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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Further Numerical Methods (6)

Further Pure Mathematics 1 · Topic 6

3 questions · 29 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
6.1 NUMERICAL SOLUTION OF DES		2 Qs · 23 marks	3
1. Q4 <i>also Topic 3</i>	Option 3A: Further Pure Mathematics 1 June2022	6	4
2. Q8 <i>also Topic 3</i>	Option 3A: Further Pure Mathematics 1 November2021	17	6
6.2 SIMPSON'S RULE		1 Qs · 6 marks	11
3. Q1	Option 3A: Further Pure Mathematics 1 June2025	6	12



QUESTIONS

3 questions · 29 marks · 2 sub-topics



SUB-TOPIC 6.1

NUMERICAL SOLUTION OF DES

Sub-topic 6.1 · 2 questions · 23 marks · lowest tariff first



(Total for Question 4 is 6 marks)









SUB-TOPIC 6.2

SIMPSON'S RULE

Sub-topic 6.2 · 1 question · 6 marks · lowest tariff first



1. (a) Given that

$$y = e^{\operatorname{cosec}^2 x}$$

complete the table below with the value of y corresponding to $x = 2$, giving your answer to 2 decimal places.

x	1.5	1.75	2	2.25	2.5
y	2.73	2.81		5.22	16.31

(1)

- (b) Use Simpson's rule, with all the values of y in the completed table, to estimate, to one decimal place, the value of

$$\int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x} dx$$

(3)

- (c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_{1.5}^{2.5} e^{\cot^2 x} dx$$

(2)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
4(a)	Identifies $t_0 = 0$, $v_0 = 0$, $\left(\frac{dv}{dt}\right)_0 = 10$ and $h = 0.5$	B1	3.4
	$v_1 = v_0 + h\left(\frac{dv}{dt}\right)_0 \Rightarrow v_1 = 0 + 0.5 \times 10 = \dots$	M1	1.1b
	$v_1 = 5$	A1	1.1b
	$\left(\frac{dv}{dt}\right)_1 = -0.1(5)^2 + 10 = \dots\{7.5\}$ $v_2 = v_1 + h\left(\frac{dv}{dt}\right)_1 \Rightarrow v_2 = 5 + 0.5 \times 7.5 = \dots$	M1	3.4
	$v_2 = 8.75$ so 8.75 ms^{-1}	A1	1.1b
		(5)	
(b)	$\frac{dv}{dt} = -0.1v^2 + A$ where $0 < A < 10$	B1	3.5c
		(1)	
(6 marks)			
Notes:			
(a) B1: Uses the model to identify the correct initial conditions and requirements for h. May be implied by use in the equation. M1: Applies the approximation formula with their values for v_0, $\left(\frac{dv}{dt}\right)_0$ and h to find a value for v_1 A1: $v_1 = 5$ M1: Uses their v_1 to find a value for $\left(\frac{dv}{dt}\right)_1$ and applies the approximation formula with their values for v_1, $\left(\frac{dv}{dt}\right)_1$ and h to find a value for v_2 A1: $v_2 = 8.75$ or 8.75 ms^{-1}			
(b) B1: Reduce the value of 10 or explains this is what needs reducing, but do not accept 0 or negative values in place of the 10. Note: “change the 10” is B0 if it does not explain how to change it.			

Question	Scheme	Marks	AOs
8(a)	At 6 hours $t = 0.25$ so “ h ” is 0.25	B1	3.1b
	At $t = 0$ $\frac{dx}{dt} = \frac{3 + \cosh 0}{3 \times 3^2 \cosh 0} - \frac{1}{3}(3) \tanh 0 = \dots \left(= \frac{4}{27} \right)$	M1	3.4
	So $x_1 \approx 3 + "0.25" \times \frac{4}{27} = \dots$	M1	1.1b
	After 6 hours concentration of the pollutant is approximately awrt 3.04 ppm (3 s.f.) or $\frac{82}{27}$ ppm	A1	3.2a
		(4)	
(b)	$\frac{du}{dt} = 3x^2 \times \frac{dx}{dt}$ or $\frac{1}{3x^2} \frac{du}{dt} = \frac{dx}{dt}$ or $\frac{du}{dt} = \frac{du}{dx} \times \frac{dx}{dt} = 3x^2 \left(\frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t \right)$	B1	2.2a
	So $\frac{dx}{dt} = \frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t \rightarrow \frac{1}{3x^2} \frac{du}{dt} = \frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t$ $\rightarrow \frac{du}{dt} = \frac{3}{\cosh t} + 1 - u \tanh t$	M1	2.1
	$\frac{dx}{dt} = \frac{3 + \cosh t}{3x^2 \cosh t} - \frac{1}{3} x \tanh t \rightarrow \frac{1}{3u^{\frac{2}{3}}} \frac{du}{dt} = \frac{3 + \cosh t}{3u^{\frac{2}{3}} \cosh t} - \frac{1}{3} u^{\frac{1}{3}} \tanh t$ $\rightarrow \frac{du}{dt} = \frac{3}{\cosh t} + 1 - u \tanh t$		
	$\frac{du}{dt} + u \tanh t = 1 + \frac{3}{\cosh t} *$	A1*	1.1b
		(3)	
(c)	I.F. = $\exp \left(\int \tanh t \, dt \right) = \exp(\ln \cosh t) = \cosh t$	B1	2.2a
	$\Rightarrow u' \cosh t = \int \cosh t' \left(1 + \frac{3}{\cosh t} \right) dt = \left\{ \int \cosh t + 3 \, dt \right\}$	M1	1.1b
	$\Rightarrow u \cosh t = \sinh t + 3t (+c)$	M1	1.1b
	$u \cosh t = \sinh t + 3t + c$ or $u = \tanh t + \frac{3t}{\cosh t} + \frac{c}{\cosh t}$ oe	A1	1.1b
		(4)	
(d)	$t = 0 \Rightarrow x = 3, u = 27 \Rightarrow c = 27 \cosh 0 - \sinh 0 - 3(0) = 27$	M1	3.4
	$\Rightarrow x = \left(\tanh t + \frac{3t + "27"}{\cosh t} \right)^{\frac{1}{3}}$	M1	3.4



	$x = \left(\tanh t + \frac{3t + 27}{\cosh t} \right)^{\frac{1}{3}}$ (oe)	A1	3.2a
		(3)	
(e)	$x(0.25) = \left(\tanh 0.25 + \frac{(0.75 + 27)}{\cosh 0.25} \right)^{\frac{1}{3}} = \dots (= 3.0055\dots)$	M1	3.4
	% error is $\frac{3.0055\dots - 3.037\dots}{3.0055\dots} \times 100 = \dots$	M1	1.1b
	Estimate in (a) is an overestimate by 1.05% (3 s.f.)	A1	3.2a
		(3)	

(17 marks)**Notes:****(a)****B1:** Identifies a correct step length for the situation – 6 hours is a quarter of a day, so $h = 0.25$ **M1:** Uses " $y_0 = x(0) = 3$ and $t = 0$ to find $\left(\frac{dy}{dx} \right)_0 = \left(\frac{dx}{dt} \right)_0$. Accept with whichever notation used, as long as it is clear they are attempting the correct things.**M1:** Applies the approximation formula with their " h " and their $\left(\frac{dy}{dx} \right)_0$ **A1:** For awrt 3.04 ppm. Accept $\frac{82}{27}$ ppm**(b)****B1:** A correct equation relating $\frac{du}{dt}$ and $\frac{dx}{dt}$ from the chain rule.**M1:** Makes a complete substitution for x and $\frac{dx}{dt}$ in equation (I) or a complete substitution for u and $\frac{du}{dt}$ in equation (II)**A1*:** Simplifies correctly to achieve the given result.**(c)****B1:** Correct integrating factor found or spotted. Allow for $e^{\ln \cosh t}$ **M1:** Applies IF to achieve $u \cosh t = \int \cosh t \left(1 + \frac{3}{\cosh t} \right) dt$ **M1:** A reasonable attempt to integrate the RHS. Need not include constant of integration. If I.F. correct allow for $\pm \sinh t + 3t (+c)$ **A1:** Correct general solution, either implicit or explicit form including the context of integration (award when first seen and isw)**(d)****M1:** Uses the initial conditions in an appropriate equation to find the constant of integration. Either $t = 0$ and $u = 27$ in the answer to (c), or $t = 0$ and $x = 3$ if substitution for x occurs first.

M1: Reverses the substitution and rearranges to find equation for x , with evaluated constant included.

A1: Correct equation, any equivalent form, but must be $x = \dots$

(e)

M1: Uses their model solution to find the value at $t = 0.25$

M1: Applies $\frac{\text{actual value} - \text{estimate}}{\text{actual value}} \times 100$ with their values.

A1: States part (a) is overestimate by 1.05%



Question	Scheme	Marks	AOs
1(a)	3.35 cao	B1	1.1b
		(1)	
(b)	Step length = 0.25	B1	1.1b
	$\frac{1}{3} \times "0.25" [2.73 + 16.31 + 2("3.35") + 4(2.81 + 5.22)]$	M1	1.1b
	=4.8	A1	1.1b
		(3)	
(c)	$\int_{1.5}^{2.5} e^{\cot^2 x} dx = \int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x - 1} dx = \frac{1}{e} \int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x} dx = \frac{1}{e} \times "4.8"$	M1	3.1a
	= awrt 1.8	A1ft	1.1b
		(2)	
(6 marks)			
Notes			
(a) B1: Correct value 3.35 only, this may be seen in the table or in the script (b) B1: Correct step length, may be seen in the application of Simpson's rule formula. Could be seen as $1.75 - 1.5$ or $\frac{2.5 - 1.5}{4}$ M1: Correct application of Simpson's rule using their step length. If no step length stated and uses $\frac{1}{3} [\text{ends} + 2(\text{evens}) + 4(\text{odd})]$ this is M0 A1: Awrt 4.8 must be using 3.35 from (a) Note calculator answer is 4.67 is no marks Note Answer only with no working is no marks, Note: If they write $\frac{1}{3} \times h [\text{ends} + 2(\text{evens}) + 4(\text{odd})]$ which leads to awrt 4.8 this gets B1M1A1 If their answer is incorrect this would be B0 (if no h stated) M0A0 (c) M1: Uses a correct identity to express $\int e^{\cot^2 x} dx$ in terms of $\int e^{\operatorname{cosec}^2 x} dx$ and then uses correct index work to allow $\int_{1.5}^{2.5} e^{\cot^2 x} dx$ to be evaluated using part (b) $\frac{1}{e} \times "4.8"$ A1ft: Awrt 1.8 or follow through $\frac{1}{e} \times$ their (b) which must be evaluated and correct for their (b) rounded to 1 d.p. If uses Simpson's rule again this is M0 A0			

