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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Centres of Mass of Plane Figures (2)

Further Mechanics 2 · Topic 2

22 questions · 200 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

22 questions · 200 marks · 4 sub-topics



SUB-TOPIC 2.1

CENTRES OF MASS OF LAMINAS AND PARTICLES

Sub-topic 2.1 · 5 questions · 40 marks · lowest tariff first



Question 1 continued

(Total for Question 1 is 6 marks)



- Three particles of masses $3m$, $4m$ and km are positioned at the points with coordinates $(2a, 3a)$, $(a, 5a)$ and $(2\mu a, \mu a)$ respectively, where k and μ are constants.

The centre of mass of the three particles is at the point with coordinates $(2a, 4a)$.

Find (i) the value of k

(ii) the value of μ

(6)

Question 1 continued

(Total for Question 1 is 6 marks)



1. Three particles of masses $3m$, $4m$ and $2m$ are placed at the points $(-2, 2)$, $(3, 1)$ and (p, p) respectively.

The value of p is such that the distance of the centre of mass of the three particles from the point $(0, 0)$ is as small as possible.

Find the value of p .

(7)



Question 1 continued

(Total for Question 1 is 7 marks)



7. [In this question, you may assume that the centre of mass of a circular arc, radius r , with angle at centre 2α , is a distance $\frac{r \sin \alpha}{\alpha}$ from the centre.]

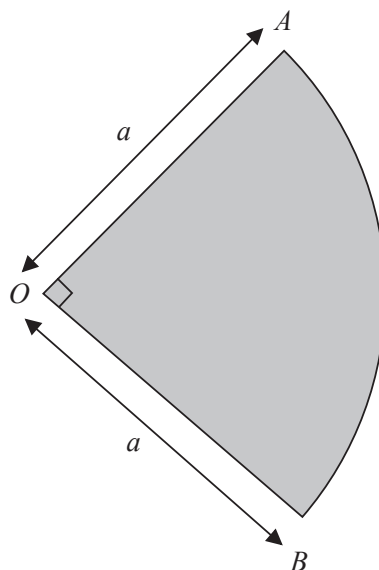


Figure 5

A thin non-uniform metal plate is in the shape of a sector OAB of a circle with centre O and radius a . The angle $AOB = \frac{\pi}{2}$, as shown in Figure 5.

The plate is modelled as a non-uniform lamina.

The mass per unit area of the lamina, at any point P of the lamina, is modelled as

$k(OP)^2$, where $k = \frac{4\lambda}{\pi a^4}$ and λ is a constant.

Using the model,

- (a) find the mass of the plate in terms of λ ,

(5)

- (b) find, in terms of a , the distance of the centre of mass of the plate from O .

(4)

Question 7 continued



Question 7 continued



Question 7 continued



Question 7 continued

(Total for Question 7 is 9 marks)



6. A uniform semicircular lamina has radius r and the midpoint of the diameter of the lamina is at O . The distance of the centre of mass of the lamina from O is d .

(a) Use algebraic integration to show that $d = \frac{4r}{3\pi}$

(4)

The uniform plane template T , shown shaded in Figure 7, is formed from a rectangle and a semicircular ring.

The rectangle has $AD = BC = 2a$ and $AB = DC = 8a$.

The midpoint of CD is O .

The semicircular ring has centre O , outer radius $OD = 4a$ and inner radius $OE = 2a$.

The template is modelled as a uniform lamina.

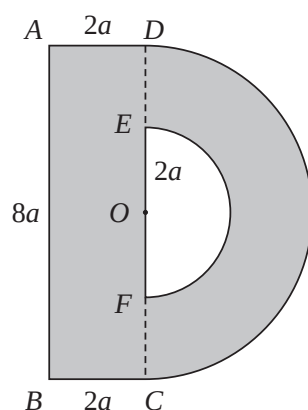


Figure 7

(b) Show that the distance of the centre of mass of T from DC is $\frac{32a}{3(8 + 3\pi)}$

(5)

The template is freely suspended from A . The weight of T is W newtons.

A horizontal force is applied to T at B so that T is held in equilibrium with AB vertical.

The force acts in the same vertical plane as T and has magnitude λW newtons.

(c) Find the value of λ

(3)

Question 6 continued



Question 6 continued



Question 6 continued

SUB-TOPIC 2.2

FRAMEWORKS AND SUSPENDED LAMINAS

Sub-topic 2.2 · 11 questions · 98 marks · lowest tariff first



2.

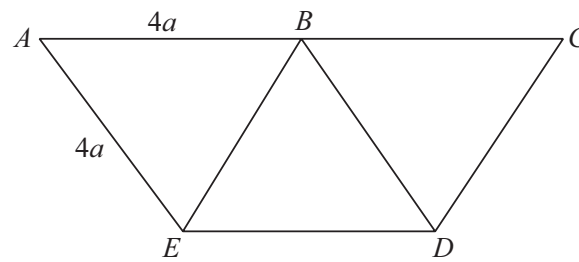


Figure 1

A uniform rod of length $28a$ is cut into seven identical rods each of length $4a$. These rods are joined together to form the rigid **framework** $ABCDEA$ shown in Figure 1.

All seven rods lie in the same plane.

The distance of the centre of mass of the framework from ED is d .

(a) Show that $d = \frac{8\sqrt{3}}{7}a$ (4)

The weight of the framework is W .

The framework is freely pivoted about a horizontal axis through C .

The framework is held in equilibrium in a vertical plane, with AC vertical and A below C , by a **horizontal force** that is applied to the framework at A .

The force acts in the same vertical plane as the framework and has magnitude F .

(b) Find F in terms of W . (3)





(Total for Question 2 is 7 marks)





Question 3 continued



(Total for Question 3 is 7 marks)

Question 5 continued

Question 5 continued



Question 5 continued

(Total for Question 5 is 7 marks)



1.

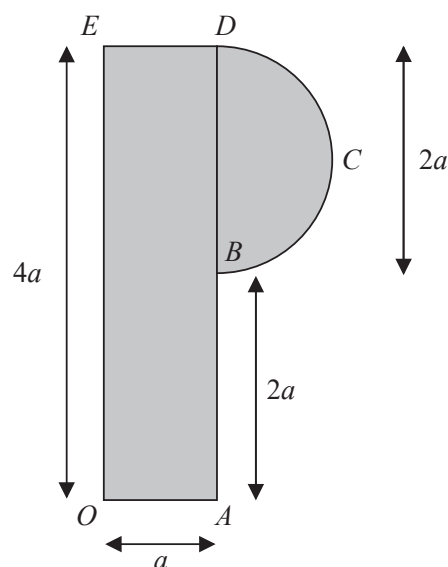


Figure 1

A letter P from a shop sign is modelled as a uniform plane lamina which consists of a rectangular lamina, $OABDE$, joined to a semicircular lamina, BCD , along its diameter BD .

$OA = ED = a$, $AB = 2a$, $OE = 4a$, and the diameter $BD = 2a$, as shown in Figure 1.

Using the model,

(a) find, in terms of π and a , the distance of the centre of mass of the letter P,

from (i) OE

(ii) OA

(6)

The letter P is freely suspended from O and hangs in equilibrium. The angle between OE and the downward vertical is α .

Using the model,

(b) find the exact value of $\tan \alpha$

(2)

Question 1 continued



Question 1 continued



Question 1 continued

(Total for Question 1 is 8 marks)



3. [In this question you may quote, without proof, the formula for the distance of the centre of mass of a uniform circular arc from its centre.]

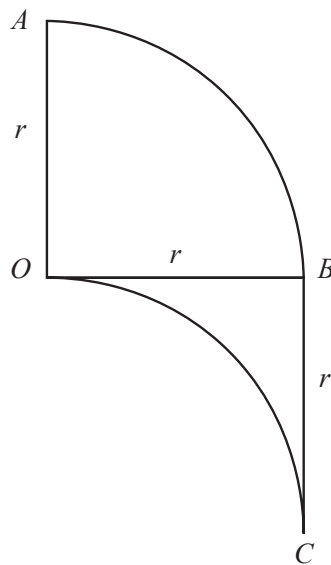


Figure 1

Five pieces of a uniform wire are joined together to form the rigid **framework** $OABCO$ shown in Figure 1, where

- OA , OB and BC are straight, with $OA = OB = BC = r$
 - arc AB is one quarter of a circle with centre O and radius r
 - arc OC is one quarter of a circle of radius r
 - all five pieces of wire lie in the same plane
- (a) Show that the centre of mass of arc AB is a distance $\frac{2r}{\pi}$ from OA .

(2)

Given that the distance of the centre of mass of the framework from OA is d ,

(b) show that $d = \frac{7r}{2(3 + \pi)}$

(4)

The framework is freely pivoted at A .

The framework is held in equilibrium, with AO vertical, by a horizontal force of magnitude F which is applied to the framework at C .

Given that the weight of the framework is W

- (c) find F in terms of W

(3)

Question 3 continued

This image shows a full page of blank, lined paper. It features approximately 28 horizontal grey lines spaced evenly apart, typical of standard notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings present.



Question 2 continued



Question 2 continued



Question 2 continued

(Total for Question 2 is 10 marks)



Question 3 continued



Question 3 continued



Question 3 continued

(Total for Question 3 is 11 marks)



Question 5 continued

Question 5 continued



Question 5 continued

(Total for Question 5 is 11 marks)

4.

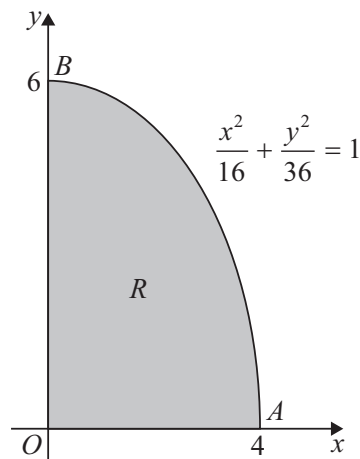


Figure 3

A uniform lamina OAB is in the shape of the region R .

Region R lies in the first quadrant and is bounded by the curve with equation

$$\frac{x^2}{16} + \frac{y^2}{36} = 1, \text{ the } x\text{-axis, and the } y\text{-axis, as shown shaded in Figure 3.}$$

The point A is the point of intersection of the curve and the x -axis.

The point B is the point of intersection of the curve and the y -axis.

One unit on each axis represents 1 m.

The area of R is 6π

The centre of mass of R lies at the point with coordinates $(\bar{x}, \bar{y})$

(a) Use algebraic integration to show that $\bar{x} = \frac{16}{3\pi}$ (5)

(b) Use algebraic integration to find the exact value of $\bar{y}$ (4)

The lamina is freely suspended from A and hangs in equilibrium with OA at angle θ° to the downward vertical.

(c) Find the value of θ (3)



Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)



1.

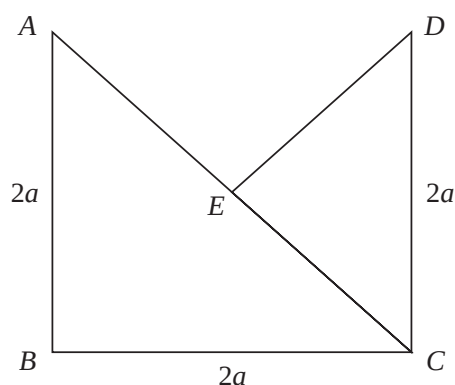


Figure 1

A uniform rod is cut into six pieces. The pieces are used to form the framework $ABCDE$ shown in Figure 1.

- All the pieces of the framework lie in the same plane.
- $AB = BC = CD = 2a$
- $AE = EC = ED = \sqrt{2}a$
- Point E is the midpoint of AC
- Angle $ABC = \text{angle } BCD = 90^\circ$

The distance of the centre of mass of the framework from AB is d

(a) Show that $d = \frac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}}a$ (4)

The mass of the framework is M . A particle of mass kM is attached to the framework at B

The distance of the centre of mass of the loaded framework from AB is a

(b) Find the value of k (3)

Question 1 continued

Question 1 continued

Question 1 continued

4.

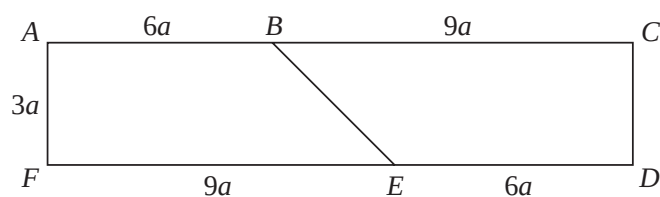


Figure 5

The uniform rectangular lamina $ACDF$ shown in Figure 5 has $AC = FD = 15a$ and $AF = CD = 3a$. The point B on AC is such that $AB = 6a$. The point E on FD is such that $ED = 6a$.

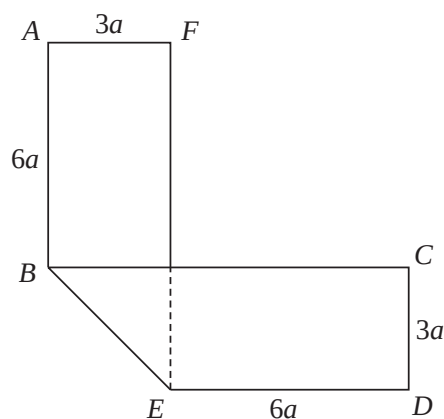


Figure 6

The rectangular lamina is folded along BE to form the folded lamina shown in Figure 6. The folded lamina has AB perpendicular to BC and the two sections, $ABEF$ and $BEDC$, of the lamina lie in the same plane.

- (a) Show that the distance of the centre of mass of the folded lamina from EF is $\frac{2}{5}a$. (5)
- (b) Explain why the centre of mass of the folded lamina lies on the perpendicular bisector of BE . (1)

The folded lamina is freely suspended from F and hangs in equilibrium.

- (c) Find the size of the angle between FE and the downward vertical. (3)

Question 4 continued



Question 4 continued



Question 4 continued

SUB-TOPIC 3.1

CENTRES OF MASS OF SOLIDS

Sub-topic 3.1 · 5 questions · 53 marks · lowest tariff first



3.

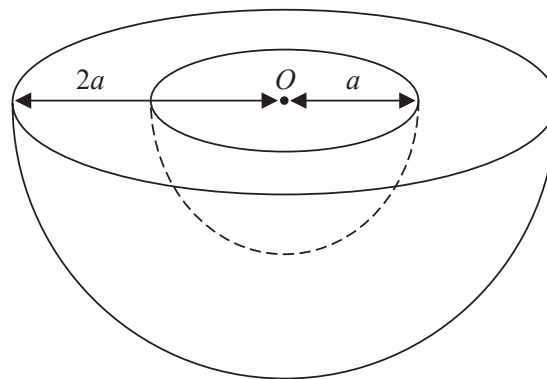


Figure 2

A uniform solid hemisphere H has radius $2a$. A solid hemisphere of radius a is removed from the hemisphere H to form a bowl. The plane faces of the hemispheres coincide and the centres of the two hemispheres coincide at the point O , as shown in Figure 2.

The centre of mass of the bowl is at the point G .

(a) Show that $OG = \frac{45a}{56}$

(4)

Figure 3 below shows a cross-section of the bowl which is resting in equilibrium with a point P on its curved surface in contact with a rough plane. The plane is inclined to the horizontal at an angle α and is sufficiently rough to prevent the bowl from slipping. The line OG is horizontal and the points O , G and P lie in a vertical plane which passes through a line of greatest slope of the inclined plane.

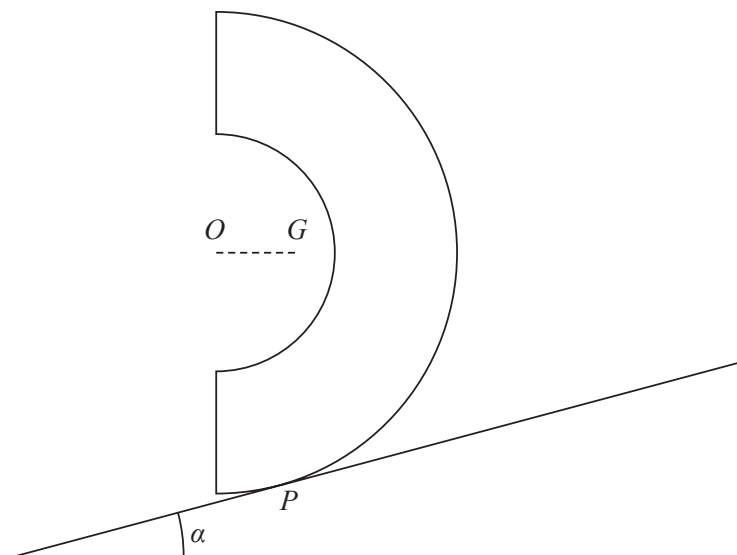


Figure 3

(b) Find the size of α , giving your answer in degrees to 3 significant figures.

(2)





(Total for Question 3 is 6 marks)





(Total for Question 6 is 10 marks)

5.

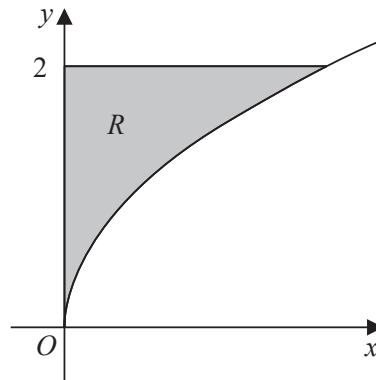


Figure 4

The region R , shown shaded in Figure 4, is bounded by part of the curve with equation $y^2 = 2x$, the line with equation $y = 2$ and the y -axis. The unit of length on both axes is one centimetre. A uniform solid, S , is formed by rotating R through 360° about the y -axis.

Given that the volume of S is $\frac{8}{5}\pi \text{ cm}^3$,

- (a) show that the centre of mass of S is $\frac{1}{3}$ cm from its plane face.

(4)

A uniform solid cylinder, C , has base radius 2 cm and height 4 cm. The cylinder C is attached to S so that the plane face of S coincides with a plane face of C , to form the paperweight P , shown in Figure 5. The density of the material used to make S is three times the density of the material used to make C .

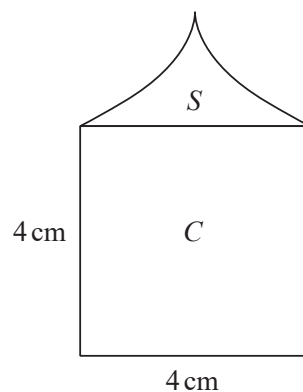


Figure 5

The plane face of P rests in equilibrium on a desk lid that is inclined at an angle θ° to the horizontal. The lid is sufficiently rough to prevent P from slipping. Given that P is on the point of toppling,

- (b) find the value of θ .

(7)







6.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

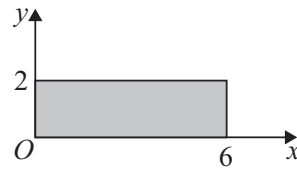


Figure 4

The shaded region, shown in Figure 4, is bounded by the x -axis, the line with equation $x = 6$, the line with equation $y = 2$ and the y -axis.

This region is rotated through 360° about the x -axis to form a solid of revolution. This solid is used to model a **non-uniform** cylinder of height 6 cm and radius 2 cm.

The mass per unit volume of the cylinder at the point (x, y, z) is $\lambda(x + 2) \text{ kg cm}^{-3}$, where $0 \leq x \leq 6$ and λ is a constant.

(a) Show that the mass of the cylinder is $120\lambda\pi \text{ kg}$.

(3)

(b) Show that the centre of mass of the cylinder is 3.6 cm from O .

(4)

The point O is the centre of one end of the cylinder. The point A is the centre of the other end of the cylinder.

A uniform solid hemisphere of radius 3 cm has density $\lambda \text{ kg cm}^{-3}$. The hemisphere is attached to the cylinder with the centre of its circular face in contact with the point A on the cylinder to form the model shown in Figure 5.

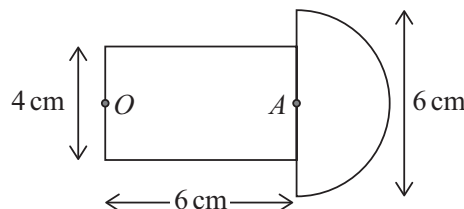


Figure 5

The model is placed with the end containing O on a rough inclined plane which is inclined at angle α° to the horizontal. The plane is sufficiently rough to prevent the model from sliding. The model is on the point of toppling.

(c) Find the value of α .

(6)

Question 6 continued



7.

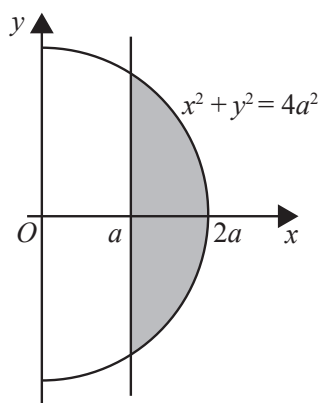


Figure 5

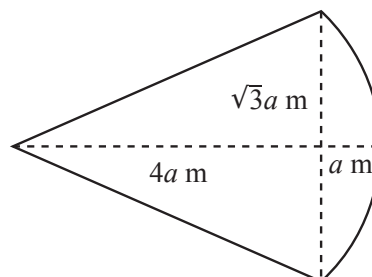


Figure 6

The shaded region shown in Figure 5 is bounded by the line with equation $x = a$ and the curve with equation $x^2 + y^2 = 4a^2$

This shaded region is rotated through 180° about the x -axis to form a solid of revolution.

This solid is used to model a dome with height a metres and base radius $\sqrt{3}a$ metres.

The dome is modelled as being non-uniform with the mass per unit volume of the dome at the point (x, y, z) equal to $\frac{\lambda}{x^2} \text{ kg m}^{-3}$, where $a \leq x \leq 2a$ and λ is a constant.

- (a) Show that the distance of the centre of mass of the dome from the centre of its plane face is $\left(4 \ln 2 - \frac{5}{2}\right)a$ metres.

(6)

A solid uniform right circular cone has base radius $\sqrt{3}a$ metres and perpendicular height $4a$ metres. A toy is formed by attaching the plane surface of the dome to the plane surface of the cone, as shown in Figure 6.

The weight of the cone is kW and the weight of the dome is $2W$

The centre of mass of the toy is a distance d metres from the plane face of the dome.

- (b) Show that $d = \frac{|k + 5 - 8 \ln 2|}{2 + k}a$

(4)

The toy is suspended from a point on the circumference of the plane face of the dome and hangs freely in equilibrium with the plane face of the dome at an angle α to the downward vertical.

Given that $\tan \alpha = \frac{1}{2\sqrt{3}}$

- (c) find the exact value of k .

(3)

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Question 7 continued



SUB-TOPIC 3.3

TOPPLING AND SLIDING

Sub-topic 3.3 · 1 question · 9 marks · lowest tariff first



4.

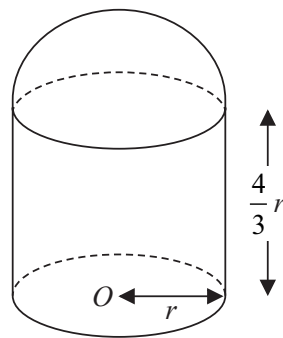


Figure 3

A uniform solid cylinder of base radius r and height $\frac{4}{3}r$ has the same density as a uniform solid hemisphere of radius r . The plane face of the hemisphere is joined to a plane face of the cylinder to form the composite solid S shown in Figure 3. The point O is the centre of the plane face of S .

- (a) Show that the distance from O to the centre of mass of S is $\frac{73}{72}r$

(4)

The solid S is placed with its plane face on a rough horizontal plane. The coefficient of friction between S and the plane is μ . A horizontal force P is applied to the highest point of S . The magnitude of P is gradually increased.

- (b) Find the range of values of μ for which S will slide before it starts to tilt.

(5)

Question 4 continued





(Total for Question 4 is 9 marks)

MARK SCHEMES

official mark schemes



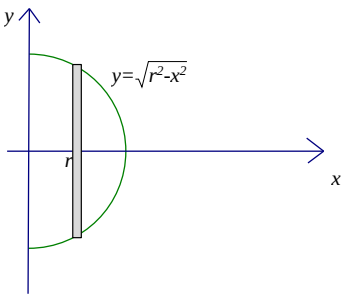
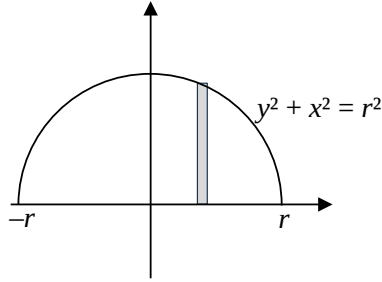
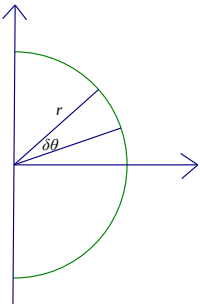
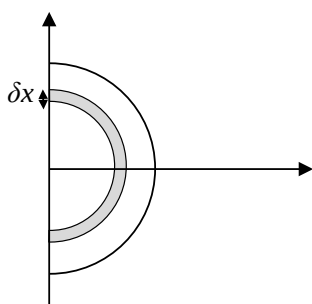
Question	Scheme	Marks	AOs
1 (a)	Moments about y-axis	M1	3.4
	$((5+k)m\bar{x} = -3kma + 6ma + 3ma) \quad \bar{x} = \frac{(9-3k)a}{5+k}$	A1	1.1b
	Moments about x-axis	M1	3.4
	$((5+k)m\bar{y} = 4kma + 4ma - 12ma) \quad \bar{y} = \frac{(4k-8)a}{5+k}$	A1	1.1b
		(4)	
	$\Rightarrow 9[(9-3k)^2 + (4k-8)^2] = (5+k)^2$ $(224k^2 - 1072k + 1280 = 0)$	M1	3.1a
	$\Rightarrow k = \frac{5}{2}, \text{ or } k = \frac{16}{7}$	A1	2.2a
		(2)	
(b)			
(6 marks)			
Notes:			
(a)			
M1	Moments equation to find $\bar{x}$ – need all terms and dimensionally correct Allow with m cancelled throughout Allow if they have a common factor of g		
A1	Correct expression for $\bar{x}$ Any equivalent form. Allow recovery		
M1	Moments equation to find $\bar{y}$ – need all terms and dimensionally correct Allow with m cancelled throughout Allow if they have a common factor of g		
A1	Correct expression for $\bar{y}$ Any equivalent form. Allow recovery		
(b)			
M1	Use their moments equations to form a quadratic equation in k only with no square root (need not simplify)		
A1	Obtain both correct values. Accept 2.5 and 2.3 or better (2.2857...)		



Question	Scheme	Marks	AOs
1	Moments about y-axis	M1	3.1a
	$2a(7+k) = 3 \times 2a + 4 \times a + k \times 2\mu a$ $(14 + 2k = 10 + 2\mu k)$	A1	1.1b
	Moments about x-axis	M1	3.1a
	$4a(7+k) = 3 \times 3a + 4 \times 5a + k \times \mu a$ $(28 + 4k = 29 + \mu k)$	A1	1.1b
	Obtain $k = 1$ or $\mu = 3$	A1	2.2a
	Obtain $k = 1$ and $\mu = 3$	A1	2.2a
		(6)	
(6 marks)			
Notes:			
M1	Dimensionally correct equation with all terms. Allow with $\bar{x}$ in place of $2a$		
A1	Correct unsimplified equation.		
M1	Dimensionally correct equation with all terms. Allow with $\bar{y}$ in place of $4a$		
A1	Correct unsimplified equation.		
	These two equations can be combined as a single vector equation.		
A1	One correct value.		
A1	Both correct values.		

Question	Scheme	Marks	AOs
1	Correct method to find an equation in $\bar{x}$	M1	1.1b
	$-3 \times 2 + 4 \times 3 + 2 \times p = 9\bar{x} \quad (6 + 2p = 9\bar{x})$	A1	1.1b
	Correct method to find an equation in $\bar{y}$	M1	1.1b
	$3 \times 2 + 4 \times 1 + 2 \times p = 9\bar{y} \quad 10 + 2p = 9\bar{y}$	A1	1.1b
	$(9\bar{x})^2 + (9\bar{y})^2 = (6 + 2p)^2 + (10 + 2p)^2$ $(= 136 + 64p + 8p^2)$	M1	1.1b
	$= 8 \left[(p + 4)^2 + 17 - 16 \right]$	M1	3.1a
	$\Rightarrow p = -4$	A1	2.2a
(7 marks)			
Notes:			
M1	Take moments about axis parallel to $x = 0$. Need all terms and dimensionally correct.		
A1	Correct unsimplified equation in $\bar{x}$. Seen or implied		
M1	Take moments about axis parallel to $y = 0$. Need all terms and dimensionally correct.		
A1	Correct unsimplified equation in $\bar{y}$. Seen or implied		
M1	Use of Pythagoras to find distance (or square of distance) from origin		
M1	Correct strategy to find value of p to minimise the distance e.g. use of calculus or complete the square		
A1	Correct answer only		

Question	Scheme		Marks	AOs
7(a)	Use of an appropriate element (quarter of a circle)		M1	2.1
	$\delta A \approx \frac{1}{2} \pi x \delta x$		A1	1.1b
	$\delta m \approx \frac{1}{2} \pi x \delta x \times \frac{4\lambda}{\pi a^4} x^2 \text{ (} = \frac{2\lambda}{a^4} x^3 \delta x \text{)}$		A1	3.4
	$M = \int_0^a \frac{2\lambda}{a^4} x^3 dx$		M1	2.1
	$M = \frac{1}{2} \lambda$		A1	1.1b
			(5)	
7(b)	Use of “ $\bar{x} = \frac{1}{M} \int x \, dm$ ”		M1	3.4
	$= \frac{1}{M} \int_0^a \left(\frac{2\sqrt{x}}{\pi} \right) \frac{2\lambda}{a^4} x^3 dx$		A1	1.1b
	Substitute for M , integrate and sub. in limits		M1	3.4
	$\bar{x} = \frac{8\sqrt{2}a}{5\pi}$		A1	1.1b
			(4)	
(9 marks)				
Notes:				
7a	M1	Use of an appropriate element (may be implied)		
	A1	Correct expression for area of element (may be implied)		
	A1	Use of proportionality model to obtain mass of element (may be implied)		
	M1	Integrating with correct limits		
	A1	cao		
7b	M1	Use the model and correct method		
	A1	Correct integral		
	M1	Use the model to complete the equation		
	A1	cao		

Question	Scheme		Marks	AOs
6(a)				
	$\left(\frac{1}{2}\pi r^2 dx = \int_0^r 2yx \, dx\right)$	$\left(\frac{1}{2}\pi r^2 dx = \int_{-r}^r \frac{1}{2}y^2 \, dx\right)$	M1	2.1
	$\int_0^r 2xy \, dx = \int_0^r 2x\sqrt{r^2 - x^2} \, dx$	$\int_{-r}^r \frac{1}{2}y^2 \, dx = \int_{-r}^r \frac{1}{2}(r^2 - x^2) \, dx$	M1	3.1a
	$= -\left[\frac{2}{3}(r^2 - x^2)^{\frac{3}{2}}\right]_0^r = \frac{2}{3}r^3$	$= \left[\frac{1}{2}\left(r^2x - \frac{1}{3}x^3\right)\right]_{-r}^r = \frac{2}{3}r^3$	A1ft	1.1b
	$\Rightarrow d = \frac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2} = \frac{4r}{3\pi} *$		A1*	2.2a
			(4)	
6(a) alt				
	CoM of “triangle” is $\frac{2}{3}r \cos \theta$ from O	CoM of “arc” is $\frac{2x}{\pi}$ from O	M1	2.1
	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}r^2 \times \frac{2}{3}r \cos \theta \, d\theta$	$\int_0^r \pi x \times \frac{2x}{\pi} \, dx$	M1	3.1a
	$= \frac{1}{3}r^3 [\sin \theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2}{3}r^3$	$= \left[\frac{2}{3}x^3\right]_0^r = \frac{2}{3}r^3$	A1ft	1.1b
	$\Rightarrow d = \frac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2} = \frac{4r}{3\pi} *$		A1*	2.2a

						(4)	
6(b)		Rectangle	Large sc	Small sc	Total		
	Mass ratio	16	8π	2π	$16 + 6\pi$	B1	1.1b
	From DC	$-a$	$\frac{16a}{3\pi}$	$\frac{8a}{3\pi}$	$\bar{x}$	B1	1.1b
	Moments about DC					M1	2.1
	$8\pi \times \frac{16a}{3\pi} - 2\pi \times \frac{8a}{3\pi} - 16a = (16 + 6\pi) \bar{x}$					A1	1.1b
	$\bar{x} = \frac{a\left(\frac{7 \times 16}{3} - 16\right)}{16 + 6\pi} = \frac{64a}{3(16 + 6\pi)} = \frac{32a}{3(8 + 3\pi)} \quad *$					A1*	2.2a
						(5)	
6(c)	Moments about A:					M1	3.1a
	$W \times \left(2a + \frac{32a}{3(8 + 3\pi)}\right) = 8a\lambda W$					A1	1.1b
	$\lambda = \left(\frac{(9\pi + 40)}{12(8 + 3\pi)}\right) = 0.33 \text{ or better } (0.3265\dots)$					A1	1.1b
						(3)	
(12 marks)							
Notes:							
(a)M1	Correct strategy to find d by integration						
M1	Integrate a function of the form $\lambda x \sqrt{r^2 - x^2}$ / $\lambda (r^2 - x^2)$. Allow without limits.						
A1ft	Follow their λ . Allow without limits.						
A1*	Obtain given answer including “ $d =$ ” from correct exact working						
(a) alt M1	Correct strategy to find d by integration						
M1	Integrate a function of the form $\lambda r^3 \cos \theta$ or λx^2 . Allow without limits.						
A1ft	Follow their λ						
A1	Obtain given answer including “ $d =$ ” from correct working						
(b)B1	Correct mass ratios						
B1	Distances from DC or a parallel axis						

M1	Moments about DC or a parallel axis. All terms required. Dimensionally correct.
A1	Correct unsimplified equation for their axis
A1*	Obtain given answer from correct working.
(c)M1	Moments about A . Dimensionally correct. Allow if a missing throughout.
A1	Correct unsimplified equation
A1	Correct only. Accept 0.33 or better (0.3265...). Condone an exact answer in terms of π if given as a single fraction.



Question	Scheme		Marks	AOs
2(a)	Moments about ED		M1	2.1
	e.g. $4 \times 4a \times 2a \cos 30^\circ + 2 \times 4a \times 4a \cos 30^\circ = 7 \times 4a \times d$		A1 A1	1.1b 1.1b
	$32\sqrt{3}a^2 = 28ad \Rightarrow d = \frac{8\sqrt{3}}{7}a$ *		A1*	1.1b
			(4)	
2(b)	Moments about C		M1	3.1a
	$8a \times F = \left(4a \cos 30^\circ - \frac{8\sqrt{3}}{7}a \right) \times W$		A1	1.1b
	$F = \frac{3\sqrt{3}}{28}W$		A1	1.1b
			(3)	
Total 7 marks				
Notes:				
2a	M1	Dimensionally correct equation with required terms. Accept use of a parallel axis. Accept equivalent mass ratio e.g. $4a$ replaced by 1		
	A1	Unsimplified equation with at most one error. Allow distances in terms of $\sin 60^\circ$ or $\cos 30^\circ$ or equivalent.		
	A1	N.B. Repeated use of an incorrect distance is only one error. Correct unsimplified equation. Allow distances in terms of $\sin 60^\circ$ or $\cos 30^\circ$ or equivalent		
	A1*	Obtain given answer from correct working including reference to d .		
2b	M1	Dimensionally correct equation with required terms and no extras Equation should be of the form $\lambda F = (\mu - d) W$		
	A1	Correct unsimplified equation		
	A1	0.19W or better (0.185576....W)		

Question	Scheme										Marks	AOs
3(a)	Moments about AC:										M1	3.1a
	rod	CD	DE	EF	FA	AB	BC	BF	DF	CF		
	Mass ratio	4	4	5	5	3	3	8	6	10		
	From AC	2a	6a	6a	2a	0	0	2a	4a	2a		
	$8a \times 4a + 2 \times 3a \times 4a + 2 \times 5a \times 2a + 10a \times 4a + 2 \times 4a \times 2a = 48a\bar{x}$									A1 A1	1.1b 1.1b	
	$(132a = 48\bar{x}) \Rightarrow \bar{x} = \frac{11}{4}a$									A1	1.1b	
										(4)		
(b)	Moments about F:										M1	3.1a
	$Mg(4a - \bar{x}) = kMg \times 4a$										A1ft	1.1b
	$\Rightarrow k = \frac{5}{16}$										A1	1.1b
											(3)	
	Moments about C:										M1	
	$4a(M + kM) = M\bar{x} + 8akM$										A1ft	
	$\Rightarrow k = \frac{5}{16}$										A1	1.1b
											(3)	
(7 marks)												
Notes:												
(a)M1	Dimensionally correct equation for moments about AC or a parallel axis. All terms needed and horizontal distances Must be using the mass ratio. Allow slips, but not consistently density and not consistently lengths. Condone without a											
A1 A1	One side of the equation correct Both sides of the equation correct											
A1	Or equivalent single term Condone if a is missing in the working and appears at the end											
(b)M1	Dimensionally correct moments equation. Accept any complete alternative method using M and kM to obtain an equation in k only. Condone if g and / or M cancelled throughout Condone incorrect distances Condone if use $M = 48$ throughout											

A1	Correct unsimplified equation (accept without g and/or M) Correct mass and distance combination for their $\bar{x}$
A1	Or 0.3125 Condone 0.31 or 0.313



Question	Scheme	Marks	AOs
5a	$\int \frac{1}{2}y^2 dx = \frac{1}{2} \int (9-x^2)^2 dx \left(= \frac{1}{2} \int (81-18x^2+x^4) dx \right)$	M1	2.1
	$= \frac{1}{2} \left[81x - 6x^3 + \frac{1}{5}x^5 \right]_0^3$	DM1	1.1b
	$= \frac{1}{2} \left(3 \times 81 - 6 \times 27 + \frac{243}{5} \right) \left(= \frac{324}{5} \right)$	A1	1.1b
	$\Rightarrow \text{distance} = \frac{64.8}{18} = 3.6 \quad *$	A1*	2.2a
		(4)	
5a alt	$\int xy dy = \int y\sqrt{9-y} dy$	M1	2.1
	$= \left[-\frac{2}{3}y(9-y)^{\frac{3}{2}} - \frac{4}{15}(9-y)^{\frac{5}{2}} \right]_0^9$	DM1	1.1b
	$= \frac{4}{15} \times 9^{\frac{5}{2}} \left(= \frac{324}{5} \right)$	A1	1.1b
	$\Rightarrow \text{distance} = \frac{64.8}{18} = 3.6(\text{m}) \quad *$	A1*	2.2a
		(4)	
5b	Moments about O:	M1	3.1a
	$3.6W = 9\lambda W$	A1	1.1b
	$\lambda = 0.4$	A1	1.1b
		(3)	
(7 marks)			
Notes:			
5(a) M1	Use of $\int \frac{1}{2}y^2 dx$ or $\int xy dy$. Ignore any limits.		
DM1	Integrate and use correct limits (0 and 3 for x, 0 and 9 for y). Usual rules for integration: powers increasing by 1		
A1	Correct unsimplified expression.		
A1*	Obtain given answer from correct working.		
5(b) M1	Complete method to obtain λ , e.g. by taking moments about O or by resolving and taking moments about a different point.		
	Allow for an equation in T_A or λ		
A1	Correct unsimplified equation in λ		
A1	Correct only.		

Question	Scheme		Marks	AOs
1.(a)	Mass ratios: $4a^2, \frac{1}{2}\pi a^2, (4a^2 + \frac{1}{2}\pi a^2)$		B1	1.2
	x: $\frac{1}{2}a, a + \frac{4a}{3\pi}, \bar{x}$			



Question	Scheme	Marks	AOs
3(a)	$\alpha = \frac{\pi}{4} \Rightarrow \frac{r \sin \alpha}{\alpha} = r \times \frac{1}{\sqrt{2}} \times \frac{4}{\pi} \left(= \frac{2\sqrt{2}r}{\pi} \right)$	B1	1.1b
	Distance from OA = $\frac{2\sqrt{2}r}{\pi} \times \cos \frac{\pi}{4} = \frac{2\sqrt{2}r}{\pi \times \sqrt{2}} = \frac{2r}{\pi}$ *	B1*	2.2a
		(2)	
3(b)	Moments about OA:	M1	3.1a
	$r \times \frac{r}{2} + r \times r + \frac{2r}{\pi} \times \frac{\pi r}{2} + \frac{2r}{\pi} \times \frac{\pi r}{2} = (3r + \pi r)d$	A1 A1	1.1b 1.1b
	$\left(\frac{7r^2}{2} = (3r + \pi r)d \right) \Rightarrow d = \frac{7r}{2(3 + \pi)}$ *	A1*	2.2a
		(4)	
3(c)	Moments about A or any other complete method to obtain F in terms of W	M1	3.1a
	$W \times \frac{7r}{2(3 + \pi)} = F \times 2r$	A1	1.1b
	$F = \frac{7}{4(3 + \pi)}W$	A1	1.1b
		(3)	
(9 marks)			
Notes:			
(a)B1	Correct use of given formula. Seen or implied.		
B1*	Obtain given result from correct working, e.g. by use of trigonometry or use of Pythagoras.		
(b)M1	Condone a dimension error in the arc length, but otherwise must be a dimensionally correct equation. Need all terms. Allow use of a parallel axis.		
A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation. An error in the arc length affects 3 terms – count it as a single error.		
A1*	Obtain given answer from correct working.		
(c)M1	Dimensionally correct equation with distances perpendicular to forces..		
A1	Correct unsimplified equation.		
A1	Any equivalent form. $0.28W$ or better ($0.28494...W$)		

Question	Scheme	Marks	AOs
2(a)	$2.4\bar{y} = \frac{1}{2} \int y^2 dx = \frac{1}{2} \int \{64e^{-2x}\} dx$	M1	2.1
	$= -16 \left[e^{-2x} \right]_{\ln 2}^{\ln 5}$	A1	1.1b
	Complete strategy to find $\bar{y}$	M1	3.1a
	$2.4\bar{y} = -16e^{-\ln 25} + 16e^{-\ln 4} = \frac{16}{4} - \frac{16}{25} = \frac{84}{25},$ $\bar{y} = \frac{84}{25} \times \frac{10}{24} \left(= \frac{7}{5} \right) = 1.4 \quad *$	A1*	2.2a
		(4)	
(b)	$2.4\bar{x} = \int (8xe^{-x}) dx$	M1	2.1
	$\left(= \left[-8xe^{-x} - 8e^{-x} \right]_{\ln 2}^{\ln 5} \right)$		
	$= -\frac{8}{5}(\ln 5 + 1) + \frac{8}{2}(\ln 2 + 1) (= 2.5974.....)$	M1	1.1b
	$\bar{x} = 1.08$	A1	1.1b
	Complete strategy to find θ	M1	3.1a
	$\tan \theta^\circ = \frac{\ln 5 - \bar{x}}{8e^{-\ln 5} - 1.4} (= 2.63.....)$	A1ft	3.4
	$\theta = 69$	A1	1.1b
		(6)	
(10 marks)			

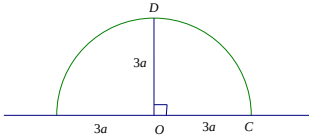
Notes:		
(a)	M1	Moments equation to obtain terms of the correct form (with or without limits) Allow if area (2.4) not seen
	A1	Correct unsimplified answer (with or without limits) Allow if area (2.4) not seen
	M1	Complete strategy for $\bar{y}$: use of moments equation with correct use of limits and division by area
	A1*	Use moments equation and given area to deduce given answer from correct working
(b)	M1	Use correct integral (with or without limits). Allow if area (2.4) not seen
	M1	Correct use of correct limits in an integral of the correct form and 2.4
	A1	Correct answer (1.0822....)
	M1	Complete strategy to find θ e.g find $\bar{x}$ and then use trig to find appropriate angle
	A1ft	Use the model to find a relevant angle. Follow their $\bar{x}$
	A1	2 s.f. or better 69.22...

Question	Scheme	Marks	AOs
3(a)	Correct strategy	M1	3.1a
	$8\bar{y} = \frac{1}{2} \int y^2 dx = \frac{1}{2} \int \left\{ \frac{(x-2)^6}{16} + (x-2)^3 + 4 \right\} dx$	M1	2.1
	$= \frac{1}{2} \left[\frac{(x-2)^7}{7 \times 16} + \frac{(x-2)^4}{4} + 4x \right]_0^4$	A1	1.1b
	$8\bar{y} = \frac{1}{2} \left[\frac{8}{7} + 4 + 16 + \frac{8}{7} - 4 - 0 \right] = \frac{64}{7}, \quad \bar{y} = \frac{8}{7} \quad *$	A1*	2.2a
		(4)	
(b)	$8\bar{x} = \int \left(\frac{x(x-2)^3}{4} + 2x \right) dx$	M1	2.1
	$= \left[\frac{x(x-2)^4}{16} - \frac{(x-2)^5}{80} + x^2 \right]_0^4$	A1	1.1b
	$= \frac{64}{16} - \frac{32}{80} + 16 - \frac{32}{80} = 19.2$	M1	1.1b
	$\bar{x} = 2.4$	A1	1.1b
	Complete strategy to find θ	M1	3.1a
	$\tan \theta = \frac{4 - \bar{x}}{4 - \frac{8}{7}} \left(= \frac{14}{25} \right)$	A1ft	3.4
	$\theta = 29.2 \text{ (Accept 29)}$	A1	1.1b
		(7)	
(11 marks)			

Question	Marks	Marking Guidance
3(a)	M1	Complete strategy for $\bar{y}$: moments equation, use of limits and division by area
	M1	Moments equation to obtain terms of the correct form (with or without limits) The integral must be in terms of x only or y only Allow if area (8) not seen Might see $\int \frac{x^6}{16} - \frac{3x^5}{4} + \frac{15x^4}{4} - 9x^3 + 9x^2 dx$

		Or $\int xy dy = \int 2y + y(4(y-2))^{\frac{1}{3}} dy$
	A1	Correct unsimplified answer (with or without limits) Allow if area (8) not seen ($\int xy dy = 12.8$)
	A1*	Use moments equation and given area to deduce given answer from correct working
	(4)	
(b)	M1	Relevant integral in terms of x only or y only (with or without limits). Allow if area (8) not seen Could start with $\int xy dx$ or $\int \frac{1}{2} x^2 dy$ Might see $\frac{x^4}{4} - \frac{6x^3}{4} + 3x^2 - 2x + 2x$
	A1	Correct unsimplified form after integration (with or without limits). Allow if area (8) not seen
	M1	Complete process to find $\bar{x}$ following relevant integral
	A1	Correct answer
	M1	Complete strategy to find θ e.g find $\bar{x}$ and then use trig to find appropriate angle
	A1ft	Use the model to find a relevant angle. Follow their $\bar{x}$
	A1	2 s.f. or better 29.2488...
	(7)	



Question	Scheme	Marks	AOs												
5(a)	Using sector: distance $OG = \frac{2 \times 3a \sin \frac{\pi}{4}}{3 \times \frac{\pi}{4}}$	B1	1.1b												
	Using Pythagoras: $2d^2 = \frac{32a^2}{\pi^2} \quad (d^2 + d^2 = OG^2)$ Or using trigonometry: Distance from $OC = OG \cos 45^\circ = OG \sin 45^\circ$	M1	2.1												
	$d = \sqrt{\frac{16a^2}{\pi^2}} = \frac{4a}{\pi} \quad *$	A1*	2.2a												
		(3)													
5(a) alt	Using semicircle of radius $3a$: $\bar{y} = \frac{4 \times 3a}{3\pi} \left(= \frac{4a}{\pi} \right)$ 	B1	1.1b												
	Moments about diameter: $\frac{9\pi a^2}{2} \times \frac{4a}{\pi} = 2 \times \frac{9\pi a^2}{4} \times d$	M1	2.1												
	$\Rightarrow d = \frac{4a}{\pi} \quad *$	A1*	2.2a												
		(3)													
(b)	<table><tr><td></td><td>ABCO</td><td>ODEF</td><td>ODC</td></tr><tr><td>Mass ratio</td><td>9</td><td>9</td><td>$\frac{9\pi}{4}$</td></tr><tr><td>From FC</td><td>$-\frac{3a}{2}$</td><td>$\frac{3a}{2}$</td><td>$\frac{4a}{\pi}$</td></tr></table>		ABCO	ODEF	ODC	Mass ratio	9	9	$\frac{9\pi}{4}$	From FC	$-\frac{3a}{2}$	$\frac{3a}{2}$	$\frac{4a}{\pi}$	B1	1.2
		ABCO	ODEF	ODC											
	Mass ratio	9	9	$\frac{9\pi}{4}$											
	From FC	$-\frac{3a}{2}$	$\frac{3a}{2}$	$\frac{4a}{\pi}$											
	Moments about FC:	M1	3.1a												
	$-9 \times \frac{3a}{2} + 9 \times \frac{3a}{2} + \frac{9\pi}{4} \times \frac{4a}{\pi} = \left(18 + \frac{9\pi}{4} \right) \bar{x} (= 9a)$	A1	1.1b												
	$\bar{x} = \frac{4a}{8 + \pi}$	A1	1.1b												
	(4)														

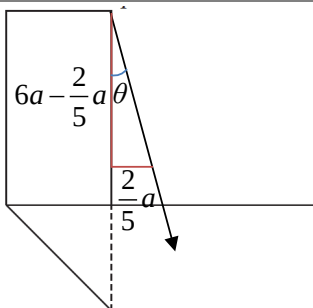
(b) alt		ABCO	ODEF	ODC		
	Mass ratio	9	9	$\frac{9\pi}{4}$		
	From BOE	0	0	$\frac{4\sqrt{2}a}{\pi}$	B1	1.2
	Moments about BOE:				M1	3.1a
	$\left(18 + \frac{9\pi}{4}\right)d = \frac{9\pi}{4} \times \frac{4\sqrt{2}a}{\pi}$				A1	1.1b
	$\bar{x} = d \cos 45^\circ = \frac{4a}{8 + \pi}$				A1	1.1b
					(4)	
(c)	$\bar{y} = \frac{4a}{8 + \pi}$ from OD or $\bar{y} = 3a + \frac{4a}{8 + \pi}$ from FE				B1ft	1.1b
	Complete method to find a relevant angle				M1	3.1a
	$\theta^\circ = \tan^{-1}\left(\frac{\bar{x}}{3a + \bar{y}}\right) = \tan^{-1}\left(\frac{4a}{28a + 3\pi a}\right)$				A1ft	1.1b
	$\theta = 6.1$				A1	1.1b
					(4)	
(11 marks)						
Notes:						
(a)B1	Correct application of standard result from formula booklet. Must substitute for α but need not simplify Implied if you see $\left(= \frac{4\sqrt{2}a}{\pi} \right)$					
M1	Correct strategy to find the distance for the quadrant Need to see use of $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ somewhere in the solution					
A1*	Obtain the given result from correct working.					
(b)B1	Correct masses and distance from FC or a parallel axis or BOE Seen or implied (a bright candidate might realise that if taking moments about FC then the two squares cancel each other).					
M1	Moments about FC or a parallel axis or BOE. All terms required, and dimensionally correct. Condone sign errors. Accept as part of a vector equation.					

A1	Correct unsimplified equation for their axis
A1	Or equivalent with no errors seen Accept $0.36a$ or better ($0.3590\dots a$)
(c)B1ft	Allow use of symmetry seen or implied. Accept $\bar{y} = \bar{x}$ (From FE, $\bar{y} = \frac{28a + 3\pi a}{8 + \pi}$) Accept + / -
M1	Correct strategy to find a relevant angle (θ or $90 - \theta$) Need to substitute their values of $\bar{x}$ and distance from $F \neq \frac{4a}{\pi}$.
A1ft	Correct unsimplified expression for a relevant angle. Follow their $\bar{x}$ and $\bar{y}$
A1	6.1 or better (6.10067...) The question defines θ as measured in degrees. 0.106 can score B1M1A1ftA0 Do not ISW

Question	Scheme	Marks	AOs
4a	Moments about the y-axis: $\int (\rho) xy dx$	M1	3.1a
	$= (\rho) \int x \sqrt{36 - \frac{9x^2}{4}} dx = k \left(36 - \frac{9x^2}{4} \right)^{\frac{3}{2}}$	M1	2.1
	$= \left[-\frac{4}{27} \left(36 - \frac{9x^2}{4} \right)^{\frac{3}{2}} (\rho) \right]_0^4 (= 32(\rho))$	A1	1.1b
	$\bar{x} = \frac{\int xy dx}{6\pi}$	DM1	3.1a
	$\bar{x} = \frac{32}{6\pi} = \frac{16}{3\pi} \quad *$	A1*	2.2a
		(5)	
4b	Moments about the x-axis: $\int \frac{1}{2} y^2 (\rho) dx \left(= \frac{1}{2} (\rho) \int 36 - \frac{9x^2}{4} dx \right)$	M1	3.1a
	$= \frac{1}{2} (\rho) \left[36x - \frac{3}{4} x^3 \right]_0^4 \left(= \frac{1}{2} (\rho) (144 - 48) = 48(\rho) \right)$	A1	1.1b
	$\bar{y} = \frac{\int \frac{1}{2} y^2 dx}{6\pi}$	DM1	2.1
	$= \frac{48}{6\pi} \left(= \frac{8}{\pi} \right)$	A1	2.2a
		(4)	
4c	Correct use of trigonometry	M1	3.1a
	$\tan \theta^\circ = \frac{\text{their } \bar{y}}{4 - \frac{16}{3\pi}} \left(= \frac{6}{3\pi - 4} \right)$	A1ft	1.1b
	$\theta = 47.9 \quad (48 \text{ or better})$	A1	1.1b
		(3)	
Total 12 marks			
Notes:			
4a	M1	Correct method for moments about the x-axis: $\int (\rho) xy dx$ or $\int (\rho) \frac{1}{2} x^2 dy$ Integrand should be in one variable only.	
	M1	Integrate to obtain $k(A - Bx^2)^{\frac{3}{2}}$ Ignore limits and / or constant of integration	

	A1	Correct integration with correct limits seen or implied.
	DM1	Complete method to obtain $\bar{x}$. Dependent on first M1.
	A1*	Obtain given answer from correct working
4b	M1	Correct method for moments about the y -axis: $\int (\rho) xy \, dy$ or $\int (\rho) \frac{1}{2} y^2 \, dx$
	A1	Correct integration with correct limits seen or implied.
	DM1	Complete method to obtain $\bar{y}$. Dependent on previous M1.
	A1	Correct exact equivalent
4c	M1	Correct use of trigonometry to find a relevant angle
	A1ft	Correct unsimplified expression for $\tan \theta$ or its reciprocal. Follow their $\bar{y}$
	A1	48 or better (47.8823...) 0.836 radians is A0

Question	Scheme	Marks	AOs
1(a)	Correct method to find d	M1	3.1a
	$(2a \times 0) + 2a \times a + 2a \times 2a + 2\sqrt{2}a \times a + \sqrt{2}a \times \frac{3a}{2} = d(6a + 3\sqrt{2}a)$	A1 A1	1.1b 1.1b
	$\Rightarrow d = \frac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}}a$ *	A1*	2.2a
		(4)	
1(b)	Correct method to form an equation in k	M1	3.1a
	$Mg \times \frac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}}a = (kMg + Mg)a$	A1	1.1b
	$k = \frac{\sqrt{2}}{12 + 6\sqrt{2}}$ or equivalent	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a) M1	Take moments about AB or an axis parallel to AB . Need all terms and dimensionally correct (condone invisible brackets).		
A1	Unsimplified equation with at most one error (repeated errors only count once; condone invisible brackets)		
A1	Correct unsimplified equation (condone invisible brackets)		
A1*	Obtain given answer including “ $d =$ ” from correct exact working (A0 if there are missing brackets in the working)		
(b)M1	Take moments about AB or an axis parallel to AB . Accept M consistently replaced by, e.g. $6+3\sqrt{2}$		
A1	Correct unsimplified equation. Allow without any of M , g and a .		
A1	Accept 0.069 or better $\left(\frac{-1 + \sqrt{2}}{6}\right)$ Condone any exact form if given as a single fraction.		

Question	Scheme					Marks	AOs
4(a)		Left rectangle	2 triangles	Right rectangle	Total	B1	1.1b
	Mass ratio	$2 \left(18a^2\right)$	$\frac{1}{\left(2 \times \frac{9}{2} a^2\right)}$	$2 \left(18a^2\right)$	$5 \left(45a^2\right)$	B1	1.1b
	From EF	$-1.5a$	$-a$	$3a$	d		
	Moments about EF					M1	2.1
	$2 \times -1.5a + 1 \times -a + 2 \times 3a = 5d$					A1	1.1b
	Distance $= \frac{2}{5} a$ *					A1*	2.2a
						(5)	
4(b)	The perpendicular bisector of BE is a line of symmetry					B1	2.4
						(1)	
4(c)							
	Vertical distance of c of m from $F = \frac{28a}{5}$					B1	3.1a
	$\tan \theta = \frac{2}{28}$					M1	1.1b
	$\theta = \tan^{-1} \left(\frac{1}{14} \right) = 4.09^\circ \quad (4.1^\circ) \quad (0.071 \text{ rads})$					A1	1.1b
						(3)	
(9 marks)							
Notes:							
(a)B1	Correct mass ratios						
B1	Correct distances from EF or a parallel axis						
M1	Moments about EF or a parallel axis. Need all terms. Dimensionally correct. Condone sign errors.						
A1	Correct unsimplified moments equation (E.g. $45d = 18 \times 1.5a + 9 \times 2a + 18 \times 6a$)						

A1*	Obtain given answer from correct working.
(b)B1	Or equivalent reasoning mentioning symmetry with no incorrect statements.
(c)B1	Correct vertical distance using symmetry or a second moments equation
M1	Use of trig to find a relevant angle. Must be dimensionally correct. Award if using an incorrectly evaluated vertical distance of the form $6a \pm \bar{y}$, where $\bar{y} = \frac{2a}{5}$ or is calculated from a moments equation.
A1	Correct only. 4.1° or better (4.08561678...)



Question	Scheme		Marks	AOs
3(a)	Mass ratios: $\frac{2}{3}\pi(2a)^3, \frac{2}{3}\pi a^3, \frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3$ (8, 1, 7)		B1	1.2
	Distances: $\frac{3}{8}(2a), \frac{3}{8}a, \bar{x}$		B1	1.2
	$\left(\frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3\right)\bar{x} = \frac{2}{3}\pi(2a)^3 \times \frac{3}{8}(2a) - \frac{2}{3}\pi a^3 \times \frac{3}{8}a$		M1	3.1a
	$\bar{x} = \frac{45a}{56}$ *		A1*	2.2a
			(4)	
3(b)	Use of appropriate trig ratio e.g. $\sin \alpha = \frac{\frac{45a}{56}}{2a}$		M1	3.1a
	$\alpha = 23.7^\circ$ (3 sf)		A1	1.1b
			(2)	
(6 marks)				
Notes:				
3a	B1	Correct unsimplified (8, 1, 7)		
	B1	Correct unsimplified but distances could be measured from a parallel axis		
	M1	All terms needed and must be dimensionally correct		
	A1*	Correct answer correctly derived		
3b	M1	Must be using an appropriate trig ratio		
	A1	Cao to 3SF		

Question	Scheme	Marks	AOs
6(a)	Mass of cone = $\int_0^9 \pi y^2 \lambda x \, dx = \pi \lambda \int_0^9 \frac{x^3}{9} \, dx$	M1	3.4
	$= \pi \lambda \left[\frac{x^4}{36} \right]_0^9 \left(= \frac{729\pi\lambda}{4} (\text{kg}) \right)$	A1	1.1b
	Moments: $\int_0^9 \pi y^2 \lambda x \times x \, dx = \pi \lambda \int_0^9 \frac{x^4}{9} \, dx$	M1	3.4
	$= \frac{\pi\lambda}{45} \left[x^5 \right]_0^9 \left(= \frac{\pi\lambda}{5} \times 9^4 \right)$	A1	1.1b
	$\Rightarrow d = \frac{\frac{\pi\lambda}{5} \times 9^4}{\frac{\pi\lambda}{4} \times 9^3}$	DM1	2.1
	$d = \frac{36}{5} = 7.2 (\text{cm})$	A1	1.1b
		(6)	
(b)	Remains at rest $\Rightarrow$ centre of mass at centre of plane surface	B1	2.1
	Moments about diameter of plane surface:	M1	3.1b
	$(9-d)W \left\{ = \left(9 - \frac{36}{5} \right) W \right\} = \frac{3}{8} \times 3 \times kW$	A1ft	1.1b
	$k = \frac{8}{5}$	A1	1.1b
		(4)	
(10 marks)			
Notes:			
(a)	NB: Some candidates are confusing the mass and the volume. For the first M1A1: - If they have a correct method for the mass and they tell you that this is mass, award the marks. - If they have a correct method for the mass say nothing, but use it correctly, award the marks. - If they have a correct method for the mass, say nothing, and use it as the moment, then M0 because this implies that they do not think it is the mass.		
M1	Use the model to find the mass of the cone. Allow without limits.		
A1	Correct integration. Correct limits seen or implied Substitution not required.		

	Allow 2/2 if π not seen and consistent with (b) if attempted
M1	Use the model to find the moment of the cone (usual rules for integration) Allow without limits
A1	Correct integration. Correct limits seen or implied Substitution not required. Allow 2/2 if π not seen and consistent with (a)
M1	Complete method to find the distance of the centre of mass from the vertex. A complete method requires the two preceding M marks. They need to get as far as a value for d . If they have a method that comes directly to this stage you might not see the λ or π
A1	Correct only If all you see is $\Rightarrow d = \frac{9^5}{45} \div \frac{9^4}{36}$ or even $\Rightarrow d = \frac{9}{5} \times 4$ then award 6/6 Allow 6/6 if π not seen throughout but otherwise correct
(b)B1	Correct deduction for location of c of m Stated or implied by their moments equation
M1	Moments about diameter of plane face(s) M0 if the moments equation contradicts the centre of mass being on the interface M0 if using volume in place of mass
A1ft	Correct unsimplified equation. Follow their 7.2 Alternative moments equations: Using vertex: $W\bar{x} + kW\left(9 + \frac{3}{8} \times 3\right) = (W + kW) \times 9$ Using base: $W(12 - \bar{x}) + kW\left(3 - 3 \times \frac{3}{8}\right) = (W + kW) \times 3$ If they are working with the axis at an angle they will possibly have trig terms which should cancel.
A1	Correct only

Question	Scheme			Marks	AOs
5(a)	$\int \pi x^2 y dy = \pi \int \frac{y^5}{4} dy = \pi \left[\frac{y^6}{24} \right]_0^2$			M1	3.4
	$= \frac{64\pi}{24} \left(= \frac{8\pi}{3} \right)$			A1	1.1b
	$\Rightarrow \bar{y} = \left(\text{their } \frac{8\pi}{3} \right) \times \frac{5}{8\pi} \left(= \frac{5}{3} \right)$			M1	3.1a
	Distance from plane face = $2 - \frac{5}{3} = \frac{1}{3} *$			A1*	2.2a
				(4)	
(b)		Mass ratio	c of m from base	B1	1.2
	S	$3 \times \frac{8\pi}{5}$	$\frac{13}{3}$		
	cylinder	16π	2		
	M	$(16 + 4.8)\pi$	d		
	Moments about diameter of the base:			M1	2.1
	$\frac{24}{5} \times \frac{13}{3} + 32 = \frac{104}{5} d \left(= \frac{264}{5} \right)$			A1	1.1b
	$d = \frac{264}{104} \left(= \frac{33}{13} \right) \text{ (cm)}$			A1	1.1b
	Complete strategy for θ			M1	3.1a
	About to topple: $\tan \theta = \frac{2}{d} \left(= \frac{26}{33} \right)$			A1ft	3.4
	$\theta = 38.2 \quad (\text{or } 38)$			A1	2.2a
				(7)	
	(11 marks)				

Question	Marks	Marking Guidance
5(a)	M1	Use the model to find the moment about the base. Usual rules for integration. Correct limits
	A1	Any equivalent form
	M1	Use the model and volume to find $\bar{y}$: their moments and $\frac{8\pi}{5}$ used correctly

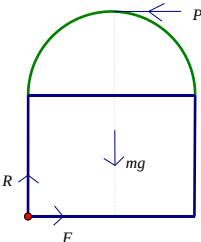
	A1*	Obtain given answer from correct working
	(4)	
(b)	B1	Correct mass ratios seen or implied
	M1	Moments equation. Dimensionally correct. Allow for their mass ratios
	A1	Correct unsimplified equation
	A1	Any equivalent form (2.54 cm)
	M1	Complete strategy to find the position of the centre of mass of the composite body and use trig. to find θ
	A1ft	Trig ratio for a relevant angle. Follow their d
	A1	Correct angle (38 or better) (38.2338....)
	(7)	

Question	Scheme	Marks	AOs
6a	$\text{Mass} = \int_0^6 \pi 2^2 \lambda (x+2) dx$	M1	3.4
	$= 4\pi\lambda \left[\frac{x^2}{2} + 2x \right]_0^6$	A1	1.1b
	$= 4\pi\lambda \left(\frac{36}{2} + 12 \right) = 120\lambda\pi \text{ (kg)} \quad *$	A1*	2.2a
		(3)	
6b	Moment about y-axis $= \int_0^6 4\pi x \lambda (x+2) dx$	M1	2.1
	$= 4\pi\lambda \left[\frac{x^3}{3} + x^2 \right]_0^6 \quad (= 4\lambda\pi(72+36) = 432\lambda\pi)$	A1	1.1b
	Distance from O $= \frac{\text{their } 432\lambda\pi}{120\lambda\pi}$	DM1	3.1b
	$= \frac{432}{120} = 3.6 \text{ (cm)} \quad *$	A1*	2.2a
		(4)	
6c	Use of $\frac{3}{8} \times 3$	B1	1.2
	Moments about a diameter of the base	M1	3.1b
	$120\lambda\pi \times 3.6 + \left(6 + \frac{3}{8} \times 3 \right) \times \frac{2}{3} \pi (3)^3 \lambda = (120+18)\pi\lambda d$	A1 A1	1.1b 1.1b
	$\left(d = \frac{747}{184} = 4.0597... \right)$		
	$\tan \alpha^\circ = \frac{2}{\text{their } d}$	M1	2.1
	$\alpha = 26.2$	A1	1.1b
		(6)	
Total 13 marks			
Notes:			
6a	M1	Correct method for total mass	
	A1	Correct integration with limits seen or implied	
	A1*	Obtain given answer from correct working	
6b	M1	Correct method for moments about y-axis (condone missing π and λ)	

	A1	Correct unsimplified integral
	DM1	Correct method to obtain distance from O ; dependent on first M.
	A1*	Obtain given answer from correct working
6c	B1	Use of correct formula for c of m of a hemisphere, seen or implied
	M1	Condone use of a parallel axis. Require relevant terms and dimensionally correct. Condone common factors cancelled throughout
	A1 A1	Unsimplified equation with at most one error. Incorrect volume of hemisphere is only one error Correct unsimplified equation
	M1	Correct use of trigonometry to obtain α
	A1	$26(26.2265\dots)$ or better

Question	Scheme	Marks	AOs
7(a)	Mass of dome = $\int_a^{2a} \pi y^2 \frac{\lambda}{x^2} dx = \pi \lambda \int_a^{2a} \frac{4a^2 - x^2}{x^2} dx = \pi \lambda \int_a^{2a} \left(\frac{4a^2}{x^2} - 1 \right) dx$	M1	3.4
	$= \pi \lambda \left[\frac{-4a^2}{x} - x \right]_a^{2a} \left(= \pi \lambda \left[\frac{-4a^2}{2a} - 2a + \frac{4a^2}{a} + a \right] = \pi \lambda a (\text{kg}) \right)$	A1	1.1b
	Moments: $\int_a^{2a} \pi y^2 \frac{\lambda}{x^2} \times x dx = \pi \lambda \int_a^{2a} \left(\frac{4a^2}{x} - x \right) dx$	M1	3.4
	$= \pi \lambda \left[4a^2 \ln x - \frac{1}{2} x^2 \right]_a^{2a} \left(= \pi \lambda \left(4a^2 \ln 2 - \frac{3a^2}{2} \right) \right)$	A1	1.1b
	$\Rightarrow \text{distance} = \frac{\pi \lambda \left(4a^2 \ln 2 - \frac{3a^2}{2} \right)}{\pi \lambda a}$	DM1	2.1
	$\Rightarrow \text{distance} = 4a \ln 2 - \frac{3a}{2} - a = \left(4 \ln 2 - \frac{5}{2} \right) a \text{ (m)} \quad *$	A1*	2.2a
		(6)	
(b)	Centre of mass of cone lies a m from the plane surface	B1	1.1b
	Moments about a diameter of the plane face	M1	3.1b
	$a \times kW - a \left(4 \ln 2 - \frac{5}{2} \right) \times 2W = (2+k)Wd$	A1	1.1b
	$d = \frac{ k+5-8 \ln 2 }{2+k} a *$	A1*	2.2a
		(4)	
(c)	Use of trigonometry	M1	3.1b
	$\tan \alpha = \frac{\left(\frac{k+5-8 \ln 2}{2+k} \right) a}{\sqrt{3}a} \left(= \frac{1}{2\sqrt{3}} \right)$	A1	1.1b
	$\Rightarrow k = 16 \ln 2 - 8$	A1	1.1b
		(3)	
(13 marks)			
Notes:			

(a)M1	Use the model to find the mass of the dome. Allow without limits.
A1	Correct integration. Correct limits seen or implied.
M1	Use the model to take moments (usual rules for integration). Allow without limits.
A1	Correct integration. Correct limits seen or implied.
DM1	Complete method to find the distance of the centre of mass from the centre of the plane face.
A1*	Obtain the given answer from full and correct working.
(b)B1	Seen or implied
M1	Moments equation. Need all terms. Dimensionally correct. Allow use of a parallel axis.
A1	Correct unsimplified equation in d
A1*	Obtain given answer from correct working. Condone if modulus signs not used
(c)M1	Complete method to find the required angle, e.g. by use of tangent. Condone if they ignore the modulus signs.
A1	Correct unsimplified equation.
A1	Any equivalent exact form.

Question	Scheme	Marks	AOs
4(a)	$\frac{4}{3}\pi r^3 \times \frac{2}{3}r + \frac{2}{3}\pi r^3 \times \left(\frac{4}{3}r + \frac{3}{8}r\right) = \left(\frac{4}{3} + \frac{2}{3}\right)\pi r^3 \times d$	M1	2.1
	$\left(\frac{8}{9}r + \frac{8}{9}r + \frac{1}{4}r = 2d\right)$	A1	1.1b
	$\left(\frac{73}{36}r = 2d\right)$	A1	1.1b
	$\Rightarrow d = \frac{73}{72}r$ *	A1*	2.2a
		(4)	
(b)			
	Resolving: $\leftrightarrow F = P$, $\uparrow R = Mg$, $F_{\max} = \mu R = \mu Mg$	M1	1.1b
	Slides if $P > \mu Mg$	A1	1.2
	Moments: $\frac{7}{3}rP = rMg$ Tilts if $P > \frac{3}{7}Mg$	B1	1.1b
	Comparison of restrictions to determine values of μ	M1	3.1a
	Slides first if $\mu Mg < \frac{3}{7}Mg$, $(0 <) \mu < \frac{3}{7}$	A1	2.2a
		(5)	
(9 marks)			

Notes:

(a)	M1	Moments equation. Dimensionally correct.
	A1	Unsimplified equation with at most one slip
	A1	Correct unsimplified equation
	A1*	Obtain given result from correct working.
(b)	M1	Resolve and use $F = \mu R$ to find values of P for sliding
	A1	Use the model to form the correct inequality
	B1	Correct inequality for tilting
	M1	Correct comparison of when it tilts and when it slides
	A1	Correct conclusion