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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Groups (1)

Further Pure Mathematics 2 · Topic 1

8 questions · 66 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

8 questions · 66 marks · 4 sub-topics



SUB-TOPIC 1.1

GROUP AXIOMS

Sub-topic 1.1 · 3 questions · 26 marks · lowest tariff first



6. (i) A binary operation $*$ is defined on positive real numbers by

$$a * b = a + b + ab$$

Prove that the operation $*$ is associative.

(4)

- (ii) The set $G = \{1, 2, 3, 4, 5, 6\}$ forms a group under the operation of multiplication modulo 7

- (a) Show that G is cyclic.

(2)

The set $H = \{1, 5, 7, 11, 13, 17\}$ forms a group under the operation of multiplication modulo 18

- (b) List all the subgroups of H .

(3)

- (c) Describe an isomorphism between G and H .

(3)

Question 6 continued

Question 6 continued

SUB-TOPIC 1.2

EXAMPLES OF GROUPS AND CAYLEY TABLES

Sub-topic 1.2 · 3 questions · 23 marks · lowest tariff first



1. The group S_4 is the set of all possible permutations that can be performed on the four numbers 1, 2, 3 and 4, under the operation of composition.

For the group S_4

- (a) write down the identity element,

(1)

- (b) write down the inverse of the element a , where

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

(1)

- (c) demonstrate that the operation of composition is associative using the following elements

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

(2)

- (d) Explain why it is possible for the group S_4 to have a subgroup of order 4. You do not need to find such a subgroup.

(2)



Question 1 continued

(Total for Question 1 is 6 marks)



6.

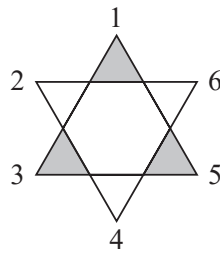


Figure 3

Figure 3 shows a plane shape made up of a regular hexagon with an equilateral triangle joined to each edge and with alternate equilateral triangles shaded.

The symmetries of this shape are the rotations and reflections of the plane that preserve the shape and its shading.

The symmetries of the shape can be represented by permutations of the six vertices labelled 1 to 6 in Figure 3. The set of these permutations with the operation of composition form a group, G .

(a) Describe geometrically the symmetry of the shape represented by the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{pmatrix}$$

(2)

(b) Write down, in similar two-line notation, the remaining elements of the group G .

(4)

(c) Explain why each of the following statements is false, making your reasoning clear.

(i) G has a subgroup of order 4

(ii) G is cyclic.

(2)

Diagram 1, on page 23, shows an unshaded shape with the same outline as the shape in Figure 3.

(d) Shade the shape in Diagram 1 in such a way that the group of symmetries of the resulting shaded shape is isomorphic to the cyclic group of order 6

(2)





Question 6 continued

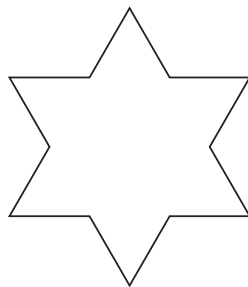
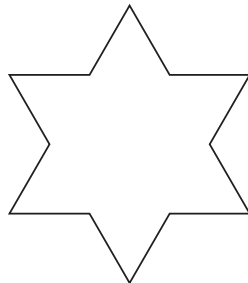


Diagram 1

Spare copy of Diagram 1



Only use this diagram if you need to redraw your answer to part (d).

(Total for Question 6 is 10 marks)

1. The set $S = \{1, 3, 5, 9, 11, 13\}$ forms the group G , under the operation multiplication modulo 14

(a) Complete the Cayley table below for the group G

$\times_{14}$	1	3	5	9	11	13
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11			
9	9	13		11		
11	11	5			9	
13	13	11				1

A spare table can be found on page 5 if you need to rewrite your Cayley table.

(3)

- (b) Write down a subgroup of G of order 2

(1)

The group H is defined by the Cayley table below.

$*$	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	t	u	r	s	p
r	r	u	t	q	p	s
s	s	r	q	p	u	t
t	t	s	p	u	r	q
u	u	p	s	t	q	r

- (c) Show that G and H are isomorphic.

(3)



Question 1 continued



Question 1 continued



Question 1 continued

Only use this grid if you need to rewrite your Cayley table.

$\times_{14}$	1	3	5	9	11	13
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11			
9	9	13		11		
11	11	5			9	
13	13	11				1

(Total for Question 1 is 7 marks)



SUB-TOPIC 1.3

ORDER AND SUBGROUPS

Sub-topic 1.3 · 1 question · 10 marks · lowest tariff first







SUB-TOPIC 1.4

LAGRANGE'S THEOREM

Sub-topic 1.4 · 1 question · 7 marks · lowest tariff first



4. Let G be a group of order $46^{46} + 47^{47}$

Using Fermat's Little Theorem and explaining your reasoning, determine which of the following are possible orders for a subgroup of G

(i) 11

(ii) 21

(7)



(Total for Question 4 is 7 marks)

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
7 (a)	$x \bullet e = x \Rightarrow 3(x + e + 1) + 2xe = x$ $\Rightarrow 3e + 2xe = x - 3x - 3 \Rightarrow e = \dots$ Or Let $e = -1 \Rightarrow e \bullet y = 3(-1 + y + 1) + 2(-1)y = \dots$	M1	2.1
	$\Rightarrow e = -\frac{2x+3}{2x+3} \Rightarrow e = -1$ Or $e \bullet y = 3(-1 + y + 1) + 2(-1)y = y$ therefore $e = -1$	A1	2.2a
		(2)	
(b)	Let the inverse of x be y then	M1	3.1a
	$x \bullet y = e \Rightarrow 3(x + y + 1) + 2xy = -1$ $\Rightarrow 3y + 2xy = -1 - 3x - 3 \Rightarrow y = \dots$	M1	2.1
	$\Rightarrow x^{-1} = -\frac{3x+4}{2x+3}$ or such as $-\frac{3}{2} + \frac{1}{4x+6}$	A1	2.2a
		(3)	
(c)	E.g. for $x = -\frac{3}{2}$ the identity property would fail since we cannot divide by $2x + 3 = 0$ or shows $x^{-1} = -\frac{1}{0}$ so no inverse or can't divide by 0 Or $-\frac{3}{2} \bullet x = 3\left(-\frac{3}{2} + x + 1\right) - 3x = -\frac{3}{2}$ so the identity property fails as not unique as any x would be an identity for $-\frac{3}{2}$.	B1	2.4
		(1)	
(6 Marks)			
Notes:			
(a) M1: Sets up and solves a correct equation for the identity element. Alternatively spots the identity is -1 and attempts to show this satisfies the identity property. E.g. $-1 \bullet x = 3(-1 + x + 1) + 2(-1)x = \dots$ A1: Correct identity / correct demonstration that -1 is the identity with appropriate conclusion. (Note only one side needs to be checked as we are told G is a group so one side is sufficient by uniqueness). Special case: states $e = -1$ with no working scores M1A0			

(b)

M1: Sets up a correct equation for the inverse using their identity element. May use x^{-1} throughout, or may use y or other variable.

M1: Expands, rearranges and by factorising out the inverse element from two terms to find an expression for the inverse element.

A1: Correct inverse element. Accept equivalent forms.

(c)

B1: Explains that the identity property would fail for $x = -\frac{3}{2}$. Alternatively accept a similar

argument that $x = -\frac{3}{2}$ could not have an inverse due to dividing by zero. May refer back to work in

(b), if done there, without need to repeat it.

Just saying undefined is B0 without a reason why.

Note: Reasons saying closure or associativity would fail are incorrect so B0, as both of these properties are satisfied on all of $\mathbb{R}$



Question	Scheme		Marks	AOs
2(a)	For $m, n \in \mathbb{Z}_0^+$ we have $m - n \in \mathbb{Z}$ (difference of integers is an integer) and so $ m - n \in \mathbb{Z}_0^+$, hence closed under $\star$.		B1	2.4
			(1)	
(b)	For $m \in \mathbb{Z}_0^+$, $0 \star m = 0 - m = -m = m$ and $m \star 0 = m - 0 = m = m$ Hence 0 is an identity*.	Checks either side	M1	1.1b
		Checks both sides and makes conclusion.	A1*	2.1
			(2)	
(c)	For $m \in \mathbb{Z}_0^+$, we need $ m - n = 0 \Rightarrow n = \dots$ or shows $ m - m = 0 = 0$		M1	2.2a
	As $ m - m = 0$ for all $m \in \mathbb{Z}_0^+$ each m is self-inverse.		A1	2.1
			(2)	
(d)	Checks associativity – ie evaluates $m \star (n \star p)$ and $(m \star n) \star p$ with letter or numbers.		M1	1.2
	E.g, $1 \star (2 \star 3) = 1 \star 2 - 3 = 1 \star 1 = 0$ but $(1 \star 2) \star 3 = 1 - 2 \star 3 = 1 \star 3 = 1 - 3 = 2$		M1	3.1a
	$1 \star (2 \star 3) \neq (1 \star 2) \star 3$ hence not associative so not a group.		A1	2.4
			(3)	
(8 marks)				
Notes:				
(a) B1: Checks difference of two non-negative integers is an integer and hence its modulus is a non-negative integer and concludes closure. “Always positive” as a conclusion is B0 without consideration of the equal zero case.				
(b) M1: Checks that 0 is a left or a right identity. A1*: Checks 0 works both sides as an identity and makes conclusion it is an identity.				
(c) M1: Realises m must be its own inverse for each m – accept if just stated m is self-inverse with no proof, or if an attempt is made to show it is self-inverse, or for an attempt to solve $ m - n = 0$ A1: Each element is self-inverse with a full proof given.				
(d) M1: Realises associativity must be checked in some way – may be by producing a counter example, or by attempting to evaluate both sides of the associativity axiom for a general case. A statement of the correct identity is sufficient for the mark to be awarded. M1: Produces a suitable counter example and evaluates both sides of associativity equation. Attempts at algebraic proofs are unlikely to succeed but allow the method for e.g consideration of $m > n > p$ giving $ m - n - p = m - n - p $ and $ m - n - p = m - n + p $ but must have a correct reason to disambiguate the inner moduli. If in doubt use review. A1: Must have provided a counter example. Deduces associativity does not hold and concludes $\mathbb{Z}_0^+$ is not a group under $\star$				



Question	Scheme	Marks	AOs														
6(i)	$(a*b)*c=(a+b+ab)*c=a+b+ab+c+(a+b+ab)c$	M1	2.1														
	$a*(b*c)=a*(b+c+bc)=a+b+c+bc+a(b+c+bc)$	M1	2.1														
	$\underline{a+b+ab+c+(a+b+ab)c=a+b+c+bc+ab+ac+abc}$ $\qquad\qquad\qquad=\underline{a+b+c+bc+a(b+c+bc)}$	A1	2.2a														
	so $(a*b)*c=a*(b*c)$ which means * is associative	A1	2.4														
		(4)															
(ii)(a)	$3^2=2 \quad 3^3=6 \quad 3^4=4 \quad 3^5=5 \quad 3^6=1$ or $5^2=4 \quad 5^3=6 \quad 5^4=2 \quad 5^5=3 \quad 5^6=1$	M1	2.1														
	Or special case for M1A0 if powers not shown: 3 has order 6 so generates the group																
	3 (or 5) has order 6 and so generates the group so G is cyclic	A1	2.4														
		(2)															
(b)	$\{1\}, H$	B1	1.1b														
	$\{1, 17\}$ or $\{1, 7, 13\}$	M1	1.1b														
	$\{1, 17\}$ and $\{1, 7, 13\}$ (and no others)	A1	1.1b														
		(3)															
(c)	<table border="1"><tr><td>G</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>H</td><td>1</td><td>7</td><td>5</td><td>13</td><td>11</td><td>17</td></tr></table>	G	1	2	3	4	5	6	H	1	7	5	13	11	17	M1	3.1a
	G	1	2	3	4	5	6										
	H	1	7	5	13	11	17										
	or	A1	1.1b														
	<table border="1"><tr><td>G</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>H</td><td>1</td><td>13</td><td>11</td><td>7</td><td>5</td><td>17</td></tr></table>	G	1	2	3	4	5	6	H	1	13	11	7	5	17	A1	1.1b
G	1	2	3	4	5	6											
H	1	13	11	7	5	17											
	(3)																
(12 marks)																	
Notes																	
(i) M1: Begins proof by correctly expanding $(a*b)*c$ or $a*(b*c)$ to an expression in a, b and c . Note they may expand as $(a*b)*c=(a*b)+c+(a*b)c=a+b+ab+c+(a+b+ab)c$ which is equally fine. M1: Makes progress towards the required result by attempting to expand both $(a*b)*c$ and $a*(b*c)$, but be generous with the attempts for this method. May achieve this by working from left to right, so look for arriving at the other expression through a chain of equalities. A1: For both underlined expressions (but accept eg. $c(a+b+ab)$ for $(a+b+ab)c$) and a correct expansion seen for each independently or part of a chain as shown. The expansion may have terms in different orders. A1: Explains that $(a*b)*c=a*(b*c)$ means that * is associative. Depends on both M marks and a correct expression having been found.																	

(ii)(a)

M1: Demonstrates understanding of the term cyclic by either attempting all the powers of 3 or 5.

Accept for this a statement $\langle 3 \rangle = \{3, 2, 6, 4, 5, 1\}$ which shows the elements list in order of powers.

A1: Must have evaluated all powers of 3 or 5 correctly and explains why the group is cyclic.

Accept as 3 generates the group, or as 3 has the same order of G as reason. Must refer to cyclic in conclusion.**Special case:** Allow M1A0 for a correct explanation of why G is cyclic if the order of 3 (or 5) is stated as 6 without justification – but must include reference to either being a generator or having the same order as G .

(b) (You may ignore references to the operation for this part)

B1: Identifies $\{1\}$ and H as subgroupsM1: Identifies $\{1, 17\}$ **or** $\{1, 7, 13\}$ as a subgroupA1: Identifies $\{1, 17\}$ **and** $\{1, 7, 13\}$ as subgroups and no others

(c)

M1: Attempts to identify an isomorphism between the groups – may be implied by

- identifying at least 2 correct non-identity pairings or
- by attempting to rearrange group tables to have the same structure, or
- by attempting to map powers of a generator to powers of a generator e.g. $(\text{their } 3)^k \rightarrow (\text{their } 5)^k$ or
- by matching of non-trivial proper subgroups to each other.

A1: Identifies 4 correct pairings, or sets up a mapping with one correct generator

A1: All pairings correct, or sets up a mapping with generators of each group correct, eg. $3^k \rightarrow 5^k$ 

Question	Scheme	Marks	AOs
1(a)	$\{e = \} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$	B1	1.1b
		(1)	
(b)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$	B1	1.1b
		(1)	
(c)	Demonstrates that, for example: $[a \circ b] \circ c = \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \right] \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ $a \circ [b \circ c] = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \right]$ $= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$	M1	2.1
	So $[a \circ b] \circ c = a \circ [b \circ c]$ or associative	A1	2.4
		(2)	
(d)	The order of the group is 24 or 4!	B1	1.1b
	4 is a factor of 24 or 4/24 therefore it is possible for a subgroup to have order 4.	B1ft	2.4
		(2)	

(6 marks)

Notes:
(a)
B1: See scheme

(b)
B1: See scheme

(c)

M1: Shows two calculations in an attempt to show associative, e.g, $[a \circ b] \circ c$ and $a \circ [b \circ c]$. There must be an intermediate line of working with evidence of using the permutations. Condone the wrong order for this mark.

A1: Correct calculations leading to $[a \circ b] \circ c = a \circ [b \circ c]$ or states associative

Note Incorrect order scores M1 A0

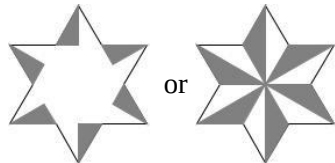
$$[a \circ b] \circ c = \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \right] \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

(d)
B1: Order is 24 or 4!

B1ft: Follow through on their order of the group, draws the correct conclusion


Question	Scheme		Marks	AOs
6(a)	A rotation ...		M1	1.1b
	... about the centre of the shape, though an angle 120° anticlockwise.		A1	1.1b
			(2)	
(b)	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 2 & 3 & 4 \end{pmatrix}$		B1	1.1b
	One of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$ or $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix}$		M1	1.1b
	Two of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$ & $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix}$		A1	1.1b
	All the above and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$ and no extra symmetries given. Two line form required for this mark.		A1	2.5
			(4)	
(c)	G has order 6 so can have no subgroup of order 4 by Lagrange’s Theorem.		B1	2.4
	There is no element of order 6 that generates the group.		B1	2.4
			(2)	
(d)	E.g.  or	M1 – breaks the reflections	M1	3.1a
		A1 – keeps all rotations and no reflections	A1	1.1b
			(2)	
(10 marks)				

Notes:**(a)****M1:** For identifying the permutation as representing a rotation.**A1:** A complete description including rotation, centre, angle (degrees or radians) and direction.**(b)** *NB accept in cycle form for the first two marks in (b).***B1:** For giving the other rotation of order 3 (rotation through 240° anticlockwise).**M1:** For one of the three reflections correctly given.**A1:** For one of the other two reflections.**A1:** For all reflections and the identity, and no extra symmetries given, and all in two-line notation.**(c)****B1:** Correct reason given. Stating there are no elements of order 4 is **not** sufficient. A longer, valid method would be to state a subgroup of order 4 would need to comprise of the identity and the three elements of order 2 (as it cannot contain any of order 3), but this is not closed as the composite of any two reflection is a rotation.**B1:** Refers to there being no element (of order 6) to generate the group. May reason using orders of elements or via geometric restrictions (e.g. cannot generate a rotation from a single reflection and vice versa).**(d)****M1:** Realises that the shape needs only the rotations preserved so shades in a way that breaks the reflection symmetries, but which preserves at least one non-trivial rotation.**A1:** Any correctly shaded shape, e.g. the ones shown above. There are many variations!

Question	Scheme							Marks	AOs
1(a)	x_{14}	1	3	5	9	11	13	M1 A1 A1	1.1b 1.1b 1.1b
	1	1	3	5	9	11	13		
	3	3	9	1	13	5	11		
	5	5	1	11	3	13	9		
	9	9	13	3	11	1	5		
	11	11	5	13	1	9	3		
	13	13	11	9	5	3	1		
									(3)
(b)	{1, 13}							B1	2.2a
								(1)	
(c)	States the groups are cyclic or identifies a generator for each. E.g. $3^2 = 9, 3^3 = 13, 3^4 = 11, 3^5 = 5, 3^6 = 1$ so 3 generates G or $q^2 = t, q^3 = s, q^4 = r, q^5 = u, q^6 = p$ so q generates H							M1	3.1a
	E.g. $(3^2 = 9, 3^3 = 13, 3^4 = 11, 3^5 = 5, 3^6 = 1)$ 3 (or 5) generates G and $(q^2 = t, q^3 = s, q^4 = r, q^5 = u, q^6 = p)$ q (or u) generates H							A1	1.1b
	Both groups are cyclic of order 6 and so are isomorphic.							A1	2.1
								(3)	
Alt I	G	1	3	5	9	11	13	M1 A1	3.1a 1.1b
	H	p	q	u	t	r	s		
	Or H	p	u	q	r	t	s		
	Also $3^2 = 9$ and $\theta(3)^2 = q^2 = t = \theta(9) = \theta(3^2)$ so structure is preserved and therefore groups are isomorphic							A1	2.1
							(3)		
Alt II	G	1	3	5	9	11	13	M1 A1	3.1a 1.1b
	Order	identity	6	6	3	3	2		
	H	p	q	r	s	t	u		
	Order	identity	6	3	2	3	6		
	Since orders match and group is order 6 (so one of only two types) therefore the groups are isomorphic isomorphic							A1	2.1
								(3)	
(7 marks)									



Notes:

(a)

M1: At least 3 correct entries.

A1: At least 6 correct entries.

A1: All 12 entries correct.

NB for the final A all entries must be given modulo 14. Do not accept e.g. 17 for 3.

(b)

B1: Deduces the correct subgroup. This is the only subgroup of order 2. Accept with set brackets or round brackets as long as both elements are listed. Accept $\langle 13 \rangle$ as long as the angled brackets are clear.

(c)

M1: State the groups are cyclic, or identifies a generator for each group.

A1: Correctly generator identified for each group.

A1: Correct work for showing a generator for each group and conclude both cyclic and hence isomorphic.

Alt I

M1: Attempts to identify an isomorphism between the elements – may be implied by

- identifying at least 4 correct pairings
- by attempting to rearrange group tables to have the same structure

A1: For a correct pairing of all items for an isomorphism. Note that $3^2 = 9$ while $q^2 = t$ and this can be used to see if they have a fully correct pairing (check if $3 \leftrightarrow q$ then you need $9 \leftrightarrow t$, or if $3 \leftrightarrow u$ then you need $9 \leftrightarrow r$)

A1: All correct pairings with demonstration of structure being preserved for at least one of the possible choices made, or via matching of group tables to show same structure, and concludes isomorphic.

NB there are two choices for isomorphism, $1 \leftrightarrow p$ and $13 \leftrightarrow s$ are forced, 3 and 5 must map to q and u either way round, but then 9 and 11 need to be correctly selected for the order of 3 and 5 – and this must be shown for fully marks. They may likely choose the pairings by considering orders (as in Alt II) but still need a reason for the final choice.

Alt II

M1: Attempts to find the order for each element of a both groups – at least four correct for at least one of the groups.

A1: All orders correct for all elements of both groups.

A1: All orders correct, reason referring to small group order and therefore groups are **isomorphic**.

Note if they go on to give an incorrect pairing it is wrong reasoning so A0.

Note: In general matching orders of all elements is a necessary but not sufficient requirement for an isomorphism (e.g. for groups of order 16 there are distinct isomorphism classes which have the same number of elements of the same orders). However, for small groups (up to order 15) with few isomorphism classes this criterion is sufficient but for a fully correct answer some reference to the group order must be given.



Question	Scheme	Marks	AOs
7(a)	$\mathbf{B}^2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \{=\mathbf{E}\}$ Accept $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ as long as $2 \equiv 0 \pmod{2}$ is used later. Or $\mathbf{B}^3 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ Accept $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ as long as $2 \equiv 0 \pmod{2}$ and $3 \equiv 1 \pmod{2}$ is used later.	M1	2.5
	$\mathbf{B}^3 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ (and $\mathbf{B}$ is not the identity) hence $\mathbf{B}$ has order 3 or $ \mathbf{B} = 3$	A1	2.1
		(2)	
(b)	$\mathbf{I}$ {is the identity so} has order 1, and $\mathbf{E}$ {is $\mathbf{B}^{-1}$ so also} has order 3.	B1	1.1b
	Finds at least one of the remaining orders e.g. $\mathbf{A}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ or $\mathbf{C}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ or $\mathbf{D}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$	M1	1.1b
	(So $\mathbf{I}$ has order 1, $\mathbf{B}$ and $\mathbf{E}$ order 3 and) $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{D}$ each have order 2	A1	1.1b
		(3)	
(c)	(i) There is no element of order 6 in G {so cannot be isomorphic to C_6 }	B1	2.4
	(ii) Either - There are 12 symmetries of a regular hexagon - Hexagon has more than 6 symmetries - Hexagon has 2 rotations of order 6, could give an example - Hexagon has 2 rotations of order 3 - Hexagon has 1 rotation of order 2 - Hexagon has 6 reflections of order 2 - Group of symmetries of a hexagon has an order greater than 6	B1	2.4
	B0 for - The group of symmetries of a hexagon is also cyclic - The groups are not of the same order - Any incorrect orders of element stated - Comments on the order of the whole group - The group of symmetries has order 6		
		(2)	
(d)	Any two correct matchings	M1	2.2a
	Completes the matching with at least 4 correct	M1	3.1a

	Hence an isomorphism is											A1	2.1	
	I	A	B	C	D	E	I	A	B	C	D			E
	e	c	a	d	f	b	e	c	b	f	d			a
	e	d	a	f	c	b	e	d	b	c	f			a
	e	f	a	c	d	b	e	f	b	d	c			a
												(3)		
(10 marks)														
Notes:														
(a)														
M1: Squares B to begin the process of finding the order, or uses a calculator to find B ³														
A1: Shows that B ³ = I and concludes that order must be 3.														
(b)														
B1: States the order for I and gives the order for E (which is the inverse of B).														
M1: Finds the order of at least one of the other elements to get an element of order 2. May use a calculator														
A1: Correct orders for the three elements of order 2.														
(c)														
B1: Correct reason given, identifies there is no element of order 6. It is not sufficient to say that it does not have a group generator, they need to give a reason.														
B1: Any correct reason given.														
Considers the total number of elements of the group, order of the group is more than 6														
Considers the orders of either reflections or rotations														
(d)														
M1: Any two correct matchings														
M1: Completes the matching with at least 4 correct														
A1: A fully specified correct isomorphism.														

Question	Scheme	Marks	AOs
4(i)	(Order of a subgroup must divide the order of a group by Lagrange's Theorem), so need to check if 11 (and/or 21) divides $46^{46} + 47^{47}$ and by FLT, e.g. $a^{11-1} = a^{10} \equiv 1 \pmod{11}$, so	M1	1.1b
	$46^{46} + 47^{47} \equiv 2^{4 \times 10 + 6} + 3^{4 \times 10 + 7} \equiv 2^6 + 3^7 \equiv 64 + (3^3)^2 \times 3 \equiv 9 + 5^2 \times 3 \equiv 84 \equiv 7 \pmod{11}$	M1	3.1a
	Hence 11 is not a divisor of $46^{46} + 47^{47}$ so not a possible order for a subgroup.	A1	2.2a
(ii)	$21 = 7 \times 3$ so need to check for factors of 7 and 3, using $a^2 \equiv 1 \pmod{3}$ and $a^6 \equiv 1 \pmod{7}$	M1	3.1a
	$46^{46} + 47^{47} \equiv 1^{46} + 2^{47} \equiv 1 + 2^{2 \times 23 + 1} \equiv 1 + 2^1 \equiv 3 \equiv 0 \pmod{3}$	M1	1.1b
	$46^{46} + 47^{47} \equiv 4^{46} + (-2)^{47} \equiv 4^{6 \times 7 + 4} + (-2)^{6 \times 7 + 5} \equiv 4^4 + (-2)^5 \equiv 16^2 - 32 \equiv 9^2 - 4 \equiv 81 - 4 \equiv 77 \equiv 0 \pmod{7}$	M1	2.1
	As $46^{46} + 47^{47}$ divisible by both 3 and 7 it is divisible by 21 and hence this is a possible order for a subgroup.	A1	2.4
		(7)	
(7 marks)			
Notes:			
(i) M1: For an attempt to apply a correct Fermat's Little theorem at least once in the question with either $p = 11$, $p = 7$ or $p = 3$ on either the 46^{46} or 47^{47} term. M1: Applies FLT and congruence arithmetic fully to find the residue of $46^{46} + 47^{47}$ modulo 11. There will be lots of different routes, so look for an attempt to apply FLT that leads to determining if 11 is a divisor or not. A1: $46^{46} + 47^{47} \equiv 7 \pmod{11}$ (accept equivalents as long as it is clear it is not congruent to 0) and deduces it is not a possible order for a subgroup.			
(ii) M1: Applies checks for both 7 and 3 as divisors of $46^{46} + 47^{47}$ via similar strategy. M1: Applies FLT with $p = 3$ to find a smaller residue modulo 3. Other routes are possible. M1: Applies FLT with $p = 7$ to find a smaller residue modulo 7. Other routes are possible. A1: Shows $46^{46} + 47^{47}$ congruent to 0 modulo 3 and modulo 7, and deduces 21 divides $46^{46} + 47^{47}$ hence it is a possible order for a subgroup. Alt: M1: Reduces the bases modulo 21 and applies a power reduction technique using congruences for at least one of the power of 46 or 47 M1: Reduces fully by congruence arithmetic either the 46^{46} or 47^{47} term. M1: Reduces fully by congruence arithmetic both the 46^{46} and 47^{47} terms A1: Shows $46^{46} + 47^{47}$ congruent to 0 modulo 21, and deduces 21 divides $46^{46} + 47^{47}$ hence it is a possible order for a subgroup.			