



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Differential Equations (3)

Further Pure Mathematics 1 · Topic 3

10 questions · 117 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

 guidemygrades.com

 @Guidemygrades

 @Guidemygrades

 @Guidemygrades

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
3.1 TAYLOR SERIES SOLUTIONS OF DES		3 Qs · 25 marks	3
1. Q4 <i>also Topic 2</i>	Option 3A: Further Pure Mathematics 1 June2024	8	4
2. Q3	Option 3A: Further Pure Mathematics 1 June2019	9	8
3. Q2	Option 3A: Further Pure Mathematics 1 June2025	8	12
3.2 DES REDUCIBLE BY SUBSTITUTION		6 Qs · 86 marks	16
4. Q10	Option 3A: Further Pure Mathematics 1 June2024	12	17
5. Q9	Option 3A: Further Pure Mathematics 1 June2022	13	22
6. Q2	Option 3A: Further Pure Mathematics 1 June2023	16	27
7. Q6	Option 3A: Further Pure Mathematics 1 June2019	17	31
8. Q8 <i>also Topic 6</i>	Option 3A: Further Pure Mathematics 1 November2021	17	37
9. Q7	Option 3A: Further Pure Mathematics 1 June2025	11	42
6.1 NUMERICAL SOLUTION OF DES		1 Qs · 6 marks	46
10. Q4 <i>also Topic 6</i>	Option 3A: Further Pure Mathematics 1 June2022	6	47



QUESTIONS

10 questions · 117 marks · 3 sub-topics



SUB-TOPIC 3.1

TAYLOR SERIES SOLUTIONS OF DES

Sub-topic 3.1 · 3 questions · 25 marks · lowest tariff first



Question 4 continued



Question 3 continued

Lined area for writing the answer to Question 3 continued.



(Total for Question 3 is 9 marks)



SUB-TOPIC 3.2

DES REDUCIBLE BY SUBSTITUTION

Sub-topic 3.2 · 6 questions · 86 marks · lowest tariff first



10. The motion of a particle P along the x -axis is modelled by the differential equation

$$t^2 \frac{d^2 x}{dt^2} - 2t(t+1) \frac{dx}{dt} + 2(t+1)x = 8t^3 e^t \quad (\text{I})$$

where P has displacement x metres from the origin O at time t minutes, $t > 0$

(a) Show that the transformation $x = tu$ transforms the differential equation (I) into the differential equation

$$\frac{d^2u}{dt^2} - 2\frac{du}{dt} = 8e^t \quad (4)$$

Given that P is at O when $t = \ln 3$ and when $t = \ln 5$

(b) determine the particular solution of the differential equation (I)

(8)







Question 10 continued

(Total for Question 10 is 12 marks)

9. A particle P moves along a straight line.

At time t minutes, the displacement, x metres, of P from a fixed point O on the line is modelled by the differential equation

$$t^2 \frac{d^2 x}{dt^2} - 2t \frac{dx}{dt} + 2x + 16t^2 x = 4t^3 \sin 2t \quad (\text{I})$$

(a) Show that the transformation $x = ty$ transforms equation (I) into the equation

$$\frac{d^2 y}{dt^2} + 16y = 4 \sin 2t \quad (5)$$

(b) Hence find a general solution for the displacement of P from O at time t minutes.





(Total for Question 9 is 13 marks)



Question 6 continued

Lined area for writing the answer to Question 6.

Question 6 continued

Question 6 continued



Question 6 continued



Question 8 continued





Question 8 continued

(Total for Question 8 is 17 marks)



SUB-TOPIC 6.1

NUMERICAL SOLUTION OF DES

Sub-topic 6.1 · 1 question · 6 marks · lowest tariff first



(Total for Question 4 is 6 marks)

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
4(a)	$\cos\left(\frac{\pi}{4}\right)\frac{d^2y}{dx^2} + (1)^2(0) + \sin\left(\frac{\pi}{4}\right) = 0 \Rightarrow \frac{d^2y}{dx^2} = \dots$	M1	3.1a
	$\frac{d^2y}{dx^2} = -1 < 0$ therefore a (local) maximum	A1	2.4
		(2)	
(b)	Achieves $\pm \sin x \frac{d^2y}{dx^2} + \cos x \frac{d^3y}{dx^3} + \alpha y \left(\frac{dy}{dx}\right)^{1 \text{ or } 2} + y^2 \frac{d^2y}{dx^2} \pm \cos x = 0$ or $\frac{d^2y}{dx^2} = -\tan x - y^2 \sec x \frac{dy}{dx} \rightarrow$ $\frac{d^3y}{dx^3} = \pm \sec^2 x \pm \alpha y \sec x \left(\frac{dy}{dx}\right)^{1 \text{ or } 2} \pm y^2 \sec x \tan x \frac{dy}{dx} \pm y^2 \sec x \frac{d^2y}{dx^2}$	M1	1.1b
	$-\sin x \frac{d^2y}{dx^2} + \cos x \frac{d^3y}{dx^3} + 2y \left(\frac{dy}{dx}\right)^2 + y^2 \frac{d^2y}{dx^2} + \cos x = 0$ or $\frac{d^3y}{dx^3} = -\sec^2 x - 2y \sec x \left(\frac{dy}{dx}\right)^2 - y^2 \sec x \tan x \frac{dy}{dx} - y^2 \sec x \frac{d^2y}{dx^2}$	A1	1.1b
	$-\sin\left(\frac{\pi}{4}\right)(-1) + \cos\left(\frac{\pi}{4}\right)\frac{d^3y}{dx^3} + 2(1)(0)^2 + (1)^2(-1) + \cos\left(\frac{\pi}{4}\right) = 0 \rightarrow \frac{d^3y}{dx^3} = \dots$ Alt: $\frac{d^3y}{dx^3} = -2 - 2(1)\sqrt{2}(0) - 1^2\sqrt{2}(1)(0) - 1^2\sqrt{2}(-1)$	M1	1.1b
	$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \frac{d^3y}{dx^3} - 1 + \frac{\sqrt{2}}{2} = 0 \Rightarrow \frac{\sqrt{2}}{2} \frac{d^3y}{dx^3} = 1 - \sqrt{2}\right)$ $\frac{d^3y}{dx^3} \left(= \frac{2 - 2\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2} - 4}{2} \right) = \sqrt{2} - 2^*$	A1*	2.1
		(4)	
(c)	$(y=)1 + \left(x - \frac{\pi}{4}\right)(0) + \frac{\left(x - \frac{\pi}{4}\right)^2}{2!}(\text{their } -1') + \frac{\left(x - \frac{\pi}{4}\right)^3}{3!}(\sqrt{2} - 2) + \dots$	M1	2.5
	$(y=)1 - \frac{\left(x - \frac{\pi}{4}\right)^2}{2} + \frac{\left(x - \frac{\pi}{4}\right)^3}{6}(\sqrt{2} - 2) + \dots$	A1	1.1b
		(2)	
(8 marks)			



Notes:**(a)**

M1: A complete method to find the value for $\frac{d^2y}{dx^2}$. Substitutes into the differential equation

$x = \frac{\pi}{4}$, $y = 1$, $\frac{dy}{dx} = 0$ and rearranges to find a value for $\frac{d^2y}{dx^2}$.

A1: $\frac{d^2y}{dx^2} = -1 < 0$ therefore a maximum. Must achieve the correct value, or at least a correct expression and give reason “<0” (oe) and conclusion and no contradictory statements.

(b)

M1: Differentiates to the form $\pm \sin x \frac{d^2y}{dx^2} + \cos x \frac{d^3y}{dx^3} + \alpha y \left(\frac{dy}{dx} \right)^{1 \text{ or } 2} + y^2 \frac{d^2y}{dx^2} \pm \cos x = 0$.

Alternatively, divides through by $\cos x$ first and differentiates to achieve the form shown in scheme.

A1: Correct differentiation, any alternative form is acceptable.

M1: Substitutes in $x = \frac{\pi}{4}$, $y = 1$, $\frac{dy}{dx} = 0$, their value of $\frac{d^2y}{dx^2} = -1$ as appropriate for their expression,

and rearranges to find a value for $\frac{d^3y}{dx^3}$ (if not already done so).

A1*: Shows that $\frac{d^3y}{dx^3} = \sqrt{2} - 2$ from correct working with at least one suitable intermediate line –

being either an unsimplified $\frac{d^3y}{dx^3} = \dots$ or $\alpha \frac{d^3y}{dx^3} = \dots$, before the final answer. Must have come from a correct derivative expression. Note that in the Alt the correct form comes out much easier, so look for at least one unsimplified line before the final answer.

(c)

M1: Uses the Taylor’s series expansion with the correct values for $y_{\frac{\pi}{4}} = 1$, $\frac{dy}{dx} = 0$, $\frac{d^3y}{dx^3} = \sqrt{2} - 2$ (or

their $\frac{d^3y}{dx^3}$ if they had a different answer to (b)) and their value of $\frac{d^2y}{dx^2}$. Must include the 1 at the start.

A1: Correct simplified expansion. The “y=” may be missing. Accept with $0\left(x - \frac{\pi}{4}\right)$ as a second term. ISW after a correct simplified expression if further attempts to simplify are made.



Question	Scheme	Marks	AOs		
Q11: $\frac{1}{3} \times \text{area 123.54...}$ or for $\frac{1}{3} \times \text{area 123.54...}$ leading to a value and where h has clearly been defined earlier.					
Dependent on both previous method marks					
A1: Awt 16.5					
Note that a minimum we would expect to see for full marks is:					
$h = 0.4$					
	y_0	y_1	y_2	y_3	y_4
x	0.4	0.8	1.2	1.6	2
y	$e^{0.16}$	$e^{0.64}$	$e^{1.44}$	$e^{2.56}$	e^4
	1.173...	1.896...	4.220...	12.935...	54.598...
$A \approx \frac{h}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + y_4] = 16.5$					
(Note that a calculator gives 16.030...for the area)					

Question	Scheme	Marks	AOs
2	$u = x^3 \Rightarrow \frac{du}{dx} = 3x^2, \frac{d^2u}{dx^2} = 6x, \frac{d^3u}{dx^3} = 6$	M1	1.1b
	$v = \sin kx \Rightarrow \frac{dv}{dx} = k \cos kx, \frac{d^2v}{dx^2} = -k^2 \sin kx, \frac{d^3v}{dx^3} = -k^3 \cos kx,$ $\frac{d^4v}{dx^4} = k^4 \sin kx, \frac{d^5v}{dx^5} = k^5 \cos kx$	M1	2.1
	$\frac{d^5y}{dx^5} = x^3 k^5 \cos kx + 5 \times 3x^2 \times k^4 \sin kx + \frac{5 \times 4}{2} \times 6x \times (-k^3 \cos kx) +$ $\frac{5 \times 4 \times 3}{3!} \times 6 \times (-k^2 \sin kx)$	M1	2.1
	$= (k^2 x^2 - 60) k^3 x \cos kx + 15 (k^2 x^2 - 4) k^2 \sin kx$	A1	1.1b
		(4)	

(4 marks)

Notes

 M1: Differentiates $u = x^3$ three times. Need to see $x^3 \rightarrow \dots x^2 \rightarrow \dots x \rightarrow k$

 M1: Uses $v = \sin kx$ to establish the form of the derivatives. Need to see at least alternating $k \dots \sin kx$ and $k \dots \cos kx$ with increasing powers of k for at least 3 derivatives.

M1: Uses a correct formula with 2 and 3! (or 6) with terms shown to disappear after the fourth term. This needs to be a correct application of the theorem so that the correct binomial coefficients need to go with the correct pairings of their derivatives. If there is any doubt, at least 3 terms should have the correct structure. Allow equivalent notation for the binomial coefficients

 e.g. $\binom{5}{0}, \binom{5}{1}$ etc. or ${}^5C_0, {}^5C_1$ etc.

 A1: Correct expression in the required form with correct values of A, B and C . Apply isw if necessary e.g. if a correct expression is followed by $A = 60, B = 15, C = -4$
 (NB $A = -60, B = 15, C = -4$)

If there is no use Leibnitz's theorem e.g. repeated differentiation of products, this scores no marks.


Question	Scheme	Marks	AOs
3(a)	$\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx} \Rightarrow \frac{d^3y}{dx^3} = -2y \frac{d^2y}{dx^2} - 2 \left(\frac{dy}{dx} \right)^2$	M1 A1	1.1b 1.1b
	$\frac{d^4y}{dx^4} = -2 \frac{dy}{dx} \frac{d^2y}{dx^2} - 2y \frac{d^3y}{dx^3} - 4 \frac{dy}{dx} \frac{d^2y}{dx^2} = -6 \frac{dy}{dx} \frac{d^2y}{dx^2} - 2y \frac{d^3y}{dx^3}$	dM1	2.1
	$\frac{d^5y}{dx^5} = -6 \frac{dy}{dx} \frac{d^3y}{dx^3} - 6 \left(\frac{d^2y}{dx^2} \right)^2 - 2y \frac{d^4y}{dx^4} - 2 \frac{dy}{dx} \frac{d^3y}{dx^3}$ $= -2y \frac{d^4y}{dx^4} - 8 \frac{dy}{dx} \frac{d^3y}{dx^3} - 6 \left(\frac{d^2y}{dx^2} \right)^2$	A1	2.1
		(4)	
(b)	$x = 0, y = 1 \Rightarrow \left(\frac{dy}{dx} \right)_0 = 0 - 1^2 = -1$	B1	2.2a
	$\left(\frac{d^2y}{dx^2} \right)_0 = 1 - 2(1)(-1) = 3, \left(\frac{d^3y}{dx^3} \right)_0 = -2(1)(3) - 2(-1)^2 = -8$ $\left(\frac{d^4y}{dx^4} \right)_0 = -6(-1)(3) - 2(1)(-8) = 34,$ $\left(\frac{d^5y}{dx^5} \right)_0 = -2(1)(34) - 8(-1)(-8) - 6(3)^2 = -186$	M1 A1	1.1b 1.1b
	$y = y(0) + x \left(\frac{dy}{dx} \right)_0 + \frac{x^2}{2!} \left(\frac{d^2y}{dx^2} \right)_0 + \frac{x^3}{3!} \left(\frac{d^3y}{dx^3} \right)_0 + \frac{x^4}{4!} \left(\frac{d^4y}{dx^4} \right)_0 + \frac{x^5}{5!} \left(\frac{d^5y}{dx^5} \right)_0 + \dots$ With their values	M1	2.5
	$(y =) 1 - x + \frac{3}{2}x^2 - \frac{8}{6}x^3 + \frac{34}{24}x^4 - \frac{186}{120}x^5 + \dots$ $(y =) 1 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3 + \frac{17}{12}x^4 - \frac{31}{20}x^5 + \dots$	A1ft	1.1b
		(5)	
(9 marks)			
Notes			
(a) M1: Attempts to find the second and third derivatives: This requires $\frac{d^2y}{dx^2} = 1 \pm 2y \frac{dy}{dx}$ or $\frac{d^2y}{dx^2} = \pm 2y \frac{dy}{dx}$ followed by $\frac{d^3y}{dx^3} = \pm 2y \frac{d^2y}{dx^2} \pm \dots$ or $\frac{d^3y}{dx^3} = \pm \dots \pm 2 \left(\frac{dy}{dx} \right)^2$ A1: Correct second and third derivatives. dM1: Continues to differentiate to reach the 5 th derivative. This is dependent on the first method mark but there is no need to check the detail and the mark can be awarded as long as the 5th derivative is reached. A1: Completes the process, collecting terms if necessary, to obtain the correct expression (NB $a = -2, b = -8, c = -6$)			

Allow dash/dot notation for the derivatives but the final answer must be in the correct form.

Note that if $\frac{d^2y}{dx^2} = \pm 2y \frac{dy}{dx}$ is obtained initially, allow a full recovery in (a).

Note that (a) can be found using Leibnitz's theorem and the following scheme should be applied:

M1: $\frac{d^2y}{dx^2} = 1 \pm 2y \frac{dy}{dx}$ or $\frac{d^2y}{dx^2} = \pm 2y \frac{dy}{dx}$ followed by an attempt to differentiate y 3 times and $\frac{dy}{dx}$ 3 times.

A1: All correct

dM1: $\frac{d^5y}{dx^5} = -2 \frac{dy}{dx} \frac{d^4y}{dx^4} - 3 \times 2 \frac{dy}{dx} \frac{d^3y}{dx^3} - 3 \times 2 \left(\frac{d^2y}{dx^2} \right)^2 - 2y \frac{d^3y}{dx^3} \frac{dy}{dx}$ (correct application of Leibnitz)

A1: $= -2y \frac{d^4y}{dx^4} - 8 \frac{dy}{dx} \frac{d^3y}{dx^3} - 6 \left(\frac{d^2y}{dx^2} \right)^2$

As in the main scheme, if $\frac{d^2y}{dx^2} = \pm 2y \frac{dy}{dx}$ is obtained initially, allow a full recovery in (a).

Alternative for (a):

M1: $\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx} = 1 - 2y(x - y^2) = 1 - 2xy + 2y^3 \Rightarrow \frac{d^3y}{dx^3} = -2y - 2x \frac{dy}{dx} + 6y^2 \frac{dy}{dx}$

Score for the second derivative form as in the main scheme and then an attempt at the third derivative with at least 2 terms correct.

A1: Fully correct

Then as main scheme.

(b)

B1: Deduces the correct value for $y'(0)$

M1: Finds the values of all the other derivatives at $x = 0$ up to 5^{th} . There is no need to check their values as long as there is no obvious incorrect work, but values for all the derivatives up to the 5^{th} must be found

A1: All values correct (as single values – e.g. do not allow unsimplified)

M1: Applies the correct Maclaurin series for their values including the factorials up to the term in x^5

A1ft: Correct expansion, follow through their values for the derivatives. This does not have to be simplified but the factorials need to be evaluated. **Once a correct, or correct follow through expression is seen apply isw.**

Question	Scheme	Marks	AOs
----------	--------	-------	-----

Question	Scheme	Marks	AOs
2(a)	$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + Ax\frac{dy}{dx} + By = 0 \quad A, B \neq 0$ $\frac{d^3y}{dx^3} = -3\frac{d^2y}{dx^2} + Ax\frac{dy}{dx} + By \quad A, B \neq 0$	M1	2.1
	$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 2y = 0$ or $\frac{d^3y}{dx^3} = -3\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y$	A1	1.1b
	$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - 2x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 2\frac{dy}{dx} = 0 \Rightarrow \frac{d^4y}{dx^4} = \dots$ Or $\frac{d^4y}{dx^4} = -3\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2\frac{dy}{dx}$ or $\frac{d^4y}{dx^4} = -3\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 4\frac{dy}{dx}$	M1	1.1b
	$\frac{d^4y}{dx^4} = 4\frac{dy}{dx} + 2x\frac{d^2y}{dx^2} - 3\frac{d^3y}{dx^3} \text{ cso}$	A1	2.2a
		(4)	
(b)	$x = 2, y = 1, \Rightarrow \frac{dy}{dx} = 1 \Rightarrow \left(\frac{d^2y}{dx^2}\right)_2 = 4 + 4 - 3 = 5$	B1	2.2a
	$\left(\frac{d^3y}{dx^3}\right)_2 = 2(2)(1) + 2(1) - 3(5) = -9$ $\left(\frac{d^4y}{dx^4}\right)_2 = 4(1) + 2(2)(5) - 3(-9) = 51$	M1	1.1b
	$y = y(2) + (x-2)\left(\frac{dy}{dx}\right)_2 + \frac{(x-2)^2}{2!}\left(\frac{d^2y}{dx^2}\right)_2 +$ $\frac{(x-2)^3}{3!}\left(\frac{d^3y}{dx^3}\right)_2 + \frac{(x-2)^4}{4!}\left(\frac{d^4y}{dx^4}\right)_2 + \dots$	M1	2.5
	$y = 1 + (x-2) + \frac{5}{2}(x-2)^2 - \frac{3}{2}(x-2)^3 + \frac{17}{8}(x-2)^4 + \dots$ Or $y = x - 1 + \frac{5}{2}(x-2)^2 - \frac{3}{2}(x-2)^3 + \frac{17}{8}(x-2)^4 + \dots$	A1	1.1b
		(4)	
(8 marks)			

Notes

(a)

M1: Attempts to differentiate including an attempt at the product rule to achieve the correct form in any order

A1: Correct differentiation

M1: Continues to differentiate using the product rule again to reach the 4th derivative to achieve

the correct form $\frac{d^4 y}{dx^4} = A \frac{d^3 y}{dx^3} + Bx \frac{d^2 y}{dx^2} + C \frac{dy}{dx} + D \frac{dy}{dx}$ in any order

A1: Completes the process to obtain the correct expression ($a = 4, b = 2, c = -3$) with no errors seen and correct notation throughout and $= 0$ where applicable cso

(b)

B1: Deduces the correct value for $y''(2)$

M1: Finds the values of the derivatives at $x = 2$. There must be evidence of the correct values of

$x = 2, y = 1, \frac{dy}{dx} = 1$ being used at least once

M1: Uses the correct Taylor series expansion with their derivatives. Must be using $y = 1, \frac{dy}{dx} = 1$

in their series expansion. If no method is shown they must have all terms correct for their derivatives

A1: Correct expansion must include $y =$ or $f(x) =$ somewhere



Question	Scheme	Marks	AOs
10 (a)	$x = tu \Rightarrow \frac{dx}{dt} = \frac{du}{dt}t + u$ or $\frac{du}{dt} = \frac{t \frac{dx}{dt} - x}{t^2}$ (oe)	B1	2.2a
	$\frac{dx}{dt} = \frac{du}{dt}t + u \Rightarrow \frac{d^2x}{dt^2} = \frac{d^2u}{dt^2}t + 2\frac{du}{dt}$ or e.g. $\frac{d^2u}{dt^2} = -\frac{1}{t^2} \frac{dx}{dt} + \frac{1}{t} \frac{d^2x}{dt^2} - \frac{1}{t^4} \left(t^2 \frac{dx}{dt} - 2tx \right)$	B1	1.1b
	$t^2 \left[\frac{d^2u}{dt^2}t + 2\frac{du}{dt} \right] - 2t(t+1) \left[\frac{du}{dt}t + u \right] + 2(t+1)tu = 8t^3e^t$ $t^3 \frac{d^2u}{dt^2} + 2t^2 \frac{du}{dt} - 2t(t+1) \frac{du}{dt}t - 2t(t+1)u + 2t(t+1)u = 8t^3e^t$ $t^3 \frac{d^2u}{dt^2} + 2t^2 \frac{du}{dt} - 2t^3 \frac{du}{dt} - 2t^2 \frac{du}{dt} - 2t(t+1)u + 2t(t+1)u = 8t^3e^t$	M1	1.1b
	$\frac{d^2u}{dt^2} - 2\frac{du}{dt} = 8e^t *$	A1*	2.1
		(4)	
(b)	$m^2 - 2m = 0 \Rightarrow m = 0, 2$	M1	1.1b
	$u = A + Be^{2t}$	A1	1.1b
	PS $\Rightarrow u = \lambda e^t$	B1	2.2a
	$\frac{du}{dt} = \lambda e^t, \frac{d^2u}{dt^2} = \lambda e^t \Rightarrow \lambda e^t - 2\lambda e^t = 8e^t \Rightarrow \lambda = \dots \{-8\}$ $\Rightarrow u = \text{PS} + \text{CF}$	M1	3.4
	$u = "A + Be^{2t}" - 8e^t$ or $x = ("A + Be^{2t}" - 8e^t)t$	A1ft	2.2a
	$x = (A + Be^{2t} - 8e^t)t$ $x = 0, t = \ln 3$ and $t = \ln 5$ $0 = A + Be^{2\ln 3} - 8e^{\ln 3} \Rightarrow 0 = A + 9B - 24$ $0 = A + Be^{2\ln 5} - 8e^{\ln 5} \Rightarrow 0 = A + 25B - 40$	M1	3.4
	$\left. \begin{matrix} A + 9B = 24 \\ A + 25B = 40 \end{matrix} \right\} \Rightarrow A = \dots \{15\}, B = \dots \{1\}$	dM1	3.1a
	$x = (15 + e^{2t} - 8e^t)t$ (oe)	A1	1.1b
		(8)	
(12 marks)			

Notes:**(a)**

B1: Deduces a correct first derivative of $x = tu$ connecting $\frac{dx}{dt}$ and $\frac{du}{dt}$. Most likely the main version shown, but alternatives are possible.

B1: Correct second derivative of $x = tu$ connecting $\frac{d^2x}{dt^2}$ and $\frac{d^2u}{dt^2}$. There may be alternatives.

M1: Substitutes their first and second derivatives into the equation and makes some attempt to expand the brackets (need not reach the final answer). (Note if you see answers substituting into the second equation that you feel are worth merit use review.)

A1*: Fully correct proof with no errors and omissions. There must be a clear line of working where the relevant terms cancel.

(b)

M1: Forms and solves the quadratic auxiliary equation $m^2 - 2m = 0$ (may be implied by the correct CF).

A1: Correct CF.

B1: Deduces the correct form of the PI.

M1: Uses the model to find the general solution. Differentiates the correct form of the PI twice (coefficient slips only permitted) and substitutes into the differential equation to find the value of λ , leading to $u = \text{PI} + \text{CF}$

A1ft: Deduces the general solution for u or for x , follow through their CF.

M1: Uses the model and $x = 0$, $t = \ln 3$ and $t = \ln 5$ to form two simultaneous equations (need not be fully correct, but correct substitution must be seen at least once in each equation, and allow if slips are made rearranging). May undo the substitution first and use x and t , or may use $x = 0 \Rightarrow u = 0$ and substitute to find the constants in their equation for u , but they must form suitable equations using correct conditions.

dM1: Dependent on the previous method mark. Proceeds to solve to find the values of the constants in their equation.

A1: Correct particular solution $x = (15 + e^{2t} - 8e^t)t$ (may be expanded).



Question	Scheme	Marks	AOs
9(a)	Use of $x = ty$ to give $\frac{dx}{dt} = y + t \frac{dy}{dt}$ or $y = \frac{x}{t} \rightarrow \frac{dy}{dt} = -\frac{x}{t^2} + \frac{1}{t} \frac{dx}{dt}$ oe	B1	1.1b
	$\frac{d^2x}{dt^2} = \frac{dy}{dt} + \frac{dy}{dt} + t \frac{d^2y}{dt^2}$ or $\frac{d^2y}{dt^2} = -\frac{1}{t^2} \frac{dx}{dt} + \frac{2x}{t^3} + \frac{1}{t} \frac{d^2x}{dt^2} - \frac{1}{t^2} \frac{dx}{dt}$ oe	M1 A1	2.1 1.1b
	$t^2 \left[\text{their } \frac{d^2x}{dt^2} \right] - 2t \left[\text{their } \frac{dx}{dt} \right] + 2[ty] + 16t^2[ty] = 4t^3 \sin 2t$ Or $\left[\text{their } \frac{d^2y}{dt^2} \right] + 16 \frac{x}{t} = 4 \sin 2t$	dM1	2.1
	$t^2 \left[\frac{dy}{dt} + \frac{dy}{dt} + t \frac{d^2y}{dt^2} \right] - 2t \left[y + t \frac{dy}{dt} \right] + 2[ty] + 16t^2[ty] =$ $4t^3 \sin 2t \quad t^3 \frac{d^2y}{dt^2} + 16t^3 y = 4t^3 \sin 2t \Rightarrow \frac{d^2y}{dt^2} + 16y = 4 \sin 2t$ * (oe in reverse)	A1	1.1b
		(5)	
(b)	Solves $m^2 + 16 = 0$ to give $m = \dots$	M1	1.1b
	$(y =) A \cos 4t + B \sin 4t$	A1	1.1b
	Particular integral $(y =) \lambda \sin 2t + \mu \cos 2t$	B1	2.2a
	$\frac{dy}{dt} = 2\lambda \cos 2t - 2\mu \sin 2t$ and $\frac{d^2y}{dt^2} = -4\lambda \sin 2t - 4\mu \cos 2t$	M1	1.1b
	Substitutes into the differential equation and finds values for λ and μ $[-4\lambda \sin 2t - 4\mu \cos 2t] + 16[\lambda \sin 2t + \mu \cos 2t] = 4 \sin 2t$ $\Rightarrow \lambda = \dots, \mu = \dots$	dM1	2.1
	$y = "A \cos 4t + B \sin 4t" + \frac{1}{3} \sin 2t$	A1ft	1.1b
	$x = t[\text{their } y]$	M1	3.4
	$x = t \left[A \cos 4t + B \sin 4t + \frac{1}{3} \sin 2t \right]$	A1	2.2a
		(8)	
(13 marks)			
Notes:			
(a) B1: For a correct suitable first derivative expression linking $\frac{dy}{dt}$ and $\frac{dx}{dt}$. M1: Uses the product rule to find an equation linking second derivatives from their first derivative expression. A1: A correct second derivative expression. dM1: Substitutes the first and second derivatives and replaces x with ty to obtain a differential equation in y and t only. Alternatively, may go in reverse and replace y with $\frac{x}{t}$ etc in the second equation to obtain a differential equation in x and t only.			

A1*: Simplifies their expression with a correct intermediate stage/working to reach the printed answer. Alternatively, correct working in the other direction to achieve equation (I) from the final equation.

(b)

M1: Forms the correct auxiliary equation and attempts to solve (any values after the correct AE seen)

A1: Correct complementary function. Accept for this mark if they give it in terms of x - you are looking for the correct form for the CF.

B1: Deduces a correct form of the particular integral (must include at least $\lambda \sin 2t$ but may be no more than this). SC if by error the CF includes $\sin 2t$ allow B1 for a PI of form $\lambda t \sin 2t + \mu t \cos 2t$

M1: Differentiates the PI twice.

dM1: Dependent on the previous method mark. Substitutes y and $\frac{d^2y}{dt^2}$ into the differential equation leading to values for the constant(s).

A1ft: Correct general equation for y following through their CF, which must be a (non-constant) function of t . Must be in terms of t and start $y = \dots$

M1: Links the solution to the solution of the model equation to find the general solution for the displacement.

A1: Deduces the correct general solution for the displacement. Must be $x = \dots$



Question	Scheme	Marks	AOs
2(a)(i)	$t = e^x \Rightarrow \frac{dt}{dh} = e^x \frac{dx}{dh} \Rightarrow \frac{dh}{dt} = e^{-x} \frac{dh}{dx}$	M1	1.1b
	$\Rightarrow t \frac{dh}{dt} = \frac{dh}{dx} *$	A1*	2.1
(ii)	$t \frac{dh}{dt} = \frac{dh}{dx} \Rightarrow t \frac{d^2h}{dt^2} + \frac{dh}{dt} = \frac{d^2h}{dx^2} \frac{dx}{dt}$	M1	1.1b
	$t \frac{d^2h}{dt^2} + \frac{dh}{dt} = \frac{d^2h}{dx^2} \frac{dx}{dt} \Rightarrow t \frac{d^2h}{dt^2} = \frac{1}{t} \frac{d^2h}{dx^2} - \frac{dh}{dt}$ $\Rightarrow t^2 \frac{d^2h}{dt^2} = \frac{d^2h}{dx^2} - t \frac{dh}{dt} = \frac{d^2h}{dx^2} - \frac{dh}{dx} *$	A1*	2.1
		(4)	
(b)	$t^2 \frac{d^2h}{dt^2} - 2t \frac{dh}{dt} + 2h = t^3 \Rightarrow \frac{d^2h}{dx^2} - \frac{dh}{dx} - 2 \frac{dh}{dx} + 2h = e^{3x}$ $\Rightarrow \frac{d^2h}{dx^2} - 3 \frac{dh}{dx} + 2h = e^{3x} *$	B1*	1.1b
		(1)	
(c)	$m^2 - 3m + 2 = 0 \Rightarrow m = 1, 2$	M1	1.1b
	$(h =) Ae^x + Be^{2x}$	A1ft	1.1b
	PI is $C = ke^{3x}$	B1	2.2a
	$\frac{dh}{dx} = 3ke^{3x}, \frac{d^2h}{dx^2} = 9ke^{3x} \Rightarrow 9k - 9k + 2k = 1 \Rightarrow k = \frac{1}{2} \Rightarrow \text{PI: } (h =) \frac{1}{2} e^{3x}$	M1 A1	1.1b 1.1b
	$h = Ae^x + Be^{2x} + \frac{1}{2} e^{3x} \Rightarrow h = At + Bt^2 + \frac{1}{2} t^3 *$	A1*	2.2a
		(6)	
(d)	$t = 1, h = 2.5 \Rightarrow 2.5 = A + B + \frac{1}{2}$	M1	3.4
	$\frac{dh}{dt} = A + 2Bt + \frac{3}{2} t^2 \Rightarrow -1 = A + 4B + 6$	M1	3.4
	$A = 5, B = -3 \Rightarrow h = 5t - 3t^2 + \frac{1}{2} t^3$	A1	1.1b
	$t = 5 \Rightarrow h = 5 \times 5 - 3 \times 5^2 + \frac{5^3}{2}$	M1	1.1b
	height is 12.5m (including units)	A1	3.2a
		(5)	
(16 marks)			
Notes			
(a)(i) M1: Uses the chain rule (must be a clear statement of this before substitution or clear implication of it use) with an attempt to differentiate $t = e^x$ to form an equation linking $\frac{dh}{dt}$ and $\frac{dh}{dx}$ (or their reciprocals) in terms of either x or t . Note they may take $\ln t$ first, which is fine. Note that $\frac{dh}{dx} = e^x \frac{dh}{dt}$ with no supporting working is M0. A1*: Correct proof with no errors. (ii)			

M1: Differentiates again with the product and chain rule, fully and correctly on at least one product, in order to establish any second derivative equation linking $\frac{d^2h}{dt^2}$ and $\frac{d^2h}{dx^2}$

A1*: Correct proof with no errors.

(b)

B1*: Shows clearly the substitution of the given results into the differential equation and obtains the given transformed equation with no errors.

(c)

M1: Forms and solves a quadratic auxiliary equation.

A1ft: Correct form for the CF for their AE solutions which must be $\pm 1, \pm 2$ (ie accept sign errors solving the AE). The $h =$ is not needed and condone e.g. $Ae^x + Be^{2x} = 0$

B1: Deduces the correct form for the PI. Must be in terms of x for the equation II.

M1: Differentiates their PI twice and substitutes their derivatives into the DE to find “ k ”.

A1: Correct PI (for equation II) seen or implied by working. May or may not be combined with the CF for this mark.

A1*: Forms the correct GS for h in terms of x and deduces the correct GS for the height in terms of t with no errors. This is a given answer, so must have seen the GS in terms of h before proceeding to this answer. Accept with the constants A and B either way round (or with other constants).

(d)

M1: Uses the conditions of the model ($t = 1, h = 2.5$) to form an equation in A and B ,

M1: Differentiates and shows of evidence of using the conditions of the model $\left(t = 2, \frac{dh}{dt} = -1\right)$ to form another equation in A and B .

A1: Solves simultaneously to obtain correct constants and hence a correct equation connecting h with t .

M1: Substitutes $t = 5$. If substitution is not seen you will need to check their answer matches their expression.

A1: Obtains 12.5 m using the model. Must include the units.

Question	Scheme	Marks	AOs
6(a)	Examples: $t = e^x \Rightarrow \frac{dt}{dC} = e^x \frac{dx}{dC} \text{ or } \frac{dC}{dx} = t \frac{dC}{dt} \text{ or } \frac{dC}{dt} = e^{-x} \frac{dC}{dx} \text{ or } \frac{dC}{dt} = \frac{1}{t} \frac{dC}{dx}$	M1	1.1b
	E.g. $\frac{dC}{dx} = t \frac{dC}{dt} \Rightarrow \frac{d^2C}{dx^2} \times \frac{dx}{dt} = t \frac{d^2C}{dt^2} + \frac{dC}{dt}$	dM1 A1	2.1 1.1b
	$\frac{d^2C}{dx^2} \times \frac{1}{t} = t \frac{d^2C}{dt^2} + \frac{1}{t} \frac{dC}{dx} \Rightarrow t^2 \frac{d^2C}{dt^2} = \frac{d^2C}{dx^2} - \frac{dC}{dx}$ $t^2 \frac{d^2C}{dt^2} - 5t \frac{dC}{dt} + 8C = \frac{d^2C}{dx^2} - \frac{dC}{dx} - 5 \frac{dC}{dx} + 8C$	dM1	2.1
	$\frac{d^2C}{dx^2} - 6 \frac{dC}{dx} + 8C = e^{3x} *$	A1*	1.1b
		(5)	
	<u>Mark (b) and (c) together and ignore labelling</u>		
(b)	$m^2 - 6m + 8 = 0 \Rightarrow m = 2, 4$	M1	1.1b
	$(C =) Ae^{4x} + Be^{2x}$	A1ft	1.1b
	PI is $C = ke^{3x}$	B1	2.2a
	$\frac{dC}{dx} = 3ke^{3x}, \frac{d^2C}{dx^2} = 9ke^{3x} \Rightarrow 9k - 18k + 8k = 1 \Rightarrow k = -1$	M1	1.1b
	$C = Ae^{4x} + Be^{2x} - e^{3x}$	A1	1.1b
	$t = e^x \Rightarrow C = \dots$	M1	3.4
	$C = At^4 + Bt^2 - t^3$	A1	2.2a
		(7)	
(c)	$t = 6, C = 0 \Rightarrow 1296A + 36B - 216 = 0$	M1	3.4
	$\frac{dC}{dt} = 4At^3 + 2Bt - 3t^2 \Rightarrow -36 = 864A + 12B - 108$	M1	3.4
	$A = 0, B = 6 \Rightarrow C = 6t^2 - t^3$	A1	1.1b
	$\frac{dC}{dt} = 12t - 3t^2 = 0 \Rightarrow t = 4 \Rightarrow C = \dots$	ddM1	1.1b
	$C = 6(4)^2 - (4)^3 = 32 \mu\text{g L}^{-1}$	A1	3.2a
		(5)	
(17 marks)			
Notes			
(a) M1: Uses $t = e^x$ to obtain a correct equation in terms of $\frac{dC}{dx}$, $\frac{dC}{dt}$ and t (or e^x) or their reciprocals			



dM1: Differentiates again **correctly** with the product rule and chain rule in order to obtain an equation involving $\frac{d^2C}{dt^2}$ and $\frac{d^2C}{dx^2}$. **This needs to be fully correct calculus work allowing sign errors only.**

A1: Correct equation.

dM1: Shows clearly their substitution into the differential equation (or equivalent work) in order to form the new equation. **Dependent on the first method mark and dependent on having obtained two terms for the second derivative.**

Allow substitution for $\frac{dC}{dx}$ and $\frac{d^2C}{dx^2}$ into equation (II) to achieve equation (I)

A1*: Fully correct proof with no errors

(b)

M1: Forms and solves a quadratic auxiliary equation $m^2 - 6m + 8 = 0$

A1ft: Correct form for the CF for their AE solutions **which must be distinct and real**

B1: Deduces the correct form for the PI (ke^{3x})

M1: Differentiates their PI, **which is of the correct form**, and substitutes their derivatives into the DE to find “ k ”

A1: Correct GS for C in terms of x (**this must be seen explicitly unless implied by subsequent work**)

M1: Links the solution to DE (II) to the solution of the model to find the concentration at time t

A1: Deduces the correct GS for the concentration

If a correct GS is fortuitously found in (b) (e.g. from an incorrect PI form, allow full recovery in (c).

(c)

M1: Uses the conditions of the model ($t = 6$, $C = 0$) to form an equation in A and B .

***Note that is acceptable to use their C in terms of x for this mark as long as they use $x = \ln 6$ when $C = 0$

M1: Uses the conditions of the model $\left(t = 6, \frac{dC}{dt} = -36\right)$ to form another equation in A and B .

***Note that it is **not** acceptable to use $\frac{dC}{dx} = -36$ with $x = \ln 6$, as it is necessary to use

$$\frac{dC}{dt} = \frac{dC}{dx} \frac{dx}{dt} \text{ e.g. } -36 = (4Ae^{4\ln 6} + 2Be^{2\ln 6} - 3e^{3\ln 6}) \times e^{-\ln 6} \text{ or } -216 = 4Ae^{4\ln 6} + 2Be^{2\ln 6} - 3e^{3\ln 6}$$

A1: Correct equation connecting C with t

ddM1: Uses a suitable method to find the maximum concentration. E.g. solves $\frac{dC}{dt} = 0$ for t and

substitutes to find C . Allow a solution that solves $\frac{dC}{dx} = 0$ for x and uses this correctly to find C .

Dependent on both previous method marks.

A1: Obtains $32 \mu\text{g L}^{-1}$ using the model. Units are required but allow e.g.

- micrograms per litre
- $\mu\text{g/L}$
- $\mu\text{g/l}$
- $\mu\text{g l}^{-1}$



Question	Scheme	Marks	AOs
8(a)	At 6 hours $t = 0.25$ so “ h ” is 0.25	B1	3.1b
	At $t = 0$ $\frac{dx}{dt} = \frac{3 + \cosh 0}{3 \times 3^2 \cosh 0} - \frac{1}{3}(3) \tanh 0 = \dots \left(= \frac{4}{27} \right)$	M1	3.4
	So $x_1 \approx 3 + "0.25" \times \frac{4}{27} = \dots$	M1	1.1b
	After 6 hours concentration of the pollutant is approximately awrt 3.04 ppm (3 s.f.) or $\frac{82}{27}$ ppm	A1	3.2a
		(4)	
(b)	$\frac{du}{dt} = 3x^2 \times \frac{dx}{dt}$ or $\frac{1}{3x^2} \frac{du}{dt} = \frac{dx}{dt}$ or $\frac{du}{dt} = \frac{du}{dx} \times \frac{dx}{dt} = 3x^2 \left(\frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t \right)$	B1	2.2a
	So $\frac{dx}{dt} = \frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t \rightarrow \frac{1}{3x^2} \frac{du}{dt} = \frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t$ $\rightarrow \frac{du}{dt} = \frac{3}{\cosh t} + 1 - u \tanh t$	M1	2.1
	$\frac{dx}{dt} = \frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t \rightarrow \frac{1}{3u^{\frac{2}{3}}} \frac{du}{dt} = \frac{3 + \cosh t}{3u^{\frac{2}{3}} \cosh t} - \frac{1}{3} u^{\frac{1}{3}} \tanh t$ $\rightarrow \frac{du}{dt} = \frac{3}{\cosh t} + 1 - u \tanh t$		
	$\frac{du}{dt} + u \tanh t = 1 + \frac{3}{\cosh t} *$	A1*	1.1b
		(3)	
(c)	I.F. = $\exp \left(\int \tanh t \, dt \right) = \exp(\ln \cosh t) = \cosh t$	B1	2.2a
	$\Rightarrow u' \cosh t = \int \cosh t' \left(1 + \frac{3}{\cosh t} \right) dt = \left\{ \int \cosh t + 3 \, dt \right\}$	M1	1.1b
	$\Rightarrow u \cosh t = \sinh t + 3t (+c)$	M1	1.1b
	$u \cosh t = \sinh t + 3t + c$ or $u = \tanh t + \frac{3t}{\cosh t} + \frac{c}{\cosh t}$ oe	A1	1.1b
		(4)	
(d)	$t = 0 \Rightarrow x = 3, u = 27 \Rightarrow c = 27 \cosh 0 - \sinh 0 - 3(0) = 27$	M1	3.4
	$\Rightarrow x = \left(\tanh t + \frac{3t + "27"}{\cosh t} \right)^{\frac{1}{3}}$	M1	3.4



	$x = \left(\tanh t + \frac{3t + 27}{\cosh t} \right)^{\frac{1}{3}}$ (oe)	A1	3.2a
		(3)	
(e)	$x(0.25) = \left(\tanh 0.25 + \frac{(0.75 + 27)}{\cosh 0.25} \right)^{\frac{1}{3}} = \dots (= 3.0055\dots)$	M1	3.4
	% error is $\frac{3.0055\dots - 3.037\dots}{3.0055\dots} \times 100 = \dots$	M1	1.1b
	Estimate in (a) is an overestimate by 1.05% (3 s.f.)	A1	3.2a
		(3)	

(17 marks)

Notes:
(a)
B1: Identifies a correct step length for the situation – 6 hours is a quarter of a day, so $h = 0.25$
M1: Uses " $y_0 = x(0) = 3$ and $t = 0$ to find $\left(\frac{dy}{dx} \right)_0 = \left(\frac{dx}{dt} \right)_0$. Accept with whichever notation used, as long as it is clear they are attempting the correct things.

M1: Applies the approximation formula with their " h " and their $\left(\frac{dy}{dx} \right)_0$
A1: For awrt 3.04 ppm. Accept $\frac{82}{27}$ ppm

(b)
B1: A correct equation relating $\frac{du}{dt}$ and $\frac{dx}{dt}$ from the chain rule.

M1: Makes a complete substitution for x and $\frac{dx}{dt}$ in equation (I) or a complete substitution for u and $\frac{du}{dt}$ in equation (II)

A1*: Simplifies correctly to achieve the given result.

(c)
B1: Correct integrating factor found or spotted. Allow for $e^{\ln \cosh t}$
M1: Applies IF to achieve $u \cosh t = \int \cosh t \left(1 + \frac{3}{\cosh t} \right) dt$
M1: A reasonable attempt to integrate the RHS. Need not include constant of integration. If I.F. correct allow for $\pm \sinh t + 3t (+c)$
A1: Correct general solution, either implicit or explicit form including the context of integration (award when first seen and isw)

(d)
M1: Uses the initial conditions in an appropriate equation to find the constant of integration. Either $t = 0$ and $u = 27$ in the answer to (c), or $t = 0$ and $x = 3$ if substitution for x occurs first.


M1: Reverses the substitution and rearranges to find equation for x , with evaluated constant included.

A1: Correct equation, any equivalent form, but must be $x = \dots$

(e)

M1: Uses their model solution to find the value at $t = 0.25$

M1: Applies $\frac{\text{actual value} - \text{estimate}}{\text{actual value}} \times 100$ with their values.

A1: States part (a) is overestimate by 1.05%



Question	Scheme	Marks	AOs
7(a)	Examples: $x = \frac{1}{P^2} \Rightarrow \frac{dx}{dt} = -\frac{2}{P^3} \frac{dP}{dt}$ or $\frac{dP}{dt} = \frac{dP}{dx} \times \frac{dx}{dt} = -\frac{P^3}{2} \frac{dx}{dt}$ or $P = x^{-\frac{1}{2}} \Rightarrow \frac{dP}{dt} = -\frac{1}{2} x^{-\frac{3}{2}} \frac{dx}{dt}$ or $\frac{dP}{dx} = -\frac{1}{2} x^{-\frac{3}{2}} \Rightarrow \frac{dx}{dt} = \frac{dx}{dP} \times \frac{dP}{dt} = -2x^{\frac{3}{2}} \times (4tP^3 - P)$ or $x = \frac{1}{P^2} \Rightarrow xP^2 = 1 \Rightarrow P^2 \frac{dx}{dt} + 2Px \frac{dP}{dt} = 0$ or $x = \frac{1}{P^2} \Rightarrow P^2 = \frac{1}{x} \Rightarrow 2P \frac{dP}{dt} = -\frac{1}{x^2} \frac{dx}{dt}$	M1	3.1a
	$\frac{1}{P} \times -\frac{P^3}{2} \frac{dx}{dt} = \frac{4t}{x} - 1 \Rightarrow -\frac{1}{2x} \frac{dx}{dt} = \frac{4t}{x} - 1$ or $x^{\frac{1}{2}} \times -\frac{1}{2} x^{-\frac{3}{2}} \frac{dx}{dt} = \frac{4t}{x} - 1 \Rightarrow -\frac{1}{2x} \frac{dx}{dt} = \frac{4t}{x} - 1$ or $\frac{dx}{dt} = -2x^{\frac{3}{2}} \times (4tP^3 - P) = -2x^{\frac{3}{2}} \times \left(\frac{4t}{x^2} - \frac{1}{x^{\frac{1}{2}}} \right)$ or $\frac{1}{P} \times -\frac{P}{2x} \frac{dx}{dt} = \frac{4t}{x} - 1 \Rightarrow -\frac{1}{2x} \frac{dx}{dt} = \frac{4t}{x} - 1$ or $\frac{1}{P} \times -\frac{1}{2Px^2} \frac{dx}{dt} = \frac{4t}{x} - 1 \Rightarrow -\frac{1}{2x} \frac{dx}{dt} = \frac{4t}{x} - 1$	M1	1.1b
	$\frac{dx}{dt} - 2x = -8t \quad * \text{ cso}$	A1*	2.1
		(3)	
(b)	$I = e^{\int -2dt} = e^{-2t}$	B1	2.2a
	$xe^{-2t} = \int -8te^{-2t} \{dt\}$	M1	1.1b
	$= 4te^{-2t} - \int 4e^{-2t} \{dt\}$	M1	3.1a
	$= 4te^{-2t} + 2e^{-2t} + k$	A1	1.1b
	$xe^{-2t} = 4te^{-2t} + 2e^{-2t} + k \Rightarrow \frac{1}{0.5^2} = 2 + k \Rightarrow k = \dots \{2\}$ or	M1	3.4

	$xe^{-2t} = 4te^{-2t} + 2e^{-2t} + k \Rightarrow x = 4t + 2 + ke^{2t} \Rightarrow \frac{1}{P^2} = 4t + 2 + ke^{2t}$ $P = 0.5, t = 0 \Rightarrow \frac{1}{0.5^2} = 2 + k \Rightarrow k = \dots\{2\}$		
	$\frac{1}{P^2} = 4t + 2 + 2e^{2t} \Rightarrow P^2 = \frac{1}{4t + 2 + 2e^{2t}} \text{ cso}$	A1	2.1
		(6)	
(c)	$t = 3 \Rightarrow P^2 = \frac{1}{14 + 2e^6} \Rightarrow P = 0.03490326\dots$	M1	3.4
	This value is close to 0.034 so the model is reliable. - Between 0.0323 and 0.0357 good model - Between 0.0306 and 0.0323 or 0.0357 and 0.0374 either a good or bad model with reasoning - Less than 0.0306 or more than 0.0374 bad model	A1ft	3.5a
		(2)	
(11 marks)			
Notes			
(a) M1: Identifies and applies a correct strategy for the differentiation. This may be seen when $\frac{dP}{dt}$ is substituted into the equation. M1: Substitutes into the given differential equation and proceeds to an equation in x and t only. A1*: Correct proof with sufficient working shown and no errors, cso (b) B1: Deduces the correct integrating factor M1: Applies their integrating factor I to obtain $Ix = \int \pm 8It \{dt\}$ condone missing dt M1: Recognises that integration by parts is required and applies this correctly to reach the form $= Ate^{-2t} - \int Be^{-2t} \{dt\}$ A1: Correct integration including an arbitrary constant M1: Uses the conditions given in the model to find the constant of integration. This may be seen before rearranging to get $P^2 = \dots$, condone a slip A1: Correct equation with no errors seen, cso, missing dt during working would lose this mark Note that the first 4 marks in (b) can also be obtained as follows: B1: AE: $m - 2 = 0 \Rightarrow m = 2 \Rightarrow x = Ae^{2t}$ (Correct CF) M1: PI: $x = at + b \Rightarrow \frac{dx}{dt} = a$ (Selects the correct PI form and differentiates) M1: $a - 2at - 2b = -8t \Rightarrow a = \dots(4), b = \dots(2)$ (Substitutes and compares coefficients to find a and b) A1: $x = 4t + 2 + Ae^{2t}$ (Correct expression) (c) M1: Uses the model with $t = 3$ to find P, where $P^2 > 0$ or compares their value of P^2 with $0.001156 = 0.034^2$ A1ft: Compares their value of P with 0.034 and makes a suitable conclusion, making any comparison with P^2 is A0			

Question	Scheme	Marks	AOs
4(a)	Identifies $t_0 = 0$, $v_0 = 0$, $\left(\frac{dv}{dt}\right)_0 = 10$ and $h = 0.5$	B1	3.4
	$v_1 = v_0 + h\left(\frac{dv}{dt}\right)_0 \Rightarrow v_1 = 0 + 0.5 \times 10 = \dots$	M1	1.1b
	$v_1 = 5$	A1	1.1b
	$\left(\frac{dv}{dt}\right)_1 = -0.1(5)^2 + 10 = \dots\{7.5\}$ $v_2 = v_1 + h\left(\frac{dv}{dt}\right)_1 \Rightarrow v_2 = 5 + 0.5 \times 7.5 = \dots$	M1	3.4
	$v_2 = 8.75$ so 8.75 ms^{-1}	A1	1.1b
		(5)	
(b)	$\frac{dv}{dt} = -0.1v^2 + A$ where $0 < A < 10$	B1	3.5c
		(1)	
(6 marks)			
Notes:			
(a) B1: Uses the model to identify the correct initial conditions and requirements for h. May be implied by use in the equation. M1: Applies the approximation formula with their values for v_0, $\left(\frac{dv}{dt}\right)_0$ and h to find a value for v_1 A1: $v_1 = 5$ M1: Uses their v_1 to find a value for $\left(\frac{dv}{dt}\right)_1$ and applies the approximation formula with their values for v_1, $\left(\frac{dv}{dt}\right)_1$ and h to find a value for v_2 A1: $v_2 = 8.75$ or 8.75 ms^{-1}			
(b) B1: Reduce the value of 10 or explains this is what needs reducing, but do not accept 0 or negative values in place of the 10. Note: “change the 10” is B0 if it does not explain how to change it.			