:	M1	2.1
	$\frac{2mgx}{a} = mg \Rightarrow x = \frac{a}{2} \quad *$	A1*	1.1b
	Alternative for the first M1A1:		
	Use the work-energy equation to obtain $\frac{dV^2}{dx}$ and set the derivative equal to zero	M1	
	$\frac{1}{a} \times 2x - 1 = 0 \Rightarrow x = \frac{a}{2}$	A1	
	Use work-energy equation to find max speed:	M1	3.4
	$\frac{2mgx^2}{2a} + mg \times (2a - x) + \frac{1}{2}mV^2 = \frac{2mg(2a)^2}{2a}$	A1	1.1b
	$\left(\frac{ag}{4} + \frac{3ag}{2} + \frac{1}{2}V^2 = 4ag \right)$	A1	1.1b
7(c)	$V = 3\sqrt{\frac{ag}{2}}$	A1	2.2a
		(6)	
7(c)	e.g. for B1 Need to include the GPE of the spring The extension of the spring at equilibrium will be different The spring will have KE You would need to include the KE of the spring in the energy equation You would need to include the GPE of the spring in the energy equation The GPE of the system changes It would take work to raise the spring so the package would have less KE If the spring has mass then GPE of the spring would need to be included	B1	3.5b



		(1)	
(Total 12 Marks)			
Notes			
(a) M1	Correct method for EPE seen or implied Need something of the form $\frac{1}{2}kx^2$ where $k = \frac{\lambda}{a}$ Must be using the formula for EPE correctly at least once		
M1	Require all terms. Dimensionally correct. Condone their EPE. Condone sign errors		
A1	Unsimplified equation with at most one error. A repeated error in EPE formula is one error		
A1	Correct unsimplified equation.		
A1*	Obtain given answer from correct working		
(b) M1	Use correct method for tension to find the extension at equilibrium. Need to see the formula for tension used . Allow verification with an appropriate conclusion If they use SHM they must use $F = ma$ to prove that P is moving with SHM, otherwise 0/2.		
A1*	Correct answer from correct work Allow verification with an appropriate conclusion		
Alt:M1	Or an equivalent method for finding the turning point of a quadratic		
Alt:A1*	Correct answer from correct work		
M1	Use given x to form work-energy equation. Need all terms, and dimensionally correct. Condone sign errors. Accept with values of λ and x not substituted		
A1	Unsimplified equation with at most one error. Need given λ and given x substituted at some point. A repeated error in the formula for EPE is one error.		
A1	Correct unsimplified equation with given λ and given x substituted at some point		
A1	Use correct method for tension to find the extension at equilibrium. Any equivalent form. $2.1\sqrt{ag}$ or better		
(c) B1	Any valid response. B0 if answer includes an additional incorrect factor. Must be specific e.g. not just “the GPE changes”, but the GPE of the system changes is OK. Must relate to an effect on the energy equation E.g. for B0 The extension changes AB will increase The tension/energy/GPE/work done etc would increase The KE/GPE/EPE/acceleration/extension/velocity changes The mass of the spring would drag down and the EPE would change The EPE/KE/GPE etc would be variable		
	There would be tension in the spring as well It has weight The velocity would decrease as energy is converted		



Question	Scheme	Marks	AOs
4(a)	$T = \frac{4mge}{2a}$	B1	3.3
	$T = mg$	M1	3.1a
	$e = \frac{1}{2}a$	A1	1.1b
	$OE = \frac{5a}{2}$	A1	1.1b
		(4)	
4(b)	GPE term, $\pm mga$	B1	3.4
	Work done against resistance, $\pm \frac{1}{4}mga$	B1	3.4
	Use of EPE formula once.	M1	3.4
	$\pm \frac{4mg}{2 \times 2a} \{(2a)^2 - a^2\}$	A1	1.1b
	Work energy equation:	M1	3.1a
	$\frac{1}{4}mga = \frac{4mg}{2 \times 2a} \{(2a)^2 - a^2\} - mga - \frac{1}{2}mv^2$	A1	1.1b
	$v = \sqrt{\frac{7ag}{2}}$ oe	A1	1.1b
		(7)	
4(c)	$mg - T - \frac{1}{4}mg = 0$	M1	3.1a
	$mg - \frac{4mgx}{2a} - \frac{1}{4}mg = 0$	A1	1.1b
	$x = \frac{3a}{8}$	A1	1.1b
	$OB = \frac{19a}{8}$ oe	A1	1.1b
		(4)	
(15 marks)			
Notes:			

(a)	
B1	Hooke's Law seen with $4mg$ and $2a$ substituted.
M1	Resolving vertically. Correct number of terms.
A1	cao for extension.
A1	cao for OE . Note that if the extension is $(OE - 2a)$ in their equation, OE can be found directly and both A's can be earned together.
(b)	
B1	GPE term seen, ignore sign.
B1	Work term seen $\frac{mga}{4}$, ignore sign. Allow B1 for the case where $WD = \frac{5mga}{4}$. This is a special case where the work done against resistance is included within the term. $\frac{5mga}{4} = \text{WD against resistance} + \text{WD against weight}$.
M1	Use of EPE formula. Accept EPE in the form $\frac{\lambda x^2}{ka}$
A1	Difference between two correct EPE terms seen, unsimplified.
M1	Work-energy equation is formed with all relevant terms and no extras.: KE, GPE, 2EPE, WD. Condone sign errors. M0: For work-energy equation with $WD = \frac{5mga}{4}$ and a GPE term. This is because weight is considered twice and so the equation contains an extra term.
A1	Correct unsimplified equation
A1	Correct answer in terms of a and g , do not allow 9.8 for g $v = \sqrt{\frac{7ag}{2}}$, $v = \frac{1}{2}\sqrt{14ag}$
(c)	
M1	Vertical equilibrium equation or equation of motion with $a = 0$. Condone sign errors. Correct no. of terms - all 3 forces must be included although $\left(mg \pm \frac{mg}{4}\right)$ may already be simplified. Hooke's Law does not need to be substituted but M0 if the equilibrium position from (a) is used.
A1	Correct equation in one unknown.
A1	cao
A1	cao Note that if the extension is $(OB - 2a)$ in their equation, OB can be found directly and both A's can be earned together.



4(c) Alt 1	Using differentiation with a Work - energy equation from the point of release
M1	Forming work-energy equation with the usual rules: all relevant terms to be included and of the correct form and no extra terms. $\frac{1}{2}mv^2 = mgh - \frac{4mg(h-2a)^2}{2(2a)} - \frac{mgh}{4}$
A1	Correct equation for v^2 and h (may use a different letter)
A1	Correct equation after differentiating v^2 or v with respect to h and setting it equal to zero. $\frac{d}{dh}(v^2) = 0 \rightarrow \frac{3g}{2} = \frac{4g(h-2a)}{a} \text{ oe}$
A1	Correct answer $OB = \frac{19a}{8}$



Question	Scheme	Marks	AOs	
6(a)	Correct use of Hooke's law $T = \frac{\frac{8}{3}mge}{2a} \quad \text{or} \quad T = \frac{\frac{8}{3}mg(AB - 2a)}{2a}$	M1	3.4	
	Resolve parallel to slope	M1	3.1b	
	$T = mg \sin \theta \left(= \frac{1}{3}mg \right)$	A1	1.1b	
	$\Rightarrow e = \frac{2a}{8} \left(= \frac{a}{4} \right)$ $AB = \frac{9}{4}a \quad *$	$\Rightarrow AB - 2a = \frac{2a}{8} \left(= \frac{a}{4} \right)$ $AB = \frac{9}{4}a \quad *$	A1*	2.2a
			(4)	
(b)	Loss in EPE	M1	3.3	
	$\frac{\lambda(3a - 2a)^2}{2(2a)} - \frac{\lambda \left(\frac{9a}{4} - 2a \right)^2}{2(2a)}$	A1	1.1b	
	$= \frac{5mga}{8}$	A1	1.1b	
			(3)	
(c)	Energy equation	M1	3.1b	
	$"\frac{5mga}{8}" = mg \left(3a - \frac{9a}{4} \right) \sin \theta + \frac{1}{2}mv^2$	A1ft	1.1b	
	$v = \sqrt{\frac{3ga}{4}}$	A1	1.1b	
			(3)	
(10 marks)				
Notes:				
(a)				
M1	Correct use of Hooke's law with $\frac{8mg}{3}$ and $2a$ substituted.			
M1	Resolve parallel to the slope with all required terms and no extras. Dimensionally correct. Weight must be resolved but condone sin/cos confusion. T does not need to be replaced.			
A1	Correct unsimplified equilibrium equation, T does not need to be replaced.			
A1*	A complete and correct method using Hooke's law with the parallel equilibrium equation to obtain the given answer. There must be at least one line of working between the initial equations and the given answer. Must see ' $AB = \dots$ '. Accept fractions $\frac{9a}{4}$ or $\frac{9}{4}a$.			

(b)	
M1	Correct method for difference in EPE at <i>B</i> and <i>C</i> , allow either way round. Dimensionally correct and of the correct structure. No need to substitute λ . For M mark, condone denominator of $2a$. Must use extensions $(3a - 2a)$ and $\left(\frac{9a}{4} - 2a\right)$.
A1	Correct unsimplified expression for change in EPE. Allow $\pm$
A1	Correct answer, o.e. ISW. Accept $0.625mga$ and $0.63mga$. Must be positive but allow a negative expression to change to a positive expression without justification.
(c)	
M1	Use of conservation of energy principle from <i>C</i> to <i>B</i> to form an equation with all terms of the correct structure and dimensionally correct. Condone sign errors. Condone sin/cos confusion on vertical height. For GPE, $mg\left(3a - \frac{9a}{4}\right)\sin\theta$ o.e for example $mg\frac{3a}{4}\sin\theta$, $mg\frac{3a}{4}\left(\frac{1}{3}\right)$, $mg\frac{a}{4}$ For EPE change, may use their answer from (b) if dimensionally correct (of the form kma where k is a constant) or start again with 2 EPE terms: $\frac{\frac{8mg}{3}(3a - 2a)^2}{2(2a)} \text{ and } \frac{\frac{8mg}{3}\left(3a - \frac{9}{4}a\right)^2}{2(2a)}.$ For M mark, condone denominator of $2a$.
A1ft	Correct unsimplified equation. Terms must be correct when the equation is formed but follow their answer to (b) for EPE if used.
A1	Correct answer in terms of $\sqrt{ag}$ ISW. Accept eg $\frac{1}{2}\sqrt{3ag}$, $0.87\sqrt{ag}$ or better. N.B. If the final mark in (b) is A0 due to substituting $g = 9.8 \text{ ms}^{-2}$, do not penalise again for the same reason here.