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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Elastic Collisions in One Dimension (4)

Further Mechanics 1 · Topic 4

25 questions · 287 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

25 questions · 287 marks · 4 sub-topics



SUB-TOPIC 4.1

NEWTON'S LAW OF RESTITUTION

Sub-topic 4.1 · 4 questions · 37 marks · lowest tariff first



3. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. It collides directly with a particle Q of mass m that is moving on the plane with speed $2u$ in the opposite direction to P .

The coefficient of restitution between P and Q is e , where $e > \frac{4}{5}$

- (a) Show that the speed of Q immediately after the collision is $\frac{(4 + 10e)u}{3}$ (6)

After the collision Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q . The coefficient of restitution between Q and the wall is f .

- (b) Find, **in terms of e** , the set of values of f for which there will be a second collision between P and Q .
- (4)**



Question 3 continued

Lined area for writing the answer to Question 3 continued.

(Total for Question 3 is 10 marks)



5. Two particles, P and Q , are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

The mass of P is $3m$ and the mass of Q is $4m$.

Immediately before the collision the speed of P is $2u$ and the speed of Q is u .

The coefficient of restitution between P and Q is e .

- (a) Show that the speed of Q immediately after the collision is $\frac{u}{7}(9e + 2)$

(6)

After the collision with P , particle Q collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of Q .

The coefficient of restitution between Q and the wall is $\frac{1}{2}$

- (b) Find the complete range of possible values of e for which there is a second collision between P and Q .

(4)







Question 5 continued

(Total for Question 5 is 10 marks)







Question 5 continued

(Total for Question 5 is 11 marks)



SUB-TOPIC 4.2

SUCCESSIVE DIRECT IMPACTS

Sub-topic 4.2 · 4 questions · 56 marks · lowest tariff first



Question 2 continued



Question 2 continued

(Total for Question 2 is 14 marks)

[illegible]

(Total for Question 5 is 14 marks)





Question 5 continued



Question 5 continued

SUB-TOPIC 5.1

OBLIQUE IMPACTS BETWEEN SPHERES

Sub-topic 5.1 · 8 questions · 96 marks · lowest tariff first



Question 4 continued

Question 4 continued



Question 4 continued

(Total for Question 4 is 9 marks)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued



Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)



5.

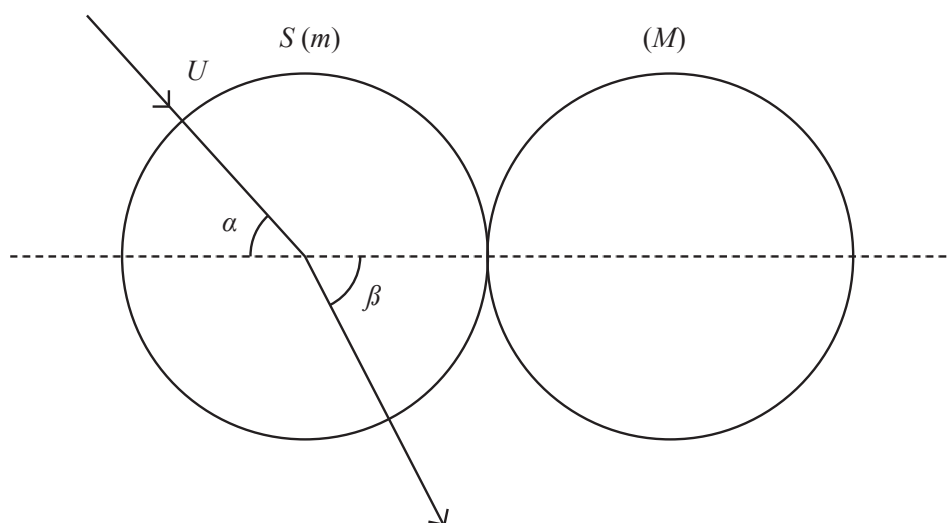


Figure 1

A smooth uniform sphere S of mass m is moving with speed U on a smooth horizontal plane. The sphere S collides obliquely with another uniform sphere of mass M which is at rest on the plane. The two spheres have the same radius.

Immediately before the collision the direction of motion of S makes an angle α , where $0 < \alpha < 90^\circ$, with the line joining the centres of the spheres.

Immediately after the collision the direction of motion of S makes an angle β with the line joining the centres of the spheres, as shown in Figure 1.

The coefficient of restitution between the spheres is e .

(a) Show that $\tan \beta = \frac{(m + M) \tan \alpha}{(m - eM)}$ (8)

Given that $m = eM$,

(b) show that the directions of motion of the two spheres immediately after the collision are perpendicular. (2)

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)





Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)



Question 3 continued

Question 3 continued



Question 3 continued

(Total for Question 3 is 14 marks)



- (3)

Question 5 continued

Question 5 continued

(Total for Question 5 is 14 marks)



Question 7 continued



Question 7 continued



Question 7 continued

(Total for Question 7 is 15 marks)



7.

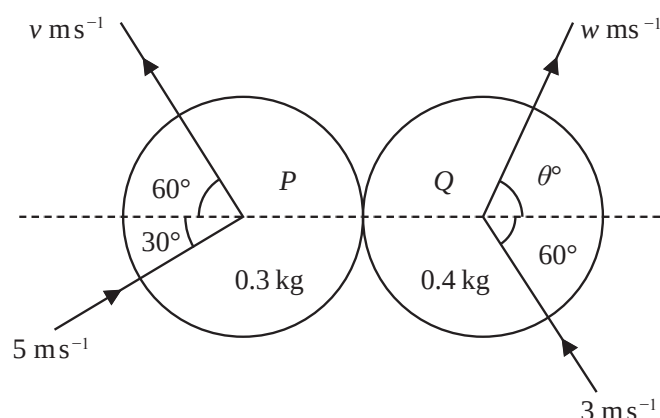


Figure 4

Two uniform spheres P and Q have equal radii. The mass of P is 0.3 kg and the mass of Q is 0.4 kg . The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately **before** the collision,

- P is moving with speed 5 ms^{-1} at 30° to the line of centres of the spheres
- Q is moving with speed 3 ms^{-1} at 60° to the line of centres of the spheres

Immediately **after** the collision,

- P is moving with speed $v \text{ ms}^{-1}$ at 60° to the line of centres of the spheres
- Q is moving with speed $w \text{ ms}^{-1}$ at θ° to the line of centres of the spheres

as in the plan view shown in Figure 4.

(a) Show that $v = \frac{5\sqrt{3}}{3}$ (3)

(b) Find (i) the value of w
 (ii) the value of θ (7)

(c) Find the coefficient of restitution between P and Q . (3)

Question 7 continued

Question 7 continued



SUB-TOPIC 5.2

OBLIQUE IMPACTS WITH WALLS AND PLANES

Sub-topic 5.2 · 9 questions · 98 marks · lowest tariff first



Question 1 continued



Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)



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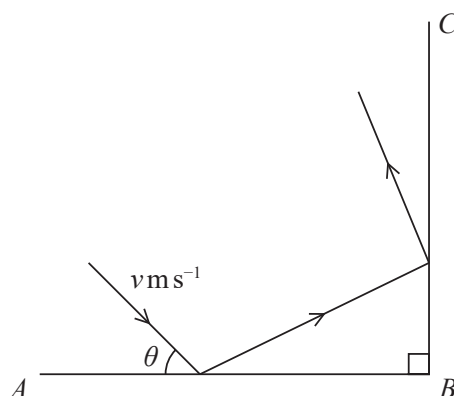


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls, with AB perpendicular to BC .

A small ball is projected along the floor towards the wall AB . Immediately before hitting the wall AB the ball is moving with speed $v \text{ ms}^{-1}$ at an angle θ to AB .

The ball hits the wall AB and then hits the wall BC .

The coefficient of restitution between the ball and the wall AB is $\frac{1}{3}$.

The coefficient of restitution between the ball and the wall BC is e .

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall AB .

(a) Find the exact value of $\cos \theta$.

(5)

The ball loses half of its remaining kinetic energy in the impact with the wall BC .

(b) Find the exact value of e .

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)



Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)



Question 8 continued

Question 8 continued

Question 8 continued



Question 8 continued

(Total for Question 8 is 10 marks)



Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)



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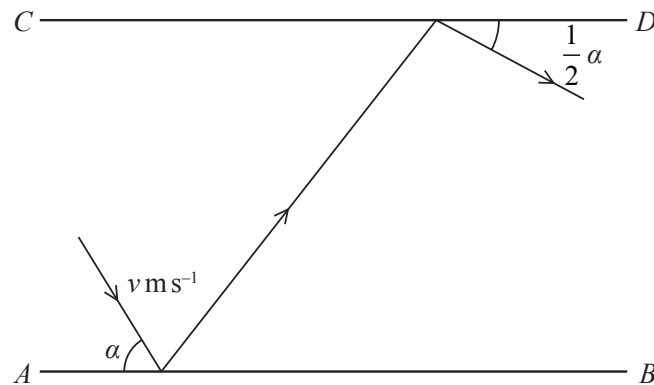


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD .

A small ball is projected along the floor towards wall AB . Immediately before hitting wall AB , the ball is moving with speed $v \text{ m s}^{-1}$ at an angle α to AB , where $0 < \alpha < \frac{\pi}{2}$.

The ball hits wall AB and then hits wall CD .

After the impact with wall CD , the ball is moving at angle $\frac{1}{2}\alpha$ to CD .

The coefficient of restitution between the ball and wall AB is $\frac{2}{3}$.

The coefficient of restitution between the ball and wall CD is also $\frac{2}{3}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that $\tan\left(\frac{1}{2}\alpha\right) = \frac{1}{3}$ (7)

(b) Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts. (4)

Question 7 continued



Question 7 continued



Question 7 continued

(Total for Question 7 is 11 marks)



Question 6 continued



Question 6 continued

(Total for Question 6 is 12 marks)



7.

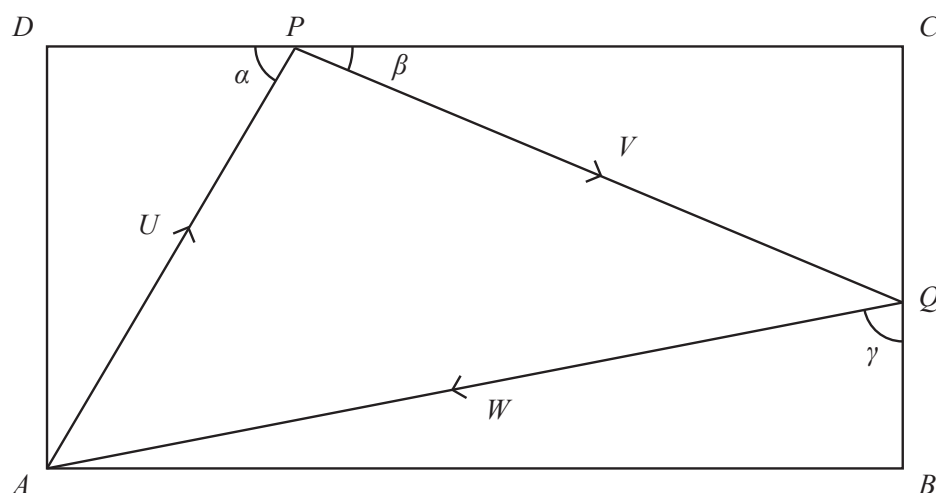


Figure 2

A small smooth snooker ball is projected from the corner A of a horizontal rectangular snooker table $ABCD$.

The ball is projected so it first hits the side DC at the point P , then hits the side CB at the point Q and then returns to A .

Angle $APD = \alpha$, Angle $QPC = \beta$, Angle $AQB = \gamma$

The ball moves along AP with speed U , along PQ with speed V and along QA with speed W , as shown in Figure 2.

The coefficient of restitution between the ball and side DC is e_1

The coefficient of restitution between the ball and side CB is e_2

The ball is modelled as a particle.

Use the model to answer all parts of this question.

(a) Show that $\tan \beta = e_1 \tan \alpha$ (4)

(b) Hence show that $e_1 \tan \alpha = e_2 \cot \gamma$ (3)

(c) By considering (angle APQ + angle AQP) or otherwise, show that it would be possible for the ball to return to A only if $e_2 > e_1$ (6)

If instead $e_1 = e_2$, the ball would **not** return to A .

Given that $e_1 = e_2$

(d) use the result from part (b) to describe the path of the ball after it hits CB at Q , explaining your answer. (1)

Question 7 continued



Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)



8.

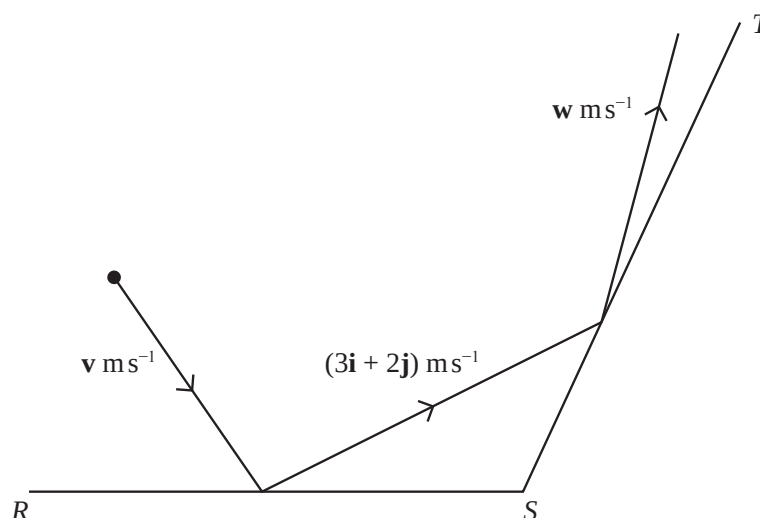


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where RS and ST are smooth fixed vertical walls.

The vector $\overrightarrow{RS}$ is in the direction of the vector $\mathbf{i}$.

The vector $\overrightarrow{ST}$ is in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$.

A small ball B of mass 0.25 kg is projected across the floor towards RS .

Immediately before the impact with RS , the velocity of B is $\mathbf{v} \text{ ms}^{-1}$

Immediately after the impact with RS , the velocity of B is $(3\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$

The coefficient of restitution between B and RS is $\frac{1}{3}$

The ball is modelled as a particle.

(a) Show that the kinetic energy lost by B in the impact with RS is 4 J .

(6)

Immediately after the impact with ST , the velocity of B is $\mathbf{w} \text{ ms}^{-1}$

Given that the coefficient of restitution between B and ST is $\frac{1}{3}$

(b) find $\mathbf{w}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(6)

Question 8 continued



Question 8 continued



Question 8 continued

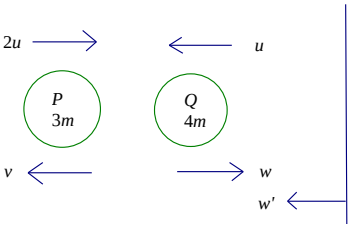
MARK SCHEMES

official mark schemes

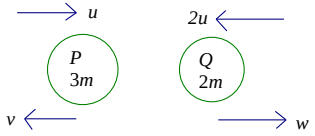


Question	Scheme	Marks	AOs
3(a)	If it helps the candidate, ignore their diagram. 		
	CLM:	M1	3.1a
	$2m \times 3u - m \times 2u = 2mv_P + mv_Q \quad (4u = 2v_P + v_Q)$	A1	1.1b
	Impact Law:	M1	3.4
	$5ue = -v_P + v_Q$	A1	1.1b
	Attempts to solve for v_Q	dM1	2.1
	$v_Q = \frac{(4+10e)u}{3} *$	A1*	2.2a
		(6)	
3(b)	$v_P = \frac{(4-5e)u}{3}$ oe	M1	1.1b
	Correct rebound speed or velocity of Q seen $\pm \frac{f(4+10e)u}{3}$	B1	3.4
	States a correct inequality Eg 2 nd collision if $\frac{f(4+10e)u}{3} > -\frac{(4-5e)u}{3}$ $\frac{f(4+10e)u}{3} > \frac{(5e-4)u}{3}$ $\frac{(4-5e)u}{3} > -\frac{f(4+10e)u}{3}$ $-\frac{(5e-4)u}{3} > -\frac{f(4+10e)u}{3}$	M1	3.1a
	$(1 \geq) f > \frac{5e-4}{4+10e}$	A1	1.1b
		(4)	
(10 marks)			
Notes:			
(a)			

M1	CLM used. Dimensionally correct, mass $\times$ velocity. All terms required. Condone sign errors. Condone consistent g 's or cancelled m 's.
A1	Correct unsimplified equation
M1	NEL used correctly with e appearing on the correct side of the equation. Condone sign errors, must have the correct number of terms.
A1	Correct unsimplified equation. Direction of v_Q and v_P must be consistent with their CLM equation.
dM1	Use their correctly formed equations to solve for v_Q . At least one line of working should be seen.
A1*	Correct given answer correctly obtained $v_Q = \frac{(4+10e)u}{3}$ * Also accept: $\frac{1}{3}(4+10e)u$ $\frac{u(4+10e)}{3}$ $\frac{u}{3}(4+10e)$ Do not accept the 4 and 10e reversed eg $\frac{(10e+4)u}{3}$ is A0*
(b)	
M1	Attempt to solve for v_P. If v_P is found in (a) it must be used in (b) to score this mark. Note that if P is assumed to reverse direction in (a) then $v_P = \frac{(5e-4)u}{3}$ oe
B1	Correct expression seen for speed or velocity of Q after rebound $\pm \frac{f(4+10e)u}{3}$. This may appear on a diagram.
M1	Correct unsimplified inequality seen. The inequality must be correct, accepts cancelled u 's and/or 3's
A1	Correct inequality, do not ISW. Allow $1 \geq f$ to be omitted but do not allow the strict inequality $1 > f$.

Question	Scheme	Marks	AOs
5a			
	Using CLM:	M1	3.4
	$6mu - 4mu = -3mv + 4mw \quad (2u = -3v + 4w)$	A1	1.1b
	Use of impact law	M1	3.1a
	$w + v = e \times 3u$	A1	1.1b
	Complete method to find w	M1	2.1
	$\begin{cases} 3w + 3v = 9eu \\ -3v + 4w = 2u \end{cases} \Rightarrow 7w = 9eu + 2u, \quad w = \frac{u}{7}(9e + 2) \quad *$	A1*	2.2a
		(6)	
5b	$w' = \frac{1}{2} \times \frac{u}{7}(9e + 2) \quad \left(= \frac{u}{14}(9e + 2) \right)$	B1	1.1b
	$v = \frac{u}{7}(12e - 2)$	B1	1.1b
	For a second collision: $w' > v$	M1	3.3
	$9e + 2 > 2(12e - 2), \quad 0 < e < \frac{2}{5}$	A1	1.1b
		(4)	
(Total 10 marks)			
Notes			
(a) M1	Use of CLM. Need all terms. Must be dimensionally correct. Condone sign errors. Accept consistent cancelling of m		
A1	Correct unsimplified equation for CLM. They can have v in either direction		
M1	Correct use of the impact law (used the right way round) Condone sign errors in finding speed of approach and speed of separation.		
A1	Correct unsimplified equation. Signs consistent with equation for CLM.		
M1	Complete method to find w e.g. by forming simultaneous equations using CLM and Impact Law and solving. This requires both of the preceding M marks		
A1*	Obtain given answer from correct working. Accept with $2 + 9e$ in place of $9e + 2$		

	Check that the answer does follow from the working.
(b) B1	Speed of Q after impact with the wall. Any equivalent form. Correct speed can be implied by a correct negative velocity.
B1	Speed of P after impact with Q . Accept $\pm$. Any equivalent form in u and e (seen or implied)
M1	Form correct inequality using their v and w' . A correct inequality has P and Q both moving away from the wall
A1	Correct interval only. Accept unsimplified fraction. Need both ends of the interval. Must be strict inequality at both ends.

Question	Scheme	Marks	AOs	Notes
5(a)				
	Use of CLM	M1	3.1a	Use of CLM. All terms required. Must be dimensionally correct. Condone sign errors
	$3mu - 4mu = 2mw - 3mv \quad (-u = -3v + 2w)$	A1	1.1b	Correct unsimplified equation
	Use of impact law	M1	3.4	Use of impact law. Must be dimensionally correct and used correctly. Condone sign errors
	$w + v = 3ue$	A1	1.1b	Correct unsimplified equation Signs consistent with CLM equation
	Correct strategy to form equation in w and find critical value of $e \in (0,1)$ $(5w = u(9e - 1))$	M1	3.1a	Correct overall strategy to find the critical value of e in $(0,1)$ in e eg by using CLM and impact law to form equation or inequality in w and solve for e .
	$w > 0 : e > \frac{1}{9}$	A1	1.1b	One inequality for e correct Condone $e \geq \frac{1}{9}$
	Complete strategy to justify the range of values of e $(5v = u(1 + 6e)) \quad v > 0 : \text{true for all } e$	M1	3.1a	Correct strategy to find the range of possible value of e . i.e find second speed and form second inequality
	Therefore $\frac{1}{9} < e \leq 1$	A1	2.2a	Correct final conclusion
		(8)		

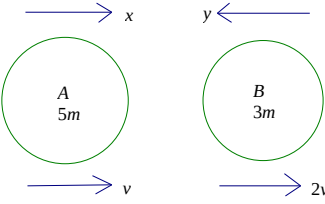


Question	Scheme	Marks	AOs	Notes
5(b)	Final KE = 25% of initial KE	M1	3.1a	Use KE to form equation in e . 25% should be used correctly Condone if mass cancelled throughout
	$\frac{1}{2} \times 2m \times \frac{u^2(9e-1)^2}{25} = \frac{1}{4} \times \frac{1}{2} \times 2m \times 4u^2 \quad \left(\text{or } w = \frac{1}{2} \times 2u \right)$	A1ft	1.1b	Correct unsimplified equation – follow their w
	$\Rightarrow (9e-1)^2 = 25, \quad e = \frac{2}{3} \text{ only}$	A1	1.1b	Or equivalent. Correct conclusion ISW after correct answer.
		(3)		
(11marks)				



	Either For B: $20mv = 3m(3v - (-ku))$ Or CLM: $4m(u) - 3m(ku) = 3m(3v) - 4m(2v)$	A1ft	1.1b
	$k = \frac{11}{9}$	A1	1.1b
		(3)	
(6 marks)			
Notes: Working in parts (a) and (b) may be marked together.			
(a)			
M1	Use of $I = m(v - u)$ on A or equivalent complete method to form an equation in terms of u and v (and m) only (not k). Must consider change in momenta but condone in the wrong order or velocity sign errors. Might see $(2v + u)$ rather than $(2v - u)$. Dimensionally correct equation with correct mass and velocities pairings. If CLM equation is formed in (a), then an impulse-equation must also be used to eliminate k for this mark. Allow consistent omission of m .		
A1	Correct unsimplified equation in u and v (and m)		
A1	Correct only, ISW after $u = 3v$ A0 for only $v = \frac{u}{3}$		
(b)			
M1	Correct use of impulse-momentum equation or CLM to form an equation in k , u , v (and m). All terms required, allow consistent omission of m . Dimensionally correct.		
A1ft	Correct unsimplified equation in k , u and v . No need to replace u and v (follow through their positive u if substituted)		
A1	Correct exact value. Accept $1\frac{2}{9}$ or recurring decimal $1.\dot{2}$ with correct notation.		

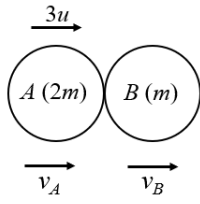
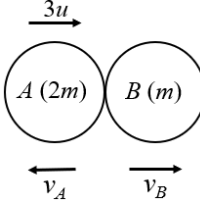


Question	Scheme	Marks	AOs
2(a)			
	Use of CLM	M1	3.1a
	$5mv + 6mv (= 11mv) = 5mx - 3my \quad (11v = 5x - 3y)$	A1	1.1b
	Use of impact law	M1	3.1a
	$v = e(x + y)$	A1	1.1b
	$\begin{cases} 11ev = 5ex - 3ey \\ 3v = 3ex - 3ey \end{cases} \Rightarrow x = \frac{v}{8e}(11e + 3)$	M1	3.1a
	$y = \frac{v}{8e}(5 - 11e)$	A1	1.1b
	$e > 0 \quad (\Rightarrow x > 0) \Rightarrow 5 - 11e > 0$	M1	3.4
	$\Rightarrow 0 < e < \frac{5}{11}$	A1	2.2a
	(8)		
(b)	Form equation for KE	M1	2.1
	$\frac{1}{2} \times 5m \times v^2 = \frac{16}{100} \times \frac{1}{2} \times 5m \times \frac{v^2}{64e^2} (11e + 3)^2$	A1ft	1.1b
	$(4(11e + 3) = (\pm)80e) \quad e = \frac{1}{3}$	A1	1.1b
	Impulse = $-5m(v - x)$	M1	3.1a
	$= -5m \left(v - \frac{11v}{8} - \frac{3v}{8e} \right)$ Or: $3m \left(2v + \frac{5v}{8e} - \frac{11v}{8} \right)$	A1ft	1.1b
	Magnitude = $\frac{15}{2}mv$	A1	2.2a
	(6)		
Alt(b)	Form equation for KE	M1	2.1
	$\frac{1}{2} \times 5m \times v^2 = \frac{16}{100} \times \frac{1}{2} \times 5m \times x^2$	A1	1.1b
	$\Rightarrow x = \frac{5v}{10} \quad v = \frac{v}{2} \quad \Rightarrow e = \frac{1}{2}$	A1	1.1b

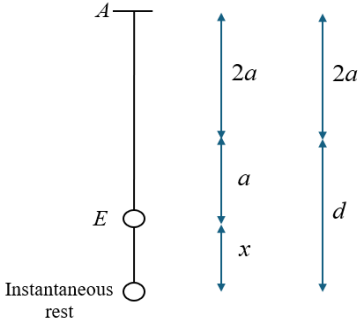
	Impulse = $-5m(v - x)$	M1	3.1a
	$= -5m\left(v - \frac{5v}{2}\right)$ Or: $3m\left(2v + \frac{v}{2}\right)$	A1	1,16
	Magnitude = $\frac{15}{2}mv$	A1	2.2a
		(6)	
(14 marks)			
Notes:			
(a)M1	All terms required. Dimensionally correct. Condone sign errors		
A1	Correct unsimplified equation		
M1	Used correctly. Condone sign errors		
A1	Correct unsimplified equation		
M1	Use their correctly formed equations to solve for v or w or a multiple of v or w		
A1	Both velocities correct		
M1	Use their velocities (in general form – not by considering one specific value) to form inequality for both moving in the same direction.		
A1	Correct only.		
(b)M1	Dimensionally correct. Condone 16% on wrong side Allow M or $5m$		
A1ft	Or equivalent. Correct unsimplified equation. Follow their x Allow M or $5m$		
A1	Correct answer only Allow M or $5m$		
M1	Correct use of $I = mv - mu$. Must be subtracting.		
A1ft	Accept $\pm$ Follow their x, y, e		
A1	Correct only. Must be positive.		

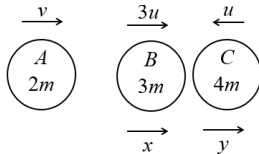
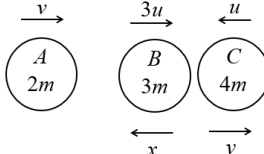
Question	Scheme	Marks	AOs
3(a)	Taking left to right as positive,		
	CLM:	M1	3.1a
	$6mu + 4mu (= 10mu) = 3mv + 4mw \quad (10u = 3v + 4w)$	A1	1.1b
	Impact Law:	M1	3.4
	$w - v = e(2u - u) (= eu)$	A1	1.1b
	Solve for v or w	M1	2.1
	$w = \frac{u}{7}(10 + 3e)$	A1	1.1b
	$v = \frac{u}{7}(10 - 4e)$	A1	1.1b
	$0 \leq e \leq 1 \Rightarrow 10 + 3e > 0$ and $10 - 4e > 0$ hence both particles still travelling in the original direction. *	A1*	2.2a
	(8)		
(b)	CLM: $4mw = 4mx + 2my \quad (2w = 2x + y)$	M1	3.1a
	Impact: $y - x = ew$	M1	3.4
	$\Rightarrow w(2 - e) = 3x, \quad x = \frac{u}{21}(10 + 3e)(2 - e)$	M1	1.1b
	Consider $v - x$ i.e. $\frac{u}{7}(10 - 4e) - \frac{u}{21}(10 + 3e)(2 - e)$ $(3e^2 - 8e + 10)$	M1	2.1
	Show that $v - x > 0 \quad \forall e$	M1	1.1b
	Complete correct argument and conclusion *	A1*	2.2a
	(6)		
(14 marks)			
Notes:			
(a)M1	All terms required. Condone sign errors.		
A1	Correct unsimplified equation		

M1	Law used correctly. Condone sign errors
A1	Correct unsimplified equation
M1	Use their correctly formed equations to solve for v or w
A1	Either velocity correct
A1	Both velocities correct
A1*	Use possible values of e to justify given result from correct working.
(b)M1	All terms required. Condone sign errors
M1	Correct use of impact law. Condone sign errors
M1	Use their correctly formed equations to find velocity of B (x)
M1	Form relevant difference for a second collision
M1	Complete correct method (e.g. differentiation or completing the square or discriminant) to determine when inequality is true
A1*	Reach correct conclusion from correct work.

Question	Scheme		Marks	AOs
4(a)				
	CLM:		M1	3.1a
	$2m \times 3u = 2mv_A + mv_B$ ($6u = 2v_A + v_B$)	$2m \times 3u = -2mv_A + mv_B$ ($6u = -2v_A + v_B$)	A1	1.1b
	Impact Law:		M1	3.4
	$3ue = -v_A + v_B$	$3ue = v_A + v_B$	A1	1.1b
	Solve for v_B		M1	2.1
	$(v_B =) 2u(1+e) *$		A1*	2.2a
			(6)	
4(b)	Solve for v_A		M1	3.1a
	$v_A = u(2-e)$	$v_A = u(e-2)$	A1	1.1b
	Complete and correct explanation: $0 \leq e \leq 1 \Rightarrow v_A > 0$ $\Rightarrow A$ continues to move towards the wall $\Rightarrow A$ will collide again with B .	Complete and correct explanation: $0 \leq e \leq 1 \Rightarrow v_A < 0$ $\Rightarrow A$ continues to move towards the wall $\Rightarrow A$ will collide again with B .	A1	2.4
			(3)	
4(c)	Rebound speed or velocity of $B = \pm \frac{1}{2} \times 2u(1+e)$		B1	3.4
	$\pm m[-u(1+e) - 2u(1+e)]$		M1	3.1a
	$3(1+e)mu$		A1	1.1b
			(3)	
4(d)	Attempt at KE loss of B		M1	3.4
	$\frac{1}{2}m[(2u(1+e))^2 - (u(1+e))^2]$		A1	3.1a
	$\frac{3mu^2(1+e)^2}{2}$		A1	1.1b
			(3)	
	(15 marks)			
	Notes:			

(a)	
M1	Use of CLM, all terms required, dimensionally correct (mass $\times$ velocity in each term). Mass and velocity paired correctly. Condone sign errors on velocities. Condone consistent extra g and/or consistent missing m (in every term).
A1	Correct unsimplified equation.
M1	Correct use of Impact Law, dimensionally correct, condone sign errors on velocity. M0 if separation and approach are on the wrong sides.
A1	Correct unsimplified equation, the direction of A must be consistent with their CLM.
M1	Use their <i>correctly formed</i> equations to solve for v_B
A1*	Given answer correctly obtained and exactly as printed. Working should include an equation in v_B only <i>before</i> reaching the given answer.
(b)	
M1	Use given answer in (a) to solve for v_A . If seen in (a) it must be used in (b) to earn this mark.
A1	Correct expression seen for velocity of A after impact.
A1*	Correct and complete explanation with no incorrect statements. Must include all of: $0 < e \leq 1$ or $0 \leq e \leq 1$ or $0 \leq e < 1$ or 'for all e' $v_A > 0$ or $v_A < 0$ (must be correct for their v_A) - Must refer to the wall. - A continues to move with unchanged direction or towards the wall (eg do not accept descriptions for direction of travel as 'to the right' or similar) - Conclude second collision between A and B Note: A0* for explanations that rely on comparing speed of v_B and v_A
(c)	
B1	Correct unsimplified expression for rebound speed or velocity of B, may be seen on diagram or elsewhere in working. Accept positive or negative.
M1	Attempt to find difference in momenta, dimensionally correct (mass $\times$ velocity). Must use given answer from (a) for V_b . Condone use of e or e' in place of $\frac{1}{2}$
A1	Correct answer, must be positive. Accept any equivalent form that is either factorised or reduced to 2 terms eg $3(1+e)mu$, $3mu + 3emu$
(d)	
M1	Attempt at KE loss of B: clear attempt at a difference in KE of B before and after impact with the wall. Must use velocity of B from (a). Condone use of e or e' in place of $\frac{1}{2}$. Dimensionally correct, condone subtraction either way round.
A1	Correct unsimplified expression for KE loss of B
A1	Any equivalent factorised form eg $\frac{3mu^2(1+e)^2}{2}$, $\frac{3mu^2(1+2e+e^2)}{2}$, oe

Question	Scheme	Marks	AOs
	 The diagram shows a vertical line representing a string or rod. At the top is a horizontal line labeled A. A vertical line descends from A to a point labeled E. Below E is another point labeled "Instantaneous rest". To the right of the vertical line, there are two vertical double-headed arrows. The first arrow starts at the level of A and ends at the level of E; it is labeled $2a$ in the middle. The second arrow starts at the level of E and ends at the level of "Instantaneous rest"; it is labeled x in the middle. To the left of the vertical line, there are two vertical double-headed arrows. The first arrow starts at the level of A and ends at the level of "Instantaneous rest"; it is labeled d in the middle. The second arrow starts at the level of E and ends at the level of "Instantaneous rest"; it is labeled a in the middle.		

E · Q · (b) · Question 3C: Use of NEL · Use of ME1 · June 2025 · Q5			M1	3.4		
	$3u - v = e \times 5u$	$3u + v = e \times 5u$	A1	1.1b		
	$\Rightarrow e = \frac{1}{2}$		A1	1.1b		
			(3)			
(c)						
	Use of CLM		M1	3.4		
	$3m(3u) + 4m(-u) = 3mx + 4my$	$3m(3u) + 4m(-u) = 3m(-x) + 4my$	A1	1.1b		
	Use of NEL		M1	3.4		
	$y - x = f \times (3u - -u)$	$y + x = f \times (3u - -u)$	A1	1.1b		
	E.g. $\begin{cases} 3x + 4y = 5u \\ y - x = 4uf \end{cases}$ $\Rightarrow x = \frac{u}{7}(5 - 16f) \text{ o.e.}$	E.g. $\begin{cases} -3x + 4y = 5u \\ y + x = 4uf \end{cases}$ $\Rightarrow x = \frac{u}{7}(16f - 5) \text{ o.e.}$	M1	1.1b		
	$v > x$ $\Rightarrow \frac{1}{2}u > \frac{u}{7}(5 - 16f)$	$v > -x$ $\Rightarrow \frac{1}{2}u > -\frac{u}{7}(16f - 5)$	dM1	3.1b		
	$\frac{3}{32} < f \leq 1$		A1	2.2a		
			(7)			
(13 marks)						



	M0 if separation and approach are on the wrong sides.
A1	Correct unsimplified equation.
A1	Correct only.
(c)	
M1	Use CLM, dimensionally correct, with correct mass and velocity pairings. Must have all non-zero velocities. Condone sign errors. Follow their directions for the unknown velocities of B and C after impact. (Ignore the diagram if it benefits the candidate.)
A1	Correct unsimplified equation.
M1	Correct use of NEL for B and C , dimensionally correct. Condone sign errors on velocity. Must have all non-zero velocities. Condone use of another letter for f . M0 if separation and approach are on the wrong sides.
A1	Correct unsimplified equation, the directions of B and C after impact must be consistent with their CLM equation.
M1	Solve simultaneous equations with two unknowns to find an expression for the velocity of B after the second collision in terms of f and u . Condone use of another letter for f . Must use CLM and NEL to reach $x = \dots$ or a multiple of $x = \dots$
dM1	Dependent on 3 previous M's. Complete method to find the values of f for a second collision. Must consider A and B moving in the same direction and the speed of $A >$ speed of B .
A1	$0.094 < f \leq 1$ (0.094 or better) Both ends required with correct inequality signs and must use f as coefficient of restitution.



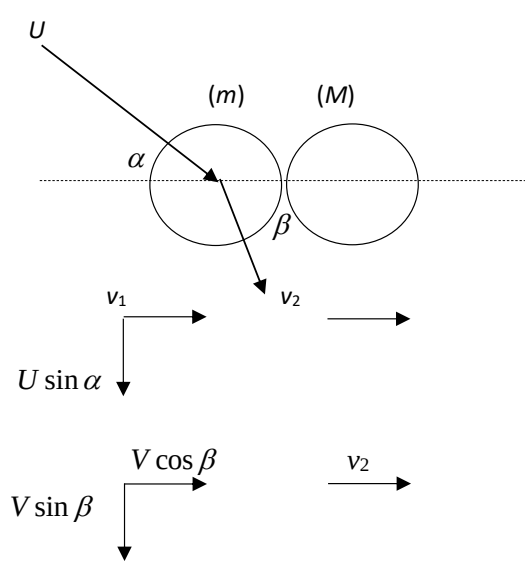
Question	Scheme	Marks	AOs
4(a)	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ or $\mathbf{v} - \mathbf{u}$	M1	2.1
	$\mathbf{I} = 0.5((\mathbf{i} + 6\mathbf{j}) - (7\mathbf{i} + 2\mathbf{j})) \quad (= (-3\mathbf{i} + 2\mathbf{j}))$	A1	1.1b
	Use of scalar product $(-3\mathbf{i} + 2\mathbf{j}) \cdot (2\mathbf{i} + 3\mathbf{j}) = -6 + 6 = 0$	M1	1.1b
	Hence impulse perpendicular to $(2\mathbf{i} + 3\mathbf{j})$, so AB must be parallel to $(2\mathbf{i} + 3\mathbf{j})$. *	A1*	2.2a
		(4)	
4(a) alt	Components of velocities parallel to $(2\mathbf{i} + 3\mathbf{j})$:	M1	2.1
	$\left(\frac{1}{\sqrt{13}}\right)(7\mathbf{i} + 2\mathbf{j}) \cdot (2\mathbf{i} + 3\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(14 + 6)$ $\left(\frac{1}{\sqrt{13}}\right)(\mathbf{i} + 6\mathbf{j}) \cdot (2\mathbf{i} + 3\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(2 + 18)$	A1	1.1b
	Simplify and compare values	M1	1.1b
	Hence component of velocity parallel to $(2\mathbf{i} + 3\mathbf{j})$ is unchanged, so AB must be parallel to $(2\mathbf{i} + 3\mathbf{j})$. *	A1*	2.2a
		(4)	
4(a) alt	Use conservation of velocity parallel to $a\mathbf{i} + b\mathbf{j}$	M1	2.1
	$(7\mathbf{i} + 2\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j}) = (\mathbf{i} + 6\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j})$ $(\Rightarrow 7a + 2b = a + 6b)$	A1	1.1b
	Find ratio of a and b to obtain direction: $\left(b = \frac{2}{3}a\right)$	M1	1.1b
	Hence AB must be parallel to $(2\mathbf{i} + 3\mathbf{j})$. *	A1*	2.2a
		(4)	
4(b)	Use scalar product to find components of velocities perpendicular to the wall	M1	3.1b
	$\left(\frac{1}{\sqrt{13}}\right)(-3\mathbf{i} + 2\mathbf{j})(7\mathbf{i} + 2\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(-21 + 4) \quad \left(= \frac{-17}{\sqrt{13}}\right)$ $\left(\frac{1}{\sqrt{13}}\right)(-3\mathbf{i} + 2\mathbf{j})(\mathbf{i} + 6\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(-3 + 12) \quad \left(= \frac{9}{\sqrt{13}}\right)$	A1 A1	1.1b 1.1b
	Use of impact law	M1	3.4
	$e = \frac{9}{17}$	A1	1.1b

		(5)	
(9 marks)			
Notes:			
(a)M1	Must be finding the difference between two momenta or two velocities		
A1	Correct unsimplified equation for the impulse or for change in velocity		
M1	Use of scalar product or equivalent. In the alt method allow full marks if $\sqrt{13}$ not used.		
A1*	Reach given conclusion from correct working		
	If working with angles, score M1 for correct method to find components parallel to the wall A1 for $\sqrt{53} \cos 40.36..^\circ$ and $\sqrt{37} \cos 24.23..^\circ$ M1 for comparing the two values A0 because the work has involved decimal approximations (since working towards an exact given answer). Alternative: Could use $e \tan 24.2..^\circ = \tan 40.36..^\circ$		
(b)M1	Condone if not using a unit vector		
A1	One correct value		
A1	Second correct values		
	If working with angles, score M1A1A1 for $\sqrt{53} \sin 40.36..^\circ$ and $\sqrt{37} \sin 24.23...^\circ$		
M1	Use their components the right way round in the impact law. Condone sign error.		
A1	0.53 or better (0.52941.....)		

Question	Scheme	Marks	AOs
7(a)	Component parallel to the wall: $\left[\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) \cdot (8\mathbf{i} + 2\mathbf{j}) \right]$	M1	2.1
	$= 5\sqrt{2}$	A1	1.1b
	Use of impact law perpendicular to wall:	M1	3.4
	Component perpendicular to wall after impact $\frac{1}{3} \left[\frac{1}{\sqrt{2}}(-\mathbf{i} + \mathbf{j}) \cdot (8\mathbf{i} + 2\mathbf{j}) \right] = -\sqrt{2}$	A1	1.1b
	For a complete method to find $\mathbf{v}$	M1	1.1b
	$\Rightarrow \mathbf{v} = (5\mathbf{i} + 5\mathbf{j}) + (-\mathbf{i} + \mathbf{j}) = (4\mathbf{i} + 6\mathbf{j})$ *	A1*	2.2a
	(6)		
(a) alt	If $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ component parallel to the wall:	M1	2.1
	$(8\mathbf{i} + 2\mathbf{j}) \cdot (\mathbf{i} + \mathbf{j}) = (a\mathbf{i} + b\mathbf{j}) \cdot (\mathbf{i} + \mathbf{j}) \quad (a + b = 10)$	A1	1.1b
	Use of impact law:	M1	3.4
	$-\frac{1}{3}(8\mathbf{i} + 2\mathbf{j}) \cdot (-\mathbf{i} + \mathbf{j}) = (a\mathbf{i} + b\mathbf{j}) \cdot (-\mathbf{i} + \mathbf{j}) \quad (2 = -a + b)$	A1	1.1b
	For a complete method to find $\mathbf{v}$	M1	1.1b
	$\Rightarrow \mathbf{v} = (4\mathbf{i} + 6\mathbf{j})$ *	A1*	2.2a
	(6)		
(a) alt 2	Angle to wall = 31° , component parallel to the wall:	M1	2.1
	$= \sqrt{68} \cos 31^\circ = 7.07$	A1	1.1b
	Component perpendicular to the wall	M1	3.4
	$= \frac{1}{3} \sqrt{68} \sin 31^\circ = 1.42$	A1	1.1b
	For a complete method to find $\mathbf{v}$	M1	1.1b
	$\Rightarrow \mathbf{v} = (\sqrt{52} \cos 56.3^\circ \mathbf{i} + \sqrt{52} \sin 56.3^\circ \mathbf{j}) = (4\mathbf{i} + 6\mathbf{j})$	A1*	2.2a
	(6)		
(b)	$\mathbf{I} = 0.25(4\mathbf{i} + 6\mathbf{j}) - 0.25(8\mathbf{i} + 2\mathbf{j})$ $(\mathbf{I} = 0.25(-\mathbf{i} + \mathbf{j}) - 0.25(3\mathbf{i} - 3\mathbf{j})) \quad (\mathbf{I} = (-\mathbf{i} + \mathbf{j}))$	M1	3.1b

	Use of Pythagoras	M1	1.1b
	$ \mathbf{I} = \sqrt{2} \text{ (Ns)}$	A1	1.1b
		(3)	
(9 marks)			
Notes:			
(a)M1	Use of scalar product or equivalent. Allow M1 if not using unit vector		
A1	Correct unsimplified expression for component parallel to wall		
M1	Correct use of impact law perpendicular to the wall. Condone sign error		
A1	Correct unsimplified expression for component perpendicular to wall		
M1	Complete method to solve for $\mathbf{v}$		
A1*	Obtain given result from correct working		
(b) M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ with velocities or perpendicular components of velocities. Must be subtracting but allow subtraction in either order.		
M1	Correct use of Pythagoras to find modulus		
A1	Accept 1.4 Ns or better		



Question	Scheme	Marks	AOs
5(a)	If it helps the candidate, ignore their diagram. 		
	$U \sin \alpha$ seen as velocity component of S, perpendicular to line of centres after impact.	B1	3.4
	CLM along line of centres	M1	3.1b
	$mU \cos \alpha = mv_1 + Mv_2$	A1	1.1b
	NEL used along line of centres	M1	3.3
	$eU \cos \alpha = -v_1 + v_2$	A1	1.1b
	$\tan \beta = \frac{U \sin \alpha}{v_1}$	dM1	2.1
	Solve to produce an equation for $\tan \beta$ in m, M, e and α	dM1	1.1b
	$\tan \beta = \frac{(m+M) \tan \alpha}{(m-eM)}$ *	A1*	1.1b
	(8)		
5(a) alt1	$U \sin \alpha$ seen as velocity cpt of S, perpendicular to line of centres after impact.	B1	3.4
	CLM along line of centres	M1	3.1b
	$mU \cos \alpha = mV \cos \beta + Mv_2$	A1	1.1b
	NEL used along line of centres	M1	3.3
	$eU \cos \alpha = -V \cos \beta + v_2$	A1	1.1b

	$\tan \beta = \frac{U \sin \alpha}{V \cos \beta}$ or $V \sin \beta = U \sin \alpha$	dM1	2.1
	Solve to produce an equation for $\tan \beta$ in m, M, e and α	dM1	1.1b
	$\tan \beta = \frac{(m+M) \tan \alpha}{(m-eM)}$ *	A1*	1.1b
		(8)	
5(b)	Use the given condition to find the direction of S after impact. Eg $m = eM \Rightarrow \tan \beta = \infty$ or $\tan \beta$ is undefined so $\beta = 90^\circ$ oe $m = eM \Rightarrow v_1 = 0$ so velocity component of S parallel to line of centres is zero.	M1	3.1b
	Conclusion: After the collision, S moves perpendicular to the line of centres and the other sphere moves parallel to the line of centres i.e. they move at right angles oe *	A1*	2.4
		(2)	
(10 marks)			
Notes:			
(a)			
B1	$U \sin \alpha$ or $U \cos(90 - \alpha)$ used as the perpendicular velocity component of S after impact. Must be seen in working for (a) or on a velocity diagram.		
M1	CLM along the line of centres. Dimensionally correct, correct no. of terms, condone sin/cos confusion and sign errors.		
A1	Correct equation.		
M1	NEL used correctly along the line of centres with e appearing on the correct side of the equation. Condone sin/cos confusion as long as it is consistent with their CLM. Condone sign errors but must have the correct number of terms.		
A1	Correct equation (the signs of v_1 and v_2 must be consistent with their CLM)		
dM1	Use of the fact that S moves at β to the line of centres after the collision. Use of their components after the collision to form an equation in β . Dependent on both previous M's.		
dM1	Eliminate v_1 to produce an equation for $\tan \beta$ in m, M, e and α . Dependent on first two M's in (b) Note: $v_1 = u \cos \alpha \left(\frac{m - eM}{m + M} \right)$		
A1*	Given answer correctly obtained. Must match printed answer EXACTLY.		
5(a) alt1			



B1	$U \sin \alpha$ or $U \cos(90 - \alpha)$ used as the perpendicular velocity component of S after impact. Must be seen in (a) or on a velocity diagram.
M1	CLM along the line of centres. Dimensionally correct, correct no. of terms, condone sin/cos confusion and sign errors.
A1	Correct equation
M1	NEL used correctly along the line of centres with e appearing on the correct side of the equation. Condone sin/cos confusion as long as it is consistent with their CLM. Condone sign errors but must have the correct number of terms.
A1	Correct equation (signs and sin/cos must be consistent with their CLM)
dM1	Use of the fact that S moves at β to the line of centres after the collision. Use of their components after the collision to form an equation β . Dependent on both previous M's.
dM1	Eliminate $V \cos \beta$ to produce an equation for $\tan \beta$ in m, M, e and α . Dependent on first two M's in (b) Note: $V \cos \beta = u \cos \alpha \left(\frac{m - eM}{m + M} \right)$
A1*	Given answer correctly obtained. Must match printed answer EXACTLY.
(b)	
M1	Use of given condition to deduce that $\beta = 90^\circ$ or that velocity component parallel to line of centres is zero.
A1*	Correct explanation using given information. Must refer correctly to the direction of both particles, eg perpendicular, at right angles, parallel and perpendicular to the line of centres, Do not accept horizontally and vertically since the surface is defined as horizontal.

Question	Scheme	Marks	AOs	Notes
6(a)				
	Perpendicular to line of centres: $2\mathbf{j}$	B1	3.4	Use the model to find the component perpendicular to the line of centres. Correct value seen or implied
	CLM parallel to the line of centres	M1	3.1b	Use of CLM parallel to line of centres. Need all terms and dimensionally correct. Condone sign errors
	$0.2 \times 3 - 0.4 \times 4 = 0.4w - 0.2v \quad (-5 = 2w - v)$	A1	1.1b	Correct unsimplified equation.
	Impact law parallel to the line of centres	M1	3.4	Correct use of impact law parallel to the line of centres. Condone sign errors
	$7e = v + w \Rightarrow 3 = v + w$	A1	1.1b	Correct equation with $\frac{3}{7}$ used.
	Complete strategy to find $\mathbf{v}_A$	M1	3.1b	Complete strategy to find components parallel and perpendicular to line of centres, eg by using CLM and impact law
	$\mathbf{v}_A = -\frac{11}{3}\mathbf{i} + 2\mathbf{j} \text{ (ms}^{-1}\text{) follow their } 2\mathbf{j}$	A1ft	1.1b	$\mathbf{v}_A$ correct, follow their $a\mathbf{j}$ for $2\mathbf{j} (a \neq 0)$
	(7)			
6(b)	Magnitude of impulse on A: $0.2\left(\frac{11}{3} - (-3)\right)$	M1	3.1b	Evidence of use of $m(v-u)$ parallel to the line of centres
	$= 0.2\left(\frac{11}{3} + 3\right) = \frac{4}{3} \text{ (Ns)}$	A1	1.1b	1.3 (Ns) or better
	(2)			

Question	Scheme	Marks	AOs	Notes
6(c)	Use of scalar product to find the angle	M1	3.1a	Complete method for finding the required angle. Allow for $\tan^{-1} \frac{3}{2}$ or $\tan^{-1} \frac{2}{3}$ and $\tan^{-1} \frac{6}{11}$ or $\tan^{-1} \frac{11}{6}$
	$\cos \theta = \frac{(3\mathbf{i} + 2\mathbf{j}) \cdot \left(-\frac{11}{3}\mathbf{i} + 2\mathbf{j}\right)}{\sqrt{13} \times \sqrt{\frac{157}{9}}}$	A1ft	1.1b	A correct unsimplified expression Follow their $\mathbf{V}_A$. Do not ISW
	$\theta = 118^\circ$	A1	1.1b	Correct answer only. (Q asks for the nearest degree) Do not ISW
	Alternative method: $180^\circ - \tan^{-1} \frac{2}{3} - \tan^{-1} \frac{6}{11}$ Or $\tan^{-1} \frac{3}{2} + \tan^{-1} \frac{11}{6}$			62° probably scores M1A0A0
		(3)		
(Total 12 marks)				



Question	Scheme	Marks	AOs
3(a)			
	For P after: Component in $\mathbf{j}$ direction = 2	B1	3.4
	Deflected through 90° so velocity after = $\left(-\frac{4}{u}\mathbf{i} + 2\mathbf{j}\right)(\text{ms}^{-1})$	B1	3.4
	CLM parallel to line of centres:	M1	3.1a
	$0.3\left(u + \frac{4}{u}\right) = 0.5(4 + w)$	A1ft	1.1b
	Impact law parallel to line of centres:	M1	3.1a
	$w + \frac{4}{u} = \frac{3}{5}(u + 4)$	A1ft	1.1b
	$\begin{cases} 3u + \frac{12}{u} = 20 + 5w \\ \frac{20}{u} + 5w = 3u + 12 \end{cases}$	M1	1.1b
(b)	$\left(\Rightarrow \frac{32}{u} = 32,\right) \quad u = 1$	A1	2.2a
		(8)	
	For Q after: $w = -1$	B1	1.1b
	$\mathbf{v} = -\mathbf{i} + 3\mathbf{j}$	B1ft	1.1b
	Find relevant angle between directions	M1	3.1a
	$\alpha^\circ = \tan^{-1} 3 - \tan^{-1} \frac{3}{4}$ or $\alpha^\circ = \cos^{-1} \left(\frac{4+9}{5 \times \sqrt{10}} \right)$	A1ft	1.1b
(c)	$\alpha = 34.7 \quad (35)$	A1	1.1b
		(5)	
	The line of centres is parallel to the surface the spheres are moving on, so the impulse acts parallel to the surface.	B1	3.5a
		(1)	
(14 marks)			

Notes:	
(a) B1	Correct only Check the diagram
B1	Correct only. Seen or implied.
M1	Correct use of CLM. Need all terms. Condone sign errors
A1ft	Follow their components of velocity of P , with or without a value for the $\mathbf{i}$ component.
M1	Correct use of the impact law. Condone sign errors
A1ft	Follow their components of velocity of P , with or without a value for the $\mathbf{i}$ component.
M1	Solve their correctly formed simultaneous equations to obtain value of u .
A1	Correct only
(b) B1	Correct only
B1ft	Follow their w
M1	Correct method to find a relevant angle between the directions
A1ft	Correct unsimplified expression. Follow their $\mathbf{v}$
A1	35 or better (34.695...) 0.61 radians
(c)B1	Or equivalent that explains that the line of centres is parallel to the surface



Question	Scheme	Marks	AOs
5(a)			
	Components perpendicular to the line of centres after the collision: $\mathbf{v}_{Pj} = 2\mathbf{j}(\text{ms}^{-1})$, $\mathbf{v}_{Qj} = \mathbf{j}(\text{ms}^{-1})$	B1	3.4
	Kinetic energy:	M1	3.1a
	$\frac{1}{2} \times 0.2 \times (v^2 + 1) = \frac{1}{2} \times \frac{1}{2} \times 0.2 \times (9 + 1)$	A1	1.1b
	CLM parallel to line of centres:	M1	3.1a
	$0.3 \times 4 - 0.2 \times 3 = 0.2v - 0.3u \quad (6 = 2v - 3u)$	A1	1.1b
	Impact law parallel to line of centres	M1	3.1a
	$v + u = e(4 + 3)$	A1	1.1b
	Solve for $\mathbf{v}_P$, $\mathbf{v}_Q$ or e	M1	1.1b
	$\mathbf{v}_P = \frac{2}{3}\mathbf{i} + 2\mathbf{j}(\text{ms}^{-1})$ and $\mathbf{v}_Q = 2\mathbf{i} + \mathbf{j}(\text{ms}^{-1})$	A1	1.1b
	$e = \frac{4}{21}$	A1	1.1b
	If $v = -2 \Rightarrow u = -\frac{10}{3} \Rightarrow P$ and Q have passed through each other: impossible, so solution is unique *	A1*	2.4
		(11)	
(b)	Use trig to find angle between velocities	M1	3.1a
	$\cos \theta = \frac{\frac{8}{3} + 4}{\sqrt{20}\sqrt{4\frac{4}{9}}}$ or $\theta = \tan^{-1} \frac{2}{2/3} - \tan^{-1} \frac{1}{2}$	A1ft	1.1b
	$\theta = 45^\circ \left(\frac{\pi}{4} \text{ rads} \right)$	A1	1.1b
		(3)	
(14 marks)			
Notes:			

(a)B1	Seen or implied. Correct only
M1	Equation for KE of Q. Dimensionally correct. Condone $\frac{1}{2}$ on the wrong side.
A1	Correct unsimplified equation in v^2
M1	Equation for CLM. Correct terms required. Condone sign errors. Dimensionally correct.
A1	Correct unsimplified equation
M1	Correct use of impact law. Condone sign errors
A1	Correct unsimplified equation.
M1	Complete method to solve for $\mathbf{v}_p, \mathbf{v}_Q$ or e
	(Working in e gives $v = \frac{1}{5}(6 + 21e)$ and $441e^2 + 252e - 64 = 0$)
A1	Both velocities correct. Need to see answers in the form $a\mathbf{i} + b\mathbf{j}$ or equivalent
A1	Correct only. 0.19 or better (0.19047....)
A1*	Or equivalent justification of given result. e.g. a negative value for e is not possible
(b) M1	Use of trig or equivalent to find a relevant angle between two velocities e.g by scalar product or difference between angles.
A1ft	Correct unsimplified equation in θ . Follow their $\mathbf{v}_p$
A1	Correct only. (0.785... radians) Do not ISW



Question	Scheme		Marks	AOs
7(a)	CLM along line of centres		M1	3.1b
	$mU \cos \alpha = -mv_A + 3mv_B$	$mU \cos \alpha = -mV \sin \alpha + 3mv_B$	A1	1.1b
	Impact Law used along line of centres		M1	3.3
	$eU \cos \alpha = v_A + v_B$	$eU \cos \alpha = V \sin \alpha + v_B$	A1	1.1b
	Solve for v_B		DM1	2.1
	$\frac{1}{4}(1+e)U \cos \alpha$ *		A1*	2.2a
			(6)	
7(b)	Unsimplified expression for v_A seen	Unsimplified expression for $V \sin \alpha$ seen	M1	2.1
	$= \frac{1}{4}(3e-1)U \cos \alpha$ oe seen		A1	1.1b
	Solve $v_A > 0$ to find an inequality for e	Solve $V \sin \alpha > 0$ to find an inequality for e	M1	3.1b
	Correct and complete reasoning leading to $e > \frac{1}{3}$ * Must include: correct inequality for v_A , $0^\circ < \alpha < 90^\circ$ or α is acute, $\cos \alpha > 0$.		A1*	2.2a
			(4)	
7(c)	Velocity component of A, perpendicular to the line of centres, after collision = $U \sin \alpha$		B1	3.3
	Use of 90° deflection: $\tan \alpha = \frac{\frac{1}{4}(3e-1)U \cos \alpha}{U \sin \alpha}$		M1	2.1
	Correct equation in e and $\tan \alpha$ $\tan^2 \alpha = \frac{1}{4}(3e-1)$ oe		A1	1.1b

	$e \leq 1 \Rightarrow \tan^2 \alpha \leq \frac{1}{2}$	DM1	3.1b
	$0 < \tan \alpha \leq \frac{1}{\sqrt{2}}$ since $0^\circ < \alpha \leq 90^\circ$ *	A1*	2.2a
		(5)	
(15 marks)			
Notes:			
(a)			
M1	Use of CLM along the line of centres with mass and velocities paired correctly. Correct no. of terms, condone sin/cos confusion and sign errors on velocities. Allow consistent missing m 's and/or consistent extra g 's (every term).		
A1	Correct unsimplified equation		
M1	Use of Impact Law along the line of centres. Correct no. of terms, condone sin/cos confusion and sign errors but e must be on the correct side of the equation		
A1	Correct unsimplified equation (signs consistent with CLM equation)		
DM1	Dependent on both previous M's. Solve for their v_B		
A1*	Given answer correctly obtained and EXACTLY as printed. Working should include an equation in v_B only before reaching given answer.		
(b)			
M1	Use the given expression for v_B to find an unsimplified expression for v_A or $V \sin \alpha$ Note: If seen in (a) it must be used in (b) to achieve the mark.		
A1	Correct unsimplified expression for v_A or $V \sin \alpha$ seen		
M1	Solve $v_A > 0$ or $V \sin \alpha > 0$ to find an inequality for e		
A1*	Given answer complete and correctly obtained with no incorrect statements. Complete explanation must include: • correct inequality for v_A • $0^\circ < \alpha < 90^\circ$ or α is acute • $\cos \alpha > 0$ Leading to $e > \frac{1}{3}$		
(c)			
B1	Velocity component of A, perpendicular to the line of centres, after collision is $U \sin \alpha$ May be seen labelled on the diagram or written as an expression early on in their working. Does not need to be used here.		
M1	Use of perpendicular deflection to form an equation in α and e (and U). Allow reciprocal. Alternative methods include: • scalar product of vectors, eg $\begin{pmatrix} U \cos \alpha \\ U \sin \alpha \end{pmatrix} \cdot \begin{pmatrix} -v_A \\ U \sin \alpha \end{pmatrix} = 0$ with v_A substituted		

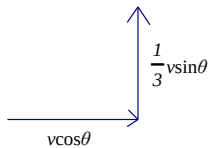
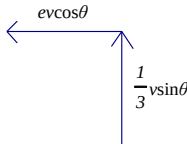
	• $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ with $\alpha + \beta = 90 \Rightarrow 1 - \tan \alpha \tan \beta = 0 \Rightarrow \dots$
A1	A correct equation in $\tan \alpha$ and e only eg $\tan^2 \alpha = \frac{1}{4}(3e - 1)$ or $e = \frac{4 \tan^2 \alpha + 1}{3}$ oe
DM1	Dependent on previous M. Clear explanation using $e \leq 1$ to form an inequality in $\tan \alpha$. Condone, use of $\max e = 1 \Rightarrow \max \tan^2 \alpha = \dots$
A1*	Given answer obtained from correct and complete working with no incorrect statements. Complete explanation must justify both sides of inequality using both $e \leq 1$ and $0^\circ < \alpha < 90^\circ$ (allow α is acute)

		(3)	
(b)	Velocity of Q perpendicular to the line of centres unchanged	M1	3.4
	Either $w \sin \theta^\circ = 3 \sin 60^\circ \left(= \frac{3\sqrt{3}}{2} \right)$ or $a = 3 \sin 60^\circ \left(= \frac{3\sqrt{3}}{2} \right)$	A1	1.1b
	CLM parallel to the line of centres	M1	3.4
	Either $0.3 \times 5 \cos 30^\circ - 0.4 \times 3 \cos 60^\circ = 0.4w \cos \theta^\circ - 0.3v \cos 60^\circ$ Or $0.3 \times 5 \cos 30^\circ - 0.4 \times 3 \cos 60^\circ = 0.4b - 0.3x$	A1	1.1b
	Solve for w or θ using $\begin{cases} w \sin \theta^\circ = \frac{3\sqrt{3}}{2} \\ w \cos \theta^\circ = \frac{5\sqrt{3}-3}{2} \end{cases}$ or $\begin{cases} a = \frac{3\sqrt{3}}{2} \\ b = \frac{5\sqrt{3}-3}{2} \end{cases}$	dM1	3.1a
	$\theta = 43 \quad (42.552\dots)$ or $w = 3.8 \quad (3.84182\dots)$	A1	2.2a
	$\theta = 43 \quad (42.552\dots)$ and $w = 3.8 \quad (3.84182\dots)$	A1	2.2a
		(7)	
(c)	NEL parallel to the line of centres	M1	3.1b
	$v \cos 60^\circ + w \cos \theta^\circ = e(5 \cos 30^\circ + 3 \cos 60^\circ)$ $x + b = e(5 \cos 30^\circ + 3 \cos 60^\circ)$	A1ft	1.1b
	$e = 0.73 \quad (0.73300\dots)$	A1	1.1b
		(3)	
(13 marks)			
Notes:			

	(b) Use the model for components of Q perpendicular to the line of centres. Condone sin/cos confusion if clearly working perpendicular to the line of centres. M0 if speeds are used instead of components.
M1	
A1	Correct unsimplified equation.
M1	Correct use of CLM parallel to the line of centres. Dimensionally correct. Need all terms. No need to replace v . Condone sign errors. Condone sin/cos confusion consistent with perpendicular components. Must have correct mass-velocity pairings. M0 if speeds are used instead of components.
A1	Correct unsimplified equation (no need to replace v).
dM1	Dependent on both previous M marks. Complete method using two correctly formed equations to obtain w or θ . i.e. form an equation in w or θ only and solve to reach $w = \dots$ or $\theta = \dots$ If numerical values appear without working out, a complete method using a calculator may be implied by correct answers following correct equations.
A1	One value correct to 2 sf or better, $\theta = 43$ (42.552...) or $w = 3.8$ (3.84182....) θ must be in degrees.
A1	Both values correct to 2 sf or better, $\theta = 43$ (42.552...) and $w = 3.8$ (3.84182....) θ must be in degrees.
(c)	
M1	Correct use of NEL parallel to the line of centres. Must form an equation in e and their components parallel to the line of centres. Condone sign errors. Condone sin/cos confusion consistent with perpendicular components. If seen in earlier work, NEL must be used in (c) to earn the marks here. M0 if separation and approach are on the wrong side. M0 if speeds are used instead of velocity components.
A1ft	Correct unsimplified equation (in e and their components parallel to the line of centres).
A1	Correct answer, 2 sf or better.

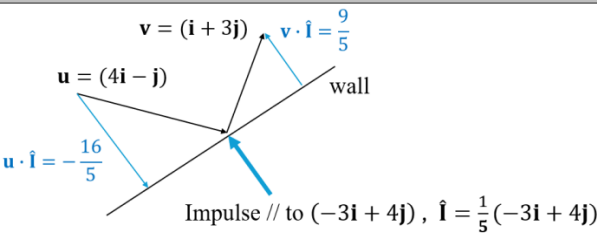
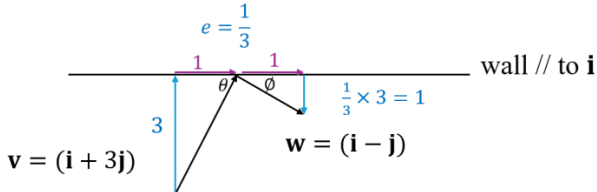
Question	Scheme	Marks	AOs	Notes
1a	Speed after first impact $= \frac{2}{3}u$	B1	3.4	Correct use of impact law, seen or implied. Allow +/-
	Speed after second impact $= \frac{4}{9}u$	B1	3.4	Correct use of impact law a second time, seen or implied. Allow +/-
	Correct method for total time	M1	2.1	Use of $t = \frac{d}{v}$ or equivalent for at least 2 of the 3 parts added
	$T = \frac{d}{u} + \frac{3}{\frac{2}{3}u} + \frac{3-d}{\frac{4}{9}u}$	A1ft A1ft	1.1b 1.1b	Unsimplified expression for T with all 3 terms and at most one error. Follow their speeds. Correct unsimplified expression for T . Follow their speeds
	$= \frac{4d + 18 + 27 - 9d}{4u} = \frac{45 - 5d}{4u} \quad *$	A1*	2.2a	Obtain given answer from correct working
		(6)		
1b	- Least T when d is maximum - Furthest distance at highest speed - Highest average speed - Sketch graph of function	B1	2.4	Correct reasoning
	i.e. $d = 3$, least $T = \frac{30}{4u} = \frac{15}{2u}$	B1	2.2a	Correct answer only. Any equivalent form. $\left(\frac{7.5}{u}\right)$
		(2)		
(8 marks)				

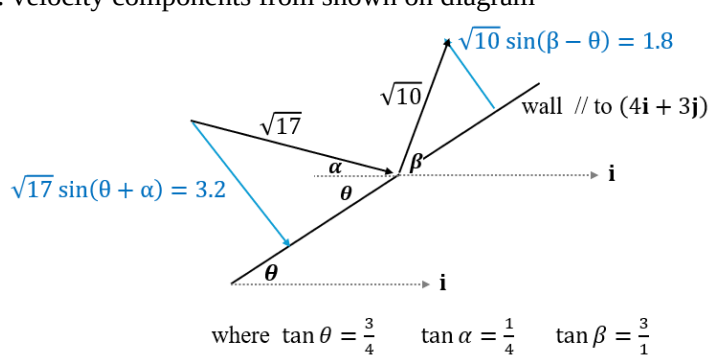
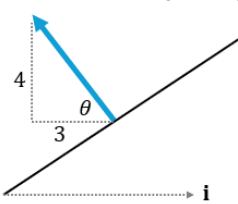


Question	Scheme	Marks	AOs
5(a)	Use the model to find components of velocity after first impact: 	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{2} \times \frac{1}{2} m v^2 = \frac{1}{2} m \left(v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$	M1	3.1b
	$\frac{1}{2} = \frac{1}{9} + \frac{8}{9} \cos^2 \theta$	M1	1.1b
	$\frac{7}{16} = \cos^2 \theta, \quad \cos \theta = \frac{\sqrt{7}}{4}$	A1	1.1b
	(5)		
(a) alt	Working with initial velocity $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$, after impact $\mathbf{v} = x\mathbf{i} + \frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
	KE: $\frac{1}{2} \times \frac{1}{2} m (x^2 + y^2) = \frac{1}{2} m \left(x^2 + \frac{1}{9} y^2 \right)$	M1	3.1b
	$y^2 = \frac{9}{7} x^2, \quad \frac{y}{x} = \tan \theta = \frac{3}{\sqrt{7}}$	M1	1.1b
	$\cos \theta = \frac{\sqrt{7}}{4}$	A1	1.1b
	(5)		
(b)	Use the model to find components of velocity after second impact: 	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{4} \times \frac{1}{2} m v^2 = \frac{1}{2} m \left(e^2 v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$ or $\frac{1}{2} \times \frac{1}{2} m \left(v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right) = \frac{1}{2} m \left(e^2 v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$	M1	3.1b
	$\frac{1}{4} = \frac{7}{16} e^2 + \frac{1}{9} \times \frac{9}{16}$	M1	1.1b
	$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
	(5)		
(b) alt	After second impact $\mathbf{v} = -e x \mathbf{i} + \frac{1}{3} y \mathbf{j}$	B1 B1	1.1b 3.4



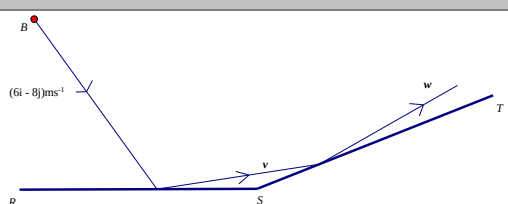
	$\text{KE: } \frac{1}{4} \times \frac{1}{2} m(x^2 + y^2) = \frac{1}{2} m\left(e^2 x^2 + \frac{1}{9} y^2\right)$	M1	3.1b
	$4e^2 x^2 + \frac{4}{9} y^2 = x^2 + y^2, \quad 4e^2 = 1 + \frac{5}{9} \left(\frac{y}{x}\right)^2$	M1	1.1b
	$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
(10 marks)			
Notes:			
(a) B1 B1	Parallel component correct Perpendicular component correct Check the diagram		
M1	Equation for KE in v, θ . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side		
M1	Form and solve equation in $\cos \theta$		
A1	Or exact equivalent		
(b) B1 B1	Parallel component correct Perpendicular component correct		
M1	Equation for KE in x, y . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side		
M1	Use their $\cos \theta$ to form and solve equation in e		
A1	Or exact equivalent		

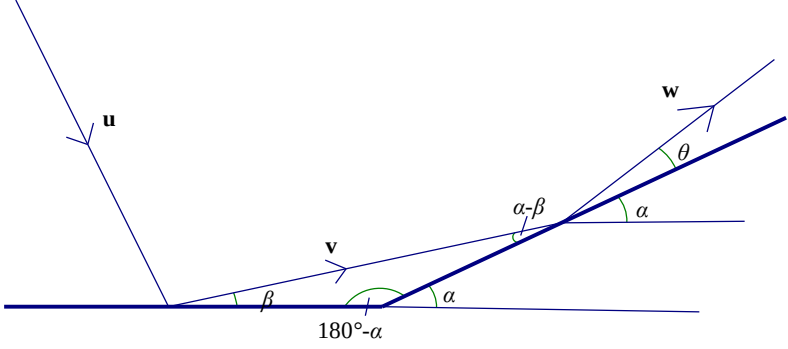
Question	Scheme	Marks	AOs
6(a)			
	Complete method to find direction of normal (perpendicular to wall) ($\pm m$) $((\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - \mathbf{j}))$	M1	2.1
	$(-3\mathbf{i} + 4\mathbf{j})$ or any parallel vector	A1	1.1b
	Resolve both velocities parallel to impulse	M1	3.1a
	Approach: $(4\mathbf{i} - \mathbf{j}) \cdot \left(\frac{1}{5}\right)(-3\mathbf{i} + 4\mathbf{j}) = -\frac{16}{5}$ N.B. $\frac{1}{5}$ is not required	A1	1.1b
	Separation: $(\mathbf{i} + 3\mathbf{j}) \cdot \left(\frac{1}{5}\right)(-3\mathbf{i} + 4\mathbf{j}) = \frac{9}{5}$ N.B. $\frac{1}{5}$ is not required		
	$e = \text{separation speed} / \text{approach speed}$	DM1	3.3
	$e = \frac{9}{16}$ (0.56 or better)	A1	1.1b
	(6)		
6(b)			
	Complete method to find both components of rebound velocity // to wall ($\mathbf{i}$), unchanged - Perp ($\mathbf{j}$), $\frac{1}{3} \times 3$	M1	2.1
	Both velocity components correct, may be given if seen on a diagram.	A1	1.1b
	Complete method to find the angle of deflection, α° Eg - Using trig $\alpha^\circ = \theta + \phi = \tan^{-1}\left(\frac{3}{1}\right) + \tan^{-1}\left(\frac{1}{1}\right)$ - Using scalar product $\alpha^\circ = \cos^{-1}\left(\frac{(\mathbf{i} + 3\mathbf{j}) \cdot (\mathbf{i} - \mathbf{j})}{\sqrt{1^2 + 3^2} \sqrt{1^2 + 1^2}}\right) = \cos^{-1}\left(\frac{-2}{\sqrt{10} \sqrt{2}}\right)$ Note: $\tan \phi = \frac{1}{3} \tan \theta$ may be used in working	DM1	3.1a
	$\alpha = 117$ (nearest whole number)	A1	1.1b

		(4)	
(10 marks)			
Notes: Allow column vectors throughout the question			
(a)			
M1	Complete method to find the direction of the normal (perpendicular to the wall). Eg - Main scheme: use of impulse equation with difference in momenta, m is not required so may see only the difference in velocities. - Find the direction of the wall first by equating parallel components then find a perpendicular direction using the scalar product or otherwise. Eg $(4\mathbf{i} - \mathbf{j}) \cdot (x\mathbf{i} + y\mathbf{j}) = (\mathbf{i} + 3\mathbf{j}) \cdot (x\mathbf{i} + y\mathbf{j}) \Rightarrow 3x = 4y$ Wall direction $\parallel (4\mathbf{i} + 3\mathbf{j}) \Rightarrow$ Impulse direction $\parallel (-3\mathbf{i} + 4\mathbf{j})$		
A1	Any correct vector parallel to impulse $(-3\mathbf{i} + 4\mathbf{j})$		
M1	Complete method to resolve both velocities parallel to impulse - Using the scalar product (main scheme) - Using trig: velocity components from shown on diagram  where $\tan \theta = \frac{3}{4}$ $\tan \alpha = \frac{1}{4}$ $\tan \beta = \frac{3}{1}$ - Using rotation matrix : velocity components shown on RHS of matrix equation  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 3.2 \\ 2.6 \end{pmatrix}$ $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1.8 \\ 2.6 \end{pmatrix}$		
A1	Both correct velocity components parallel to impulse, allow positive or negative values in both cases.		
DM1	Dependent on previous M. Use of Impact Law parallel to impulse. Ratio of speeds correct way up (separation $\div$ approach)		
A1	Correct answer (0.56 or better)		
(b)			

M1	Method to find velocity after impact with second wall: i component unchanged, j component: $\frac{1}{3}$ (3) . Condone sign errors.
A1	Correct velocity components after impact, may be given if seen on a diagram.
DM1	Complete method to find the angle of deflection for their velocity after second impact. Dependent on previous M.
A1	Correct answer, must be rounded to nearest degree.

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
8a			
	$\mathbf{v} = 6\mathbf{i} + \dots$	B1	3.4
	$\dots 8\mathbf{j}$	B1	3.4
	impact with $ST \Rightarrow \frac{8e}{6} < \frac{1}{2}, 0 < e < \frac{3}{8}$	B1	3.1b
	(3)		
8b	Perpendicular to ST : direction $\pm\mu(-\mathbf{i} + 2\mathbf{j})$	B1	1.2
	Component parallel to ST : $(6\mathbf{i} + 2\mathbf{j}) \cdot \lambda(2\mathbf{i} + \mathbf{j})$	M1	3.1b
	$= \left((6\mathbf{i} + 2\mathbf{j}) \cdot \frac{1}{\sqrt{5}}(2\mathbf{i} + \mathbf{j}) \right) = \frac{1}{\sqrt{5}}(12 + 2)$	A1	1.1b
	Component perpendicular to ST : $\pm \left(\frac{1}{2}(6\mathbf{i} + 2\mathbf{j}) \cdot \gamma(-\mathbf{i} + 2\mathbf{j}) \right)$	M1	3.4
	$= \frac{1}{2\sqrt{5}}(-6 + 4)$	A1	1.1b
	$\mathbf{w} = \frac{14}{\sqrt{5}} \frac{1}{\sqrt{5}}(2\mathbf{i} + \mathbf{j}) + \frac{1}{\sqrt{5}} \frac{1}{\sqrt{5}}(-\mathbf{i} + 2\mathbf{j})$	M1	3.1b
	$\mathbf{w} = \left(\frac{28}{5} - \frac{1}{5} \right) \mathbf{i} + \left(\frac{14}{5} + \frac{2}{5} \right) \mathbf{j} = \left(\frac{27}{5} \mathbf{i} + \frac{16}{5} \mathbf{j} \right) \text{ or } (5.4\mathbf{i} + 3.2\mathbf{j}) \text{ (m s}^{-1}\text{)}$	A1	2.2a
	(7)		
8b alt 1	Perpendicular to ST : direction $\pm\mu(-\mathbf{i} + 2\mathbf{j})$	B1	1.2
	$\mathbf{w} = a\mathbf{i} + b\mathbf{j} \Rightarrow (6\mathbf{i} + 2\mathbf{j}) \cdot (2\mathbf{i} + \mathbf{j}) = (a\mathbf{i} + b\mathbf{j}) \cdot (2\mathbf{i} + \mathbf{j})$	M1	
	$14 = 2a + b$	A1	
	$\pm \frac{1}{2}(6\mathbf{i} + 2\mathbf{j}) \cdot (-\mathbf{i} + 2\mathbf{j}) = (a\mathbf{i} + b\mathbf{j}) \cdot (-\mathbf{i} + 2\mathbf{j})$	M1	
	$2b - a = \pm 1$	A1	
	Solve simultaneous equations for a and b	M1	
	$\mathbf{w} = \left(\frac{27}{5} \mathbf{i} + \frac{16}{5} \mathbf{j} \right) \text{ or } (5.4\mathbf{i} + 3.2\mathbf{j}) \text{ (m s}^{-1}\text{)}$	A1	
	(7)		
8balt 2	Perpendicular to ST : direction $\pm\mu(-\mathbf{i} + 2\mathbf{j})$	B1	1.2
	$\mathbf{v} = 6\mathbf{i} + 2\mathbf{j} = p(2\mathbf{i} + \mathbf{j}) + q(-\mathbf{i} + 2\mathbf{j})$	M1	3.1b

	$6 = 2p - q, \quad 2 = p + 2q \quad \left(p = \frac{14}{5}, \quad q = \frac{-2}{5} \right)$	A1	1.1b
	Component perpendicular to $ST \pm \frac{1}{2} \times q(-\mathbf{i} + 2\mathbf{j})$	M1	3.4
	$\pm \frac{1}{2} \times q(-\mathbf{i} + 2\mathbf{j})$	A1	1.1b
	Solve for p and q to obtain velocity $\mathbf{w} = \frac{14}{5}(2\mathbf{i} + \mathbf{j}) + \frac{1}{2} \times \frac{2}{5}(-\mathbf{i} + 2\mathbf{j})$	M1	3.1b
	$\mathbf{w} = \left(\frac{27}{5}\mathbf{i} + \frac{16}{5}\mathbf{j} \right) \text{ or } (5.4\mathbf{i} + 3.2\mathbf{j}) \quad (\text{m s}^{-1})$	A1	2.2a
		(7)	
8balt 3			
	$\alpha - \beta = 8.1...^\circ$	B1	
	Component of $\mathbf{w}$ parallel to ST is $ \mathbf{v} \cos(\alpha - \beta)$	M1	
	$= \sqrt{40} \cos(\alpha - \beta) \left(= \sqrt{40} \times \frac{7}{\sqrt{50}} = 6.26.. \right)$	A1	
	Component of $\mathbf{w}$ perpendicular to ST is $\frac{1}{2} \mathbf{v} \sin(\alpha - \beta)$	M1	
	$\pm \frac{1}{2} \times \sqrt{40} \sin(\alpha - \beta) \left(= \frac{\sqrt{40}}{2} \times \frac{1}{\sqrt{50}} = 0.447... \right)$	A1	
	$\mathbf{w} = \mathbf{w} \cos(\alpha + \theta) \mathbf{i} + \mathbf{w} \sin(\alpha + \theta) \mathbf{j}$	M1	
	$\mathbf{w} = \left(\frac{27}{5}\mathbf{i} + \frac{16}{5}\mathbf{j} \right) \text{ or } (5.4\mathbf{i} + 3.2\mathbf{j}) \quad (\text{m s}^{-1})$	A1	
		(7)	
(Total 10 marks)			
8a			
B1	Component parallel to the wall unchanged. Could be on a diagram or implied if they use $e \tan \alpha = \tan \beta$		
B1	Use of impact law perpendicular to the wall. Could be on a diagram or implied if they use $e \tan \alpha = \tan \beta$		



B1	Use the direction to determine the range for e . (could come via $e \tan \alpha = \tan \beta < 1/2$)
8b	
B1	Correct vector perpendicular to ST seen or implied μ can have any scalar value
M1	Use scalar product to find component of $\mathbf{v}$ parallel to ST . λ can have any scalar value
A1	Correct unsimplified expression for the magnitude
M1	Use scalar product and impact law perpendicular to ST to find magnitude of component perpendicular to the wall. For their perpendicular vector Must clearly be using $e = \frac{1}{2}$. γ can have any scalar value.
A1	Correct unsimplified expression for the perpendicular component. Allow $\pm$
M1	Combine the magnitudes and directions to obtain the velocity. The perpendicular should now be in the correct direction.
A1	Correct simplified velocity.
8b alt	
B1	Correct vector perpendicular to ST seen or implied. μ can have any scalar value
M1	Correct method for component parallel to ST
A1	Correct equation in a and b
M1	Correct method for component perpendicular to ST Allow $\pm$ For their perpendicular vector
A1	Correct equation in a and b
M1	Solve for a and b to obtain velocity. Using the correct direction for the perpendicular component
A1	Correct simplified answer.
8b alt2	
B1	Correct vector perpendicular to ST seen or implied. μ can have any scalar value
M1	Split $\mathbf{v}$ into components parallel and perpendicular to ST
A1	Two equations in p and q
M1	Use the impact law perpendicular to ST For their perpendicular vector
A1	Correct unsimplified perpendicular component. With q or their q

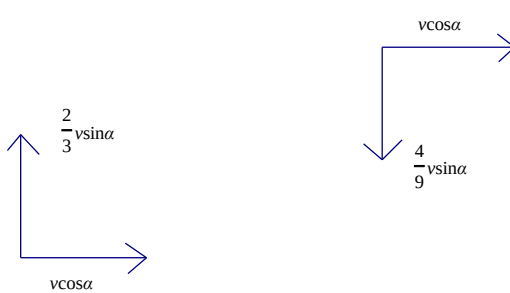
M1	Solve for p and q to obtain velocity Using the correct direction for the perpendicular component
A1	Correct simplified total.
8balt3	
B1	Seen or implied. $\sin(\alpha - \beta) = \frac{1}{\sqrt{50}}, \cos(\alpha - \beta) = \frac{7}{\sqrt{50}},$ $\tan(\alpha - \beta) = \frac{1}{7}$
M1	Correct use of their $ \mathbf{v} $ and their $\alpha - \beta$
A1	Correct unsimplified
M1	Correct use of $\frac{1}{2}$, their $ \mathbf{v} $ and their $\alpha - \beta$
A1	Correct unsimplified
M1	Use of Pythagoras and correct method for $\theta + \alpha$. $\cos(\alpha + \theta) = \frac{27}{\sqrt{5}\sqrt{197}}, \sin(\alpha + \theta) = \frac{16}{\sqrt{5}\sqrt{197}}$ $\alpha + \theta = 30.65^\circ$
A1	Correct simplified total.

Question	Scheme	Marks	AOs	Notes
2(a)	After hit AB : $\rightarrow 6 \cos \alpha (=v \cos \beta) (=3.6)$	B1	3.1b	Use model to find component parallel to the wall
	Use of impact law:	M1	3.4	Use model and impact law perpendicular to the wall
	$\uparrow 6e \sin \alpha (=v \sin \beta) \left(= \frac{24e}{5} \right) (=4.8e)$	A1	1.1b	Correct perpendicular component
	$\tan \beta = \frac{1}{3} = \frac{6e \sin \alpha}{6 \cos \alpha} \left(= \frac{24e}{5} \div \frac{18}{5} \right)$	M1	2.1	Use $\frac{1}{3}$ and their components to form equation in e $\left(v = \frac{6\sqrt{10}}{5} = 3.79 \right)$
	$e = \frac{18}{3 \times 24} = \frac{1}{4} *$	A1*	2.2a	Correct answer from correct exact working
				If only see $e \tan \alpha = \tan \beta$ with no explanation of where it comes from then score 0/5
		(5)		
(b)	After hit BC : $\uparrow \frac{6}{5}$	B1	1.1b	First component correct
	$\rightarrow \frac{1}{2} \times \frac{18}{5} \left(= \frac{9}{5} \right)$	B1	3.4	Second component correct
				Alternative: B1 for speed of impact with $BC = 3.79 \dots$ B1 for path on leaving BC at $56.3 \dots^\circ$ to BC
	Speed $= \frac{3}{5} \sqrt{2^2 + 3^2}$	M1	1.1b	Use Pythagoras' theorem or trigonometry to find the speed
	$= \frac{3\sqrt{13}}{5} (\text{m s}^{-1})$	A1	1.1b	Any equivalent form. 2.2 or better (2.1633....)
		(4)		



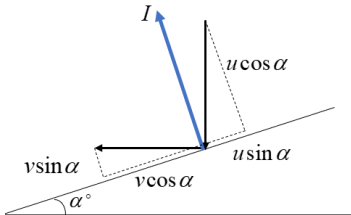
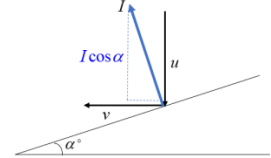
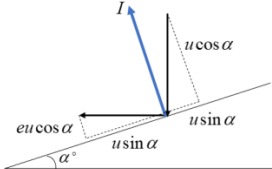
(c)	An appropriate refinement	B1	3.5c	Two independent refinements relating to the modelling e.g. • Include friction between the floor and the ball • Include friction between the ball and the walls • Give the ball dimensions . • Consider air resistance • Spin / rotation Do not accept comments about mass / gravity / levels / perpendicularity
	A second independent appropriate refinement and no incorrect refinements	B1	3.5c	
		(2)		
(11 marks)				

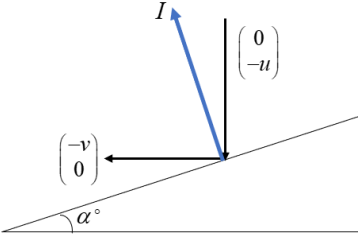


Question	Scheme	Marks	AOs
7(a)	Use model to find components of velocity after the impacts: 	B1	3.1b
		B1	3.4
		B1	3.1b
		B1	3.4
	$\tan \frac{\alpha}{2} = \frac{\frac{4}{9} v \sin \alpha}{v \cos \alpha} \left(= \frac{4}{9} \tan \alpha \right)$	M1	3.1b
(b)	$t = \tan \frac{\alpha}{2} \Rightarrow t = \frac{4 \times 2t}{9(1-t^2)}$	M1	1.1b
	$1-t^2 = \frac{8}{9}, \quad t = \frac{1}{3} *$	A1*	2.2a
		(7)	
	$\tan \alpha = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$	B1	1.1b
	change in KE $\frac{1}{2}mv^2 - \frac{1}{2}m \left(v^2 \cos^2 \alpha + \left(\frac{4}{9}v \right)^2 \sin^2 \alpha \right)$	M1	3.1b
	$\% \text{ of KE lost} = 100 \left(1 - \frac{\frac{1}{2}mv^2 \left(\frac{16}{25} + \frac{16}{81} \times \frac{9}{25} \right)}{\frac{1}{2}mv^2} \right)$	M1	1.1b
	$= 28.888...(\%)$	A1	1.1b
		(4)	
(11 marks)			
Notes:			
(a)B1 B1 B1 B1	One mark for each component correct.		
M1	Form expression for $\tan \frac{\alpha}{2}$ in terms of $\tan \alpha$		

M1	Form and solve equation in $\tan \frac{\alpha}{2}$
A1*	Obtain given answer from correct working
	NB: This is a “Show that ..” question. A candidate who assumes, without proof, that $\tan \frac{\alpha}{2} = e^2 \tan \alpha$ can only score the last two marks.
(b)B1	Correct use of $t = \frac{1}{3}$ Must be seen / used in part (b)
M1	Dimensionally correct expression for change in KE NB note that they may not show component parallel to the wall
M1	Dimensionally correct expression for the percentage of KE lost.
A1	Accept 29(%) or better Accept $\frac{260}{9}$



Question	Scheme	Marks	AOs
6(a)			
	CLM along the plane:	M1	3.1a
	$(m)u \sin \alpha = (m)v \cos \alpha$	A1	1.1b
	Impulse-momentum perp to the plane:	M1	3.1a
	$I = m(v \sin \alpha - (-u \cos \alpha))$	A1	1.1b
	$I = m\left(\frac{u \sin^2 \alpha}{\cos \alpha} + u \cos \alpha\right) = \frac{mu}{\cos \alpha}(\sin^2 \alpha + \cos^2 \alpha) = mu \sec \alpha^*$	A1*	2.2a
	(5)		
6(a) alt1		M1	3.1a
	Impulse-momentum vertically.	M1	3.1a
	$I \cos \alpha = m(0 - -u)$	A1	1.1b
		A1	1.1b
	$I = mu \sec \alpha^*$	A1*	2.2a
	(5)		
6(a) alt 2	Introduce and use an expression for e 		
	CLM along the plane:	M1	3.1a
	$u \sin \alpha$ unchanged	A1	1.1b
	Finds an expression for e together with Impulse-momentum perpendicular to the plane $\tan \alpha = \frac{eu \cos \alpha}{u \sin \alpha} \Rightarrow e = \tan^2 \alpha$ and $I = m(eu \cos \alpha - (-u \cos \alpha))$	M1	3.1a
	$I = m(u \cos \alpha \tan^2 \alpha - (-u \cos \alpha))$	A1	1.1b
	$I = m\left(\frac{u \sin^2 \alpha}{\cos \alpha} + u \cos \alpha\right) = \frac{mu}{\cos \alpha}(\sin^2 \alpha + \cos^2 \alpha) = mu \sec \alpha^*$	A1*	2.2a
	(5)		

6(a) alt 3	Use a vector approach and magnitude of impulse			
	CLM along the plane:		M1	3.1a
	$(m)u \sin \alpha = (m)v \cos \alpha$ (this leads to $v = u \tan \alpha$)		A1	1.1b
	Impulse-momentum as a vector equation followed by Pythagoras to find the magnitude. $I = m \begin{pmatrix} -v \\ u \end{pmatrix}$ and $ I = m\sqrt{v^2 + u^2}$		M1	3.1a
	$ I = m\sqrt{u^2 \tan^2 \alpha + u^2}$		A1	1.1b
	$I = m\sqrt{u^2(1 + \tan^2 \alpha)} = m\sqrt{u^2 \sec^2 \alpha} = mu \sec \alpha^*$		A1*	2.2a
			(5)	
6(b)	NEL: $eu \cos \alpha = v \sin \alpha$		M1	3.4
	Squaring and adding their expressions for $v \sin \alpha$ and $v \cos \alpha$.		M1	1.1b
	$v^2 = u^2(\sin^2 \alpha + e^2 \cos^2 \alpha)^*$		A1*	1.1b
			(3)	
6(c)	KE loss = $\frac{1}{2}mu^2 - \frac{1}{2}mu^2(\sin^2 \alpha + e^2 \cos^2 \alpha)$.		M1	2.1
	Use $\sin^2 \alpha + \cos^2 \alpha = 1$ to give KE loss = $\frac{1}{2}mu^2(1 - e^2)\cos^2 \alpha^*$		A1*	1.1b
			(2)	
	Use $\tan^2 \alpha = e$ or to eliminate α in given expression from (c)		M1	3.1a
6(d)	KE Loss = $\frac{1}{2}mu^2(1 - e)$ or $\frac{1}{2}mu^2 \frac{1}{1 + e}(1 - e^2)$		A1	1.1b
			(2)	
(12 marks)				
Notes:				
(a)				
M1	Correct no. of terms, dimensionally correct, mass $\times$ velocity, condone sin/cos confusion.			
A1	Correct equation			
M1	Dimensionally correct. Must be subtracting, but condone subtracting in the wrong order and sin/cos confusion			

A1	Correct unsimplified equation
A1*	Given answer correctly obtained. Must be EXACT factorisation.
(b)	
M1	Attempt at NEL
M1	Squaring and adding their expressions for $v \sin \alpha$ and $v \cos \alpha$ to obtain v^2 .
A1*	Given answer correctly obtained. Must be EXACT.
(c)	
M1	Expression for difference of KE in terms of m , u , α and e
A1*	Given answer correctly obtained. Factorisation must be EXACT.
(d)	
M1	Complete method to eliminate α e.g. using $\tan^2 \alpha = e$ to eliminate α Any trig identity used must be correct eg $\sec^2 \alpha = 1 + e$ or $\cos^2 \alpha = \frac{1}{1+e}$
A1	Correct answer.

Question	Scheme	Marks	AOs
	Note: The diagram below is an aide for marking. In reality, the velocity components cannot be represented by the side lengths of the snooker table. The magnitude of PC is not the magnitude of $U \cos \alpha$		

Question	Scheme	Marks	AOs
7(a)	$(V \sin \beta =) e_1 U \sin \alpha$	B1	3.4
	$(V \cos \beta =) U \cos \alpha$	B1	3.4
	Eliminate U and V from two equations	M1	1.1b
	$\tan \beta = e_1 \tan \alpha$ *	A1*	2.2a
		(4)	
7(b)	Form a correct equation for γ , β and e_2 $\tan \gamma = e_2 \tan(90^\circ - \beta)$ $\tan \gamma = e_2 \cot \beta$ $\cot \gamma = \frac{\tan \beta}{e_2}$	B1	1.1b
	$\tan \gamma = e_2 \times \frac{1}{\tan \beta} = e_2 \times \frac{1}{e_1 \tan \alpha}$	M1	3.1b
	$e_1 \tan \alpha = e_2 \cot \gamma$ *	A1*	2.2a
		(3)	
7(c)	(angle APQ + angle AQP) = $(180^\circ - \alpha - \beta) + \{180^\circ - (90^\circ - \beta) - \gamma\} = 270^\circ - \alpha - \gamma$ Otherwise: • angle $PAQ = \alpha + \gamma - 90$	M1	1.1b
	To return to A , (angle APQ + angle AQP) < 180° , since APQ is a triangle Otherwise: • angle $PAQ > 0$	M1	3.1b
	$270^\circ - \alpha - \gamma < 180^\circ \Rightarrow \alpha > 90^\circ - \gamma$ oe	A1	1.1b
	$\tan \alpha > \tan(90^\circ - \gamma)$ oe See notes for completion using addition formulae.	M1	1.1b

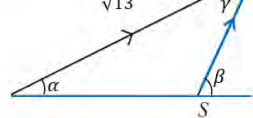
	$\frac{e_2 \cot \gamma}{e_1} > \cot \gamma$	M1	1.1b
	$e_2 > e_1^*$	A1*	2.2a
		(6)	
7(d)	From (b), $\alpha = 90^\circ - \gamma$, so it moves parallel to AP or Eg parallel to the initial velocity	B1	2.4
(14 marks)			
Notes:			
(a)			
B1	$e_1 U \sin \alpha$ seen from a relevant equation or on a diagram.		
B1	$U \cos \alpha$ seen in a relevant equation or on a diagram.		
M1	A clear method using two equations to eliminate U and V .		
A1*	GIVEN answer correctly obtained. Must include two equations showing how to reach both $\tan \beta$ and $e_1 \tan \alpha$. It is not sufficient to use the side lengths of the snooker eg using $\tan \beta = \frac{CQ}{PC}$ or is not sufficient. Accept $\tan \beta = e_1 \tan \alpha$ or $e_1 \tan \alpha = \tan \beta$		
(b)	This part states 'hence' so β must be used.		
B1	Form a correct expression for $\tan \gamma$ or $\cot \gamma$ in terms of e_2 and β or $(90 - \beta)$. May quote result from (a) or obtain again.		
M1	Use result from (a) to eliminate $\tan \beta$ and form an equation in α, γ, e_1, e_2		
A1*	Given answer correctly obtained. The solution must include the replacement of $\tan \beta$ and rearrangement to the correct form. Accept $e_1 \tan \alpha = e_2 \cot \gamma$ or $e_2 \cot \gamma = e_1 \tan \alpha$		
(c)			
M1	Clear attempt to find angle sum (condone slips) or another relevant starting point eg an expression for angle PAQ		
M1	Clear statement to form an inequality eg - the correct angle sum < 180 is acceptable - angle $PAQ > 0$		
A1	Correct simplified inequality in correct form		
M1	Correct method to form an inequality in $\tan$ or $\cot$		
M1	Using part (b) to eliminate the angles		

A1*	Given answer correctly obtained
7(c) alt	Use of trig identity
M1	$(\text{angle } APQ + \text{angle } AQP) = (180^\circ - \alpha - \beta) + \{180^\circ - (90^\circ - \beta) - \gamma\} = 270 - \alpha - \gamma$
M1	To return to A, $(\text{angle } APQ + \text{angle } AQP) < 180^\circ$, since APQ is a triangle
A1	$\tan(\alpha + \gamma) = \frac{\tan \alpha + \tan \gamma}{1 - \tan \alpha \tan \gamma} \quad \text{and} \quad \tan \alpha = \frac{e_2 \cot \gamma}{e_1} \quad \text{or} \quad \tan \alpha = \frac{e_2}{e_1 \tan \gamma}$ Leads to $\tan(\alpha + \gamma) = \frac{e_2 + e_1 \tan^2 \gamma}{e_1 \tan \gamma - e_2 \tan \gamma} \quad \text{oe}$
M1	$180 > (\alpha + \gamma) > 90 \Rightarrow \tan(\alpha + \gamma) < 0 \Rightarrow \frac{e_2 + e_1 \tan^2 \gamma}{e_1 \tan \gamma - e_2 \tan \gamma} < 0$ Condone if '180 >' is not stated again.
M1	Since numerator > 0 $e_1 \tan \gamma - e_2 \tan \gamma < 0$
A1	$e_2 > e_1$ *
(d)	
B1	Use the given information in (b) to make any equivalent statement with a correct reason and no incorrect statements. $\alpha = 90^\circ - \gamma$, so it moves parallel to AP $\alpha = 90^\circ - \gamma$, so it moves parallel to the initial velocity Do not accept 'it moves parallel to the initial speed'.

	KE loss	M1	3.1b
	$= \frac{1}{2} \times 0.25(3^2 + 6^2) - \frac{1}{2} \times 0.25(3^2 + 2^2)$	A1	1.1b
	$= 4 \text{ (J)}$ *	A1*	2.2a
		(6)	
(b) ALT 1	Method [1] using scalar product and components of $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$ and $(3\mathbf{i} + 2\mathbf{j})$ parallel to ST $(3\mathbf{i} + 4\mathbf{j})$. Allow without $\frac{1}{5}$.	M1	3.1b
	$\frac{1}{5}(3\mathbf{i} + 4\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j}) = \frac{1}{5}(3\mathbf{i} + 4\mathbf{j}) \cdot (3\mathbf{i} + 2\mathbf{j})$ $\{3a + 4b = 17\}$	A1	1.1b
	Method [1] using scalar product and components of $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$ and $(3\mathbf{i} + 2\mathbf{j})$ perpendicular to ST , $(-4\mathbf{i} + 3\mathbf{j})$. Any multiple of $(-4\mathbf{i} + 3\mathbf{j})$. Must have $\pm \frac{1}{3}$. Allow without $\frac{1}{5}$.	M1	3.1b
	$\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j}) = -\frac{1}{3} \left[\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j}) \cdot (3\mathbf{i} + 2\mathbf{j}) \right]$ $\{-4a + 3b = 2\}$	A1	1.1b
	Complete method [1] to solve for a and b and form $\mathbf{w}$ $\begin{cases} 3a + 4b = 17 \\ -4a + 3b = 2 \end{cases} \Rightarrow \mathbf{w} = a\mathbf{i} + b\mathbf{j}$	dM1	2.1
	$= \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}$ ($\mathbf{i}$ - $\mathbf{j}$ notation required)	A1	2.2a
		(6)	
(b)	Method [2] using scalar product and component of $(3\mathbf{i} + 2\mathbf{j})$ with a unit vector parallel to ST $(3\mathbf{i} + 4\mathbf{j})$	M1	3.1b



(b) ALT 3	Method [3] finding coefficients to express $(3\mathbf{i} + 2\mathbf{j})$ as multiples of $(3\mathbf{i} + 4\mathbf{j})$ and $(-4\mathbf{i} + 3\mathbf{j})$.	M1	3.1b
	$3\mathbf{i} + 2\mathbf{j} = p(3\mathbf{i} + 4\mathbf{j}) + q(-4\mathbf{i} + 3\mathbf{j})$	A1	1.1b
	Equate components and solve for p and q : eg $\begin{cases} 3 = 3p - 4q \\ 2 = 4p + 3q \end{cases}$	M1	3.1b
	$p = \frac{17}{25} \quad q = -\frac{6}{25}$	A1	1.1b
	Complete method [3] to combine correctly with $\pm \frac{1}{3}$ and form $\mathbf{w}$ $\mathbf{w} = p(3\mathbf{i} + 4\mathbf{j}) + -\frac{1}{3} \times q(-4\mathbf{i} + 3\mathbf{j})$	dM1	2.1
	$= \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j} \quad (\mathbf{i}\text{-}\mathbf{j} \text{ notation required})$	A1	2.2a
		(6)	
(b) ALT 4	Method [4] using rotation matrix on $3\mathbf{i} + 2\mathbf{j}$	M1	
	$\begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{17}{5} \\ -\frac{6}{5} \end{pmatrix}$	A1	
	Method [4] using $\pm \frac{1}{3}$ with the correct component to find velocity relative to ST .	M1	
	$\begin{pmatrix} \frac{17}{5} \\ \frac{2}{5} \end{pmatrix}$	A1	
	Complete method [4] to reverse rotation and form $\mathbf{w}$	dM1	


 Method [5] using speed and direction parallel to ST

M1

3.1b

$$w \cos \delta = \sqrt{13} \cos \gamma \quad (= 3.4)$$

A1

1.1b

 Method [5] using speed and direction perpendicular to ST with $\pm \frac{1}{3}$

M1

3.1b

$$w \sin \delta = \frac{1}{3} \sqrt{13} \sin \gamma \quad (= 0.4)$$

A1

1.1b

 Complete method [5] to combine modulus with angle $(\beta + \delta)$ and form $\mathbf{w}$

dM1

2.1

$$\mathbf{w} = \frac{\sqrt{293}}{5} \cos(\beta + \delta) \mathbf{i} + \frac{\sqrt{293}}{5} \sin(\beta + \delta) \mathbf{j}$$

$$= \frac{43}{25} \mathbf{i} + \frac{74}{25} \mathbf{j} \quad (\mathbf{i}\text{-}\mathbf{j} \text{ notation required})$$

A1

2.2a

(6)

(12 marks)

Notes:

(a)

B1

Correct only, 3i. Accept a magnitude of 3, if seen with correct direction indicated. Accept trig components with 3 and direction indicated

$$\text{E.g. } v \cos \theta = \sqrt{13} \cos \alpha = \sqrt{13} \times \frac{3}{\sqrt{13}} = 3$$

M1

 Correct use of the impact law perpendicular to RS with $\frac{1}{3}$ and 2. Allow $\pm$.
M0 if separation and approach are on the wrong side.

A1*	NB - If the given answer is obtained by only considering the KE of the perpendicular components ie $\frac{1}{2} \times 0.25(6^2) - \frac{1}{2} \times 0.25(2^2)$, the method is incomplete. Max score B1M1A1M1A1A0* - If the given answer is obtained by considering the KE of the perpendicular components, $\frac{1}{2} \times 0.25(6^2) - \frac{1}{2} \times 0.25(2^2)$ and states that the KE parallel to RS is unchanged, all marks are available. - If the solution only consists of the energy equation, this is insufficient working for a given answer $\frac{1}{2} \times 0.25(3^2 + 6^2) - \frac{1}{2} \times 0.25(3^2 + 2^2)$. Max score B1M0A0M1A1A0*
(b) ALT 1	Method [1] Accept column vectors throughout working but must be in i-j form for final A mark.
M1	Method [1] using scalar product and components of $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$ and $(3\mathbf{i} + 2\mathbf{j})$ parallel to ST $(3\mathbf{i} + 4\mathbf{j})$ i.e. any multiple of $(3\mathbf{i} + 4\mathbf{j})$ to form an equation in a and b. Allow without $\frac{1}{5}$.
A1	Correct equation in a and b.
M1	Method [1] using scalar product and components of $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$ and $(3\mathbf{i} + 2\mathbf{j})$ perpendicular to ST $(-4\mathbf{i} + 3\mathbf{j})$ i.e. any multiple of $(-4\mathbf{i} + 3\mathbf{j})$ to form an equation in a and b. Must have $\pm \frac{1}{3}$. Allow without $\frac{1}{5}$. N.B. It is possible to form 2 simultaneous equations using 2 perpendicular equations instead of 1 parallel and 1 perpendicular NEL: $\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j}) = -\frac{1}{3} \left[\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j}) \cdot (3\mathbf{i} + 2\mathbf{j}) \right] \Rightarrow -4a + 3b = 2$ Perp.: $\mathbf{I} = 0.25 \begin{pmatrix} a-3 \\ b-2 \end{pmatrix}$ and $\mathbf{I} \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 0 \Rightarrow 3a + 4b = 17$
A1	Correct equation in a and b.

	A1	Correct expression.
		Dependent on previous 2 M's. Complete method [2] to combine and form w . Must see
	dM1	$\pm \frac{1}{3}$ with $(-4\mathbf{i} + 3\mathbf{j})$ component.
	A1	Correct i-j form. Accept $\frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i} + 2.96\mathbf{j})$
	(b) ALT 3	Method [3] Accept column vectors throughout working but must be in i-j form for final A mark.
	M1	Method [3] finding coefficients to express $(3\mathbf{i} + 2\mathbf{j})$ as multiples of $(3\mathbf{i} + 4\mathbf{j})$ and $(-4\mathbf{i} + 3\mathbf{j})$
	A1	Correct expression
	M1	Solve to obtain the coefficients.
	A1	Correct only.
	dM1	Dependent on previous 2 M's. Complete method [3] to w . Must see $\pm \frac{1}{3}$ with $(-4\mathbf{i} + 3\mathbf{j})$.
	A1	Correct i-j form. Accept $\frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i} + 2.96\mathbf{j})$
	(b) ALT 4	Method [4] Accept column vectors throughout working but must be in i-j form for final A mark.
	M1	Method [4] using rotation matrix on $3\mathbf{i} + 2\mathbf{j}$
	A1	Correct matrix expression
	M1	Method [4] using $\pm \frac{1}{3}$ with the correct component to find velocity relative to ST .
	A1	Correct only.
	dM1	Dependent on previous 2 M's. Complete method [4] by reversing rotation to form w



A1	$w \sin \delta = \frac{1}{3} \sqrt{3^2 + 2^2} \sin \gamma \quad (= 0.4)$
dM1	Dependent on previous 2 M's. Complete method [5] to combine the modulus with the relevant angle and form $\mathbf{w}$ $\mathbf{w} = \frac{\sqrt{293}}{5} \cos(\beta + \delta) \mathbf{i} + \frac{\sqrt{293}}{5} \sin(\beta + \delta) \mathbf{j}$
A1	Correct $\mathbf{i}$-$\mathbf{j}$ form. Accept $\frac{43}{25} \mathbf{i} + \frac{74}{25} \mathbf{j}$ or $(1.72\mathbf{i} + 2.96\mathbf{j})$

