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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Number Theory (5)

Further Pure Mathematics 2 · Topic 5

12 questions · 105 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
1.1 GROUP AXIOMS		1 Qs · 12 marks	3
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2. Q7 <i>also Topic 1 & 3</i>	Option 4A: Further Pure Mathematics 2 June2024	10	9
5.2 BEZOUT'S IDENTITY		3 Qs · 24 marks	13
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QUESTIONS

12 questions · 105 marks · 7 sub-topics



SUB-TOPIC 1.1

GROUP AXIOMS

Sub-topic 1.1 · 1 question · 12 marks · lowest tariff first



Question 6 continued

Lined area for writing the answer to Question 6 continued.

Question 6 continued

Lined area for writing the answer to Question 6.

SUB-TOPIC 1.3

ORDER AND SUBGROUPS

Sub-topic 1.3 · 1 question · 10 marks · lowest tariff first





(Total for Question 7 is 10 marks)



SUB-TOPIC 5.2

BEZOUT'S IDENTITY

Sub-topic 5.2 · 3 questions · 24 marks · lowest tariff first



- (2)

(3)

(2)



3. (a) Use the Euclidean Algorithm to find integers a and b such that

$$125a + 87b = 1$$

(5)

(b) Hence write down a multiplicative inverse of 87 modulo 125

(1)

(c) Solve the linear congruence

$$87x \equiv 16 \pmod{125}$$

(2)







(Total for Question 3 is 8 marks)



(2)

(3)

(1)

(3)

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(Total for Question 4 is 9 marks)



SUB-TOPIC 5.3

MODULAR ARITHMETIC

Sub-topic 5.3 · 1 question · 11 marks · lowest tariff first



3.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

- (a) Use the Euclidean Algorithm to determine the highest common factor h of 234 and 96

(3)

- (b) Hence determine integers a and b such that

$$234a + 96b = h$$

(3)

- (c) Solve the congruence equation

$$96x \equiv 36 \pmod{234}$$

(5)





(Total for Question 5 is 11 marks)



SUB-TOPIC 5.5

DIVISIBILITY TESTS

Sub-topic 5.5 · 1 question · 4 marks · lowest tariff first



1.

In this question you must show detailed reasoning.

Without performing any division, explain why $n = 20210520$ is divisible by 66

(4)



(Total for Question 1 is 4 marks)

SUB-TOPIC 5.6

SOLVING CONGRUENCES

Sub-topic 5.6 · 2 questions · 18 marks · lowest tariff first



8. The four digit number $n = abcd$ satisfies the following properties:

- (1) $n \equiv 3 \pmod{7}$
- (2) n is divisible by 9
- (3) the first two digits have the same sum as the last two digits
- (4) the digit b is smaller than any other digit
- (5) the digit c is even

(a) Use property (1) to explain why $6a + 2b + 3c + d \equiv 3 \pmod{7}$

(2)

(b) Use properties (2), (3) and (4) to show that $a + b = 9$

(4)

(c) Deduce that $c \equiv 5(a - 1) \pmod{7}$

(2)

(d) Hence determine the number n , verifying that it is unique. You must make your reasoning clear.

(4)







Question 8 continued

(Total for Question 8 is 12 marks)

SUB-TOPIC 5.7

COMBINATORICS

Sub-topic 5.7 · 3 questions · 26 marks · lowest tariff first











MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs														
6(i)	$(a*b)*c=(a+b+ab)*c=a+b+ab+c+(a+b+ab)c$	M1	2.1														
	$a*(b*c)=a*(b+c+bc)=a+b+c+bc+a(b+c+bc)$	M1	2.1														
	$\underline{a+b+ab+c+(a+b+ab)c=a+b+c+bc+ab+ac+abc}$ $\qquad\qquad\qquad =\underline{a+b+c+bc+a(b+c+bc)}$	A1	2.2a														
	so $(a*b)*c=a*(b*c)$ which means * is associative	A1	2.4														
		(4)															
(ii)(a)	$3^2=2 \quad 3^3=6 \quad 3^4=4 \quad 3^5=5 \quad 3^6=1$ or $5^2=4 \quad 5^3=6 \quad 5^4=2 \quad 5^5=3 \quad 5^6=1$	M1	2.1														
	Or special case for M1A0 if powers not shown: 3 has order 6 so generates the group																
	3 (or 5) has order 6 and so generates the group so G is cyclic	A1	2.4														
		(2)															
(b)	$\{1\}, H$	B1	1.1b														
	$\{1, 17\}$ or $\{1, 7, 13\}$	M1	1.1b														
	$\{1, 17\}$ and $\{1, 7, 13\}$ (and no others)	A1	1.1b														
		(3)															
(c)	<table><tr><td>G</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>H</td><td>1</td><td>7</td><td>5</td><td>13</td><td>11</td><td>17</td></tr></table>	G	1	2	3	4	5	6	H	1	7	5	13	11	17	M1	3.1a
	G	1	2	3	4	5	6										
	H	1	7	5	13	11	17										
	or																
	<table><tr><td>G</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>H</td><td>1</td><td>13</td><td>11</td><td>7</td><td>5</td><td>17</td></tr></table>	G	1	2	3	4	5	6	H	1	13	11	7	5	17	A1	1.1b
G	1	2	3	4	5	6											
H	1	13	11	7	5	17											
	A1	1.1b															
	(3)																
(12 marks)																	
Notes																	
(i) M1: Begins proof by correctly expanding $(a*b)*c$ or $a*(b*c)$ to an expression in a, b and c . Note they may expand as $(a*b)*c=(a*b)+c+(a*b)c=a+b+ab+c+(a+b+ab)c$ which is equally fine. M1: Makes progress towards the required result by attempting to expand both $(a*b)*c$ and $a*(b*c)$, but be generous with the attempts for this method. May achieve this by working from left to right, so look for arriving at the other expression through a chain of equalities. A1: For both underlined expressions (but accept eg. $c(a+b+ab)$ for $(a+b+ab)c$) and a correct expansion seen for each independently or part of a chain as shown. The expansion may have terms in different orders. A1: Explains that $(a*b)*c=a*(b*c)$ means that * is associative. Depends on both M marks and a correct expression having been found.																	

(ii)(a)

M1: Demonstrates understanding of the term cyclic by either attempting all the powers of 3 or 5.

Accept for this a statement $\langle 3 \rangle = \{3, 2, 6, 4, 5, 1\}$ which shows the elements list in order of powers.

A1: Must have evaluated all powers of 3 or 5 correctly and explains why the group is cyclic.

Accept as 3 generates the group, or as 3 has the same order of G as reason. Must refer to cyclic in conclusion.**Special case:** Allow M1A0 for a correct explanation of why G is cyclic if the order of 3 (or 5) is stated as 6 without justification – but must include reference to either being a generator or having the same order as G .

(b) (You may ignore references to the operation for this part)

B1: Identifies $\{1\}$ and H as subgroupsM1: Identifies $\{1, 17\}$ **or** $\{1, 7, 13\}$ as a subgroupA1: Identifies $\{1, 17\}$ **and** $\{1, 7, 13\}$ as subgroups and no others

(c)

M1: Attempts to identify an isomorphism between the groups – may be implied by

- identifying at least 2 correct non-identity pairings or
- by attempting to rearrange group tables to have the same structure, or
- by attempting to map powers of a generator to powers of a generator e.g. $(\text{their } 3)^k \rightarrow (\text{their } 5)^k$ or

- by matching of non-trivial proper subgroups to each other.

A1: Identifies 4 correct pairings, or sets up a mapping with one correct generator

A1: All pairings correct, or sets up a mapping with generators of each group correct, eg. $3^k \rightarrow 5^k$ 

Question	Scheme	Marks	AOs
7(a)	$\mathbf{B}^2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \{=\mathbf{E}\}$ Accept $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ as long as $2 \equiv 0 \pmod{2}$ is used later. Or $\mathbf{B}^3 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ Accept $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ as long as $2 \equiv 0 \pmod{2}$ and $3 \equiv 1 \pmod{2}$ is used later.	M1	2.5
	$\mathbf{B}^3 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ (and $\mathbf{B}$ is not the identity) hence $\mathbf{B}$ has order 3 or $ \mathbf{B} = 3$	A1	2.1
		(2)	
(b)	$\mathbf{I}$ {is the identity so} has order 1, and $\mathbf{E}$ {is $\mathbf{B}^{-1}$ so also} has order 3.	B1	1.1b
	Finds at least one of the remaining orders e.g. $\mathbf{A}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ or $\mathbf{C}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$ or $\mathbf{D}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \{=\mathbf{I}\}$	M1	1.1b
	(So $\mathbf{I}$ has order 1, $\mathbf{B}$ and $\mathbf{E}$ order 3 and) $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{D}$ each have order 2	A1	1.1b
		(3)	
(c)	(i) There is no element of order 6 in G {so cannot be isomorphic to C_6 }	B1	2.4
	(ii) Either - There are 12 symmetries of a regular hexagon - Hexagon has more than 6 symmetries - Hexagon has 2 rotations of order 6, could give an example - Hexagon has 2 rotations of order 3 - Hexagon has 1 rotation of order 2 - Hexagon has 6 reflections of order 2 - Group of symmetries of a hexagon has an order greater than 6	B1	2.4
	B0 for - The group of symmetries of a hexagon is also cyclic - The groups are not of the same order - Any incorrect orders of element stated - Comments on the order of the whole group - The group of symmetries has order 6		
		(2)	
(d)	Any two correct matchings	M1	2.2a
	Completes the matching with at least 4 correct	M1	3.1a



	Hence an isomorphism is												A1	2.1
	I	A	B	C	D	E	I	A	B	C	D	E		
	e	c	a	d	f	b	e	c	b	f	d	a		
	e	d	a	f	c	b	e	d	b	c	f	a		
	e	f	a	c	d	b	e	f	b	d	c	a		
													(3)	
(10 marks)														
Notes:														
(a)														
M1: Squares B to begin the process of finding the order, or uses a calculator to find B ³														
A1: Shows that B ³ = I and concludes that order must be 3.														
(b)														
B1: States the order for I and gives the order for E (which is the inverse of B).														
M1: Finds the order of at least one of the other elements to get an element of order 2. May use a calculator														
A1: Correct orders for the three elements of order 2.														
(c)														
B1: Correct reason given, identifies there is no element of order 6. It is not sufficient to say that it does not have a group generator, they need to give a reason.														
B1: Any correct reason given.														
Considers the total number of elements of the group, order of the group is more than 6														
Considers the orders of either reflections or rotations														
(d)														
M1: Any two correct matchings														
M1: Completes the matching with at least 4 correct														
A1: A fully specified correct isomorphism.														

Question	Scheme	Marks	AOs	
4(a)	$124 = 17 \times 7 + 5$ $17 = 3 \times 5 + 2$ $5 = 2 \times 2 + 1$ $\{2 = 2 \times 1\}$	M1	2.1	
	Since the gcd is 1 , 124 and 17 are relatively prime (coprime)	A1	2.4	
		(2)		
(b)	$1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 1 = 7 \times 5 - 2 \times 17$ $1 = 7(124 - 17 \times 7) - 2 \times 17$ $\Rightarrow 1 = 7 \times 124 - 51 \times 17$	M1 A1	2.1 1.1b	
	$\Rightarrow 10 = 70 \times 124 - 510 \times 17$ $x = 70, y = -510$	A1	2.2a	
	Alternative For example $124 = 17 \times 7 + 5$ $5 = 124 - 7 \times 17$	$1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 4$ $\qquad\qquad\qquad = 2 \times 17 - 6 \times 5$ $4 = 2 \times 17 - 6(124 - 17 \times 7)$ $\Rightarrow 4 = 44 \times 17 - 6 \times 124$	M1 A1	2.1 1.1b
	$\Rightarrow 10 = 2 \times 124$ $\qquad\qquad\qquad - 14$ $\qquad\qquad\qquad \times 17$ $x = 2, y = -14$	$\Rightarrow 10 = 110 \times 17 - 15 \times 124$ $x = -15, y = 110$	A1	2.2a
		(3)		
(c)	A complete method using modulo arithmetic to achieve $x \equiv \dots \pmod{17}$ For example $7 \times 124x \equiv 7 \times 6 \pmod{17} \Rightarrow x \equiv \dots \pmod{17}$	M1	2.1	
	$x \equiv 8 \pmod{17}$ o.e.	A1	1.1b	
		(2)		
(7 marks)				
Notes:				
(a) M1: Complete attempt at the Euclidean algorithm to find the gcd (hcf) of 124 and 17, condone a numerical slip. A1: A fully correct application of the Euclidean algorithm and draws the conclusion that since the gcd/hcf = 1 therefore 124 and 17 are relatively prime (coprime) .				
(b) M1: Attempts to find the Bezout’s identity, condone sign slips A1: Correct Bezout’s identity. A1: Deduces a set of correct values of x and y.				

Alternatively

M1: Uses the solution to (a) to find an identity of the form $a \times 124 + b \times 17 = c$

A1: Correct identity

A1: Deduces a set of correct values of x and y .

(c)

M1: A complete method to reach using modulo arithmetic to achieve $x \equiv \dots \pmod{17}$

For example

Multiplies through by their multiplicative inverse of 124 and reaches $x \equiv \dots \pmod{17}$. Bezout's identity used in part (b) follow through on their multiple of 124

A1: $x \equiv 8 \pmod{17}$ o.e.

Question	Scheme	Marks	AOs
3(a)	$125 = 87 \times 1 + 38$ $87 = 38 \times 2 + 11 \dots$	M1	1.1b
	$38 = 11 \times 3 + 5$ $11 = 5 \times 2 + 1$	M1 A1	1.1b 1.1b
	$1 = 11 - 5 \times 2$ $= 11 - (38 - 11 \times 3) \times 2 = 11 \times 7 - 38 \times 2$ $= (87 - 38 \times 2) \times 7 - 38 \times 2 = 87 \times 7 - 38 \times 16$	M1	2.1
	$1 = 87 \times 7 - (125 - 87 \times 1) \times 16 = -16 \times 125 + 23 \times 87$ (So $a = -16$ and $b = 23$)	A1	1.1b
		(5)	
(b)	From (a) $23 \times 87 \equiv 1 \pmod{125}$ so multiplicative inverse of 87 is 23.	B1ft	2.2a
		(1)	
(c)	$x \equiv 23 \times 16 \pmod{125}$	M1	1.1b
	$x \equiv 368 \equiv 118 \pmod{125}$	A1	1.1b
		(2)	
(8 marks)			
Notes:			
(a) M1: Begins the process of applying the Euclidean algorithm, with attempt at the first two steps. Allow slips. M1: Completes the process to the stage shown – if errors have been made at least three steps should have been made in reaching their final line (ending +1) to score this mark. A1: Algorithm correctly carried out – as shown. M1: Starts the process of back substitution – at least two substitutions made. A1: Completes the process and finds the correct values for a and b.			
(b) B1ft: Deduces correct multiplicative inverse. Accept 23 or anything congruent to 23 modulo 125 or follow through their b.			
(c) M1: Multiplies 16 by their multiplicative inverse or any other full method to proceed to the solution, e.g. multiplying the identity found in (a) through by 16 and reducing modulo 125. A1: $x \equiv 118 \pmod{125}$. Accept 118 or anything congruent to 118 modulo 125 as long as it is part of a correct modulo statement. (Do not accept just 368 on its own.)			

Question	Scheme	Marks	AOs
4(a)	$168 = 66 \times 2 + 36$ $66 = 36 \times 1 + 30$ $36 = 30 \times 1 + 6$ $\{30 = 5 \times 6 + 0\}$	M1	2.1
	{Last non-zero remainder is 6} so highest common factor is 6.*	A1*	2.4
		(2)	
(b)	$6 = 36 - 30 \times 1$ $= 36 - (66 - 36 \times 1) = 2 \times 36 - 1 \times 66$ $= 2(168 - 66 \times 2) - 1 \times 66$	M1 A1	2.1 1.1b
	$\Rightarrow 6 = 168 \times 2 - 66 \times 5$ so $a = 2, b = -5$	A1	2.2a
		(3)	
(c)	As $168x + 66y$ will always be an integer multiple of 6, but 10 is not a multiple of 6 , there can be no integer solutions. or $\gcd(168, 66) = 6$ and as $6 \nmid 10$	B1	2.4
		(1)	
(d)	From (b) we deduce $1 = 28 \times 2 - 11 \times 5$	B1	2.2a
	Hence $8 \times 11 \times -5 \equiv -5 \times 8 \pmod{28}$	M1	1.1b
	$v \equiv -40 \equiv 16 \pmod{28}$	A1	1.1b
		(3)	
	Alternative 1 Forms an expression of the form $8 = 28A + 11B$	M1	1.1b
	e.g. $8 = 5 \times 28 - 12 \times 11$ $8 = 16 \times 28 - 40 \times 11$	B1	2.2a
	e.g. $v \equiv -12 \pmod{28}$ $v \equiv -40 \pmod{28}$	A1	1.1b
		(3)	
	Alternative 2 $11v \equiv 8 \pmod{28} \Rightarrow 44v \equiv 32 \pmod{28} \Rightarrow 11v \equiv 8 \pmod{7}$	B1	1.1b
	$v \equiv 11^5 \times 8 \pmod{7} \Rightarrow \{v \equiv 2 \pmod{7}\}$	M1	1.1b
	$v \equiv 16 \pmod{28}$	A1	2.2a
		(3)	
	(9 marks)		

Notes:**(a)****M1:** Attempt at the Euclidean algorithm to find the gcd of 168 and 66, condone a numerical slip.**A1*:** A fully correct application of the Euclidean algorithm and explains that the highest common factor is 6.**(b)****M1:** Attempts to apply back substitution to find suitable values.**A1:** Correct expression in just the numbers 168 and 66 reached (all remainders eliminated). Need not be fully simplified.**A1:** Deduces the correct values of a and b . Allow if only seen in the expression.**(c)****B1:** Correct explanation. Must refer to 10 not being a multiple of 6.**(d)****B1:** Uses the result in (b) and divides through by 6 to form a correct statement between 11 and 28. Alternatively uses Euclidean algorithm for 28 and 11 to form an equation for 1 as multiplies of 28 and 11. Allow this mark for a correct unsimplified equation.**M1:** Multiplies through in their equation by 8 or the original equation through by -5 (or other multiplicative inverse).**A1:** Deduces the correct solutions. Accept either $-40 \pmod{28}$ or $16 \pmod{28}$, or any other correct congruence.**Alternative 1: note the order of marks has changed****M1:** forms an equation for 8 in terms of multiplies of 28 and 11**B1:** Correct equation, allow this mark for a correct unsimplified equation.**A1:** Correct solution**Alternative 2:****B1:** Correct equation**M1:** Uses Fermat's little theorem**A1:** Correct solution**Trial and error approach****(d)** – likely to score B0M1A1.**B1:** Gives some kind of proof that the solution is unique. E.g. by reference to $\text{hcf}(11,28) = 1$ or trial of every possible value.**M1:** Applies trial and improvement method to find one solution.**A1:** Correct solution found.

Correct answer stated with no supportive working scores B0M1A1



Question	Scheme	Marks	AOs
3(a)	$234 = 2 \times 96 + 42$	M1	1.1b
	$96 = 2 \times 42 + 12; \quad 42 = 3 \times 12 + 6; \quad 12 = 2 \times 6(+0)$	M1	1.1b
	Hence $h = 6$	A1	2.2a
		(3)	
(b)	Using back substitution $6 = 42 - 3 \times 12$	M1	1.1b
	$= 42 - 3(96 - 2 \times 42) = 7 \times 42 - 3 \times 96$ $= 7 \times (234 - 2 \times 96) - 3 \times 96$	dM1	1.1b
	$= 7 \times 234 - 17 \times 96 \quad (\text{so } a = 7 \text{ and } b = -17)$	A1	1.1b
		(3)	
(c)	As 6 divides 36 the congruence is equivalent to $16x \equiv 6(\text{mod } 39)$	B1	2.2a
	From (b) we deduce $1 = 7 \times 39 - 17 \times 16$ so -17 (or 22) is a multiplicative inverse of 16 modulo 39 or $16x \equiv 6(\text{mod } 39) \Rightarrow 2 \times 8x \equiv 2 \times 3(\text{mod } 39) \Rightarrow 8x \equiv 3(\text{mod } 39)$ Finds multiplicative inverse of 8 e.g. $1 = 5 \times 8 - 39$	M1	3.1a
	Hence $22 \times 16x \equiv 22 \times 6(\text{mod } 39)$ or $-17 \times 16x \equiv -17 \times 6(\text{mod } 39)$ or Hence $5 \times 8x \equiv 5 \times 3(\text{mod } 39)$ Leading to $x \equiv \dots(\text{mod } 39)$	M1	1.1b
	$x \equiv 132 \equiv 15(\text{mod } 39) \quad \text{Accept just 15}$	A1	1.1b
	So the solution is $x \equiv 15(\text{mod } 39)$ or $x \equiv 15, 54, 93, 132, 171 \text{ or } 210 \text{ (mod 234)}$	A1	2.3
		(5)	
	(11 marks)		
Notes:			
(a)			
M1: Starts the process of using the algorithm, with attempt at $234 = p \times 96 + q$.			
M1: Continues the process until remainder zero is reached.			
A1: Deduces the correct highest common factor from correct work.			
(b)			



M1: Begins the process of back substitution by rearranging their equation with least positive remainder.

dM1: Completes the process.

A1: Correct expression or values of a and b identified.

(c)

B1: Deduces the correct equivalent congruence.

M1: Chooses a suitable strategy to solve the reduced congruence. May use multiplicative inverse as per scheme or see alts for some variations, must be using their value for b

M1: Applies their multiplicative inverse to reach $x \equiv \dots \pmod{39}$

In the alternative it is for reaching $x \equiv \dots \pmod{39}$ from a succession of multiples.

A1: For 15 as a solution, need not have the $\pmod{39}$. (Accept one correct solution if an alternative method is used.)

A1: For stating the solution is $15 \pmod{39}$ as the answer (not just in working) **or** for listing all the solutions modulo 234. Either way of expressing the answer is fine (single answer mod 39, or all 6 mod 234), but must realise it is more than just the 15, so do not accept just 15 with no indication of the modulus being considered.

ISW if they achieve the correct answer and then try to list solutions

SPECIAL CASE: B1 For one correct value of x found

Alts for (c)

(c) Way 2	From (b) we deduce $1 = 7 \times 39 - 17 \times 16$ so -17 (or 22) is a multiplicative inverse of 96 modulo 239	B1	2.2a
	Hence $-17 \times 96x \equiv -17 \times 36 \pmod{234}$ or $6x \equiv -612 \pmod{234}$	M1	3.1a
	As 6 divides the congruence $6x \equiv 90 \pmod{234}$ to reach $x \equiv \dots \pmod{39}$	M1	1.1b
	As 6 divides the congruence $6x \equiv -612 \pmod{234}$ to reach $x \equiv \dots \pmod{39}$		
	$x \equiv 15 \pmod{39}$ Accept just 15	A1	1.1b
	So the solution is $x \equiv 15 \pmod{39}$ or $x \equiv 15, 54, 93, 132, 171$ or $210 \pmod{234}$	A1	2.3
		(5)	
	Notes B1: Deduces the multiplicative inverse of 96 M1: Multiplies through by the multiplicative inverse of 96 M1: For using reaching $x \equiv \dots \pmod{39}$. A1: For 15 as a solution, need not have the $\pmod{39}$. (Accept one correct solution if an alternative method is used.)		



	A1: For stating the solution is $15 \pmod{39}$ as the answer (not just in working) or listing all the solutions modulo 234. As per main scheme do not accept just 15 with no indication of the modulus being considered.		
(c) Way 3	As 6 divides 36 the congruence is equivalent to $16x \equiv 6 \pmod{39}$	B1	2.2a
	$5 \times 16x \equiv 5 \times 6 \pmod{39} \Rightarrow 2x \equiv 30 \pmod{39}$	M1	3.1a
	$\Rightarrow 20 \times 2x \equiv 20 \times 30 \pmod{39} \Rightarrow 1x \equiv 15 \pmod{39}$	M1	1.1b
	$x \equiv 15 \pmod{39}$ Accept just 15	A1	1.1b
	So the solution is $x \equiv 15 \pmod{39}$ or $x \equiv 15, 54, 93, 132, 171 \text{ or } 210 \pmod{234}$	A1	2.3
		(5)	

Notes

B1: Deduces the correct equivalent congruence.

M1: Tries multiplying by various numbers in an attempt to reduce the coefficient to 1. One example shown above but others are possible. Allow the M for an attempt at starting such a process.

M1: For using reaching $x \equiv \dots \pmod{39}$ from a succession of multiples.

A1: For 15 as a solution, need not have the $\pmod{39}$. (Accept one correct solution if an alternative method is used.

A1: For stating the solution is $15 \pmod{39}$ as the answer (not just in working) or listing all the solutions modulo 234. As per main scheme do not accept just 15 with no indication of the modulus being considered.

(c) Way 4	As 6 divides 36 the congruence is equivalent to $16x \equiv 6 \pmod{39}$	B1	2.2a
	So $16x \equiv 6, 45, 84, 123, \dots$	M1	3.1a
	$16x \equiv \dots, 240$	M1	1.1b
	$240 = 16 \times 15 \Rightarrow x \equiv 15 \pmod{39}$ Accept just 15	A1	1.1b
	So the solution is $x \equiv 15 \pmod{39}$ or $x \equiv 15, 54, 93, 132, 171 \text{ or } 210 \pmod{234}$	A1	2.3
		(5)	

Notes

B1: Deduces the correct equivalent congruence.

M1: Works out the possibilities for $16x$ to try and find one that is divisible by 16. Look for the first few evaluated.

M1: For reaching a value for $16x$ that is a multiple of 16.

A1: For 15 as a solution, need not have the $\pmod{39}$. (Accept one correct solution if an alternative method is used.



Question	Scheme	Marks	AOs
1	Identifies either 3 or 11 as a prime divisor of 66 and proceeds to apply the divisibility test for this prime number.	M1	3.1a
	Either $2-0+2-1+0-5+2-0=0=0\times 11$ hence n is divisible by 11 Or $2+0+2+1+0+5+2+0=12=4\times 3$ hence n is divisible by 3	A1	2.2a
	Both $2-0+2-1+0-5+2-0=0=0\times 11$ hence n is divisible by 11 And $2+0+2+1+0+5+2+0=12=4\times 3$ hence n is divisible by 3	A1	2.2a
	As also n is even, it is divisible by 2, and hence as divisible by 2, 3 and 11, is divisible by $2\times 3\times 11=66$	A1	2.4
		(4)	
(4 marks)			
Notes:			
M1: Identifies one of the odd prime factors of 66 and proceeds to check divisibility for it. A1: Correct method and deduction for either divisibility by 3 or by 11 A1: Correct method and deduction for both divisibility by 3 and by 11 A1: Notes also divisibility by 2 and explains why divisibility by 66 follows. The explanation may have been given in a preamble “$66 = 2\times 3\times 11$ so divisible by 66 if divisible by 2, 3 and 11” The must be a correct reason for divisibility by 2, ie “it is even” or “last digit is even”. Do not accept “last digit is 0” with no reason given. NB There is no divisibility test for 6, so attempting such will result in the loss of the last two A marks. (E.g. sum of digits being a multiple of 6 is an incorrect test, as for instance 15 satisfies $1+5 = 6$ but is not a multiple of 6.)			

Question	Scheme	Marks	AOs
8(a)	The integer n can be written as $n = 1000a + 100b + 10c + d$	M1	1.1b
	As $1000 = 142 \times 7 + 6$, $100 = 14 \times 7 + 2$ and $10 = 7 + 3$, reducing coefficients modulo 7 gives $n \equiv 6a + 2b + 3c + d \pmod{7}^*$	A1*	2.4
		(2)	
(b)	n divisible by 9 means $a + b + c + d = 9k$ for some integer k	B1	1.1b
	$a + b = c + d \Rightarrow 2(a + b) = 9k$, hence k even	M1	3.1a
	But $a + b + c + d$ must be at least 3 and at most 35 (as b smaller than all other numbers).	B1	2.1
	So since k must be even, only possibility is $k = 2$ to keep $a + b + c + d$ in range 0-36, hence $a + b = 9^*$	A1*	2.2a
		(4)	
(c)	Combining (a) and (b) gives $3 \equiv 2(a + b) + 4a + (c + d) + 2c \equiv 4a + 2c + 27 \pmod{7}$	M1	3.1a
	$\Rightarrow 2c \equiv -4a - 24 \equiv -4(a + 6) \equiv 3(a - 1) \pmod{7}$ $\Rightarrow 8c \equiv 12(a - 1) \pmod{7} \Rightarrow c \equiv 5(a - 1) \pmod{7}^*$	A1*	2.1
		(2)	
(d)	As $b < a$ and $a + b = 9$ we must have $a \leq 5$	M1	3.1a
	If $a = 9$ then $c \equiv 5 \pmod{7} \Rightarrow c = 5$, contradicting c is even.	dM1 A1	1.1b 1.1b
	If $a = 8$ then $c \equiv 0 \pmod{7} \Rightarrow c = 0$ or 7 but 7 not even, and $c > b$ so can't be zero.		
	If $a = 7$ then $c \equiv 2 \pmod{7} \Rightarrow c = 2$, but also $b = 2$ for $a = 7$, and $b < c$ so not possible.		
	If $a = 6$ then $c \equiv 4 \pmod{7} \Rightarrow c = 4$, $d = 5$ and $b = 3$ which works.		
	If $a = 5$ then $c \equiv 6 \pmod{7} \Rightarrow c = 6$, $d = 3$ and $b = 4$ but then $d < b$, not allowed.		
	Hence $n = 6345$ (only)	B1	2.2a
		(4)	
(12 marks)			



Notes: Allow credit for any part for relevant work seen throughout.

(a)

M1: Writes $n = abcd$ as sum of multiples of the digits, as shown in scheme.

A1*: Explains how each coefficient reduces modulo 7 to give the stated answer.

(b)

B1: Applies the divisibility test for 9, either as in scheme or stating $a + b + c + d$ is a multiple of 9 (or similar). $a + b + c + d \equiv 0 \pmod{9}$ is fine.

M1: Uses the third fact in combination with the second to eliminate c and d and deduce k even.

Alternatively, $a + b + c + d \equiv 2(a + b) \equiv 0 \pmod{9} \Rightarrow 10(a + b) \equiv 5 \times 0 \pmod{9} \Rightarrow a + b \equiv 0 \pmod{9}$

B1: Eliminates the possibilities that $k = 0$ or 4 using the fourth fact. Alt: reasons as $b < a$ then $a + b$ cannot be 0 or 18 .

A1*: Completes the proof, with all steps explained. Allow if the previous B has not been earned.

(c)

M1: Combines the results of (a) and (b) to eliminate b and d from the result in (a).

A1*: Correct completion to the given statement. There will be different ways to achieve this, one example is shown. Check work carefully!

(d)

M1: There may be many approaches here. Scored for beginning a process of eliminating possibilities for one of the digits, e.g. deducing a is at least 5 as shown. (May be done by individually eliminating the lower values.)

dM1: Continues the process eliminating at least two more cases.

A1: Complete argument, leading to one solution only (and no possibility of others). Need not be expressed as eloquently as above, but should be clear all cases have been considered.

B1: For $n = 6345$ obtained in any way.



Question	Scheme	Marks	AOs
7(i)	$\gcd(36, 21) = 3$	M1	1.1b
	3 does not divide into 11 therefore no solutions	A1	2.4
		(2)	
(ii)	$12 \times 3y \equiv 12 \times 16 \pmod{40}$	M1	1.1b
	$\gcd(12, 40) = 4$ and divides through by 4	M1	1.1b
	$3y \equiv 16 \pmod{10} \Rightarrow 3y \equiv 6 \pmod{10}$ $\gcd(3, 10) = 1$	dM1	2.1
	$y \equiv 2 \pmod{10}$ or $y \equiv 2, 12, 22, 32 \pmod{40}$	A1	2.2a
		(4)	
	Alternative $36y \equiv 192 \equiv 32 \pmod{40}$	M1	1.1b
	$\text{hcf}(36, 40) = 4 \Rightarrow 9y \equiv 8 \pmod{10}$	M1	1.1b
	E.g. $9y \equiv 18 \pmod{10} \Rightarrow y \equiv 2 \pmod{10}$	dM1	2.1
	$y \equiv 2 \pmod{10}$ or $y \equiv 2, 12, 22, 32 \pmod{40}$	A1	2.2a
		(4)	

(6 marks)

Notes:

(i)

M1: Finds the gcd of 36 and 21 or in some way considers the hcf of these two numbers.**A1:** States that the gcd is 3 and does not divide into 11 therefore no solutions

(ii)

M1: Identifies $36y$ as kay and 192 as kb (may be implied by cancelling).**M1:** Finds the gcd of k and 40. Then divides through by the gcd of k and 40, Modulus must be reduced to 10.**dM1:** Completes the solution by appropriate means. There will be many variations on how they can do this. Accept just $y = 2$ as an attempt to find the solution. Note the multiplicative inverse of 9 modulo 10 is 9. If a multiplicative inverse is attempted, it must be from a correct method.**A1:** Deduces the correct solution

Alternative I

M1: Reduces RHS modulo 40 to give $36y \equiv 32 \pmod{40}$ **M1:** Finds the gcd of 36 and 40 and divides through by 4. This must include the modulus being reduced to 10.**dM1:** Depends on previous M. Finds a value for y by any appropriate means. There will be many variations on how they can do this. Accept just $y = 2$ as an attempt to find the solution.**A1:** Deduces the correct solution

Note: Candidates to start by finding the hcf of 36 and 40 and divide through to get $9y \equiv 48 \pmod{10}$ will score M2 at the start of the solution.

(b) Alt II	$36y \equiv 192 \pmod{40} \Rightarrow 36y = 192 + 40k$	M1	1.1b
	$\Rightarrow 9y = 48 + 10k$	M1	1.1b
	$\Rightarrow 10y - y = 50 - 2 + 10k \Rightarrow y = 10y - 50 - 10k + 2$ $\Rightarrow y = 2 + 10(y - 5 - k)$	dM1	2.1
	$y \equiv 2 \pmod{10}$ or $y \equiv 2, 12, 22, 32 \pmod{40}$	A1	2.2a
		(4)	

(6 marks)

Notes:

(b) Alt II

M1: Writes the congruence as a linear equation with $40k$

M1: Divides the equation through by 4

dM1: Completes the solution by appropriate means. One example shown in scheme.

A1: Deduces the correct solution.



Question	Scheme	Marks	AOs
5(i)	(a) $5^4 \times \dots$ or $\dots \times 4 \times 3 \times 2$ or $\dots \times {}^4P_3$	M1	1.1b
	$5^4 \times 4 \times 3 \times 2 = 15000$	A1	1.1b
	(b) The structure is $_N_N_N_N_$, where three $_$'s are letters, so have ${}^5C_3$ choices for the letters, hence new number of combinations is ${}^5C_3 \times 15000$ or 10×15000	M1	3.1a
	So increase in number of codes is 135000	A1ft	1.1b
		(4)	
(ii)	(a) Divisible by 9 $\Rightarrow a + b + c + d = 9k$ or e.g. attempt $1+3+5+7=16$, not a multiple of 9, so reject. E.g. $1+3+5+7+9=25$ so $k=1$ or 2, but $1+3+5+7=16 > 9$, so $k=2$ hence $\Rightarrow 25 - (a + b + c + d) = 25 - 18 = 7$ so 7 missing. Or considers options in turn and finds only $1+3+5+9=18$ works	M1	3.1a
	So the digits are 1, 3, 5 and 9	A1	1.1b
	(b) Combination is either 1359 or 9531 and $1359 \pmod{13} \equiv 59 \pmod{13} \equiv 7$, $9531 \pmod{95} \equiv 31$	M1	1.1b
	Combination is 1359	A1	2.2a
		(4)	
(8 marks)			
Notes:			
(i)(a) M1: $5^4 \times \dots$ or $\dots \times 4 \times 3 \times 2$ or $\dots \times {}^4P_3$ A1: For 15000 (b) M1: Correct strategy to find the new number of codes seen or implied. A1ft: Increase in codes is 135000. Follow through $9 \times$ their 15000			
(ii)(a) M1: Full strategy to deduce the correct set of digits. A1: Digits are 1,3, 5 and 9 Writes down the correct values scores M1A1 (b) M1: Checks which of the two possible combinations satisfies the final property. A1: Combination is 1359 Writes down the correct code scores M1A1			



Question	Scheme	Marks	AOs
7(i) (a)	120	B1	1.1b
		(1)	
(i) (b)	300	B1	1.1b
		(1)	
(ii)(a)	As divisible by 11 implies $a - b + c = 11p$ where p is an integer Due to the restrictions on a, b and c $\Rightarrow a - b + c = 11$ or 0	M1	2.4
	For example $a + b + c$ is even $\Rightarrow (a + b + c) - (2b)$ is even $\Rightarrow a - b + c$ is even or Sum of digits is even implies that $a + b + c = 2q$ $(a - b + c = (a + b + c) - 2b = 2q - 2b = 2n)$ Alternative approach 1 $a + b + c = 2q$ where $q \in \mathbb{Z}$ either $a + b + c = 2q$ and $a - b + c = 0 \Rightarrow 2b = 2q \Rightarrow b = q$ valid $a + b + c = 2q$ and $a - b + c = 11 \Rightarrow 2b = 2q - 11 \Rightarrow b = q - 5.5$ not valid Or $a + b + c = 2q$ and $a - b + c = 0 \Rightarrow 2a + 2c = 2q$ valid $a + b + c = 2q$ and $a - b + c = 11 \Rightarrow 2a + 2c = 2q + 11$ not valid as $2(a + c)$ even or This approach may be in words Alternative approach 2 If sum $a - b + c$ is odd/11 then either one or all three numbers odd. This would mean sum $a + b + c$ would be odd/not even Alternative approach 3 If $a - b + c = 11$ this implies $a + b + c \equiv 1 \pmod{2}$ (contradiction) Conclusion therefore $a - b + c = 0$ c.s.o	A1*	2.1
		(2)	
(ii)(b)	$N = 100a + 10b + c \Rightarrow a + b + c \equiv 8 \pmod{9}$ o.e. or $N + 1 \equiv 0 \pmod{9}$ ($N + 1$ is a multiple of 9) o.e.	B1	1.1b

	$\Rightarrow a + b + c + 1 \equiv 0 \pmod{9}$ ($a + b + c + 1$ is a multiple of 9) o.e		
	Solves $a - b + c = 0$ and $a + b + c \equiv 8 \pmod{9}$ either ($a + b + c \equiv 8$ or 26) to find a value for b ($= 4$) and uses this value to form an equation $a + c = \dots (4)$ Or $2a + 2c \equiv 8 \pmod{9} \Rightarrow a + c \equiv 4 \pmod{9}$ leading to $a + c = \dots (4)$ or $2a + 2c \equiv 8 \pmod{9} \Rightarrow a + c \equiv 4 \pmod{9} \Rightarrow b \equiv 4 \pmod{9}$ leading to $b = \dots (4)$ Or $a - b + c = 0$, $b = a + c$ so $2b + 1$ is a multiple of 9, $2b + 1 = 9$ or 18 leading to $b = \dots (4)$	M1	2.1
	$N = 143, 242, 341, 440$	M1 A1	1.1b 2.2a
		(4)	
	Alternative $(N) = 11n \equiv 8 \pmod{9}$	B1	1.1b
	Finds multiplicative inverse of 11 using $5 \times 9 - 4 \times 11 = 1$ $-4 \times 11n \equiv -4 \times 8 \pmod{9} \Rightarrow n \equiv -32 \pmod{9} \Rightarrow n \equiv 4 \pmod{9}$ So $N = 11(9n + 4)$	M1	2.1
	$N = 143, 242, 341, 440$	M1	1.1b
		A1	2.2a
		(4)	
(8 marks)			
Notes:			
(i)(a) B1: See scheme			
(i)(b) B1: See scheme			
(ii) (a) M1: Uses the divisibility rule for 11 and the restrictions on the values a, b and c leading to $a - b + c = 0$ or 11 only A1*: Uses the information that the sum of the digits is even and that $a - b + c = 0$ or 11 from the divisibility rule for 11 to show that $a - b + c = 0$ cso			
(ii) (b) B1: States $a + b + c \equiv 8 \pmod{9}$ or $a + b + c + 1 \equiv 0 \pmod{9}$			

M1: Uses their equations to find a value for $a + c = \dots$ or a value for b

M1: Uses their values to find at least two correct values for N

A1: Deduces all four correct values of N and no extra values

Alternative

B1: States $(N) = 11n \equiv 8 \pmod{9}$

M1: Finds the multiplicative inverse of 11 and uses this to find an expression for N

M1: Uses their expression for N to find at least two values for N

A1: Deduces all four correct values of N and no extra values

Trial and error

B1: For identifying $a + c = b$

M1: For one correct value, this implies B1

M1: For two correct values

A1: Deduces all four correct values of N and no extra values

Question	Scheme	Marks	AOs
5(i)(a)	Any two out of 26 or $10 \times 9 \times 8$ or 4 seen multiplied together	M1	1.1b
	$26 \times 10 \times 9 \times 8 \times 4 = 74880$	A1	1.1b
		(2)	
(i) (b)	their '74880' $\times \frac{5!}{3!} = 1497600$	B1ft	1.1b
		(1)	
(ii)	$4^{12} \equiv 1 \pmod{13}$ or $4^{13} \equiv 4 \pmod{13}$	B1	1.2
	$(4^{12})^4 \times 4^2 \equiv 1 \times 4^2 \pmod{13}$ or $(4^{13})^3 \times 4^{11} \equiv 4^3 \times 4^{11} \equiv 4^{14} \equiv 4^{13} \times 4^1 \equiv \dots \pmod{13}$	M1	2.1
	$4^{50} \equiv 3 \pmod{13}$	A1	2.2a
		(3)	
(iii)	The number of different 1-digit odd numbers that do not contain the digit 2 is 5	B1	2.2a
	The number of different 2-digit odd numbers that do not contain the digit 2 is $8 \times 5 \{= 40\}$ The number of different 3-digit odd numbers that do not contain the digit 2 is $8 \times 9 \times 5 \{= 360\}$	M1	1.1b
	There are 500 odd numbers therefore $500 - (5 + 40 + 360)$	dM1	2.1
	$= 95$	A1	1.1b
		(4)	
Alt 1	Identifies that 2 can only appear in the first and/or second position	B1	2.2a
	First digit is 2, second digit 10 possible, third digit 5 possible $= 10 \times 5 = \dots \{50\}$ First digit not 2, 9 options (including 0), second digit is 2, third digit 5 possible $= 9 \times 5 = \dots \{45\}$	M1	1.1b
	Therefore total number of digits is "50" + "45" = ...	dM1	2.1
	$= 95$	A1	1.1b
		(4)	
Alt 2	1 digit number = 0 ways	B1	2.2a
	2 digit number = $1 \times 5 = 5$ ways 3 digit number starting with 2 = $1 \times 10 \times 5 = 50$ ways Other 3 digit number with middle number 2 = $8 \times 1 \times 5 = 40$ ways	M1	1.1b
	Total number = $0 + 5 + 50 + 40$	dM1	2.1
	$= 95$	A1	1.1b
		(4)	

(10 marks)

Notes**(i) (a)****M1:** Any two out of 26 or $10 \times 9 \times 8$ or 4 seen multiplied together**A1:** Correct answer**(b)****B1ft:** For the correct answer or follow through their answer to (a) multiplied by 20. Note they may start again and use $26 \times {}^{10}C_3 \times 4 \times 5!$ **(ii)****B1:** Recalls Fermat's Little Theorem correctly. May be implicit in the working rather than stated separately. Accept if stated in terms of a and p .**M1:** A complete attempt to write 4^{50} as powers of 4^{12} or 4^{13} and completes the process to find the residue.**A1:** Deduces the correct residue.**(iii)****B1:** Deduces that there are 3 different 1-digit odd numbers that do not contain the digit 2. Variation: if they consider 000 to 999 allow B1 for correct option of 9 for first digit.**M1:** Finds the number of different 2-digit and 3-digit odd numbers that do not contain the digit 2. (In the variation they would do $9 \times 9 \times 5$ possibilities without 2, giving 405 directly.)**dm1:** For 500 – the sum of the number of 1, 2, 3 digits that don't contain the digit 2. Must have been full method to find all the digits without 2 first.**A1:** Correct answer**Alternative 1****B1:** Deduces that there are only two positions for 2, first and second. (May be implied)**M1:** Considers first digit is 2 and the possible digits for second and third position and considers second digit is 2 and the possible digits for first and third position. Allow if they have double counted some digits for this mark.**dm1:** Full method considering the possibilities for 2 in various positions and adds the results. They must subtract any repeats for this mark.**A1:** Correct answer**Alternative 2****B1:** Deduces that there are no single digit numbers**M1:** Considers the possible 2 and 3 digit numbers. Allow if they have double counted some digits for this mark or omit some cases as long as at least some cases are considered.**dm1:** Adds up all the possible digits following consideration of all possible combinations. They must subtract any repeats for this mark. For 3 digit numbers they may have $1 \times 10 \times 5$ starting 2, $9 \times 1 \times 5$ with 2 in middle, but need to note 5 of these are repeated (22x), and so get $5 + 50 + 45 - 5 = \dots$ **A1:** Correct answer.

Other Alternatives are possible. E.g.

B1: Considers 3 digit numbers from 000 to 999. (May be implicit)**M1:** Starting with 2 : 50 possibilities (must be odd), middle 2 but not starting 2 : $9 \times 1 \times 5$ possibilities.**dm1:** So total number of possibilities is $50 + 45 = \dots$ **A1:** Correct answer.