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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Vectors (5)

Further Pure Mathematics 1 · Topic 5

6 questions · 57 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
5.1 THE VECTOR PRODUCT		2 Qs · 15 marks	3
1. Q6	Option 3A: Further Pure Mathematics 1 June2022	6	4
2. Q3	Option 3A: Further Pure Mathematics 1 October2020	9	8
5.2 THE SCALAR TRIPLE PRODUCT		1 Qs · 10 marks	12
3. Q7	Option 3A: Further Pure Mathematics 1 June2019	10	13
5.3 APPLICATIONS TO 3D GEOMETRY		3 Qs · 32 marks	17
4. Q4	Option 3A: Further Pure Mathematics 1 November2021	7	18
5. Q7	Option 3A: Further Pure Mathematics 1 June2023	15	22
6. Q9	Option 3A: Further Pure Mathematics 1 June2025	10	27



QUESTIONS

6 questions · 57 marks · 3 sub-topics



SUB-TOPIC 5.1

THE VECTOR PRODUCT

Sub-topic 5.1 · 2 questions · 15 marks · lowest tariff first



Question 6 continued

Question 6 continued

Lined area for writing the answer to Question 6 continued.

(Total for Question 6 is 6 marks)



3. The points A, B and C , with position vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{c} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ respectively, lie on the plane Π

(a) Find $\vec{AB} \times \vec{AC}$

(b) Find an equation for Π in the form $\mathbf{r} \cdot \mathbf{n} = p$

(2)

The point D has position vector $8\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

(c) Determine the volume of the tetrahedron $ABCD$

Question 3 continued



SUB-TOPIC 5.2

THE SCALAR TRIPLE PRODUCT

Sub-topic 5.2 · 1 question · 10 marks · lowest tariff first







SUB-TOPIC 5.3

APPLICATIONS TO 3D GEOMETRY

Sub-topic 5.3 · 3 questions · 32 marks · lowest tariff first



4.

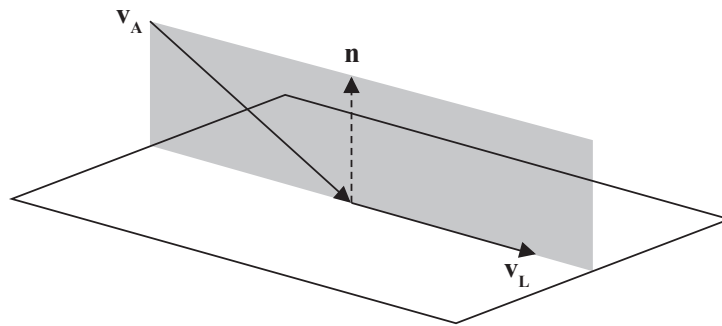


Figure 2

A small aircraft is landing in a field.

In a model for the landing the aircraft travels in different straight lines before and after it lands, as shown in Figure 2.

The vector $\mathbf{v}_A$ is in the direction of travel of the aircraft as it approaches the field.

The vector $\mathbf{v}_L$ is in the direction of travel of the aircraft after it lands.

With respect to a fixed origin, the field is modelled as the plane with equation

$$x - 2y + 25z = 0$$

and

$$\mathbf{v}_A = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

(a) Write down a vector $\mathbf{n}$ that is a normal vector to the field.

(1)

(b) Show that $\mathbf{n} \times \mathbf{v}_A = \lambda \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$, where λ is a constant to be determined.

(2)

When the aircraft lands it remains in contact with the field and travels in the direction $\mathbf{v}_L$.

The vector $\mathbf{v}_L$ is in the same plane as both $\mathbf{v}_A$ and $\mathbf{n}$ as shown in Figure 2.

(c) Determine a vector which has the same direction as $\mathbf{v}_L$.

(3)

(d) State a limitation of the model.

(1)













MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
6(a)	Finds any two vectors $\pm\overrightarrow{PQ}$, $\pm\overrightarrow{PR}$ or $\pm\overrightarrow{QR}$ $\pm\begin{pmatrix} 2 \\ 3 \\ -9 \end{pmatrix}$ or $\pm\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ or $\pm\begin{pmatrix} -1 \\ -1 \\ 8 \end{pmatrix}$	M1	1.1b
	A correct equation for the plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ $\mathbf{b}$ and $\mathbf{c}$ are any two vectors from $\pm\begin{pmatrix} 2 \\ 3 \\ -9 \end{pmatrix}$ or $\pm\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ or $\pm\begin{pmatrix} -1 \\ -1 \\ 8 \end{pmatrix}$	A1	1.1b
		(2)	
(b)	Forms two simultaneous equations by setting $y = 0$ and $z = 0$ e.g. $-2 + 3\lambda + 2\mu = 0$ $4 - 9\lambda - \mu = 0$	M1	3.1a
	Solves their simultaneous equations to find a value for μ and a value for λ e.g. $\begin{cases} -2 + 3\lambda + 2\mu = 0 \\ 4 - 9\lambda - \mu = 0 \end{cases} \Rightarrow \lambda = 0.4, \mu = 0.4$	dM1	1.1b
	Uses their values of μ and λ to find the x coordinate $x = 1 + 2\lambda + \mu = 1 + 2(0.4) + (0.4) = \dots$	ddM1	1.1b
	$(2.2, 0, 0)$	A1	1.1b
		(4)	
	Alternative $\begin{vmatrix} 2 & 3 & -9 \\ 1 & 2 & -1 \end{vmatrix} = (-3 + 18)\mathbf{i} - (-2 + 9)\mathbf{j} + (4 - 3)\mathbf{k}$	M1	3.1a
	$\begin{pmatrix} 15 \\ -7 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} = 15 + 14 + 4 = 33$ leading to $15x - 7y + z = 33$	dM1	1.1b
	$15x - 7(0) + (0) = 33 \Rightarrow x = \dots$	ddM1	1.1b
	$(2.2, 0, 0)$	A1	1.1b
		(4)	
(6 marks)			
Notes:			
Accept alternative vector forms throughout.			
(a)			



M1: Finds any two vectors $\pm \overrightarrow{PQ}$, $\pm \overrightarrow{PR}$ or $\pm \overrightarrow{QR}$ by subtracting relevant vectors. Two out of three values correct is sufficient to imply the correct method

A1: Any correct equation for the plane. Must start with $\mathbf{r} = \dots$

(b)

M1: Uses their equation for the plane to form two simultaneous equations by setting $y = 0$ and $z = 0$

dM1: Dependent on the previous method mark. Solves their simultaneous equations from the y and z coordinates to find a value for μ and a value for λ

ddM1: Depends on both method marks. Uses their value for μ and their value for λ to find the x coordinate

A1: Correct coordinates. Accept as a column vector or listed separately ($y = 0$ and $z = 0$ may be implied). Accept equivalent fractions, e.g. $\left(\frac{11}{5}, 0, 0\right)$ or $\left(\frac{33}{15}, 0, 0\right)$

Alternative

M1: Finds the cross product of the vectors $\mathbf{b}$ and $\mathbf{c}$ for their plane. Allow one slip in expansion.

dM1: Finds the Cartesian equation of the plane

ddM1: Depends on both previous method marks. Sets $y = 0$ and $z = 0$ to find the x coordinate.

A1: Correct coordinates. Accept as a column vector or listed separately ($y = 0$ and $z = 0$ may be implied). Accept equivalent fractions, e.g. $\left(\frac{11}{5}, 0, 0\right)$ or $\left(\frac{33}{15}, 0, 0\right)$



Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{AB} = -2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{AC} = -5\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$	B1	1.1b
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} -2 & 6 & 4 \\ -5 & 5 & 2 \end{vmatrix}$ $= (6 \times 2 - 4 \times 5)\mathbf{i} - (-2 \times 2 - 4 \times -5)\mathbf{j} + (-2 \times 5 - 6 \times -5)\mathbf{k}$	M1	1.1b
	$= -8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}$	A1	1.1b
		(3)	
(b)	E.g. $\mathbf{n} = -8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}$ gives $p = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = \dots$ E.g. $\mathbf{n} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ gives $p = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) = \dots$	M1	1.1b
	Equation is $\mathbf{r} \cdot (-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = 28$ or $\mathbf{r} \cdot (2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) = -7$ (oe)	A1	2.5
		(2)	
(c)	$\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = \overrightarrow{AD} \cdot (-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = \dots$	M1	1.1b
	$\overrightarrow{AD} = 5\mathbf{i} + 9\mathbf{j} + 4\mathbf{k}$	B1	1.1b
	Volume $= \frac{1}{6} \overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = \frac{1}{6} (5\mathbf{i} + 9\mathbf{j} + 4\mathbf{k}) \cdot (-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = \dots$	M1	3.1a
	$= \frac{52}{3}$ o.e. $17\frac{1}{3}$	A1	1.1b
		(4)	

(9 marks)

Notes:
(a)
B1: Both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correct.

M1: Applies the cross product to their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$. There must be at least two correct components if no method seen. Method can be implied by $\mathbf{i} \begin{vmatrix} 6 & 4 \\ 5 & 2 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -2 & 4 \\ -5 & 2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -2 & 6 \\ -5 & 5 \end{vmatrix} = \dots$ with at least one correct component.

A1: Correct vector

(b)

M1: Uses their $\mathbf{n}$ (which may be any multiple of their $\overrightarrow{AB} \times \overrightarrow{AC}$) and any point on the plane in an attempt to find p . (Use of $\overrightarrow{AB}$ or $\overrightarrow{AC}$ is M0.)

A1: Correct equation in form stated. Accept any multiples, e.g. $\mathbf{r} \cdot (-8\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}) = 28$

(c)

M1: Attempts a suitable scalar triple product, e.g. $\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$. Must include a complete method to use all necessary vectors.

B1: Correct ' $\overrightarrow{AD}$ ' if using $\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$, or all vectors correct if using a different product.

M1: Use of volume $= \frac{1}{6} |\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})|$ (oe full method to find the volume).

A1: Correct exact answer.



Question	Scheme	Marks	AOs
	$f(x) = x^4 \sin(2x) \quad u = x^4 \quad v = \sin(2x)$		



Question	Scheme	Marks	AOs
7(a)	Examples: Area $APQC$ = Area ABC – Area PBQ Area $APQC$ = Area APC + Area CPQ Area $APQC$ = Area APQ + Area AQC $\text{Area } APQC = \frac{1}{2} \mathbf{AQ} \times \mathbf{PC} $	M1	3.1a
	$\text{Line } AB: r = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 10-3 \\ -1-4 \\ 5-5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -5 \\ 0 \end{pmatrix}$ or $\text{Line } BC: r = \begin{pmatrix} 10 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 10-4 \\ -1-7 \\ 5+9 \end{pmatrix} = \begin{pmatrix} 10 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ -8 \\ 14 \end{pmatrix}$	M1	3.1a
	$4(3+7\lambda) - 8(4-5\lambda) + 5 = 2 \Rightarrow \lambda = \dots \Rightarrow P \text{ is } \dots$ or $4(10+6\mu) - 8(-1-8\mu) + 5 + 14\mu = 2 \Rightarrow \mu = \dots \Rightarrow Q \text{ is } \dots$ $\left(\text{NB } \lambda = \frac{1}{4}, \mu = -\frac{1}{2} \right)$	M1	2.1
	$P(4.75, 2.75, 5) \text{ and } Q(7, 3, -2)$	A1	1.1b
	$\text{Area } ABC = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -5 & 0 \\ 6 & -8 & 14 \end{vmatrix} = \frac{1}{2} \sqrt{70^2 + 98^2 + 26^2}$ $\text{Area } PBQ = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5.25 & -3.75 & 0 \\ 3 & -4 & 7 \end{vmatrix} = \frac{1}{2} \sqrt{26.25^2 + 36.75^2 + 9.75^2}$ $\text{Area } APQC = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 7 \\ 0.75 & -4.25 & 14 \end{vmatrix} = \frac{1}{2} \sqrt{43.75^2 + 61.25^2 + 16.25^2}$ NB: Area APQ = 7.7004, Area AQC = 30.8018, Area CPQ = 23.101, Area APC = 15.4008	M1	2.1
	Area ABC – Area PBQ = 38.5*	A1*	1.1b
		(6)	
(b)	$\overrightarrow{AB} = \begin{pmatrix} 7 \\ -5 \\ 0 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 1 \\ 3 \\ -14 \end{pmatrix}, \overrightarrow{AD} = \begin{pmatrix} k-3 \\ 0 \\ -6 \end{pmatrix}$	M1	3.1a

	$\overrightarrow{AB} \times \overrightarrow{AC} \cdot \overrightarrow{AD} = \begin{vmatrix} 7 & -5 & 0 \\ 1 & 3 & -14 \\ k-3 & 0 & -6 \end{vmatrix} = \dots$		
	$\overrightarrow{AB} \times \overrightarrow{AC} \cdot \overrightarrow{AD} = 7 \times -18 + 5(-6 + 14k - 42)$	A1	1.1b
	$7 \times -18 + 5(-6 + 14k - 42) = \pm 226 \Rightarrow k = \dots$	dM1	3.1a
	$k = 2 \text{ or } \frac{296}{35}$	A1	1.1b
		(4)	
(10 marks)			
Notes			
(a) M1: Identifies a correct strategy to determine the area of the required quadrilateral. The attempt does not need to be complete for this mark so one of the statements (or intentions) in the markscheme would be sufficient. M1: Correct attempt to find the equation of the line <i>AB</i> or the line <i>BC</i> M1: Uses at least one of their lines and the equation of the given plane to determine the value of at least one of the parameters and hence the coordinates of <i>P</i> or <i>Q</i> A1: Both coordinates correct – allow as vectors and may be implied if for example the candidate calculates the vectors e.g. <i>AP</i>, <i>AQ</i>, <i>CP</i>, <i>CQ</i> without stating the coordinates explicitly M1: Uses all the required information to calculate appropriate areas correctly leading to the area of the quadrilateral. Needs to be a complete method here. A1*: Reaches 38.5 with no errors (b) M1: Adopts a correct strategy by finding suitable vectors and forming the scalar triple product. This is often done in 2 steps e.g. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 70 \\ 98 \\ 26 \end{pmatrix} \text{ or } \overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} 30 \\ 42 \\ 5k-15 \end{pmatrix} \text{ or } \overrightarrow{AC} \times \overrightarrow{AD} = \begin{pmatrix} -18 \\ 48-14k \\ -3k+9 \end{pmatrix}$ $\overrightarrow{AB} \times \overrightarrow{AC} \cdot \overrightarrow{AD} = \begin{pmatrix} 70 \\ 98 \\ 26 \end{pmatrix} \cdot \begin{pmatrix} k-3 \\ 0 \\ -6 \end{pmatrix} = 70k - 210 - 156$ $\text{or } \overrightarrow{AB} \times \overrightarrow{AD} \cdot \overrightarrow{AC} = \begin{pmatrix} 30 \\ 42 \\ 5k-15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -14 \end{pmatrix} = 30 + 126 - 70k + 210$ $\text{or } \overrightarrow{AC} \times \overrightarrow{AD} \cdot \overrightarrow{AB} = \begin{pmatrix} -18 \\ 48-14k \\ -3k+9 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ -5 \\ 0 \end{pmatrix} = -126 + 70k - 240$ If it is not clear that the vector product is being used, at least 2 of the components should be correct.			

A1: Correct expression for the triple product in terms of k (should be $\pm(70k - 366)$)

Ignore the presence or absence of “1/6” for the first 2 marks

dM1: Realises that ± 226 is possible for the value of the triple product and attempts to solve to obtain 2 values for k . **Dependent on the previous method mark.**

A1: Correct values (must be exact)



Question	Scheme	Marks	AOs
4(a)	$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \\ 25 \end{pmatrix}$ or any non-zero scalar multiple thereof	B1	1.2
		(1)	
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 25 \\ 3 & -2 & -1 \end{vmatrix} = \begin{pmatrix} (-2)(-1) - (-2)(25) \\ -((1)(-1) - (3)(25)) \\ (1)(-2) - (3)(-2) \end{pmatrix} = \dots$	M1	1.1b
	$= \begin{pmatrix} 52 \\ 76 \\ 4 \end{pmatrix} = 4 \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$ (or correct multiple for their normal vector used)	A1	2.1
		(2)	
(c)	Landing direction is perpendicular to $\mathbf{n} \times \mathbf{v}_A$ and $\mathbf{n}$ so required direction is given by $\begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 25 \end{pmatrix} = \dots$ Alternatively recognises the landing direction is the line of intersection of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ and the plane representing the field. Finds the equation of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ $\begin{vmatrix} x & y & z \\ 1 & -2 & 25 \\ 3 & -2 & -1 \end{vmatrix} = 0$ $x - 2y + 25z = 0$ and $13x + 19y + z = 0$	M1	3.1b
	$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & 19 & 1 \\ 1 & -2 & 25 \end{vmatrix} = \begin{pmatrix} (19)(25) - (-2)(1) \\ -((13)(25) - (1)(1)) \\ (13)(-2) - (1)(19) \end{pmatrix} = \dots$ Alternative Selects a value for either x , y or z and solves simultaneously e.g $z = -5$ leading to $x - 2y = 125$ and $13x + 19y = 5$ $\Rightarrow x = \dots, y = \dots$	M1	3.4
	$= \begin{pmatrix} 477 \\ -324 \\ -45 \end{pmatrix}$ or any positive multiple thereof, e.g $\begin{pmatrix} 53 \\ -36 \\ -5 \end{pmatrix}$ or $\begin{pmatrix} 1908 \\ -1296 \\ -180 \end{pmatrix}$	A1	1.1b
		(3)	
(d)	Any acceptable reason e.g - Paths would not be linear - May have some lateral movement	B1	3.5b



	- Could be affected by cross winds - Field might not be flat		
		(1)	
(7 marks)			
Notes:			
(a) B1: Correct normal vector (any non-zero scalar multiple thereof is fine)			
(b) M1: Uses their $\mathbf{n}$ and $\mathbf{v}_A$ in cross product formula. Allow slips in coordinates as long as the intent is clear. A1: Correct work leading to a multiple of the required vector – may be a different multiple to the one shown if their $\mathbf{n}$ was different.			
(c) M1: Uses a correct strategy to find the direction, ie realises $\mathbf{v}_A$ must be perpendicular to both the vector from (b) and $\mathbf{n}$. Allow for vectors used either way round. Other methods may be possible – e.g finds the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ and solves with the plane representing the field. M1: Uses their answer to (b) with the normal vector to find a vector in the direction required. Allow for vectors either way round. Alternative find the line of intersection of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ and the field. A1: A correct direction vector, as shown or any positive multiple. For this mark the direction should be correct – so order of vectors must have been correct, or adapted to correct direction if initially incorrect.			
(d) B1: Any correct limitation given. See scheme for examples.			

Question	Scheme	Marks	AOs
7(a)	Vector A to l is $\pm \begin{pmatrix} 12 \\ 30 \\ 39 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix}$	B1	2.2a
	$\frac{ 9+7\lambda }{ 24+13\lambda } = 15 \Rightarrow (9+7\lambda)^2 + (24+13\lambda)^2 + (34+24\lambda)^2 = 15^2 \Rightarrow \lambda = \dots$	M1	3.1a
	$794\lambda^2 + 2382\lambda + 1588 = 0 \Rightarrow \lambda = -1, -2$	A1	1.1b
	$B(-2, 4, -9)$ and $C(5, 17, 15)$	A1	2.2a
	(4)		
(b)	E.g. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -5 \\ -2 \\ -14 \end{pmatrix} \times \begin{pmatrix} 2 \\ 11 \\ 10 \end{pmatrix} = \text{or}$ $\overrightarrow{AB} \times \overrightarrow{BC} = \begin{pmatrix} -5 \\ -2 \\ -14 \end{pmatrix} \times \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} = \pm \begin{pmatrix} 134 \\ 22 \\ -51 \end{pmatrix}$	M1	3.1a
	$\begin{pmatrix} 134 \\ 22 \\ -51 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} = 279$	M1	1.1b
	$134x + 22y - 51z = 279$	A1	1.1b
	(3)		
(c)	E.g. $\overrightarrow{DA} = \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ \alpha \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5-\alpha \end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 1 \\ \alpha \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ -9 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 9+\alpha \end{pmatrix}$	M1	3.1a
	$\begin{pmatrix} 134 \\ 22 \\ -51 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 5 \\ 5-\alpha \end{pmatrix} = \dots$ or $\begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} \cdot \left(\begin{pmatrix} 5 \\ 2 \\ 14 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 9+\alpha \end{pmatrix} \right) = \dots$	dM1	1.1b
	$\frac{1}{6} 525 + 51\alpha = \pm 147 \Rightarrow \alpha = \dots$	M1	3.1a
	$\alpha = 7, -\frac{469}{17}$	A1	1.1b
	(4)		
(d)	E.g. Vector connecting A and l is $\pm \begin{pmatrix} 9 \\ 24 \\ 34 \end{pmatrix}$ or D and l is $\pm \begin{pmatrix} 0 \\ -3 \\ 16 \end{pmatrix}$	B1ft	2.2a
	$\overrightarrow{AD} \times \overrightarrow{BC} = \begin{pmatrix} -5 \\ -5 \\ 2 \end{pmatrix} \times \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} = \begin{pmatrix} -146 \\ 134 \\ -30 \end{pmatrix}$	M1	1.1b



	$d = \frac{\left \begin{pmatrix} -146 \\ 134 \\ -30 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 24 \\ 34 \end{pmatrix} \right }{\sqrt{146^2 + 134^2 + 30^2}} = \dots$	M1	3.1a
	$d = \text{awrt } 4.4$	A1	1.1b
		(4)	
(15 marks)			
Notes			
Notes: Accept alternative vector forms throughout (including use of row vectors), and accept coordinates as vectors. For cross products with no working shown accept two out three correct entries to imply the method. (a) B1: Deduces the correct vector for A to l in parametric form (either direction). M1: Makes the key step of attempting the general distance from A to l, sets equal to 15 and attempts to solve for λ A1: Correct values for λ A1: Both coordinates correct (b) M1: Realises that the normal to the plane is required and applies the vector product to 2 vectors in the plane. (Many possibilities are possible, the two most common are in the scheme.) M1: Attempts scalar product using a point in the plane and their normal vector. A1: Correct equation (allow any multiple). (c) M1: Finds an appropriate vector using the coordinates of D e.g. joining D to A or B. dM1: Forms and evaluates an appropriate scalar triple product. M1: Realises that ± 147 is possible for the value of $1/6$ of the triple product and attempts to solve to obtain 2 distinct values for α. Allow this mark if one value is later rejected (or crossed out). A1: Correct values, but A0 if one is later rejected. (d) B1ft: Deduces a correct vector joining A or D and l (follow through their B or C if used). Accept in either direction (do not be concerned about the labelling). Many other vectors are possible here. M1: Calculates the vector product between the directions. M1: Fully correct strategy for the shortest distance. A1: Awrt 4.4			
(b) Alt	Plane is e.g. $\mathbf{r} = \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 2 \\ 14 \end{pmatrix} \Rightarrow \begin{cases} x = 3 + 7\lambda + 5\mu \\ y = 6 + 13\lambda + 2\mu \\ x = 5 + 24\lambda + 14\mu \end{cases}$	M1	1.1b
	$\Rightarrow \begin{cases} 5y - 2x = 24 + 51\lambda \\ 7y - z = 37 + 67\lambda \end{cases} \Rightarrow 51(7y - z - 37) - 67(5y - 2x - 24) = 0$	M1	3.1a
	$134x + 22y - 51z = 279$	A1	1.1b
		(3)	

(d) Alt	$\overrightarrow{AD} \times \overrightarrow{BC} = \begin{pmatrix} -5 \\ -5 \\ 2 \end{pmatrix} \times \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} = \begin{pmatrix} -146 \\ 134 \\ -30 \end{pmatrix}$ and need distance between planes $\mathbf{r} \cdot \begin{pmatrix} -146 \\ 134 \\ 30 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -146 \\ 134 \\ 30 \end{pmatrix} = 216 \text{ and}$ $\mathbf{r} \cdot \begin{pmatrix} -146 \\ 134 \\ 30 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -9 \end{pmatrix} \cdot \begin{pmatrix} -146 \\ 134 \\ 30 \end{pmatrix} = 1098$ (one correct B1ft on their normal vectors, M1 attempt both)	B1ft M1	2.2a 1.1b
	$d = \frac{1098 - 216}{\sqrt{146^2 + 134^2 + 30^2}}$	M1	3.1a
	$d = \text{awrt } 4.4$	A1	1.1b
		(4)	
(d) Alt 2	E.g. $\mathbf{r} = \begin{pmatrix} 9 \\ 24 \\ 34 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 5 \\ -2 \end{pmatrix}$ Sets up plane perpendicular to both direction vectors, passing through point represented by vector that joins the two lines. (Translates e.g. point A to the origin and considers parallel planes.)	B1ft	2.2a
	$\mathbf{r} \cdot \begin{pmatrix} 9 + 7\lambda + 5\mu \\ 24 + 13\lambda + 5\mu \\ 34 + 24\lambda + 2\mu \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix} = 0 \Rightarrow a + b\lambda + c\mu = 0 \text{ and}$ $\mathbf{r} \cdot \begin{pmatrix} 9 + 7\lambda + 5\mu \\ 24 + 13\lambda + 5\mu \\ 34 + 24\lambda + 2\mu \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 5 \\ -2 \end{pmatrix} = 0 \Rightarrow d + e\lambda + f\mu = 0$ Attempts scalar products of the plane with both direction vectors (finds point on second plane normal the origin).	M1	1.1b
	$\lambda = \dots, \mu = \dots \Rightarrow$ $d = \sqrt{(9 + 7\lambda + 5\mu)^2 + (24 + 13\lambda + 5\mu)^2 + (34 + 24\lambda + 2\mu)^2} = \dots$ Solves for λ and μ and proceeds to find the distance using their values.	M1	3.1a
	$d = \text{awrt } 4.4$	A1	1.1b
		(4)	

Question	Scheme	Marks	AOs
9(a)	e.g. $\overrightarrow{EF} = (7-10)\mathbf{i} + (-2+1)\mathbf{j} + (7+6)\mathbf{k}$ $\overrightarrow{EH} = (13-10)\mathbf{i} + (3+1)\mathbf{j} + (-4+6)\mathbf{k}$	M1	3.1a
	For example, $ \overrightarrow{EF} \times \overrightarrow{EH} = \left \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 13 \\ 3 & 4 & 2 \end{vmatrix} \right = -54\mathbf{i} + 45\mathbf{j} - 9\mathbf{k} = \sqrt{54^2 + 45^2 + 9^2}$	M1	2.1
	$9\sqrt{62}$ cso	A1	1.1b
		(3)	
(b)	$(\mathbf{r} - (10\mathbf{i} - \mathbf{j} - 6\mathbf{k})) \times (6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = \mathbf{0}$ or $\mathbf{r} - \begin{pmatrix} 10 \\ -1 \\ -6 \end{pmatrix} \times \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix} = \mathbf{0}$ o.e. Or $(\mathbf{r} - (10\mathbf{i} - \mathbf{j} - 6\mathbf{k})) \times (54\mathbf{i} - 45\mathbf{j} + 9\mathbf{k}) = \mathbf{0}$ or $\mathbf{r} - \begin{pmatrix} 10 \\ -1 \\ -6 \end{pmatrix} \times \begin{pmatrix} 54 \\ -45 \\ 9 \end{pmatrix} = \mathbf{0}$ o.e.	M1 A1ft	1.1b 2.2a
		(2)	
(c)	$6x - 5y + z = d \rightarrow d = 6 \times 25 - 5 \times (-4) + 13$ $54x - 45y + 9z = d \rightarrow d = 54 \times 25 - 45 \times (-4) + 9 \times 13$	M1	3.1a
	$6x - 5y + z = 183$ o.e. $54x - 45y + 9z = 1647$	A1	1.1b
	$6(10+6\lambda) - 5(-1-5\lambda) + \lambda - 6 = 183 \Rightarrow \lambda = \dots(2)$ $54(10+6\lambda) - 45(-1-5\lambda) + 9(\lambda - 6) = 1647 \Rightarrow \lambda = \dots(18)$ Or $6x - 5y + z = d \rightarrow d = 6 \times 10 - 5 \times (-1) - 6 \Rightarrow 6x - 5y + z = 59$ $\frac{183}{\sqrt{6^2 + 5^2 + 1^2}} - \frac{59}{\sqrt{6^2 + 5^2 + 1^2}} = \dots$	M1	3.1a
	$\lambda = 2 \Rightarrow V = 9\sqrt{62} \times 2(6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = 9\sqrt{62} \times 2\sqrt{6^2 + 5^2 + 1} = \dots$ or $\lambda = 2 \Rightarrow V = (-54\mathbf{i} + 45\mathbf{j} - 9\mathbf{k}) \cap (6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = \dots$	dM1	3.1a
	$= 1116$	A1	1.1b
		(5)	
	Alternative 1 $6x - 5y + z = d \rightarrow d = 6 \times 10 - 5 \times (-1) - 6$	M1	3.1a

	$54x - 45y + 9z = d \rightarrow d = 54 \times 10 - 45 \times (-1) + 9 \times (-6)$		
	$6x - 5y + z = 59$ o.e. $54x - 45y + 9z = -531$	A1	1.1b
	$\frac{ (25 \times 6) + (-4 \times -5) + (13 \times 1) \pm 59 }{\sqrt{6^2 + 5^2 + 1^2}} = \dots \left\{ \frac{124}{\sqrt{62}} \right\}$ $\frac{ (25 \times 54) + (-4 \times -45) + (13 \times 9) \pm 531 }{\sqrt{54^2 + 45^2 + 9^2}} = \dots \left\{ \frac{124}{\sqrt{62}} \right\}$	M1	3.1a
	$V = 9\sqrt{62} \times \frac{124}{\sqrt{62}}$	dM1	3.1a
	$= 1116$	A1	1.1b
		(5)	
	Alternative 2 $\begin{pmatrix} 25 \\ -4 \\ 13 \end{pmatrix} - \begin{pmatrix} 10 \\ -1 \\ -6 \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix}$	M1 A1	3.1a 1.1b
	$\frac{\begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}}{\sqrt{6^2 + 5^2 + 1^2}} = \dots \text{ or } \begin{vmatrix} 3 & 4 & 2 \\ -3 & -1 & 13 \\ 15 & -3 & 19 \end{vmatrix} \text{ or } \begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix} \bullet \begin{pmatrix} -54 \\ 45 \\ -9 \end{pmatrix}$	M1	3.1a
	$V = 9\sqrt{62} \times \frac{124}{\sqrt{62}}$ Or $\begin{vmatrix} 3 & 4 & 2 \\ -3 & -1 & 13 \\ 15 & -3 & 19 \end{vmatrix} = 3(-19 + 39) - 4(-57 - 195) + 2(9 + 15)$ Or $\begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix} \bullet \begin{pmatrix} -54 \\ 45 \\ -9 \end{pmatrix} = 15 \times -54 - 3 \times 45 + 19 \times -9$	dM1	3.1a
	$= 1116$	A1	1.1b
(10 marks)			
Notes			
(a) M1: Adopts a correct strategy by finding 2 appropriate vectors that will enable the area to be calculated, not two parallel vectors. If no method seen then 2 correct values will imply this mark.			

$$\overrightarrow{EF} = \overrightarrow{HG} = \begin{pmatrix} -3 \\ -1 \\ 13 \end{pmatrix}, \overrightarrow{EH} = \overrightarrow{FG} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}, \overrightarrow{EG} = \begin{pmatrix} 0 \\ 3 \\ 15 \end{pmatrix}, \overrightarrow{FH} = \begin{pmatrix} 6 \\ 5 \\ -11 \end{pmatrix},$$

M1: Forms the cross product of 2 non-parallel vectors and calculates the magnitude to find that required area. May split into two triangles and then add the areas. If uses the cross product of the diagonal vectors they would need to divide their answer by 2 to score this mark.

A1: Correct area cso, look out for correct signs

(b)

M1: Substitutes the position and direction vectors into the correct positions, condone the lack of brackets, and a sign slip and missing = 0

A1ft: Deduces the correct equation in the required form, follow through on their normal vector as long as both method marks scored in (a), must have correct bracketing and = 0

(c)

M1: Recognises the requirement to find the equation of Π by forming the necessary scalar product

A1: Correct equation for Π

M1: Performs the key step of finding the value of the parameter that determines the required distance or vector that will allow the volume to be calculated

Alternatively find the equation of the plane containing P and then finds the shortest distance between the two planes

dM1: Completes the strategy by attempting the product of their answer to part (a) with the length of the relevant vector or attempts the magnitude of the scalar triple product using the relevant vectors

A1: Correct volume

Alternative 1

M1: Recognises the requirement to find the equation of the plane containing the parallelogram P by forming the necessary scalar product

A1: Correct equation for the plane

M1: Finds the shortest distance between the point $(25, -4, 13)$ and the plane

dM1: Dependent on previous method. Finds the required volume by multiplying their answer to (a) multiplied by the shortest distance

A1: Correct volume

Alternative 2

M1: Find the vector between E and the position vector on Π

A1: Correct vector

M1: Finds the shortest distance between the planes or states determinant approach of 3

appropriate vectors for example $\overrightarrow{EF}, \overrightarrow{EH}, \overrightarrow{EA}$ where A is $(25, -4, 13)$ or the triple scalar product

dM1: Finds the required volume by multiplying their answer to (a) by the shortest distance or uses the determinant approach of 3 appropriate vectors or attempts the triple scalar product.

If no working is seen the answer must be correct for their vectors.

Using $\frac{1}{6}$ volume is dM0

A1: Correct volume