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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Hypothesis Testing (4)

Further Statistics 1 · Topic 4

10 questions · 123 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

10 questions · 123 marks · 4 sub-topics



SUB-TOPIC 4.1

HYPOTHESIS TESTS FOR THE POISSON MEAN

Sub-topic 4.1 · 4 questions · 46 marks · lowest tariff first



- The number of errors made by a secretary is modelled by a Poisson distribution with a mean of 2.4 per 100 words.

A 100-word piece of work completed by the secretary is selected at random.

- (a) Find the probability that

- (i) there are exactly 3 errors,
- (ii) there are fewer than 2 errors.

(2)

After a long holiday, a randomly selected piece of work containing 250 words completed by the secretary is examined to see if the rate of errors has changed.

- (b) Stating your hypotheses clearly, and using a 5% level of significance, find the critical region for a suitable test.

(4)

- (c) Find $P(\text{Type I error})$ for the test in part (b)

(1)



Question 2 continued

(Total for Question 2 is 7 marks)



Question 5 continued



Question 5 continued

(Total for Question 5 is 12 marks)



Question 1 continued



Question 1 continued



Question 1 continued

(Total for Question 1 is 13 marks)



7. A software company develops computer programs.
Its *Lovelace* program is found to glitch 7 times on average when it is run.

The company makes some alterations to improve the program.
It then tests to see if it has been successful in reducing the average number of glitches per run.
The glitches can be assumed to occur independently.

- (a) (i) State suitable hypotheses for the test.
(ii) Find the critical region for the test at a 10% level of significance.
(iii) State the size of the test.

(4)

The altered program is actually found to glitch 5 times on average when run.

- (b) Find the probability of a Type II error for the company's test.

(3)

Due to the high value of the probability of a Type II error in part (b), the company makes an entirely new version of the program.
The new version of the program is designed to have an average of fewer than k glitches per run.

The random variable T represents the number of **runs** of the program until a run with no glitches occurs.

- (c) Write down the name of a suitable distribution for T

(1)

If the new version has an average of λ glitches per run, this new version of the program will be tested using the hypotheses

$$H_0 : \lambda = k \qquad H_1 : \lambda < k$$

The company will reject H_0 if the first run without any glitches of the new version occurs within n runs.

- (d) Show that the power function of the test is given by

$$1 - (1 - e^{-\lambda})^n$$

(2)

The company requires $P(\text{Type II error}) < 0.2$
Assuming that the new version of the program has an average of 2.75 glitches per run,

- (e) find the minimum value of n

(4)

Question 7 continued



Question 7 continued

(Total for Question 7 is 14 marks)



SUB-TOPIC 4.2

HYPOTHESIS TESTS FOR THE GEOMETRIC PARAMETER

Sub-topic 4.2 · 1 question · 18 marks · lowest tariff first



5. Asha, Davinda and Jerry each have a bag containing a large number of counters, some of which are white and the rest are red. Each person draws counters from their bag one at a time, notes the colour of the counter and returns it to their bag.

The probability of Asha getting a red counter on any one draw is 0.07

- (a) Find the probability that Asha will draw at least 3 white counters before a red counter is drawn.

(2)

- (b) Find the probability that Asha gets a red counter for the second time on her 9th draw.

(2)

The probability of Davinda getting a red counter on any one draw is p .

Davinda draws counters until she gets n red counters. The random variable D is the number of counters Davinda draws.

Given that the mean and the standard deviation of D are 4400 and 660 respectively,

- (c) find the value of p .

(4)

Jerry believes that his bag contains a smaller proportion of red counters than Asha's bag. To test his belief, Jerry draws counters from his bag until he gets a red counter. Jerry defines the random variable J to be the number of counters drawn up to and including the first red counter.

- (d) Stating your hypotheses clearly and using a 10% level of significance, find the critical region for this test.

(5)

Jerry gets a red counter for the first time on his 34th draw.

- (e) Giving a reason for your answer, state whether or not there is evidence that Jerry's bag contains a smaller proportion of red counters than Asha's bag.

(2)

Given that the probability of Jerry getting a red counter on any one draw is 0.011

- (f) show that the power of the test is 0.702 to 3 significant figures.

(3)

Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 18 marks)



SUB-TOPIC 4.3

OTHER HYPOTHESIS TESTS

Sub-topic 4.3 · 4 questions · 47 marks · lowest tariff first







(Total for Question 5 is 8 marks)







Question 7 continued

(Total for Question 7 is 11 marks)



(Total for Question 5 is 13 marks)

3. In a class experiment, each day for 170 days, a child is chosen at random and spins a large cardboard coin 5 times and the number of heads is recorded. The results are summarised in the following table.

Number of heads	0	1	2	3	4	5
Frequency	3	10	45	62	38	12

Marcus believes that a $B(5, 0.5)$ distribution can be used to model these data and he calculates expected frequencies, to 2 decimal places, as follows

Number of heads	0	1	2	3	4	5
Expected frequency	r	26.56	s	s	26.56	r

- (a) Find the value of r and the value of s (3)
- (b) Carry out a suitable test, at the 5% level of significance, to determine whether or not the $B(5, 0.5)$ distribution is a good model for these data. You should state clearly your hypotheses, the test statistic and the critical value used. (6)

Nima believes that a better model for these data would be $B(5, p)$

- (c) Find a suitable estimate for p (1)

To test her model, Nima uses this value of p , to calculate expected frequencies as follows

Number of heads	0	1	2	3	4	5
Expected frequency	2.07	14.65	41.44	58.63	41.47	11.74

The test statistic for Nima's test is 1.62 (to 3 significant figures)

- (d) State,
 (i) giving your reasons, the degrees of freedom
 (ii) the critical value
 that Nima should use for a test at the 5% significance level. (3)
- (e) With reference to Marcus' and Nima's test results, comment on
 (i) the probability of the coin landing on heads,
 (ii) the independence of the spins of the coin.
 Give reasons for your answers. (2)





SUB-TOPIC 6.1

CHI-SQUARED TESTS

Sub-topic 6.1 · 1 question · 12 marks · lowest tariff first



3. A journalist uses a gender equality test on some randomly chosen films. She believes that whether or not a film passes the test is associated with the period in which the film is first released. Her data is summarised in Table 1. The journalist decides to test whether the data supports her belief.

Table 1: Observed frequencies

Test result \ Period	1990–1999	2000–2004	2005–2013	Total
Pass	45	70	65	180
Fail	75	90	55	220
Total	120	160	120	400

Some of the expected frequencies she calculates are shown in Table 2 on page 7.

- (a) Complete Table 2 showing expected frequencies. (2)
- (b) Write down hypotheses for a suitable test to assess the journalist's belief. (1)
- (c) Showing your working clearly, test the journalist's belief using a 5% level of significance. (6)
- You should state your critical value and conclusion clearly.

The journalist wants to publish her data and findings. She decides to publish the data as **percentages** of pass and fail for each period, as shown in Table 3, rather than as **frequencies**.

Table 3: Percentages

Test result \ Period	1990–1999	2000–2004	2005–2013
Pass (%)	37.5	43.75	54.17
Fail (%)	62.5	56.25	45.83
Total (%)	100	100	100

The editor suggests carrying out the hypothesis test of the journalist's belief using the percentages in Table 3.

- (d) (i) Describe the effect this would have on the test statistic, justifying your answer. (3)
- (ii) Explain whether or not the journalist should follow her editor's suggestion.

Question 3 continued

Table 2: Expected frequencies

Period Test result	1990–1999	2000–2004	2005–2013
Pass			54
Fail			66



Question 3 continued



MARK SCHEMES

official mark schemes



Qu	Scheme	Mark	AO
(a)	1 st M1 for an attempt at $E(X)$, at least the final 2 products seen <u>or</u> an answer of 2 2 nd M1 for attempting a correct expression for $E(X^2)$, at least 3 non-zero products seen A1 for 4.9 or exact equivalent		
(b)	1 st M1 for attempting a correct expression, at least 3 non-zero products seen (implied by 176.9) 2 nd M1 for a correct method, ft their 176.9 and their 8.9 but must be intending $E(X^4)$ and $E(X^2)$ A1 for 97.69 or awrt 97.7		

Qu	Scheme	Mark	AO
2 (a)	(i) 0.20901... awrt 0.209	B1	3.4
	(ii) 0.30844.. awrt 0.308	B1 (2)	1.1b
(b)	$H_0: \lambda = 2.4$ (or $\mu = 6$) $H_1: \lambda \neq 2.4$ (or $\mu \neq 6$)	B1	2.5
	$[E = \text{no. of errors}] E \sim \text{Po}(6)$	M1	3.3
	$P(E \leq 1) = 0.0174$ <u>or</u> $P(E \leq 2) = 0.0620$ and $P(E \leq 11) = 0.980$ <u>or</u> $P(E \geq 12) = 0.0201$	M1	3.4
	Critical region: $E \leq 1$ or $E \geq 12$	A1 (4)	1.1b
(c)	$[P(\text{Type I error}) = 0.0174 + 0.0201 =]$ 0.0375 (Calc gives: $0.017351... + 0.0200919... = 0.037443...$)	B1ft (1)	1.2
		(7 marks)	

Notes			
(a)	1 st B1 for awrt 0.209 2 nd B1 for awrt 0.308		
(b)	B1 for both hypotheses correct in terms of λ or μ (allow $\lambda = 6$ etc) 1 st M1 for selecting the correct model. Sight or use of $\text{Po}(6)$ 2 nd M1 for use of the correct model with two probs correct to 2.s.f. (accept $P(E \geq 12) = 0.02$) Must see attempt at lower and upper limit. Probabilities may be seen in (c). A1 for correct critical region (both parts). Allow $E \leq 1$ and $E \geq 12$ or $E \leq 1, E \geq 12$ etc Writing CR as probability statements is A0 NB: Completely correct CR implies M1M1A1 <u>SC: 1-tailed test</u> B0 as hypotheses are incorrect M1 for sight or use of $\text{Po}(6)$ M1 (dep on H_1) for sight of $P(E \leq 1) = 0.0174$ or $P(E \geq 11) = 0.0426$, in line with their H_1 A1 for CR: $E \leq 1$ or CR: $E \geq 11$, in line with their hypotheses		
(c)	B1ft for 0.0375 or 0.0374 <u>or</u> summing their two appropriate probs (ft their CR) NB: If candidate uses a 1-tailed test, this mark cannot be gained		

Qu	Scheme	Marks	AO	
5(a)	$H_0: \lambda = 2.5$ (or $\mu = 7.5$) $H_1: \lambda \neq 2.5$ (or $\mu \neq 7.5$)	B1	2.5	
	[X = no. of accidents in a 3-month period] $X \sim \text{Po}(7.5)$	M1	3.3	
	$P(X \leq 2) = 0.0203$ (calc: 0.020256...) { or $P(X \leq 3) = 0.0591$ }			
	$P(X \leq 13) = 0.9784$ so $P(X \geq 14) = 0.0216$ (calc: 0.0215646...)	M1	3.4	
	{ or $P(X \geq 15) = 0.0103$ }			
	Giving Critical region of: $X \leq 2$	A1	1.1b	
	$X \geq 14$	A1	1.1b	
		(5)		
	(b)	$[0.0203 + 0.0216] = \text{awrt } \underline{\underline{0.0419}} \quad \text{or} \quad (\text{calc: } 0.041821366... \text{ awrt } \underline{\underline{0.0418}})$	B1ft	1.2
		(1)		
(c)	[Let M = no of 3-month periods with a significant result]			
	$M \sim B(8, "0.0419")$	M1	3.3	
	$[P(M \geq 2)] = 1 - P(M \leq 1)$	M1	1.1b	
	$[= 1 - 0.9584...]$			
	$= 0.04153...(\text{calc: } 0.041394...) \quad [\underline{\underline{0.04139 \sim 0.04154}}]$	A1cso	1.1b	
		(3)		
(d)	$Y \sim \text{Po}(6.3)$	M1	3.3	
	$P(\text{Type II error}) = P(3 \leq Y \leq 13)$ or $P(Y \leq 13) - P(Y \leq 2)$	M1	3.4	
	$[= 0.9945147... - 0.049846...]$			
	$= 0.9446... \quad \text{awrt } \underline{\underline{0.945}}$	A1	1.1b	
		(3)		
		(12 marks)		
	Notes			
(a)	B1 for both hypotheses in terms of λ or μ (either way around)			
	1 st M1 for selecting the correct Po model. Sight or use of Po(7.5) may be implied by 2 nd M1			
	2 nd M1 for using the correct model to find one of these probs with correct label (2sf or better)			
	1 st A1 for one end correct	Allow any letter, even $CR \leq 2$ or set notation but not $P(X \leq 2)$ Can have $X < 3$ and $X > 13$ etc		
	2 nd A1 for a fully correct CR			
(b)	B1ft for awrt 0.0419 <u>or</u> awrt 0.0418 <u>or</u> ft addition of their two probs provided both are $0 < \text{prob} < 0.025$ (awrt 3sf)			
(c)	1 st M1 for selecting a correct binomial model, ft their answer to part (b)			
	2 nd M1 for a correct probability statement of $1 - P(M \leq 1)$ dep on a binomial selected			
	A1cso for answer in range [0.04139, 0.04154] dep on use of B(8, "0.0419") or better			
(d)	1 st M1 for selecting a Po(6.3) model			
	2 nd M1 for a correct probability statement using their Poisson model and their CR in (a) which may have just one tail.			
	A1 for awrt 0.945			

Question	Scheme	Marks	AOs
1(a)(i)	$X \sim \text{Po}(24)$	B1	3.4
	$P(X = 26) = 0.071912\dots$ awrt 0.0719	B1	1.1b
		(2)	
(ii)	$P(X \geq 21) = 1 - P(X \leq 20) [= 1 - 0.24263\dots]$	M1	3.4
	$= 0.75736\dots$ awrt 0.757	A1	1.1b
		(2)	
(b)	$H_0: \lambda = 2$ [$\mu = 16$] $H_1: \lambda < 2$ [$\mu < 16$]	B1	2.5
	$P(Y \leq 10 Y \sim \text{Po}(16)) = 0.077396\dots$ awrt 0.0774	B1	1.1b
	Not significant / Do not reject H_0 / 10 is not in the CR	M1	1.1b
	There is <u>not</u> sufficient evidence to suggest a decrease/change in the rate of <u>customers</u> entering Jeff's supermarket.	A1	2.2b
		(4)	
(c)	Use of $\text{Po}(8)$ to attempt critical region	M1	2.1
	Critical region is $Y \leq 3 / H_0$ is not rejected when $Y \geq 4$	A1	1.1b
	True distribution is $W \sim \text{Po}(4)$	B1	2.1
	$P(W \geq 4 W \sim \text{Po}(4)) = 1 - P(W \leq 3) [= 1 - 0.43347\dots]$	M1	1.1b
	$= 0.56652\dots$ awrt 0.567	A1	1.1b
		(5)	
(13 marks)			
Notes			
(a)(i) (ii)	B1: For realising the distribution is $\text{Po}(24)$ (May be seen or implied in part (ii)) B1: awrt 0.0719 M1: Writing or using $1 - P(X \leq 20)$ A1: awrt 0.757		
(b)	B1: Both hypotheses correct (must use μ or λ) B1: awrt 0.0774 Allow awrt 0.08 from a correct probability statement. allow CR: $X \leq 9$ M1: Correct non-contextual conclusion (may be implied by correct contextual conclusion). Allow a f.t. comparison of 'their p ' with 0.05 (Ignore any contradictory contextual comments for this mark) A1: A fully correct solution drawing a correct inference in context with all previous marks in (b) scored.		
(c)	M1: Use of $\text{Po}(8)$ to attempt critical region [$P(Y \leq 3) = 0.0423\dots$ $P(Y \leq 4) = 0.0996\dots$] A1: Finding critical region for the test $Y \leq 3$ which must come from $\text{Po}(8)$. B1: Identifying the need to use $\text{Po}(4)$ as the true distribution. Allow $\text{Po}(4)$ seen or used for this mark. M1: Writing or using $P(W \geq 4)$ or $1 - P(W \leq 3)$ from $\text{Po}(4)$. Allow f.t. on their identified CR but must be using $\text{Po}(4)$ A1: awrt 0.567		

Question	Scheme	Marks	AOs
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Q	Scheme		Marks	AOs
7(a)(i)	$H_0: \lambda = 7 \quad H_1: \lambda < 7$		B1	2.5
(a)(ii)	$X \sim \text{Po}(7) \quad [P(X \leq 2) = 0.0296..., P(X \leq 3) = 0.0817..., P(X \leq 4) = 0.172...]$		M1	3.4
	Critical region: $X \leq 3$		A1ft	1.1b
(a)(iii)	awrt 0.0818		B1ft	1.2
			(4)	
(b)	$Y \sim \text{Po}(5)$		M1	3.3
	$P(Y \geq '4' \mid \lambda = 5)$		M1	3.4
	$P(Y \geq 4 \mid \lambda = 5) = 0.735$ awrt 0.735		A1	1.1b
			(3)	
(c)	Geometric		B1	3.3
			(1)	
(d)	$T \sim \text{Geo}(e^{-\lambda})$		M1	2.1
	$P(T \leq n) = 1 - (1 - e^{-\lambda})^n$		A1cso*	1.1b
			(2)	
(e)	$(1 - e^{-2.75})^n < 0.2$		B1	3.4
	$n \log(1 - e^{-2.75}) < \log 0.2$		M1	2.1
	$n > \frac{\log 0.2}{\log(1 - e^{-2.75})} \approx$ awrt 24.4		A1	1.1b
	$n = 25$		A1	1.1b
			(4)	
(14 marks)				
Notes:				
(a)(i)	B1:	Both hypotheses correct in terms of λ (do not accept μ or $\bar{x}$). May be seen in (a)(ii)/(iii)		
(a)(ii)	M1:	Sight or use of $X \sim \text{Po}(7)$ [may be implied by a correct probability to 2 s.f. in (a)]		
	A1ft:	Correct CR clearly stated (A0 if given only as a probability) [ft their hypotheses from (a)(i), 2-tailed CR is $X \leq 2, X \geq 13$]		
(a)(iii)	B1ft:	awrt 0.0818, ft the probability associated with their CR		
(b)	M1:	Sight or use of $Y \sim \text{Po}(5)$, condone $\lambda = 5$ as a probability statement [implied by a correct probability e.g. 0.124..., 0.265..., 0.0404..., 0.0734...]		
	M1:	Attempt to find $P(Y \geq 4)$ with $Y \sim \text{Po}(5)$ (ft from their CR)		
	A1:	awrt 0.735 (correct answer implies full marks)		
(c)	B1:	Stating Geometric or Geo (parameter not required, ignore parameter if given)		
(d)	M1:	Sight of $T \sim \text{Geo}(e^{-\lambda})$ or $P(\text{zero glitches}) = e^{-\lambda}$ [condone $P(T = 0) = e^{-\lambda}$]		
	A1cso*:	$P(T \leq n)$ o.e. or stating that Power = $1 - P(\text{Type II error})$ leading to $1 - (1 - e^{-\lambda})^n$ [working must be fully correct with no errors]		
(e)	B1:	$(1 - e^{-2.75})^n < 0.2$ (condone use of $=$ or $\leq$ instead of $<$)		
	M1:	Correct process to reach a linear equation involving logs. May be implied by awrt 24.4 or awrt 3.38		
	A1:	Solves equation to obtain awrt 24.4		
	A1:	$(n =) 25$ ($n \geq 25$ is A0) Award full marks for 25 if not from clearly incorrect working		

Question	Scheme	Marks	AOs
5(a)	$P(\text{at least 3 whites}) = (1 - 0.07)^3$ or $1 - 0.07 - 0.93 \times 0.07 - 0.93^2 \times 0.07$	M1	1.1b
	$= 0.8043\dots$ awrt 0.804	A1	1.1b
		(2)	
(b)	$P(\text{2nd red on 9th draw}) = \binom{8}{1} 0.93^7 \times 0.07^2$	M1	3.3
	$= 0.02358\dots$ awrt 0.0236	A1	1.1b
		(2)	
(c)	$\frac{n}{p} = 4400$ and $\frac{n(1-p)}{p^2} = 660^2$	M1 A1	3.1b 1.1b
	$1 - p = 99p$ oe	M1	1.1b
	$p = 0.01$	A1	1.1b
		(4)	
(d)	$H_0: p = 0.07$ $H_1: p < 0.07$	B1	2.5
	$J \sim \text{Geo}(0.07)$	M1	3.3
	$P(J \geq c) < 0.1 \Rightarrow (1 - 0.07)^{c-1} < 0.1$	M1	3.4
	$c - 1 > \frac{\log 0.1}{\log 0.93}$	M1	1.1b
	$c > 32.72\dots \therefore \text{CR } J \geq 33$	A1	1.1b
		(5)	
(e)	34 is in the Critical region	M1	1.1b
	There is evidence to suggest that Jerry's bag contains a smaller proportion of red counters than Asha's bag.	A1	2.2b
		(2)	
(f)	Power of test = $P(J \geq 33 p = 0.011)$	M1	2.1
	$= (1 - 0.011)^{32}$ oe	M1	1.1b
	$= 0.7019\dots^*$	A1*	1.1b
		(3)	
(18 marks)			



Notes:		
(a)	M1:	A correct method to find $P(X \geq 3)$
	A1:	awrt 0.804
(b)	M1:	For selecting the appropriate model negative binomial or binomial with an extra trial
	A1:	awrt 0.0236
(c)	M1:	Forming an equation for the mean and variance. At least one correct.
	A1:	Both equations correct
		Allow M1 A1 if both equations correct with the same number subst for n
	M1:	Solving the 2 equations leading to $1 - p = 99p$ oe Allow $p - p^2 = 99p^2$ ft their 4400 and 660 Allow $1 - p = 0.15p$
	A1:	0.01
(d)	M1:	Both hypotheses correct using correct notation allow eg $p > 0.93$
	M1:	Realising the need to use Geo(0.07) ft their Hypotheses
	M1:	Using the model to find $P(J \geq c)$ Condone $(1 - 0.07)^c < 0.1$ ft their $0.07 \neq 0.93$ ALT $P(J \geq 32) = 0.1[054\dots]$ or $P(J \geq 33) = 0.09[8\dots]$ Implied by correct CR
	M1:	For a valid method to solve the inequality or $P(J \geq 32) = 0.1[054]$ and $P(J \geq 33) = 0.09[81]$ Implied by correct CR
	A1:	Correct CR(any letter) A0 if given as a probability statement. Must be integer
(e)	M1:	Comparing 34 with their CR eg $34 > 33$ $34 \geq 33$ or $P(J \geq 34) = 0.09[12]$
	A1:	Fully correct conclusion in context. Allow Jerry's belief is true. Allow probability for proportion
(f)	M1:	Realising they need to find $P(\text{their CR in (d)})$ Allow $1 - P(J \leq 32)$
	M1:	For a Correct method. Allow $1 - 0.2981\dots$ May be implied by $0.7019\dots$ If the CR is incorrect $(1 - 0.011)^{\text{CR} - 1}$ or $1 - \{1 - (1 - 0.011)^{\text{CR} - 1}\}$ must be seen
	A1*:	Only award if both method marks awarded.

Qu 5	Scheme	Marks	AO
(a)	$H_0: \mu = 330$ $H_1: \mu < 330$	B1	2.5
	$[\bar{X} \sim N\left(330, \left(\frac{8}{\sqrt{25}}\right)^2\right)]$	M1	3.3
	$P(\bar{X} < C) = 0.05 \Rightarrow \frac{C - 330}{\frac{8}{\sqrt{25}}} = -1.6449$	M1	3.4
	So $C = 327.368\dots$ and critical region is: $\bar{X} < \text{awrt } \underline{327}$	A1	1.1b
		(4)	
	(b) $\bar{Y} \sim N\left(330, \left(\frac{8}{\sqrt{55}}\right)^2\right)$ and require $2 \times P(\bar{Y} < 328)$ (o.e.)	M1	3.3
	$= 0.063732\dots$ awrt <u>0.0637</u>	A1	1.1b
		(2)	
	(c) $P(\bar{X} > "327.368\dots" \mu = 325)$ or $1 - P(\bar{X} < "327.368\dots" \mu = 325)$	M1	3.4
	$= 0.0694233\dots$ awrt <u>0.0694</u>	A1	1.1b
		(2)	
		(8 marks)	
Notes			
(a)	B1 for both hypotheses in terms of μ 1 st M1 for stating or using the correct model – may be implied by use in later line. Condone X or any letter for $\bar{X}$ 2 nd M1 for a correct equation for C Allow any z value that satisfies $1.6 < z < 1.7$ If standardisation equation not seen, this mark may be implied by CV = awrt 327 or CR: <awrt 327 A1 for a correct CR allow just "< awrt 327" Condone e.g. $X < 327$ rather than $\bar{X} < 327$ Condone $\leq$		
(b)	M1 for sight of correct model and attempt at $P(\bar{Y} < 328)$ (o.e.) Condone missing $2 \times$ A1 for awrt 0.0637 (correct answer scores 2 out of 2)		
(c)	M1 for a correct (ft) statement may be implied by sight of e.g. $Z > \frac{"327.36\dots" - 325}{\frac{8}{\sqrt{5}}} = 1.48\dots$ For $\mu = 325$ allow $\bar{X} \sim N(325, \dots)$ Allow ft from a 2-tailed test in part (a) A1 for awrt 0.0694 (correct answer scores 2 out of 2)		
SC	Sight of $P(328 < \bar{X} < 332 \mu = 325)$ or $1 - P(\bar{X} < 328 \cup \bar{X} > 332 \mu = 325)$ scores M1A0		

Question	Scheme	Marks	Aos
7(a)	$\bar{X} \sim N(1000, 90)$ (May be implied by correct prob or z value seen) $P(\bar{X} > 1020) = 0.0175\dots$ or $z = 2.108$ $0.0175\dots < 0.025$ or $z = 2.108\dots > 1.96$ therefore reject H_0 . There is evidence that the <u>mean weight</u> of the <u>flour</u> in a bag is <u>not 1000 g</u> or evidence of a <u>change in mean weight of flour</u> in a bag	M1 A1 M1 A1 cso (4)	3.3 3.4 1.1b 2.2b
(b)	$\left[\bar{Y} \sim N\left(1000, \frac{900}{n}\right) \Rightarrow \frac{c-1000}{\frac{30}{\sqrt{n}}} = 1.6449 \right]$ $c = 1000 + \frac{49.347}{\sqrt{n}}$	M1 A1 (2)	3.4 1.1b
(c)	$\frac{1000 + \frac{49.347}{\sqrt{n}} - 1020}{\frac{30}{\sqrt{n}}} = -2.5758$ $\frac{126.621}{\sqrt{n}} = 20 \quad \text{or} \quad \frac{49.34\dots}{c-1000} = \frac{-77.274}{c-1020} \quad (\text{Allow 2sf accuracy})$ $n = \underline{40}$ $c = 1007.8\dots$ awrt 1010	M1 A1ft dM1 A1 A1 (5)	3.4 1.1b 1.1b 2.1 1.1b
(11 marks)			
Notes:			
(a)	1st M1 Setting up the correct model. Normal with $\mu = 1000, \sigma^2 = 90$ or $\sigma = \sqrt{90}$ or awrt 9.49 1st A1 Using the model to find the correct z value or $P(\bar{X} > 1020) = \text{awrt } 0.0175$ Allow CR $\bar{C} \dots 1018.59\dots$ awrt 1019 [$>$ is OK] Ignore lower CR provided < 1000 2nd M1 Correct comparison <u>or</u> non-contextual conclusion. Allow comparison of 1020 with critical region. Dep on $P(\bar{X} > 1020)$ M0 if there are contradictory statements. 2nd A1 cso dep on M1A1M1 for a correct conclusion in context with underlined words Do NOT accept “mean weight has <u>increased</u> ”		
(b)	M1 For Finding the CR using the Normal distribution. Condone $\sigma = \sqrt{\frac{30}{n}}$ to score M1 $\frac{c-1000}{\frac{30}{\sqrt{n}}} = z \quad \text{where } z > 1.5$ Allow any inequality or = for M1 in (b) and M1 A1ft M1 in (c) A1 A correct equation in the form $c = \dots$ and for use of awrt 1.6449 (implied by awrt 49.3[4]) Condone $\bar{X}$ used for c (o.e.)		
(c)	1st M1 Standardising using their c (letter or expression) and equating to z ($ z > 2$) to form an equation in n or n and c . Can ft their σ used in (b) for M1A1ft here 1st A1ft Ft their “ c ” for a correct equation with -2.58 (or 1.64 or 1.65 used in (b)) 2nd dM1 Dependent 1st M1 . Isolating or eliminating either $\sqrt{n}$ or n <u>or</u> eliminating c leading to an equation for n or c 2nd A1 For 40 (allow 41) Must be an integer. With correct working. 3rd A1 For awrt 1010 from correct working	e.g. Check correct σ has been used	



Qu.	Scheme	Marks	AOs												
5(a)	$p = \frac{(0)+11+14+6+(0)+5+(0)}{6 \times 40}$	M1	2.1												
	$p = \underline{0.15}^*$	A1*cs0	1.1b												
		(2)													
(b)	$X \sim B(6, 0.15)$	M1	3.4												
	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$40 \times P(X = x)$</td><td>7.04...</td><td>1.65...</td><td>0.219...</td><td>0.015...</td><td>0.00...</td></tr></table>			x	2	3	4	5	6	$40 \times P(X = x)$	7.04...	1.65...	0.219...	0.015...	0.00...
	x			2	3	4	5	6							
	$40 \times P(X = x)$	7.04...	1.65...	0.219...	0.015...	0.00...									
	Require $40 \times P(X \geq k) > 5$ Exp. frequency for $X \geq 2 = 8.94... / X \geq 3 = 1.89...$	M1	1.1b												
	Combine last 5 cells / only 3 cells in total	A1	2.2a												
	2 is subtracted (as there are 2 restrictions) and the proportion used from data (and 1 equal totals)	B1	2.4												
	$3 - 2 = 1$ degree of freedom	A1	1.1b												
	H_0 : Binomial distribution is a suitable model H_1 : Binomial distribution is not a suitable model	B1	3.4												
	Critical value $\chi^2_{(1,0.10)} = 2.705$ or 2.706	B1ft	1.1b												
	Test statistic is not in the critical region, insufficient evidence to reject H_0 ($2.689 < 2.705/6$) Data are consistent with binomial/engineer's/suggested model.	B1ft	3.5a												
	(8)														
(c)	The total amount/proportion of defective pins remains the same.	M1	2.4												
	The cells for $X \geq 2$ are still combined in the test.	M1	1.1b												
	So there is no change to the value of the test statistic.	A1	2.2a												
		(3)													
(13 marks)															
Notes															
(a)	M1: Correct expression for p (may be seen in stages). Allow $\frac{36}{240}$ but not $\frac{6}{40}$ on its own A1*cs0: $p = 0.15$ stated and no incorrect working seen														
(b)	M1: Attempting to find expected frequencies, at least 2 correct trunc. or rounded 1dp M1: Recognising need to combine cells (Sight of awrt 8.94 implies M1M1) A1: Combining cells for $X \geq 2$ (to make 3 cells) B1: Justifying why 2 is subtracted with p being calculated from data A1: 1 degree of freedom B1: Correct hypotheses (0.15 must not be included) Allow engineer's model. B1ft: Correct critical value (ft their df) May see $\chi^2_{(2,0.10)} = 4.605$ or $\chi^2_{(3,0.10)} = 6.251$ B1ft: Correct inference (ft comparison of their CV with 2.689). Condone $p = 0.15$ included here. Do not allow contradictory statements to score here. Hypotheses must be correct way round.														
(c)	M1: Determining the number ($N=36$)/proportion ($p=0.15$) of defective pins has not changed. e.g. $11 + 12 + 9 + 4 = 36$. But not $7 + 2 + 1 = 6 + 3 + 1$ M1: Understanding the cells for $X \geq 2$ are still combined in the test A1: (dep on both M1s) Concluding that there is no change to the value of the test statistic.														

Qu 3	Scheme	Mark	AO														
(a)	$[X \sim B(5, 0.5)] P(X = 0) = P(X = 5) = 0.03125$ <u>or</u> $P(X = 2) \text{ or } P(X = 3) = 0.3125$ [multiply by 170 to get] $r = \underline{5.31}(25)$; $s = \underline{53.1}(25)$	M1 A1;A1 (3)	1.1b 1.1b(x2)														
(b)	$H_0 : B(5, 0.5)$ is a suitable model $H_1 : B(5, 0.5)$ is NOT a ...	B1	2.5														
	<table><tr><td>$\frac{(O_i - E_i)^2}{E_i}$</td><td>$\frac{(3 - 5.31)^2}{5.31}$ = 1.00...</td><td>$\frac{(10 - 26.56)^2}{26.56}$ = 10.3...</td><td>$\frac{(45 - 53.1)^2}{53.1}$ = 1.23...</td><td>$\frac{(62 - 53.1)^2}{53.1}$ = 1.48...</td><td>$\frac{(38 - 26.56)^2}{26.56}$ = 4.92...</td><td>$\frac{(12 - 5.31)^2}{5.31}$ = 8.41...</td></tr><tr><td>$\frac{O_i^2}{E_i}$</td><td>$\frac{3^2}{5.31}$ = 1.69...</td><td>$\frac{10^2}{26.56}$ = 3.76...</td><td>$\frac{45^2}{53.1}$ = 38.1...</td><td>$\frac{62^2}{53.1}$ = 72.3...</td><td>$\frac{38^2}{26.56}$ = 54.3...</td><td>$\frac{12^2}{5.31}$ = 27.1...</td></tr></table>	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{(3 - 5.31)^2}{5.31}$ = 1.00...	$\frac{(10 - 26.56)^2}{26.56}$ = 10.3...	$\frac{(45 - 53.1)^2}{53.1}$ = 1.23...	$\frac{(62 - 53.1)^2}{53.1}$ = 1.48...	$\frac{(38 - 26.56)^2}{26.56}$ = 4.92...	$\frac{(12 - 5.31)^2}{5.31}$ = 8.41...	$\frac{O_i^2}{E_i}$	$\frac{3^2}{5.31}$ = 1.69...	$\frac{10^2}{26.56}$ = 3.76...	$\frac{45^2}{53.1}$ = 38.1...	$\frac{62^2}{53.1}$ = 72.3...	$\frac{38^2}{26.56}$ = 54.3...	$\frac{12^2}{5.31}$ = 27.1...	M1	1.1b
$\frac{(O_i - E_i)^2}{E_i}$	$\frac{(3 - 5.31)^2}{5.31}$ = 1.00...	$\frac{(10 - 26.56)^2}{26.56}$ = 10.3...	$\frac{(45 - 53.1)^2}{53.1}$ = 1.23...	$\frac{(62 - 53.1)^2}{53.1}$ = 1.48...	$\frac{(38 - 26.56)^2}{26.56}$ = 4.92...	$\frac{(12 - 5.31)^2}{5.31}$ = 8.41...											
$\frac{O_i^2}{E_i}$	$\frac{3^2}{5.31}$ = 1.69...	$\frac{10^2}{26.56}$ = 3.76...	$\frac{45^2}{53.1}$ = 38.1...	$\frac{62^2}{53.1}$ = 72.3...	$\frac{38^2}{26.56}$ = 54.3...	$\frac{12^2}{5.31}$ = 27.1...											
	$\sum \frac{(O_i - E_i)^2}{E_i}$ or $\sum \frac{O_i^2}{E_i} - 170 = 27.4 \dots$ awrt <u>27.4</u> or awrt <u>27.5</u>	A1	1.1b														
	Degrees of freedom is $6 - 1 = \underline{5}$, and critical value is <u>11.07(0)</u> [Significant result] <u>Marcus' model/B(5, 0.5)</u> is not a good fit. (o.e.)	B1ft B1ft A1 (6)	1.1b(x2) 2.2b														
(c)	$\hat{p} = \left[\frac{0 \times 3 + 1 \times 10 + \dots + 5 \times 12}{170 \times 5} \right] = 0.58588 \dots$ awrt <u>0.586</u>	B1 (1)	1.1b														
(d)(i)	Need to pool (first 2) cells (0 and 1 since $E(0) < 5$) <u>and</u> use of $\hat{p}$ Degrees of freedom: 5 groups – 2 constraints = <u>3</u>	M1 A1	2.4 1.1b														
(ii)	Critical value is <u>7.815</u>	B1ft (3)	1.1b														
(e)(i)	Nima's model is a good fit (since $1.62 < '7.815'$)/Marcus' is not <u>and</u> this suggests coin is biased/probability of head approx. 0.6	B1	2.4														
(ii)	Nima's test suggests binomial is a good model <u>and</u> therefore independence of spins is a reasonable assumption	B1	2.2b														
		(2)	(15 marks)														
	Notes																
(a)	M1 for 1 correct probability which may be embedded (0.03125 or 0.3125 or 0.5^5 or $5C2 \ 0.5^2 \times 0.5^3$) 1 st A1 for $r =$ awrt 5.31 (condone $\frac{85}{16}$) 2 nd A1 for $s =$ awrt 53.1 (condone $\frac{425}{8}$)																
(b)	1 st B1 for both hypotheses mentioning B(5, 0.5) or Marcus' distribution at least once M1 for at least one correct (ft) term or expression of the test statistic (accept 2sf) 1 st A1 for awrt 27.4 or awrt 27.5 (correct value here scores M1A1) 2 nd B1 for 5 <u>or</u> ft if 'their r ' < 5, then $df (= 4 - 1) = 3$ 3 rd B1 for 11.07(0) (or better) for ft $df = 4 \rightarrow 9.488$ or $df = 3 \rightarrow 7.815$ A1 dep on 1 st M1 for a suitable conclusion in context rejecting <u>B(5, 0.5)/Marcus' model</u> Must be compatible with their test statistic and their CV. Just 'Bin is not a good fit' is A0 A0 if inconsistent comments are seen e.g. "do not reject H_0 , B(5, 0.5) is not a good fit"																
(c)	B1 for awrt 0.586 allow $\frac{498}{850}$ o.e.																
(d)(i)	M1 for both reasons, must mention pooling or show pooling or mention exp. value < 5 <u>and</u> use of estimated parameter A1 for $df = 3$ (must have scored the M1 for this mark)																
(ii)	B1 for 7.815 (or better) allow this independent of the M1 only allow ft on $df = 4 \rightarrow 9.488$																
(e)(i)	1 st B1 for stating Nima's (binomial) model is a good fit/do not reject H_0 for Nima's model/Marcus' model is not a good fit <u>and</u> suggest that coin is probably biased/ $p > 0.5$ (p closer to 'their (c)') Only comparing 1.62 with '27.4' to reach $p > 0.5$ is incorrect and scores B0																



Question	Scheme	Marks	AOs																
3(a)	$\left[E_1 = \frac{180 \times 120}{400} \quad E_2 = \frac{180 \times 160}{400} \quad E_3 = \frac{220 \times 120}{400} \quad E_4 = \frac{220 \times 160}{400} \right]$																		
	Two of: $E_1 = 54 \quad E_2 = 72 \quad E_3 = 66 \quad E_4 = 88$	B1	2.2a																
	All four of: $E_1 = 54 \quad E_2 = 72 \quad E_3 = 66 \quad E_4 = 88$	B1	1.1b																
		(2)																	
(b)	H_0 : Whether a film passes the (Gender Equality) <u>Test</u> is <u>not associated</u> with the <u>period</u> in which the film is first released H_1 : Whether a film passes the (Gender Equality) <u>Test</u> or not <u>is associated</u> with the <u>period</u> in which the film is first released	B1	2.5																
		(1)																	
(c)	Correct method for finding at least two terms of χ^2 : <table><tr><td>$\frac{(O_i - E_i)^2}{E_i}$</td><td>1.5</td><td>0.06</td><td>2.24</td><td>$\frac{O_i^2}{E_i}$</td><td>37.5</td><td>68.06</td><td>78.24</td></tr><tr><td></td><td>1.23</td><td>0.05</td><td>1.83</td><td></td><td>85.23</td><td>92.05</td><td>45.83</td></tr></table>	$\frac{(O_i - E_i)^2}{E_i}$	1.5	0.06	2.24	$\frac{O_i^2}{E_i}$	37.5	68.06	78.24		1.23	0.05	1.83		85.23	92.05	45.83	M1	1.1b
	$\frac{(O_i - E_i)^2}{E_i}$	1.5	0.06	2.24	$\frac{O_i^2}{E_i}$	37.5	68.06	78.24											
		1.23	0.05	1.83		85.23	92.05	45.83											
	$\chi^2 = \text{awrt } 6.9$	A1	1.1b																
	$v = (3 - 1)(2 - 1) = 2$	B1	1.1b																
	$\chi^2_2(5\%) = 5.991$	B1ft	1.1b																
	‘6.9’ > ‘5.991’ Sufficient evidence to reject H_0	M1	3.4																
Sufficient evidence to support <u>journalist’s belief</u> or Sufficient evidence that whether a film passes the gender equality <u>test</u> is <u>associated</u> with the <u>period</u> in which the film is first released	A1ft	2.2b																	
	(6)																		
(d)(i)	The observed frequencies for each period are more than 100 or All the O_i and E_i are lower as percentages or The difference between all the O_i and E_i is lower	M1	3.5a																
	Therefore, the test statistic would decrease (5.7...)	A1	2.2a																
(d)(ii)	The journalist <u>should not</u> follow her editor’s suggestion (with an appropriate reason). e.g. - As it may/will change the result of the test - As they should use the real data rather than percentages	B1	2.4																
		(3)																	
(12 marks)																			



Notes:		
(a)	B1:	Two of E_1, E_2, E_3, E_4 correct
	B1:	All four of E_1, E_2, E_3, E_4 correct
(b)	B1:	Correct hypotheses mentioning <u>test</u> , and <u>period/date</u> and <u>association/independence</u> mentioned in at least one hypothesis Use of correlation/correlated is B0 NB: Relationship/related is condoned if association/independence mentioned at all
(c)	M1:	Correct method (terms may be unsimplified in an expression, and hence may need to be checked) shown for finding two χ^2 terms , accept either method May be implied by a test statistic of awrt 6.9
	A1:	awrt 6.9
	B1:	2 degrees of freedom (can be implied by cv of 5.991)
	B1ft:	Correct critical value (ft their df) Must be consistent with their df.
	M1:	For a correct non-contextual conclusion based on their CV and their test statistic May be implied by a correct contextual conclusion if no contradictions
	A1ft:	For a correct contextual conclusion referring to the <u>journalist's belief</u> or mentioning (gender equality) <u>test</u> and <u>period/date</u> (ft their cv) Dependent on 1 st A1 Do not allow contradictory statements B0 for correlation/correlated Condone 'related/relationship' in part (c) NB: We ignore their hypotheses when marking (c)
(d) (i)	M1:	Mentioning that observed totals for each period are more than 100 or Mentioning that all the O_i and E_i have reduced Or Mentioning that all the differences between the O_i and E_i have reduced (must state or imply that this is true for all) or Calculating the new test statistic correctly (awrt 5.7)
	A1:	The test statistic would decrease (dependent on the previous M1)
(d) (ii)	B1:	See scheme. B0 for vague reasoning such as 'should not use percentages as it would be inaccurate' NB: Reasoning for (d)(ii) may be seen in (d)(i) Do not award final B1 if contradictions between their reasoning in (i) and (ii)

