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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Coordinate Systems (4)

Further Pure Mathematics 1 · Topic 4

12 questions · 106 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

12 questions · 106 marks · 4 sub-topics



SUB-TOPIC 4.1

CONICS: CARTESIAN AND PARAMETRIC EQUATIONS

Sub-topic 4.1 · 1 question · 9 marks · lowest tariff first







(Total for Question 5 is 9 marks)



SUB-TOPIC 4.2

FOCUS, DIRECTRIX AND ECCENTRICITY

Sub-topic 4.2 · 4 questions · 25 marks · lowest tariff first



1. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

Find

(a) the coordinates of the foci of E ,

(3)

(b) the equations of the directrices of E .

(2)



Question 1 continued

(Total for Question 1 is 5 marks)



6. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where a and b are positive constants.

Given that

- the eccentricity of H is the reciprocal of the eccentricity of E
- the coordinates of the foci of H are the same as the coordinates of the foci of E

determine

- (i) the value of a
- (ii) the value of b

(6)

Question 6 continued

(Total for Question 6 is 6 marks)



(3)

Question 1 continued

(Total for Question 1 is 7 marks)



5. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

The points S and S' are the foci of E .

- (a) Find the coordinates of S and S'

(3)

- (b) Show that for any point P on E , the triangle PSS' has constant perimeter and determine its value.

(4)



Question 5 continued

(Total for Question 5 is 7 marks)



SUB-TOPIC 4.3

TANGENTS AND NORMALS TO CONICS

Sub-topic 4.3 · 6 questions · 58 marks · lowest tariff first



4. The parabola C has equation

$$y^2 = 16x$$

The distinct points $P(p^2, 4p)$ and $Q(q^2, 4q)$ lie on C , where $p \neq 0, q \neq 0$

The tangent to C at P and the tangent to C at Q meet at the point $R(-28, 6)$.

Show that the area of triangle PQR is 1331

(8)

Question 4 continued





Question 4 continued

(Total for Question 4 is 8 marks)







5. The rectangular hyperbola H has equation $xy = 36$

(a) Use calculus to show that the equation of the tangent to H at the point $P\left(6t, \frac{6}{t}\right)$ is

$$yt^2 + x = 12t$$

(3)

The point $Q\left(12t, \frac{3}{t}\right)$ also lies on H .

(b) Find the equation of the tangent to H at the point Q .

(2)

The tangent at P and the tangent at Q meet at the point R .

(c) Show that as t varies the locus of R is also a rectangular hyperbola.

(4)

Question 5 continued

Lined area for writing the answer to Question 5 continued.

Question 5 continued



(Total for Question 3 is 9 marks)



4. The ellipse E has equation

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

(a) Determine the exact value of the eccentricity of E

(2)

The points $P(4 \cos \theta, 3 \sin \theta)$ and $Q(4 \cos \theta, -3 \sin \theta)$ lie on E where $0 < \theta < \frac{\pi}{2}$
 The line l_1 is the normal to E at the point P

(b) Use calculus to show that l_1 has equation

$$4x \sin \theta - 3y \cos \theta = 7 \sin \theta \cos \theta$$

(4)

The line l_2 passes through the origin and the point Q
 The lines l_1 and l_2 intersect at the point R

(c) Determine, in simplest form, the coordinates of R

(4)

(d) Hence show that, as θ varies, R lies on an ellipse which has the same eccentricity as ellipse E

(2)



(Total for Question 4 is 12 marks)



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued





Question 8 continued

(Total for Question 8 is 14 marks)



Question 8 continued



Question 8 continued



SUB-TOPIC 4.4

LOCI PROBLEMS

Sub-topic 4.4 · 1 question · 14 marks · lowest tariff first





Question 7 continued

Lined area for writing the answer to Question 7 continued.



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
5(a)	Equations of asymptotes of H are $y = \pm \frac{1}{2}x$ oe e.g. $y = \pm \frac{3}{6}x$ or $\frac{x}{6} = \pm \frac{y}{3}$	B1	1.1b
		(1)	
(b)	For parabola $2y \frac{dy}{dx} = 32 \Rightarrow \frac{dy}{dx} = \dots$ or $y = \sqrt{32x} \Rightarrow \frac{dy}{dx} = \dots x^{-\frac{1}{2}}$ or $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \dots$	M1	2.1
	Finds the gradient of the normal using $m_N = \frac{-1}{\text{their } \frac{dy}{dx}}$ $m_N = -\frac{y}{16}$ or $-t$ or $-\frac{\sqrt{x}}{2\sqrt{2}}$ so $m_N = (\pm)\frac{1}{2} \Rightarrow y = (\pm)8, x = 2$	M1	3.1a
	Finds the equation of either l_1 or l_2 $y - "8" = "their m_N"(x - "2")$ or $y - "-8" = "their m_N"(x - "2")$	M1	1.1b
	l_1 is $y + 8 = \frac{1}{2}(x - 2)$ and l_2 is $y - 8 = -\frac{1}{2}(x - 2)$ oe $y = \frac{1}{2}x - 9$ and $y = -\frac{1}{2}x + 9$	A1	1.1b
		(4)	
(c)	Meet $H \Rightarrow \frac{x^2}{36} - \frac{(\pm(\frac{1}{2}x - 9))^2}{9} = 1 \Rightarrow \frac{x^2}{36} - \frac{\frac{1}{4}x^2 - 9x + 81}{9} = 1 \Rightarrow x = \dots$ or $\frac{(18 \pm 2y)^2}{36} - \frac{y^2}{9} = 1 \Rightarrow \frac{81 \pm 18y + y^2}{9} - \frac{y^2}{9} = 1 \Rightarrow y = \dots$	M1	2.1
	One correct point of intersection $(10, \pm 4)$	A1	2.2a
	Area OPQ is $\frac{1}{2} \times 10 \times (4 - (-4)) = \dots$ $-\frac{1}{2} \begin{vmatrix} 10 & 4 & 0 \\ 10 & -4 & 0 \end{vmatrix} = -\frac{1}{2} \hat{=} 40 - 40 \hat{=}$ $-\frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 10 & 4 & 1 \\ 10 & -4 & 1 \end{vmatrix} = -\frac{1}{2} \hat{=} 0 + \hat{=} (10 - 4) - (10 - 4) \hat{=}$	dM1	1.1b
	$= 40$	A1	1.1b
		(4)	

(9 marks)

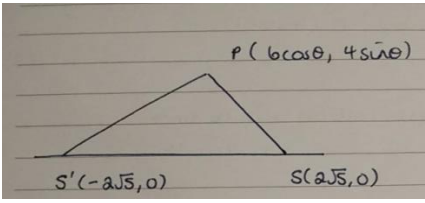
Notes:**(a)****B1:** Correct equations for the asymptotes of H seen or implied, any form and need not be simplified.**(b) Note M1 M1 A1 A1 on ePen****M1:** A correct method to find the gradient of the parabola.**M1:** Finds the gradient of the normal and sets their normal gradient equal to their asymptote gradient to obtain at least one point on C where normal is parallel to an asymptote**M1:** Finds the equation of either l_1 or l_2 **A1:** Correct equation for each normal, $y + 8 = \frac{1}{2}(x - 2)$ and $y - 8 = -\frac{1}{2}(x - 2)$. Ignore labelling.**(c)****M1:** Substitutes for x or y into the equation of the hyperbola and solves for their variable.**A1:** Achieves one correct coordinate $x = 10$ and $y = \pm 4$ **dM1:** Dependent on previous method. Correct method for the area of their triangle e.g, $\frac{1}{2} \times \text{their } 10 \times \text{twice their } 4$ or equivalent determinant methods.**A1:** Area is 40. Correct answer only.

Question	Scheme	Marks	AOs
1(a)	Uses $b^2 = a^2(1 - e^2)$ to find a value of e look for $20 = 36(1 - e^2)$	M1	1.1b
	$e = \frac{2}{3} \Rightarrow$ foci are $(\pm 6 \times \text{"their } e", 0)$	dM1	1.1b
	Foci are $(\pm 4, 0)$	A1	1.1b
		(3)	
	Alternative Sets up an equation such as $2\sqrt{p^2 + b^2} = 2a$ where p is the x coordinate of the foci $2\sqrt{p^2 + 20} = 12$	M1	1.1b
	Solves to find the value of p	dM1	1.1b
	Foci are $(\pm 4, 0)$	A1	1.1b
		(3)	
(b)	Directrices are $x = (\pm) \frac{6}{\text{their } e}$	M1	1.1b
	$x = \pm 9$ only	A1	1.1b
		(2)	
(5 marks)			
Notes:			
(a) M1: Uses $b^2 = a^2(1 - e^2)$ to obtain a value of e (allow if $-\frac{2}{3}$ also given) dM1: Uses $a = 6$ and their value of e with $0 < e < 1$, to find at least one focus using $((\pm)ae, 0)$ A1: Correct foci – both required, including y coordinates. Alternative M1: Sets up an equation using total distance from foci to point on ellipse = $2a$ dM1: Solves to find a value for the x coordinate of the foci A1: Correct foci – both required, including y coordinates.			
(b) M1: Uses $x = (\pm) \frac{a}{e}$ with $a = 6$ and their e to attempt directrices. A1: Correct directrices, both required and no other lines			



Question	Scheme	Marks	AOs
6	$9 = 25(1 - e^2) \Rightarrow e = \dots \left\{ \frac{4}{5} \right\}$	M1	1.1b
	$ae = 5 \times \frac{4}{5} = \dots \{4\}$	dM1	1.1b
	A complete method to find the value of a or b e.g. $e = \frac{1}{\text{their } \frac{4}{5}} = \dots \left\{ \frac{5}{4} \right\}$ and uses $a = \frac{\text{their '4'}}{\text{their } \frac{5}{4}} = \dots \left\{ \frac{16}{5} \right\}$	M1	3.1a
	$a = \frac{16}{5}$ or $b = \frac{12}{5}$	A1	1.1b
	E.g. Uses $b^2 = \left(\text{their } \frac{16}{5} \right)^2 \left(\left(\text{their } \frac{5}{4} \right)^2 - 1 \right)$ leading to $b = \dots$	M1	3.1a
	$a = \frac{16}{5}$ and $b = \frac{12}{5}$	A1	3.2a
		(6)	
(6 marks)			
Notes:			
M1: A complete method to find the eccentricity of E. Uses $b^2 = a^2(1 - e^2)$ to find a value for e or e^2. Ignore references to $\pm$ for the Ms. Note that identifying a and b correctly is part of the method. dM1: Dependent on the previous method mark. Finds the x coordinate of the focus of E, 5 multiplied by their value of e (or implied if working with squared forms). M1: A complete method to find the value of a. Finds the reciprocal of their eccentricity of E and uses their x coordinate of the focus of E divided by their reciprocal of e. Note they may use $e^2 = 1 + \frac{b^2}{a^2}$ and $x_F = \sqrt{a^2 + b^2}$ for the hyperbola to form and solve suitable equations. So for example $a^2 + b^2 = "4"'^2$ and $"\frac{5}{4}"'^2 = 1 + \frac{b^2}{a^2}$ leading to a value for a or b. A1: Correct value of a or b. Accept as a decimal. M1: A complete method to find a value of b or a if b found first. Uses $b^2 = a^2(e^2 - 1)$ with their values for e and a to find a value for b or vice versa, allowing slips in substitution as long as the correct formula is clear. A1: Correct values for both a and b. Accept as a decimals.			

Question	Scheme	Marks	AOs
1(a)	$b^2 = a^2(1 - e_1^2) \Rightarrow 4 = 16(1 - e_1^2) \Rightarrow e_1^2 = \dots$	M1	1.1b
	$e_1^2 = \frac{3}{4}$ or $e_1 = \frac{\sqrt{3}}{2}$	A1	1.1b
	E.g. $b^2 = a^2(e_2^2 - 1) = a^2\left(\frac{1}{e_1^2} - 1\right) = a^2\left(\frac{4}{3} - 1\right)$	dM1	2.1
	$\Rightarrow b^2 = \frac{1}{3}a^2 \Rightarrow a^2 = 3b^2$ * cso	A1*	1.1b
		(4)	
(b)	For the focus of the ellipse $(x =) 4 \times \frac{\sqrt{3}}{2}$,	M1	1.1b
	For focus of the hyperbola $(x =) a \times \frac{2}{\sqrt{3}} \Rightarrow 2\sqrt{3} = \frac{2a}{\sqrt{3}} \Rightarrow a = \dots (= 3)$ $\Rightarrow b^2 = \frac{1}{3}a^2 = \dots$	M1	3.1a
	$\frac{x^2}{9} - \frac{y^2}{3} = 1$ cso	A1	2.2a
		(3)	
(7 marks)			
Notes:			
(a) M1: Uses "$b^2 = a^2(1 - e_1^2)$" with values for a and b to find a value for e_1 or e_1^2. They may just call it e and will likely use a and b before substituted, which is fine. The formula must be correct but allow slips with a and b. A1: Correct exact value for e_1 or e_1^2. Note: allow M1A1 here if the relevant work is seen in (b). dM1: Dependent on previous method mark. Uses $e_1 \times e_2 = 1$ with their e_1 or e_1^2 to find an expression between a and b. May find an expression for $e_2^{(2)}$ and apply $e_1 \times e_2 = 1$ directly or may first substitute as per scheme. Any full method. SC: Allow M0A0dM1A0 if $b^2 = a^2(1 - e_1)$ and $b^2 = a^2(e_2 - 1)$ are used in an otherwise correct process. A1*: Achieves $a^2 = 3b^2$ with at least one intermediate unsimplified equation in a and b cso			
(b) M1: Uses/implies x coordinate of focus for the ellipse is $4 \times$ their e_1 M1: For a full process to find values for a and b or their squares. E.g. for focus of hyperbola $x = a \times$ their $e_2 = \frac{a}{e_1}$ sets equal to $4e_1$ and solves for a then attempting to use $a^2 = 3b^2$ to obtain b^2 (or b). Other methods are possible. A1: Deduces the correct equation for the hyperbola.			

Question	Scheme	Marks	AOs
5(a)	$b^2 = a^2(1 - e^2) \Rightarrow 16 = 36(1 - e^2) \Rightarrow e = \dots$	M1	1.1b
	$e^2 = \frac{20}{36}$ or $\frac{5}{9}$ or $e = \frac{\sqrt{5}}{3}$	A1	1.1b
	Foci are $(\pm 2\sqrt{5}, 0)$	A1	1.1b
		(3)	
(b)	Perimeter = $PS + PS' + SS'$ where $PS + PS' = e(PM + PM') = \dots$	M1	3.1a
	$= e \times \frac{2a}{e} = \dots$	M1	2.2a
	$\dots + 2 \times 2\sqrt{5}$	B1ft	1.1b
	$= 12 + 4\sqrt{5}$ hence perimeter is constant for any P on E .*	A1*	2.1
		M1	3.1a
	Perimeter = $PS + PS' + SS'$ $PS = \sqrt{(2\sqrt{5} - 6\cos q)^2 + (4\sin q)^2} = \dots \{6 - 2\sqrt{5}\cos q\}$ $PS' = \sqrt{(2\sqrt{5} + 6\cos q)^2 + (4\sin q)^2} = \dots \{6 + 2\sqrt{5}\cos q\}$		
	$PS + PS' = (6 - 2\sqrt{5}\cos q) + (6 + 2\sqrt{5}\cos q) = 12$	M1	2.2a
	$\dots + 2 \times 2\sqrt{5}$	B1ft	1.1b
	$= 12 + 4\sqrt{5}$ hence perimeter is constant for any P on E .*	A1*	2.1
		(4)	
(7 marks)			
Notes:			
(a)			
M1: Uses $b^2 = a^2(1 - e^2)$ with $a = 6$ and $b = 4$ to find a value for e^2 or e .			
A1: Correct value for e or e^2			
A1: Correct foci			
(b)			
M1: Forms a complete strategy to find the perimeter using general P and applies the focus directrix property to the sides PS and PS'			
M1: Deduces the length of the two sides adjacent to P is a constant			

B1ft: Uses SS' is twice their ae from (a)

A1*: Finds the value and makes conclusion that perimeter is constant for any P on E

Alternative

M1: Forms a complete strategy to find the perimeter using general P . Finds the lengths of PS and PS' using Pythagoras theorem and the general coordinate $(6\cos q, 4\sin q)$

M1: Deduces the length of the two sides adjacent to P is a constant, using trig identities.

B1ft: Uses SS' is twice their ae from (a)

A1*: Finds the value and makes conclusion that perimeter is constant for any P on E

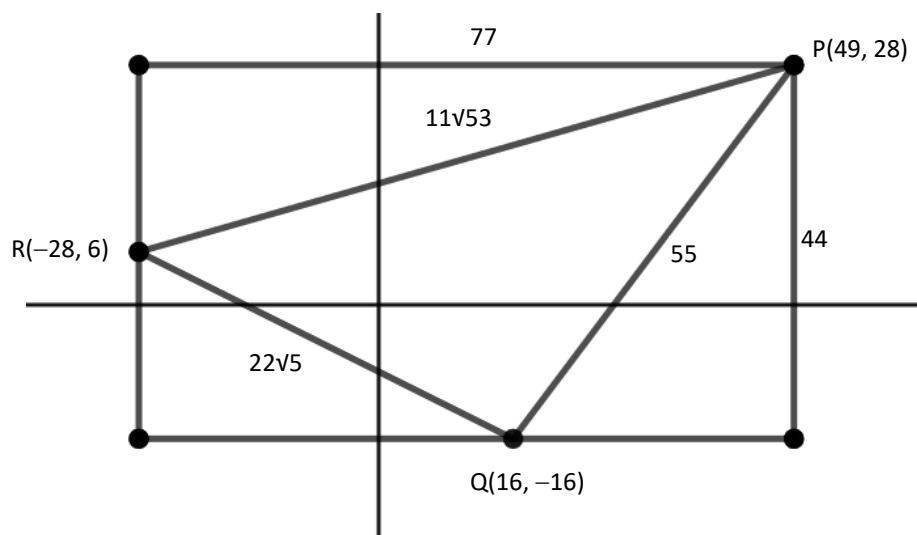
Note: Using the property that $PS + PS' = 2a$ both method marks may be awarded as long as a reason is given e.g. definition/property of an ellipse



4	$y^2 = 16x \Rightarrow 2y \frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y} = \frac{8}{4p}$ Requires $\alpha y \frac{dy}{dx} = \beta \Rightarrow \frac{dy}{dx} = f(p \text{ or } q)$ $y^2 = 16x \Rightarrow \frac{dy}{dx} = 2x^{-\frac{1}{2}} = 2(p^2)^{-\frac{1}{2}}$ Requires $\frac{dy}{dx} = \alpha x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = f(p \text{ or } q)$ $\frac{dy}{dx} = \frac{dy}{dp} \frac{dp}{dx} = \frac{4}{2p}$ Requires $\frac{dy}{dx} = \text{their } \frac{dy}{dp} \div \text{their } \frac{dx}{dp} \Rightarrow \frac{dy}{dx} = f(p \text{ or } q)$	M1	3.1a
	$\frac{dy}{dx} = \frac{8}{4p} \Rightarrow y - 4p = \frac{2}{p}(x - p^2) \text{ or } y - 4q = \frac{2}{q}(x - q^2)$	M1 A1	3.1a 1.1b
	Using $x = -28$ and $y = 6$, $6p = -56 + 2p^2 \Rightarrow p = \dots$	M1	3.1a
	Alternative for 3rd Method mark: $py = 2x + 2p^2, qy = 2x + 2q^2 \Rightarrow x = pq, y = 2(p + q)$ Using $x = -28$ and $y = 6 \Rightarrow p(\text{or } q) = \dots$		
	$p \text{ (or } q) = -4, 7$	A1	1.1b
	$(16, -16), (49, 28)$	A1	2.2a
	Way 1 $\frac{1}{2} \begin{vmatrix} -28 & 16 & 49 & -28 \\ 6 & -16 & 28 & 6 \end{vmatrix} = \frac{1}{2} 448 + 448 + 294 - 96 + 784 + 784 $ Way 2 $77 \times 44 - \frac{1}{2} \times 44 \times 22 - \frac{1}{2} \times 77 \times 22 - \frac{1}{2} \times 44 \times 33$ Way 3 $\frac{1}{2} 22\sqrt{5} \times 11\sqrt{53} \sin \left(\cos^{-1} \left(\frac{(11\sqrt{53})^2 + (22\sqrt{5})^2 - 55^2}{2 \times 11\sqrt{53} \times 22\sqrt{5}} \right) \right)$ NB angle at R is 42.5 (1dp) Way 4 $\frac{1}{2} 55 \times 11\sqrt{53} \sin \left(\cos^{-1} \left(\frac{(11\sqrt{53})^2 + 55^2 - (22\sqrt{5})^2}{2 \times 11\sqrt{53} \times 55} \right) \right)$ NB angle at P is 37.2 (1dp) Way 5 $\frac{1}{2} 55 \times 22\sqrt{5} \sin \left(\cos^{-1} \left(\frac{(22\sqrt{5})^2 + 55^2 - (11\sqrt{53})^2}{2 \times 22\sqrt{5} \times 55} \right) \right)$ NB angle at Q is 100.3 (1dp)	M1	3.1a

	Way 6 $S = \frac{55 + 22\sqrt{5} + 11\sqrt{53}}{2} \Rightarrow A = \sqrt{S(S-55)(S-22\sqrt{5})(S-11\sqrt{53})}$		
	Way 7 Line PR $y - 28 = \frac{28-6}{49+28}(x-49), x=16 \Rightarrow y = \frac{130}{7}$ $A = \frac{1}{2} \times \frac{242}{7}(28+16) + \frac{1}{2} \times \frac{242}{7}(49-16)$		
	Way 8 $\frac{1}{2} RP \times QP = \frac{1}{2} \left \begin{pmatrix} 77 \\ 22 \end{pmatrix} \times \begin{pmatrix} 33 \\ 44 \end{pmatrix} \right = \frac{1}{2}(2662)$ For such methods, a minimum of e.g. $\frac{1}{2}(2662)$ must be seen		
	$= 1331 \text{ (units}^2\text{)*}$	A1*	1.1b
		(8)	
(8 marks)			
Notes			
M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{dy}{dx}$ in terms of p or q. See scheme for requirements for this mark depending on the method chosen. (Can be implied by a correct expression) M1: Correct straight line method to find the equation of the tangent using P or Q. If using $y = mx + c$, must reach as far as $c = \dots$ A1: Obtains a correct general tangent at P or Q or both Note that if a correct tangent equation is quoted, the first 3 marks are available M1: Uses $x = -28$ and $y = 6$ with the values correctly placed in one of their tangent equations and attempts to solve the resulting 3TQ to obtain 2 values for p (or q). An alternative approach for this mark is to obtain equations for both tangents and solve simultaneously to obtain the coordinates for the intersection and then to use $x = -28$ and $y = 6$ to find values for p and q. Note that a calculator may be used for the simultaneous equations but answers must be correct for their equations if no working is shown. A1: Correct values A1: Deduces the correct coordinates of P and Q M1: Completes the problem by using a suitable complete correct method for finding the area of PQR – See examples – there will be others – in general, score M1 for a correct triangle area method for their values A1*: Correct area. Allow this mark even if the candidate reverts to decimals within their solution, providing all the working is correct.			





Generally, using midpoints of sides is unlikely to be successful, however, the line from R to the midpoint of PQ is horizontal so this is a correct approach:

$$\text{Midpoint: } \left(\frac{49+16}{2}, \frac{28-16}{2} \right) = \left(\frac{65}{2}, 6 \right) \Rightarrow \text{Area} = \frac{1}{2} \left(\frac{65}{2} + 28 \right) \times 44 = 1331$$

Question	Scheme	Marks	AOs
8(a)	$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$ or $2y \frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{4a}{2(2at)} = \frac{1}{t}$ or $y = 2\sqrt{a}\sqrt{x} \Rightarrow \frac{dy}{dx} = 2\sqrt{a} \times \frac{1}{2\sqrt{x}} = \frac{\sqrt{a}}{\sqrt{at^2}} = \frac{1}{t}$	B1	1.1b
	$y - 2at = \frac{1}{t}(x - at^2)$	M1	1.1b
	$yt = x + at^2$ *	A1*	2.1
		(3)	
(b)	Equations of the tangents $yk = x + 2k^2$ and/or $-yk = x + 2k^2$	B1	2.2a
	Solves simultaneously $\left. \begin{array}{l} yk = x + 2k^2 \\ -yk = x + 2k^2 \end{array} \right\} \Rightarrow x = \dots \{-2k^2\}, y = \dots \{0\}$	M1	3.1a
	Finds the area of the triangle, sets equal to 432 and solves to find a value for k $\frac{8k \times (2k^2 - \text{their } '-2k^2\text{'})}{2} = 432$ leading to $k = \dots$	M1	3.1a
	$k = 3$	A1	1.1b
	$(18, 12)$ and $(18, -12)$	A1	2.2a
		(5)	
(8 marks)			

Notes:**(a)**

B1: Correct derivative in terms of t from a correct calculus method. Just stating $\frac{dy}{dx} = \frac{1}{t}$ with no supporting working is B0.

M1: Uses $y - 2at = \text{their } \frac{1}{t}(x - at^2)$, or uses $y = \frac{1}{t}x + c$ with $(at^2, 2at)$ to find c . They may have just stated the $\frac{dy}{dx} = \frac{1}{t}$ for this mark.

A1*: Achieves the printed equation with no errors, but allow if the $\frac{dy}{dx} = \frac{1}{t}$ is stated without working (so B0M1A1 is possible).

(b)

B1: Deduces a correct equation for one of the tangents. May find both but accept for one correct. If working in terms of a or a and t they must identify $a = 2$ or $at^2 = -2k^2$ (oe as relevant) at some stage to gain this mark following a correct tangent in terms of a or a and t .

M1: Finds the intersection of the tangent with the x -axis, or solves both tangent equations simultaneously to find at least the x coordinate of the point of intersection. May be in terms of a and t for this mark.

Note the first 2 marks can be implied from stating a correct point of intersection

M1: A complete method to find the value of k . E.g. uses $\frac{8k \times (2k^2 - \text{their } '-2k^2')}{2} = 432$ and solves to find a value for k . Alternative approaches are possible, such as

$$432 = \frac{1}{2} |\overrightarrow{DC} \times \overrightarrow{DB}| = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4k^2 & 4k & 0 \\ 4k^2 & -4k & 0 \end{vmatrix} = \frac{1}{2} |-32k^3 \mathbf{k}| = 16k^3 \Rightarrow k = \dots \text{ or}$$

$$432 = \frac{1}{2} \begin{vmatrix} -2k & 2k & 2k & -2k \\ 0 & 4k^2 & -4k^2 & 0 \end{vmatrix} = \frac{1}{2} |(-8k^3 - 8k^3 + 0 - 0 - 8k^3 - 8k^3)| = 16k^3 \Rightarrow k = \dots$$

A1: $k = 3$

A1 Deduces the correct coordinates $(18, 12)$ and $(18, -12)$ Do not be concerned with the labelling.



Question	Scheme	Marks	AOs
9(i)(a)	$\left. \begin{aligned} 2+3\lambda &= 13+\mu \\ -3+4\lambda &= 5-2\mu \\ 1-\lambda &= 8+5\mu \end{aligned} \right\} \text{leading to } \lambda=\dots \text{ or } \mu=\dots \quad (\text{Note } \lambda=3, \mu=-2)$	M1	3.1a
	$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ -2 \end{pmatrix} \text{ or } \begin{pmatrix} 13 \\ 5 \\ 8 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ -2 \end{pmatrix}$	A1	1.1b
		(2)	
(i)(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 4 & -1 \\ 1 & -2 & 5 \end{vmatrix} = \mathbf{i}(20-2) - \mathbf{j}(15+1) + \mathbf{k}(-6-4)$	M1	3.1a
	$\pm(18\mathbf{i}-16\mathbf{j}-10\mathbf{k})$	A1	1.1b
	e.g. $\mathbf{r} \bullet \begin{pmatrix} 18 \\ -16 \\ -10 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 18 \\ -16 \\ -10 \end{pmatrix} = \dots$	M1	1.1b
	$18x-16y-10z=74$ o.e. $k(9x-8y-5z=37)$	A1	2.5
		(4)	
(ii)	$\cos^2 60^\circ + \cos^2 45^\circ + \cos^2 \theta = 1$ leading to $\cos \theta = \dots$ $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + \cos^2 \theta = 1$	M1	3.1a
	$\cos \theta = (\pm)\frac{1}{2}$	A1	1.1b
	$\frac{x}{\cos 60^\circ} = \frac{y}{\cos 45^\circ} = \frac{z}{\text{their } \cos \theta} \text{ or } \frac{x}{\left(\frac{1}{2}\right)} = \frac{y}{\left(\frac{\sqrt{2}}{2}\right)} = \frac{z}{\text{their } \cos \theta}$	M1	1.1b
	$2x = \sqrt{2}y = 2z$ and $2x = \sqrt{2}y = -2z$ o.e	A1	2.5
		(4)	
(10 marks)			
Notes:			
Note: Accept any alternative vector forms of notation throughout. (i)(a) M1: Forms and solves two equations to find the value of λ or μ. A1: Correct point of intersection. Accept as coordinates or vector (oe). (i)(b) M1: Finds the cross product of the direction vectors of the lines. If no method shown, two correct components implies the method. A1: Correct normal vector.			



M1: Dependent on having made some attempt at the cross product. Find the equation of a plane using $\mathbf{r} \cdot (\text{their cross product}) = (\text{point on the plane}) \cdot (\text{their cross product}) = \dots$ Condone minor miscopies of coordinates.

A1: Correct Cartesian equation of the plane. Accept any multiple of it.

(ii)

M1: Uses $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ to find the direction cosine to the z-axis. Allow with n or another letter in place of $\cos \theta$

A1: A correct answer for the direction cosine to the z-axis.

M1: Finds a Cartesian equation of one of the lines.

A1: Two correct Cartesian equations of the line, need not be simplified. Accept for both equation given, don't be concerned if "and" or "or" is used.

Question 9 (i) Alts

(i)(b) Alt 1	Plane is $ax + by + cz = 1$, so $\begin{cases} 2a - 3b + c = 1 \\ 13a + 5b + 8c = 1 \\ 11a + 9b - 2c = 1 \end{cases}$	M1 A1	3.1a 1.1b
	$\begin{cases} 2a - 3b + c = 1 \\ 13a + 5b + 8c = 1 \\ 11a + 9b - 2c = 1 \end{cases} \Rightarrow \begin{cases} 17a + c = 4 \\ 49a + 29c = 8 \end{cases} \Rightarrow a = \dots, b = \dots, c = \dots$	M1	1.1b
	$\frac{9}{37}x - \frac{8}{37}y - \frac{5}{37}z = 1$ or $9x - 8y - 5z = 37$ o.e.	A1	2.5
		(4)	
Notes: M1: Forms 3 equations in 3 unknowns using three points on the line (or may form four equations in four unknowns) A1: All correct relevant equations. M: Full process to solve the equations – which may be by calculator. Accept for any solutions appearing after setting up suitable equations. A1: Correct Cartesian equation of the plane.			
(i)(b) Alt 2	Let normal vector be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ Then $(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \cdot (3, 4, -1) = 0 = (a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \cdot (1, -2, 5) \Rightarrow$ $\begin{cases} 3a + 4b - c = 0 \\ a - 2b + 5c = 0 \end{cases} \Rightarrow 10b - 16c = 0 \Rightarrow b = \frac{8}{5}c, a = -\frac{9}{5}c$	M1	3.1a
	$\mathbf{n} = \pm k(9\mathbf{i} - 8\mathbf{j} - 5\mathbf{k})$	A1	1.1b
	Finds the Cartesian equation of the plane e.g. $\mathbf{r} \cdot \mathbf{n} = (11\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}) \cdot (9\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}) = \dots$	M1	1.1b
	$9x - 8y - 5z = 37$ o.e.	A1	2.5
		(4)	
Notes: M1: Sets up the normal vector in terms of variables (may set one of them as 1 or another value) and takes the dot product with both directions to form and solve equations to find the normal. A1: Correct normal vector. M1: Dependent on having made some attempt at the normal vector. Finds the equation of a plane using $\mathbf{r} \cdot (\text{their normal vector}) = (\text{point on the plane}) \cdot (\text{their normal vector}) = \dots$ Condone minor miscopies of coordinates. A1: Correct Cartesian equation of the plane. Accept any multiple of it.			



Question 9(ii) Alt

(ii) Alt	Let (unit) direction vector of line be $\mathbf{d} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ then	M1	3.1a
	$\mathbf{d} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \ \mathbf{d}\ \cos 60^\circ = \frac{\ \mathbf{d}\ }{2}$ and $\mathbf{d} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \ \mathbf{d}\ \cos 45^\circ = \frac{\ \mathbf{d}\ }{\sqrt{2}}$		
	$a = \frac{\ \mathbf{d}\ }{2}, b = \frac{\ \mathbf{d}\ }{\sqrt{2}}$	A1	1.1b
	$\mathbf{d} = \ \mathbf{d}\ \left(\frac{1}{2}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} + c\mathbf{k} \right) \Rightarrow c = \sqrt{1 - \frac{1}{4} - \frac{1}{2}} = \dots$ $\Rightarrow \mathbf{d} = \ \mathbf{d}\ \left(\frac{1}{2}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} \pm \frac{1}{2}\mathbf{k} \right) \Rightarrow \frac{x}{1/2} = \frac{y}{1/\sqrt{2}} = \frac{z}{(\pm)1/2}$	M1	1.1b
	$2x = \sqrt{2}y = 2z$ and $2x = \sqrt{2}y = -2z$ o.e	A1	2.5
		(4)	
Notes:			
(ii) M1: Sets up the direction vector for the line in unknowns (use of unit vector is fine) and applies dot product with vectors in direction of x and y axes. Note, may quote the results directly as $\cos 60^\circ = \frac{x}{\ \mathbf{d}\ }$ etc, for the projections on to the axes. A1: Correct values for/expression in a and b (the x and y components of direction vector) or multiples of them. M1: Full process to find the third ordinate for the direction vector, and proceeds to form the Cartesian equation of at least one of the lines. A1: Two correct Cartesian equations of the line, need not be simplified.			

Question	Scheme	Marks	AOs
5 (a)	$y + x \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = -\frac{\frac{6}{t}}{6t} = -\frac{1}{t^2}$ or $y = \frac{36}{x} \Rightarrow \frac{dy}{dx} = -\frac{36}{x^2} = -\frac{36}{(6t)^2} = -\frac{1}{t^2}$ or $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-6t^{-2}}{6} = -\frac{1}{t^2}$	M1	1.1b
	$y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$	M1	1.1b
	$yt^2 + x = 12t^*$	A1 *	2.1
		(3)	
(b)	$\frac{dy}{dx} = -\frac{y}{x} = -\frac{\frac{3}{t}}{12t} = -\frac{1}{4t^2}$ and $y - \frac{3}{t} = -\frac{1}{4t^2}(x - 12t)$	M1	1.1b
	$y - \frac{3}{t} = -\frac{1}{4t^2}(x - 12t)$ o.e. such as $4yt^2 + x = 24t$	A1	1.1b
		(2)	
(c)	E.g. $\begin{cases} 4yt^2 + x = 24t \\ yt^2 + x = 12t \end{cases} \Rightarrow 3yt^2 = 12t \Rightarrow y = \dots$ and $x = 12t - yt^2 = \dots$	M1	2.1
	$x = 8t$ and $y = \frac{4}{t}$	A1	1.1b
	$xy = \dots$	dM1	1.1b
	$xy = 32$ hence rectangular hyperbola	A1	2.4
		(4)	
(9 marks)			
Notes:			
(a)			
M1: Differentiates implicitly, directly or parametrically to find the gradient at the point P in terms of t . Allow slips in coefficients, as long as method is clear.			
M1: Finds the equation of the tangent at the point P using their gradient (not reciprocal etc). If using $y = mx + c$ must proceed to find c and substitute back in to equation.			
A1*: The correct equation for the tangent at the point P from correct working.			
(b)			
M1: Finds the new gradient (any method as above) and proceeds to find the equation of the tangent at the point Q . Alternatively replaces t by $2t$ in the answer to (a).			
A1: Correct equation - any form, need not be simplified and isw after a correct equation.			
(c)			
M1: Solves their simultaneous equations to find both the x and y coordinate for the point R .			
A1: Correct point of intersection, it does not need to be simplified.			
dM1: Dependent on the first method mark. Multiplies x by y to reach a constant.			
A1: Shows that $xy = 32$ and hence rectangular hyperbola			

Question	Scheme	Marks	AOs
4(a)	$b^2 = a^2(1 - e^2) \Rightarrow 9 = 16(1 - e^2) \Rightarrow e^2 = \dots$	M1	1.1b
	$e = \frac{\sqrt{7}}{4}$	A1	1.1b
		(2)	
(b)	$\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \frac{x}{8} + \frac{2yy'}{9} = 0 \Rightarrow y' = -\frac{9x}{16y} = -\frac{36\cos\theta}{48\sin\theta}$ or $x = 4\cos\theta, y = 3\sin\theta \Rightarrow \frac{dy}{dx} = -\frac{3\cos\theta}{4\sin\theta}$	M1 A1	2.1 1.1b
	$m_N = \frac{4\sin\theta}{3\cos\theta} \Rightarrow y - 3\sin\theta = \frac{4\sin\theta}{3\cos\theta}(x - 4\cos\theta)$	M1	1.1b
	$3y\cos\theta - 9\sin\theta\cos\theta = 4x\sin\theta - 16\sin\theta\cos\theta$ $\Rightarrow 4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta^*$	A1*	2.1
		(4)	
(c)	$y = -\frac{3\sin\theta}{4\cos\theta}x$ or $y + 3\sin\theta = -\frac{3\sin\theta}{4\cos\theta}(x - 4\cos\theta)$	B1	2.2a
	$4x\sin\theta + \frac{9x\sin\theta\cos\theta}{4\cos\theta} = 7\sin\theta\cos\theta \Rightarrow x = \dots$ or $-\frac{16y\sin\theta\cos\theta}{3\sin\theta} - 3y\cos\theta = 7\sin\theta\cos\theta \Rightarrow y = \dots$	M1	3.1a
	$x = \frac{28\cos\theta}{25}$ or $y = -\frac{21\sin\theta}{25}$	A1	1.1b
	$x = \frac{28\cos\theta}{25}$ and $y = -\frac{21\sin\theta}{25}$	A1	1.1b
		(4)	
(d)	$x = \frac{28\cos\theta}{25}, y = -\frac{21\sin\theta}{25} \Rightarrow a = \frac{28}{25}, b = -\frac{21}{25}$ $\left(\frac{21}{25}\right)^2 = \left(\frac{28}{25}\right)^2(1 - e^2) \Rightarrow \frac{441}{625} = \frac{784}{625}(1 - e^2) \Rightarrow e = \dots$	M1	3.1a
	Of form $(a\cos\phi, b\sin\phi)$ (where $\phi = -\theta$) so an ellipse, and $e = \frac{\sqrt{7}}{4}$ as required.	A1cso	1.1b
		(2)	
(12 marks)			
Notes			
(a) M1: Uses the correct eccentricity formula and the given equation to find a value for e or e^2 . A1: Correct exact value $\frac{\sqrt{7}}{4}$, must be simplified. Must reject the negative value, so A0 if $-\frac{\sqrt{7}}{4}$ also included.			

(b)

M1: Starts the process of establishing the gradient of the normal by adopting a suitable method to differentiate such as using implicit differentiation or parametric differentiation to obtain an expression for $\frac{dy}{dx}$ in terms of θ . The substitution for x and y may happen later, but must be in terms of θ when used in the equation for normal.

A1: Correct unsimplified $\frac{dy}{dx}$ in terms of θ . May be implied (if substitution happens later).

M1: Applies the negative reciprocal to their gradient and uses this and the coordinates of P to establish an equation of the normal. If using $y = mx + c$ they must proceed as far as finding c .

A1*: Fully correct proof with sufficient working shown. Must be an intermediate step between the initial equation and the final answer. Do not penalise minor notational slips such as missing some θ 's if the working is clear, but award A0 if they are consistently missed.

(c)

B1: Deduces the correct equation of OQ . Need not be simplified - may use point Q .

M1: Proceeds to solve their OQ line simultaneously with the given l_1 to find either the x or the y coordinate of the intersection.

A1: One correct coordinate simplified or unsimplified.

A1: Both correct simplified coordinates. (Accept decimal equivalents.)

(d)

M1: Uses their coordinates of R (of an appropriate form) correctly to determine the values of " a " and " b " for the second ellipse and uses their values with the correct eccentricity formula in an attempt to show that the eccentricities are the same.

A1cso: Must have had correct coordinates in (c), a statement or deduction (or work shown) to verify the locus of R is an ellipse (conclusion not needed, e.g. accept if the equation is given to show it is an ellipse), and proceeds to show that the value of e (or e^2 - do not penalise inclusion of the negative value a second time) is the same as that obtained in part (a) or equivalent work. Note that the correct eccentricity can arise from coordinates of R where the denominator was incorrect, but these will score A0.

Question	Scheme	Marks	AOs
8(a)	$\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow \frac{x}{8} - \frac{2yy'}{9} = 0 \Rightarrow y' = \frac{9x}{16y} = \frac{36 \cosh \theta}{48 \sinh \theta}$ or $x = 4 \cosh \theta, y = 3 \sinh \theta \Rightarrow \frac{dy}{dx} = \frac{3 \cosh \theta}{4 \sinh \theta}$	M1	3.1a
	$y - 3 \sinh \theta = \frac{3 \cosh \theta}{4 \sinh \theta} (x - 4 \cosh \theta)$	M1	3.1a
	$y = 0 \Rightarrow x = \frac{4}{\cosh \theta}$	A1	2.2a
	line l_2 has equation $x = 4$	B1	2.2a
	$x = 4 \Rightarrow y - 3 \sinh \theta = \frac{3 \cosh \theta}{4 \sinh \theta} (4 - 4 \cosh \theta)$	M1	2.1
	$y = \frac{3 \cosh \theta - 3}{\sinh \theta}$	A1	2.2a
	$M \text{ is } \left(\frac{1}{2} \left(4 + \frac{4}{\cosh \theta} \right), \frac{1}{2} \left(\frac{3 \cosh \theta - 3}{\sinh \theta} \right) \right)$	M1	1.1b
	$x = 2 + \frac{2}{\cosh \theta} \Rightarrow \cosh \theta = \frac{2}{x-2}$ $\Rightarrow y^2 = \frac{9(\cosh \theta - 1)^2}{4 \sinh^2 \theta} = \frac{9 \left(\frac{2}{x-2} - 1 \right)^2}{4 \left(\left(\frac{2}{x-2} \right)^2 - 1 \right)}$	M1	3.1a
	$= \frac{9 \left(\frac{2}{x-2} - 1 \right)^2}{4 \left(\frac{2}{x-2} - 1 \right) \left(\frac{2}{x-2} + 1 \right)} = \frac{9 \left(\frac{2}{x-2} - 1 \right)}{4 \left(\frac{2}{x-2} + 1 \right)} = \frac{9(4-x)}{4x} *$	A1*	1.1b
	Alternative for M1A1: $y^2 = \frac{9(\cosh \theta - 1)^2}{4 \sinh^2 \theta} = \frac{9(\cosh \theta - 1)^2}{4(\cosh \theta - 1)(\cosh \theta + 1)} = \frac{9(\cosh \theta - 1)}{4(\cosh \theta + 1)}$ $\frac{9(4-x)}{4x} = \frac{9 \left(4 - 2 - \frac{2}{\cosh \theta} \right)}{8 + \frac{8}{\cosh \theta}} = \frac{9(\cosh \theta - 1)}{4(\cosh \theta + 1)} \Rightarrow y^2 = \frac{9(4-x)}{4x}$		
	$p = 2$ or $q = 4$	M1	3.1a
	$p = 2$ and $q = 4$	A1	1.1b
		(11)	

(b)	$b^2 = a^2(e^2 - 1) \Rightarrow 9 = 16(e^2 - 1) \Rightarrow e = \frac{5}{4}$ Focus is at $x = ae = 4 \times \frac{5}{4} = 5$	M1	1.1b
	$d > "5" - 4 = \dots$	M1	3.1a
	$d > 1^*$	A1*	1.1b
		(3)	
(14 marks)			
Notes			
(a) M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{dy}{dx}$ in terms of θ. Allow this mark for $\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow \alpha x - \beta yy' = 0 \Rightarrow y' = \dots$ or an attempt to differentiate x and y wrt θ and then $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \dots$ M1: Correct straight line method using the coordinates of P and their gradient in terms of θ Allow the results for the first 2 M marks to be “quoted”, but any statements must be correct to score the marks. A1: Uses $y = 0$ to deduce the correct coordinates (or value of x) for the point A. Allow in any form, simplified or unsimplified (e.g. unsimplified: $x = 4 \cosh \theta - 4 \tanh \theta \sinh \theta$) B1: Deduces that the equation of l_2 is $x = 4$ (may be implied by $x = 4$ used to find y coordinate of B) M1: Realises that $x = 4$ is all that is needed for the second line and substitutes this into the first line in order to find the point B A1: Deduces the correct coordinates or y value for B (e.g. unsimplified $y = \frac{3}{\tanh \theta} - \frac{3 \cosh \theta}{\tanh \theta} + 3 \sinh \theta$) M1: Uses a correct method for the midpoint of AB (coordinates must be the right way round). This may be seen as the coordinates written separately e.g. $x = \dots, y = \dots$ M1: Having found the midpoint, identifies a correct strategy that will enable a Cartesian equation to be found. E.g. find $\cosh \theta$ in terms of x and substitutes into y or y^2 to obtain an equation in terms of y and x only. Mark positively here, so allow the mark if the candidate makes progress in eliminating θ even if there are slips in the working. A1*: Obtains the printed answer with no errors Alternative for the previous 2 marks: Substitutes the coordinates of their midpoint into both sides of the given equation in an attempt to show they are equal. Again mark positively but having made the substitution, some progress needs to be made in showing that both sides are equal. For this method there must be a minimal conclusion for the A1 e.g. tick, hence true etc. Note that these 2 marks can also be attempted by expressing the midpoint in terms of exponentials – if you are in doubt whether to award marks seek advice from your Team Leader. M1: For $p = 2$ or $q = 4$ A1: For $p = 2$ and $q = 4$			

(b)

M1: A complete method for finding the x coordinate of the focus using a correct eccentricity formula to find a value for e and then calculating $4e$

M1: Completes the problem by subtracting 4 from the x coordinate of the focus

A1*: Correct answer

If you come across correct attempts using Pythagoras to prove the result send to review.



Question	Scheme	Marks	AOs
8(a)	$\frac{x^2}{64} + \frac{y^2}{36} = 1, y = mx + c$ $\Rightarrow \frac{x^2}{64} + \frac{(mx+c)^2}{36} = 1 \Rightarrow \frac{x^2}{64} + \frac{m^2x^2 + 2cmx + c^2}{36} = 1$	M1	1.1b
	$\Rightarrow 36x^2 + 64m^2x^2 + 128cmx + 64c^2 = 2304$ $\Rightarrow (16m^2 + 9)x^2 + 32cmx + 16c^2 - 576 = 0$	A1	2.1
		(2)	
(b)	$b^2 - 4ac = 0 \Rightarrow (32cm)^2 - 4(16m^2 + 9)(16c^2 - 576) = 0$ $(\Rightarrow 36864m^2 - 576c^2 + 20736 = 0, 64m^2 - c^2 + 36 = 0)$	M1	3.1a
	$x = 10, y = 20 \Rightarrow 20 = 10m + c$ $\Rightarrow 64m^2 - (20 - 10m)^2 + 36 = 0 \text{ or } c^2 - 64\left(\frac{20 - c}{10}\right)^2 - 36 = 0$	M1	3.1a
	$36m^2 - 400m + 364 = 0$ $324m^2 - 3600m + 3276 = 0 \quad \text{o.e.}$ or $\Rightarrow 36c^2 + 2560c - 29200 = 0 \quad \text{o.e.}$	A1	1.1b
	$36m^2 - 400m + 364 = 0 \Rightarrow m = 1, \frac{91}{9} \Rightarrow c = \dots$ or $\Rightarrow 36c^2 + 2560c - 29200 = 0 \Rightarrow c = 10, -\frac{730}{9} \Rightarrow m = \dots$	dM1	1.1b
	$y = x + 10, \quad y = \frac{91}{9}x - \frac{730}{9} \quad \text{o.e.}$	A1	1.1b
		(5)	
(7 marks)			

Notes

(a)

M1: Attempts to solve simultaneously by substituting $y = mx + c$ into $\frac{x^2}{64} + \frac{y^2}{36} = 1$ and attempts to

multiply out $(mx + c)^2$ to obtain an equation in x , m and c

A1: Proceeds to obtain the equation in the required form with the correct value of k

(b)

M1: Realises that for l to be a tangent, the discriminant must be zero and applies this to their equation correctly, may contain k

M1: Uses the other condition that l passes through $(10, 20)$ to eliminate c or m to obtain an equation in c or m only following an attempt at setting their discriminant $=0$

A1: Correct 3TQ in c or m

dM1: Dependent on previous M. For a complete method to obtain at least one set of values for c and m . I.e. Finds values for m or c from a 3TQ (no need to check the roots) and attempts to find the value of the other constant

A1: Both correct equations in any form

Question	Scheme	Marks	AOs
7(a)	Gradient of $PQ = \frac{18q-18p}{9q^2-9p^2} = \frac{2}{p+q}$ $18p = 9p^2m + c$ $\& 18p - 18q = 9p^2m - 9q^2m$ $18q = 9q^2m + c$ $\& m = \frac{2}{p+q}$	B1	2.2a
	Equation of l is $y - 18p = \frac{2}{p+q}(x - 9p^2)$ $18p = 9p^2 \frac{2}{p+q} + c$ $\& c = \dots$	M1	1.1b
	Leading to $(p+q)y = 2(x+9pq)^*$	A1*	2.1
		(3)	
(b)	Complete method to find equation of both normals and attempts to solve simultaneously	M1	3.1a
	E.g. $2y \frac{dy}{dx} = 36 \Rightarrow m_T = \frac{36}{36p} \Rightarrow m_N = -p$	B1	1.1b
	Normal at P is $y - 18p = -p(x - 9p^2)$ or normal at Q is $y - 18q = -q(x - 9q^2)$ (oe)	M1	2.1
	Both normals correct $y - 18p = -p(x - 9p^2)$ or $y = -px + 9p^3 + 18p$ (o.e.) $y - 18q = -q(x - 9q^2)$ or $y = -qx + 9q^3 + 18q$ (o.e.)	A1	2.2a
	E.g. $18p - px + 9p^3 - 18q = -qx + 9q^3 \Rightarrow x = \dots$	M1	1.1b
	Need to show that $(9p^3 - 9q^3 + 18p - 18q) = (9p^2 + q^2 + pq + 2)(p - q)$ or $p^3 - q^3 = (p^2 + pq + q^2)(p - q)$ Leading to $x_A = 9(p^2 + q^2 + pq + 2)^*$	A1*	2.2a
	$y = -9p(p^2 + q^2 + pq + 2) + 9p^3 + 18p = -9p^2q - 9pq^2$ Leading to $y_A = -9pq(p+q)^*$	A1*	2.2a
		(7)	
(c)	$(12,0)$ on $l \Rightarrow pq = -\frac{4}{3}$ (oe)	B1	3.1a
	Hence $x_A = 9\left(p^2 + q^2 + \frac{2}{3}\right)$ and $y_A = 12(p+q)$	M1	1.1b
	$y^2 = 144(p^2 + q^2 + 2pq) = 144\left(\frac{x}{9} - \frac{2}{3} + 2\left(-\frac{4}{3}\right)\right)$	M1	3.1a
	$y^2 = 16(x - 30)$ or $y^2 = 16x - 480$	A1	1.1b
		(4)	

(14 marks)
Notes:
(a) B1: Deduces gradient is $\frac{2}{p+q}$. May be implied by correct simplification of equation if the unsimplified form is used to start with. M1: Correct method for the equation of the line (gradient need not be simplified/correct for this method, as long as it is clearly an attempt at the gradient). A1*: Completes to the correct equation with no errors seen.
(b) M1: A correct overall method – must find both normals and attempt to solve simultaneously. They do not need to reach $x =$ or $y =$ as long as they have eliminated one variable. B1: Correct gradient of normal found from any correct method or just stated. M1: A full correct method to find the equation of at least one of the normals with justification of the gradient shown. A1: Deduces equation of the second normal – so both correct. M1: Solves the two normal equations simultaneously leading to either $x = \dots$ or $y = \dots$ A1*: Need to show that $(9p^3 - 9q^3 + 18p - 18q) = (9p^2 + q^2 + pq + 2)(p - q)$ leading to correct x coordinate with no errors seen. This could be by long division or factorising. A1*: Correct y coordinate with no errors seen.
(c) B1: Uses the condition on l to establish the relationship between p and q M1: Uses their relationship between p and q to simplify the expressions M1: Any complete method for relating x and y independently of p and q A1: $y^2 = 16(x - 30)$ or $y^2 = 16x - 480$