



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Geometric and Negative Binomial Distributions (3)

Further Statistics 1 · Topic 3

3 questions · 25 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

 [guidemygrades.com](https://www.guidemygrades.com)

 @Guidemygrades

 @Guidemygrades

 @Guidemygrades

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
3.1 GEOMETRIC AND NEGATIVE BINOMIAL DISTRIBUTIONS		1 Qs · 7 marks	3
1. Q1	Option 3B: Further Statistics 1 June2025	7	4
3.2 MEAN AND VARIANCE OF THE GEOMETRIC DISTRIBUTION		2 Qs · 18 marks	6
2. Q5	Option 3B: Further Statistics 1 June2022	5	7
3. Q6	Option 3B: Further Statistics 1 June2025	13	9



QUESTIONS

3 questions · 25 marks · 2 sub-topics



SUB-TOPIC 3.1

GEOMETRIC AND NEGATIVE BINOMIAL DISTRIBUTIONS

Sub-topic 3.1 · 1 question · 7 marks · lowest tariff first



1. Irina is practising her serves in badminton and counts the number of her serves that are faults. She finds that 15% of her serves are faults.

Assuming that each serve is independent,

- (a) find the probability that

- (i) Irina's 4th fault comes on her 20th serve, (2)

- (ii) in 18 serves, Irina has 4 faults. (2)

With practice, Irina reduces her proportion of faults, p , so that the mean number of serves until her 4th fault is at least 32

- (b) Find the maximum value of p (3)

Question 1 continued

SUB-TOPIC 3.2

MEAN AND VARIANCE OF THE GEOMETRIC DISTRIBUTION

Sub-topic 3.2 · 2 questions · 18 marks · lowest tariff first



5. A random sample of 150 observations is taken from a geometric distribution with parameter 0.3

Estimate the probability that the mean of the sample is less than 3.45

(5)



Question 5 continued

(Total for Question 5 is 5 marks)



- 6.** A biased coin and a fair coin are thrown repeatedly.

The probability that the biased coin shows heads is $\frac{1}{3}$

Let B represent the number of throws of the biased coin until it shows heads for the first time. Let F represent the number of throws of the fair coin until it shows heads for the first time.

- (a) Find

$$(i) \quad P(B = 3) \tag{2}$$

$$(ii) \quad P(2 \leq F \leq 8) \tag{3}$$

$$(iii) \quad P(\{B = 3\} \cup \{F = 3\}) \tag{3}$$

Each day Chris invites his friend Shivani to play a game with these coins. One player's score will be X and the other player's score will be Y , where

$$X = 4F \quad \text{and} \quad Y = \frac{B^2}{2}$$

Chris and Shivani will play a large number of games and the winner will be the player with the higher **total** score.

Before their first game, Chris allows Shivani to choose whether her score for every game will be X or her score for every game will be Y

- (b) State, explaining your reasoning and showing any calculations you have made, which choice Shivani should make.
- (5)

Question 6 continued



Question 6 continued



Question 6 continued

MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
1(a)(i)	$X \sim \text{NB}(4, 0.15)$	M1	3.3
	$\binom{19}{3} (0.15)^4 (1 - 0.15)^{16} = 0.03642$ awrt 0.0364	A1	1.1b
		(2)	
(a)(ii)	$X \sim \text{B}(18, 0.15)$	M1	3.3
	$\binom{18}{4} (0.15)^4 (1 - 0.15)^{14} = 0.15920\dots$ awrt 0.159	A1	1.1b
		(2)	
(b)	Uses $\frac{4}{p}$ as an expression for the mean	B1	3.4
	$\frac{4}{p} \geq 32$	M1	1.1b
	$p = \frac{1}{8}$ or 0.125 or 12.5%	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a)(i)	M1:	selecting NB(4, 0.15) or use of a correct method, may be implied by a correct answer	
	A1:	awrt 0.0364	
(a)(ii)	M1:	selecting B(18, 0.15) or use of a correct method, may be implied by a correct answer	
	A1:	awrt 0.159	
(b)	B1:	$[E(X)] = \frac{4}{p}$	
	M1:	for an equation or inequality with their expression for mean and 32 allow mean = 32 or mean > 32 mean ≤ 32 or mean < 32 is M0	
	A1:	0.125 or exact equivalent Must not come from clearly incorrect working NB: $p \leq \frac{1}{8}$ is B1M1A0 M0 implies A0 (method mark cannot be implied here)	



Question	Scheme	Marks	AOs
5	Geo (0.3) $\mu = \frac{1}{0.3} \left[\text{or exact equivalent e.g. } \frac{10}{3} \right]$ $\sigma^2 = \frac{1-0.3}{0.3^2} \left[\text{or exact equivalent e.g. } \frac{70}{9} \right]$ CLT $\Rightarrow \bar{X} \approx N\left(\frac{10}{3}, \dots\right)$ oe $\Rightarrow \bar{X} \approx N\left(\frac{10}{3}, \frac{7}{135}\right)$ and attempt (sight of) $P(\bar{X} < 3.45)$ $= 0.69579\dots$ awrt 0.696	B1 B1 M1 M1 A1 (5)	1.1b 1.1b 2.1 3.4 1.1b
Total 5			
1st B1 2nd B1 1st M1 2nd M1 A1	correct mean correct Var may be implied by sight of $\frac{7}{135}$ in distribution of $\bar{X}$ For use of CLT (must see $\bar{X}$ and Normal with mean correct ft) <u>or</u> sight of $N\left(\frac{10}{3}, \frac{7}{135}\right)$ <u>or</u> $N\left(\frac{10}{3}, \frac{70}{9 \times 150}\right)$ with any letter Allow 3.33 or better for $\frac{10}{3}$ and 7.78 or better for $\frac{70}{9}$ May be implied by 2nd M1 Using the normal distribution to find $P(\bar{X} < 3.45)$ ft their $\frac{10}{3}$ and $\frac{70}{150}$ May be implied by correct answer. awrt 0.696 Correct answer with no incorrect working scores 5/5		
	Alternative (Use of $Y = \sum X$) $\mu = \frac{150}{0.3} [= 500]$ $\sigma^2 = \frac{150 \times 0.7}{0.3^2} \left[\frac{3500}{3} \right] = 1166.\dot{6}$ $\Rightarrow Y \approx N\left(500, \frac{3500}{3}\right)$ $P(Y < 517.5)$ $= 0.69579\dots$	B1 B1 M1 M1 A1	



Question	Scheme	Marks	AOs
6(a)(i)	$B \sim \text{Geo}\left(\frac{1}{3}\right)$	M1	3.3
	$P(B=3) = \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right) = \frac{4}{27}$ awrt 0.148	A1	1.1b
		(2)	
(a)(ii)	$F \sim \text{Geo}\left(\frac{1}{2}\right)$ and use of $P(F \leq n) = 1 - (1-p)^n$	M1	2.1
	$P(2 \leq F \leq 8) = P(F \leq 8) - P(F \leq 1)$	M1	1.1b
	$P(2 \leq F \leq 8) = 1 - \left(\frac{1}{2}\right)^8 - \left(1 - \frac{1}{2}\right) = \frac{1}{2} - \frac{1}{256} = \frac{127}{256}$ awrt 0.496	A1	1.1b
		(3)	
(a)(iii)	$P[(F=3) \cup (B=3)] = P(F=3) + P(B=3) - P(F=3) \times P(B=3)$ or $P[(F=3) \cup (B=3)] = 1 - P(F \neq 3) \times P(B \neq 3)$ or $P[(F=3) \cup (B=3)] = P(B=3) + P(F=3) \times P(B \neq 3)$	M1	2.1
	$\frac{4}{27} + \left[\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)\right] - \frac{4}{27} \times \left[\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)\right]$ or $1 - \left[1 - \frac{4}{27}\right] \times \left[1 - \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)\right]$	M1	1.1b
	$\frac{4}{27} + \frac{1}{8} - \frac{4}{27} \times \frac{1}{8} = \frac{55}{216}$ or $1 - \left[1 - \frac{4}{27}\right] \times \frac{7}{8} = \frac{55}{216}$ awrt 0.255	A1	1.1b
		(3)	
(b)	Require $E(4F)$ and $E\left(\frac{B^2}{2}\right)$	M1	3.1a
	$E(X) = E(4F) = 4E(F) = 4 \times 2 = 8$ $E(X) = 8$	B1	2.1
	$E(Y) = E\left(\frac{B^2}{2}\right) = \frac{1}{2}E(B^2) = \frac{1}{2}(\text{Var}(B) + [E(B)]^2)$	M1	1.1b
	$E\left(\frac{B^2}{2}\right) = \frac{1}{2} \left(\frac{1 - \frac{1}{3}}{\left(\frac{1}{3}\right)^2} + (3)^2 \right) = \frac{1}{2}(6 + 9)$ $E(Y) = 7.5$	A1	1.1b
	Shivani should choose X as it has a <u>higher expected score</u>	A1ft	3.2a
		(5)	
(13 marks)			

Notes:



(a)(i)	M1:	For sight or use of the correct geometric model
	A1:	$\frac{4}{27}$ or awrt 0.148 (correct answer implies full marks)
(a)(ii)	M1:	For selecting the correct geometric model and use of cumulative probability formula Implied by $\frac{1}{256}$
	M1:	Sight or use of $P(F \leq 8) - P(F \leq 1)$
	A1:	$\frac{127}{256}$ or awrt 0.496 (correct answer implies full marks)
(a)(iii)	M1:	Sight or use of $P[(F=3) \cup (B=3)] = P(F=3) + P(B=3) - P(F=3) \times P(B=3)$ or $P[(F=3) \cup (B=3)] = 1 - P(F \neq 3) \times P(B \neq 3)$ or $P[(F=3) \cup (B=3)] = P(B=3) - P(F=3) \times P(B \neq 3)$
	M1:	$' \frac{4}{27} ' + \left[\left(\frac{1}{2} \right)^2 \times \left(\frac{1}{2} \right) \right] - ' \frac{4}{27} ' \times \left[\left(\frac{1}{2} \right)^2 \times \left(\frac{1}{2} \right) \right]$ or $1 - \left[1 - ' \frac{4}{27} ' \right] \times \left[1 - \left(\frac{1}{2} \right)^2 \times \left(\frac{1}{2} \right) \right]$ ft their $P(B=3)$ from (a)(i)
	A1:	$\frac{55}{216}$ or awrt 0.255 (correct answer implies full marks)
(b)	M1:	For attempting to calculate or consider expected values/means of X and Y Implied by sight of 8 and 4.5
	B1:	for $E(X) = 8$
	M1:	Using $E(B^2) = \text{Var}(B) + [E(B)]^2$ or $E(B^2) = G_B'(1) + G_B''(1)$ with a correct PGF given for B $E(B^2) = 15$ is M1
	A1:	for $E(Y) = 7.5$ or equivalent
	A1ft:	for interpreting the outcome of their calculations in terms of the problem. Must clearly choose X or Y and refer to its expectation/average being higher Allow reference to expectation as a comparison e.g. $8 > 7.5$ ft their $E(X)$ and $E(Y)$ provided at least one of $E(X)$ or $E(Y)$ correct dependent on 1 st M1 being awarded e.g. candidates can score M1B1M0A0A1 or M1B0M1A1A1 or M1B1M1A0A1 NB: Accept expectations given as totals. E.g. $8n$ and $7.5n$, but not $7.5n^2$ etc

