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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Probability Generating Functions (7)

Further Statistics 1 · Topic 7

7 questions · 92 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

7 questions · 92 marks · 2 sub-topics



SUB-TOPIC 7.1

PROBABILITY GENERATING FUNCTIONS

Sub-topic 7.1 · 4 questions · 51 marks · lowest tariff first



6. The discrete random variable X has probability generating function

$$G_X(t) = \frac{t^2}{(3 - 2t)^2}$$

(a) Specify the distribution of X

(2)

A fair die is rolled repeatedly.

(b) Describe an outcome that could be modelled by the random variable X

(1)

(c) Use calculus and $G_X(t)$ to find

(i) $E(X)$

(ii) $\text{Var}(X)$

(7)

The discrete random variable Y has probability generating function

$$G_Y(t) = \frac{t^{10}}{(3 - 2t^3)^2}$$

(d) Find the exact value of $P(Y = 19)$

(3)





Question 6 continued



Question 6 continued

(Total for Question 6 is 13 marks)



Question 6 continued





Question 6 continued

(Total for Question 6 is 14 marks)

Question 6 continued





Question 6 continued

(Total for Question 6 is 14 marks)



Question 4 continued



Question 4 continued

SUB-TOPIC 7.2

MEAN AND VARIANCE FROM PGFS

Sub-topic 7.2 · 3 questions · 41 marks · lowest tariff first



6. The discrete random variable X has probability generating function

$$G_X(t) = k \ln \left(\frac{2}{2-t} \right)$$

where k is a constant.

- (a) Find the exact value of k **(1)**
- (b) Find the exact value of $\text{Var}(X)$ **(7)**
- (c) Find $P(X = 3)$ **(4)**

Question 6 continued





6. A discrete random variable X has probability generating function given by

$$G_x(t) = \frac{1}{64} (a + bt^2)^2$$

where a and b are positive constants.

(a) Write down the value of $P(X = 3)$

(1)

Given that $P(X = 4) = \frac{25}{64}$

(b) (i) find $P(X = 2)$

(7)

(ii) find $E(X)$

(3)

The random variable $Y = 3X + 2$

(c) Find the probability generating function of Y

(2)





(Total for Question 6 is 13 marks)

6. The random variable X has probability generating function $G_X(t)$ where

$$G_X(t) = \frac{1}{\sqrt{4-3t}}$$







MARK SCHEMES

official mark schemes



Qu6	Scheme	Marks	AO
(a)	NegBin(r, p) has pgf $\left[\frac{pt}{1-(1-p)t} \right]^r$ and identify the connection NegBin(2, $\frac{1}{3}$)	M1 A1 (2)	2.1 2.2a
(b)	e.g. no. of rolls to achieve 5 or 6 (so that $p = \frac{1}{3}$) <u>twice</u> (oe)	B1ft (1)	3.3
(c)(i)	$G'_X(t) = \frac{2t(3-2t)^2 - (-2) \times 2(3-2t)t^2}{(3-2t)^4}$ <u>or</u> $\frac{6t}{(3-2t)^3}$ $E(X) = G'_X(1) = \underline{6}$	M1 A1 A1	2.1 1.1b 1.1b
(ii)	$G''_X(t) = \frac{6(3-2t)^3 - (-2) \times 3(3-2t)^2 \times 6t}{(3-2t)^6}$ <u>or</u> $\frac{18+24t}{(3-2t)^4}$ $G''_X(1) = 42$ $\text{Var}(X) = "42" + "6" - "6"2$ $= \underline{12}$	M1 A1 A1 (7)	2.1 1.1b 1.1b 1.1b
(d)	$G_Y(t) = t^{10} \times \frac{1}{9} \left[1 - \frac{2}{3}t^3 \right]^{-2} = \frac{t^{10}}{9} \left[1 + \dots \frac{(-2)(-3)(-4)}{3!} \left(-\frac{2}{3} \right)^3 t^9 \dots \right]$ $P(Y=19) = \frac{32}{243}$	M1 A1 A1 (3)	2.1 1.1b 1.1b
ALT	Identify that $Y = 3X + 4$ $(Y = 19 \text{ requires } X = 5 \text{ so }) P(X=5) = \binom{4}{1} \binom{1}{3} \binom{2}{3}^3 \left(\frac{1}{3} \right)$	M1 A1 (13 marks)	
Notes			
(a)	M1 for identifying the NegBin distribution (allow NB for NegBin) A1 for $r = 2$ and $p = \frac{1}{3}$		
(b)	B1ft for identifying a suitable definition for X using a (fair) die, with $p = \frac{1}{3}$ and the second occurrence of the event, only ft their NegBin distribution in (a). A finite number of rolls is B0		
(c)(i)	1 st M1 for attempt to differentiate quotient or product. At least one uv' style term correct. 1 st A1 for a fully correct first derivative (needn't be simplified)		
(ii)	2 nd A1 for $E(X) = 6$ NB this A1 depends on M1 only but M1A0A1 is possible 2 nd M1 for attempt to diff' quotient or product again. At least one uv' style term correct. 3 rd A1 for 42 (may be given for incorrect G'' provided their $G''(1)$ gives 42 and M1 scored) Note all powers of $(3-2t)$ equal 1 when $t = 1$ is substituted so can be used as a check 3 rd M1 for correct use of pgf to find $\text{Var}(X)$ 4 th A1 dep on M3 for 12		
(d)	M1 for writing pgf in suitable form to carry out binomial expansion 1 st A1 for a correct expression for coefficient of t^{19} 2 nd A1 for $\frac{32}{243}$ or exact equivalent		



Question	Scheme	Marks	AOs
6(a)	$G_v(t) = \frac{9}{25}t^2 + \frac{12}{25}t^3 + \frac{4}{25}t^4$ <u>or</u> $t^2\left(\frac{9}{25} + \frac{12}{25}t + \frac{4}{25}t^2\right)$	M1	1.1b
	$= t^2\left(\frac{2}{5}t + \frac{3}{5}\right)^2$ *	A1* cso (2)	2.1
(b)(i)	$G_w'(t) = 2t\left(\frac{2}{5}t + \frac{3}{5}\right)^4 + \left(\frac{2}{5}t + \frac{3}{5}\right)^5$	M1	2.1
	$\left[G_w'(1) = \right]$ <u>3</u>	A1	1.1b
	$G_w''(t) = 2\left(\frac{2}{5}t + \frac{3}{5}\right)^4 + \frac{16}{5}t\left(\frac{2}{5}t + \frac{3}{5}\right)^3 + 2\left(\frac{2}{5}t + \frac{3}{5}\right)^4$ oe	M1	2.1
	$G_w''(1) = \frac{36}{5}$	A1	1.1b
	$\text{Var}(W) = \frac{36}{5} - \left(\frac{3}{5}\right)^2$	M1	2.1
	$= \frac{6}{5}$	A1	1.1b
		(6)	
(c)	$G_x(t) = t^2\left(\frac{2}{5}t + \frac{3}{5}\right)^2 \times t\left(\frac{2}{5}t + \frac{3}{5}\right)^5$	M1	3.1a
	$= t^3\left(\frac{2}{5}t + \frac{3}{5}\right)^7$	A1	1.1b
		(2)	
(d)	$G_Y(t) = t^3 \times \left(t^2\right)^3 \times \left(\frac{2}{5}t^2 + \frac{3}{5}\right)^7$	M1	3.1a
	$= t^9\left(\frac{2}{5}t^2 + \frac{3}{5}\right)^7$	A1	1.1b
		(2)	
(e)	$P(Y = 15)$ is coefficient of t^{15} ie $\dots + t^9 \times {}^7C_3\left(\frac{2}{5}t^2\right)^3\left(\frac{3}{5}\right)^4 + \dots$ <u>or</u> $P(X = 6)$ need coefficient of t^6 i.e. $\dots + t^3 \times {}^7C_3\left(\frac{2}{5}t\right)^3\left(\frac{3}{5}\right)^4 + \dots$	M1	1.1b
	$\left[P(Y = 15) = \right] \frac{22680}{78125} = \frac{4536}{15625} = 0.290304$	A1	1.1b
		(2)	
(14 marks)			



Notes:		
(a)	M1 A1*	A correct un-simplified pgf based on $\sum t^v P(V=v)$ cso must see an un-simplified version i.e. M1 scored and no incorrect working seen
(b) (i)	M1 A1	Differentiating using the product rule to find $G_w'(t)$ Allow un-simplified e.g. $5 \times \frac{2}{5}t$ Need two terms added and at least one correct. If they expand we need 3 correct. 3 from a correct derivative
(ii)	1stM1 1stA1 2ndM1 2ndA1	Attempt $G_w''(t)$ ft their $G_w'(t)$ [must be at least 2 terms or a product], one correct ft term, same rule for differentiating a product $\frac{36}{5}$ or 7.2 from a correct derivative $G_w''(1) + G_w'(1) - (G_w'(1))^2$ ft their $G_w''(t)$ if different from $G_w'(t)$ and $G_w(t)$ Dep on M3A2 $\frac{6}{5}$ or 1.2
(c)	M1 A1	Realising the need to use $G_x(t) = G_v(t) \times G_w(t)$ $t^3 \left(\frac{2}{5}t + \frac{3}{5} \right)^7$
(d)	M1 A1	Realising the need to multiply through by t^3 <u>or</u> substitute t^2 for t <u>or</u> sight of $t^3 G_x(t^2)$ $t^9 \left(\frac{2}{5}t^2 + \frac{3}{5} \right)^7$ <u>oe</u> Need not be in its simplest form
(e)	M1 A1	Attempting to find correct coefficient of t^n <u>or</u> identify $Y = 2J + 9$ where $J \sim B(7, 0.4)$ Need an expression can ft their $G_Y(t)$ or $G_x(t)$ of the form $t^n (at^m + b)^k$ Allow a statement that $P(Y=15) = 0$ if it follows from their pgf For a correct exact answer or allow awrt 0.2903 Allow 0.29 from correct expression

Alternative for (b)

(b)	$W = P + 1$ where $P \sim B(5, 0.4)$ so $\text{Var}(W) = \text{Var}(P)$		
(i)	$G_p'(t) = 2 \left(\frac{2}{5}t + \frac{3}{5} \right)^4$	M1	2.1
	$G_w'(1) = 2 + 1 = 3$	A1	1.1b
(ii)	$G_p''(t) = \frac{16}{5} \left(\frac{2}{5}t + \frac{3}{5} \right)^3$; $G_p''(1) = \frac{16}{5}$	M1; A1	2.1 1.1b
	$\text{Var}(W) = \frac{16}{5} - 2^2 = \frac{6}{5}$; $= \frac{6}{5}$	M1; A1	2.1 1.1b
SC	MR They use $G_v(t)$ instead of $G_w(t)$ Provided some correct differentiation seen: Award B1 for $E(V) = \frac{14}{5}$ and B1 for $\text{Var}(V) = \frac{12}{25}$ score as M0A1M0A0M0A1		



Question	Scheme	Marks	AOs
6(a)	$G_X(1) = 1$	M1	2.1
	$k \times 3^5 = 1 \therefore k = \frac{1}{243} *$	A1*cs0 (2)	1.1b
(b)	$P(X=2)$ is coefficient of t^2 so $G_X(t) = k(\dots + {}^5C_2(2t)^2 + \dots)$	M1	1.1b
	$P(X=2) = \frac{40}{243}$	A1 (2)	1.1b
(c)	$G_W(t) = \frac{t^3}{243}(1+2(t^2))^5$	M1	3.1a
	$G_W(t) = \frac{t^3}{243}(1+2t^2)^5$	A1 (2)	1.1b
(d)	$G_U(t) = \frac{1}{243}(1+2t)^5 \times \frac{t(1+2t)^2}{9}$	M1	3.1a
	$= \frac{t(1+2t)^7}{2187}$	A1 (2)	1.1b
(e)	$G_U'(t) = \frac{14t(1+2t)^6}{2187} + \frac{(1+2t)^7}{2187}$	M1	2.1
	$G_U'(1) = \frac{17}{3}$	A1ft	1.1b
	$G_U''(t) = \frac{168t(1+2t)^5}{2187} + \frac{14(1+2t)^6}{2187} + \frac{14(1+2t)^6}{2187}$	M1	2.1
	$G_U''(1) = 28$	A1	1.1b
	$\text{Var}(U) = "28" + "\frac{17}{3}" - \left("\frac{17}{3}" \right)^2$	M1	2.1
	$= \frac{14}{9}$	A1 (6)	1.1b
ALT(e)	$G_X''(t) = A(1+2t)^3$	M1	
	$G_X'(1) = \frac{10}{3}$ and $G_X''(1) = \frac{80}{9}$	A1ft	
	$G_Y''(t) = H(8+24t)$	M1	
	$G_Y'(1) = \frac{7}{3}$ and $G_Y''(1) = \frac{32}{9}$	A1	
	Using $G_U''(1) + G_U'(1) - \left(G_U'(1)\right)^2$ to find $\text{Var}(X)$, $\text{Var } Y$ and $\text{Var } U$	M1	
	$\frac{14}{9}$ or awrt 1.56	A1	
(14 marks)			

Notes:		
(a)	M1:	Stating $G_X(1) = 1$ eg $G_X(1) = k(1+2)^5 = 1$ $k(1+2)^5 = 1$ Allow Verification $\frac{1}{243} \times 3^5 = 1$
	A1*:	Fully correct proof with no errors Substituting $t=1$ Verification need therefore $G_X(1) = 1$
(b)	M1:	Attempting to find the coefficient of t^2
	A1:	$\frac{40}{243}$ or awrt 0.165
(c)	M1:	Realising the need to multiply through by t^3 or subst t^2 for t
	A1:	$\frac{t^3}{243}(1+2t^2)^5$ oe eg $\frac{t^3}{243}(1+10t^2+40t^4+80t^6+80t^8+32t^{10})$
(d)	M1:	Realising the need to use $G_U(t) = G_X(t) \times G_Y(t)$
	A1:	$\frac{t(1+2t)^7}{2187}$ oe
(e)	M1:	For an attempt to differentiate $G(u)$ e.g $G_U'(t) = At(1+2t)^6 + B(1+2t)^7$ ft their part(d) if in the form $kt(1+2t)^n$ where $n \geq 5$
	A1ft:	$\frac{17}{3}$ or awrt 5.67
	M1:	For attempting second derivative eg $G_U''(t) = Ct(1+2t)^5 + D(1+2t)^6$ ft their part(d) if in the form $kt(1+2t)^n$ where $n \geq 5$
	A1:	28
	M1:	Using $G_U''(1) + G_U'(1) - (G_U'(1))^2$ ft their values
	A1:	$\frac{14}{9}$ or awrt 1.56



Question	Scheme		M	AOs
4(a)	$G_X(1) = 1 \Rightarrow \frac{1}{1 - \frac{2}{5}} - k = 1$		M1	2.1
	$k = \frac{2}{3}$		A1	1.1b
			(2)	
(b)	$G'_X(t) = -\frac{-\frac{2}{5}}{\left(1 - \frac{2t}{5}\right)^2}$		M1	2.1
	$\left[G'_X(t) = \right] \frac{2}{5} \left(1 - \frac{2t}{5}\right)^{-2}$ may see $\left[G'_X(t) = \right] \frac{10}{(5 - 2t)^2}$		A1	1.1b
	$G'_X(1) = \frac{2}{5\left(1 - \frac{2}{5}\right)^2} \Rightarrow [E(X) =] \frac{10}{9}$		A1	1.1b
			(3)	
(c)	$G''_X(t) = (-2) \times \frac{2 \times \left(-\frac{2}{5}\right)}{5\left(1 - \frac{2t}{5}\right)^3} = \frac{8}{25\left(1 - \frac{2t}{5}\right)^3}$	Use of binomial expansion or standard power series (at least two terms or t^2 term correct)	M1	3.1a
	Use of Maclaurin series for t^2 (use of $P(X=2) = \frac{G''_X(0)}{2!}$)	$\frac{(-1)(-2)\left(-\frac{2}{5}\right)^2}{2!}$	M1	1.1b
	$\frac{1}{2} \times \frac{8}{25} = \frac{4}{25}$	$\frac{4}{25}$	A1	1.1b
			(3)	
(d)	Use of $\frac{1}{t} G_X(t)$ or $G_X(t^4)$ shown		M1	3.1a
	$G_Y(t) = \frac{1}{t} \left(\frac{1}{1 - \frac{2t^4}{5}} - \frac{2}{3} \right)$		A1ft	1.1b
			(2)	
(10 marks)				
Notes:				
(a)	M1:	For writing or using $G_X(1) = 1$		
	A1:	For finding $k = \frac{2}{3}$ (must be exact)		
(b)	M1:	Attempt to differentiate $G_X(t)$ to obtain $A\left(1 - \frac{2t}{5}\right)^{-2}$ (o.e.) for some constant A		
	A1:	Correct differential (may be unsimplified)		
	A1:	For finding $[E(X) =] \frac{10}{9}$ (must be exact, and must come from a correct differential)		
(c)	M1:	For $G''_X(t)$ in the form $A\left(1 - \frac{2t}{5}\right)^{-3}$ or a correct expression (not value) for $G''_X(0)$	Two terms correct from: $\frac{1}{3} + (-1)\left(-\frac{2}{5}t\right) + \frac{(-1)(-2)}{2}\left(-\frac{2}{5}t\right)^2$	
	M1:	For correct method to find $G''_X(t)$ and use of $\frac{G''_X(0)}{2!}$ $P(X=2) = \frac{G''_X(0)}{2!}$ (ft their $G'_X(t)$ in (b) provided $G'_X(t)$ not constant)	Correct unsimplified expression for t^2 seen (may be part of expansion)	
	A1:	For $\frac{4}{25}$ o.e.	For $\frac{4}{25}$ o.e	
(d)	M1:	Sight or use of $\frac{1}{t} G_X(t)$ or $G_X(t^4)$ stated or effected in expression		
	A1ft:	Correct expression, ft their k (accept k itself). ISW following a correct expression.		



Qu	Scheme	Marks	AO
6 (a)	$G(1) = 1 \Rightarrow k \ln 2 = 1$ so $k = \frac{1}{\ln 2}$	B1	2.1
(b)	$\left\{ G(t) = \frac{1}{\ln 2} [\ln 2 - \ln(2-t)] \right\} \Rightarrow G'(t) = \frac{1}{\ln 2} \left[\frac{1}{2-t} \right]$ or $\frac{1}{\ln 2} (2-t)^{-1}$ $[E(X) =] G'(1) = \frac{1}{\ln 2}$ $G''(t) = \frac{1}{\ln 2} \times \left[\frac{1}{(2-t)^2} \right]$ $\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2 = \frac{1}{\ln 2} + \frac{1}{\ln 2} - \left(\frac{1}{\ln 2} \right)^2$ $= \frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$	(1) M1 A1 A1 M1 A1 M1 A1	2.1 1.1b 1.1b 2.1 1.1b 2.1 1.1b
(c)	$P(X = 3) = \text{coefficient of } t^3 \text{ by Maclaurin need } G'''(0)$ $G'''(t) = \frac{1}{\ln 2} \frac{2}{(2-t)^3}$ $P(X = 3) = \frac{G'''(0)}{3!}$ $= \frac{\frac{1}{4\ln 2}}{6} = \frac{1}{24\ln 2} = 0.0601122... \text{ awrt } \underline{0.0601}$	(7) M1 A1ft M1 A1 (4)	3.1a 1.1b 3.2a 1.1b
Notes			
(a)	B1 for finding k (must be exact)		
(b)	1 st M1 for an attempt to differentiate $G(t)$ e.g. $A(2-t)^{-1}$ (o.e.) 1 st A1 for a correct first derivative (condone k or use of $\frac{1}{\ln 2} = \text{awrt } 1.44$) 2 nd A1 for correct $E(X)$ or $G'(1)$ (allow awrt 1.44 calc: 1.442695...but not k) seen anywhere 2 nd M1 for attempting second derivative (ft their $G'(t)$) 3 rd A1 for a correct 2 nd derivative (condone k or use of $\frac{1}{\ln 2} = \text{awrt } 1.44$) 3 rd M1 for a correct method for $\text{Var}(X)$ (some substitution into the correct formula) 4 th A1 for $\frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$ o.e. but must simplify i.e. collect like terms [Mark final answer – penalise incorrect log work etc] NB 0.8040211.. is A0 unless exact answer seen		
(c)	1 st M1 for a suitable strategy to solve the problem (finding link with Maclaurin) Need mention of coefficient of t^3 and $[G'''(t) \text{ or } G'''(0)]$ (condone $G'''(1)$) 1 st A1ft for 3 rd derivative, ft their 2 nd derivative in (b) (provided $G''(t)$ not const) Correct $G'''(t)$ or $G'''(0)$ scores 1 st M1 1 st A1ft 2 nd M1 for translating Maclaurin to probability (a correct expression) 2 nd A1 for $\frac{1}{24\ln 2}$ or awrt 0.0601		

Question	Scheme	Marks	AOs
6(a)	$P(X = 3) = \underline{0}$	B1	1.1b
		(1)	
(b)(i)	Coefficient of $t^4 = \frac{1}{64}b^2$	M1	2.1
	$\frac{1}{64}b^2 = \frac{25}{64}$	M1	1.1b
	$b = 5$ (reject $b = -5$ since $b > 0$)	A1	2.3
	$G_X(1) = 1$ $\frac{1}{64}(a + 5)^2 = 1$	M1	2.1
	$a = 3$ (reject $a = -13$ since $a > 0$)	A1	1.1b
	$P(X = 2) = \text{coefficient of } t^2 = \frac{1}{64}(2ab)$	M1	3.4
	$= \frac{15}{32}$	A1	1.1b
		(7)	
(ii)	$E(X) = G'_X(1)$	M1	2.1
	$G'_X(t) = \frac{2}{64}("3" + "5"t^2) \times "10"t$ or $G'_X(t) = \frac{1}{64}("60"t + "100"t^3)$	M1	1.1b
	$G'_X(1) = 2.5$	A1ft	1.1b
		(3)	
(c)	$G_Y(t) = t^2 G_X(t^3) [= \frac{t^2}{64}(a + b(t^3)^2)^2]$	M1	3.1a
	$G_Y(t) = \frac{t^2}{64}("3" + "5"t^6)^2$	A1ft	1.1b
		(2)	
(13 marks)			
Notes			
(a)	B1: 0 (Since there is no term in t^3)		
(b)(i)	M1: Realising that $\frac{1}{64}b^2$, the coefficient of t^4 , is needed M1: Equating their coefficient of t^4 to $\frac{25}{64}$ with an attempt to find b A1: $b = 5$ only M1: Realising that $G_X(1) = 1$ is required A1: $a = 3$ only M1: Finding coefficient of t^2 with their $a > 0$ and $b > 0$ A1: $\frac{15}{32}$ (condone awrt 0.469)		
(b)(ii)	M1: Realising $G'_X(1)$ is needed M1: Attempt to differentiate $G_X(t)$ with their values of a and b A1ft: 2.5 (ft (3sf) their values of a and b , $a > 0$ and $b > 0$) $E(X) = \frac{ab+b^2}{16}$ Alternative: M1: Realising $X = 0, 2$ and 4 only M1: $[0 \times P(X = 0)] + 2 \times P(X = 2) + 4 \times P(X = 4)$		
(c)	M1: either $G_X(t^3)$ or $\times t^2$ or using $Y = 2, 8, 14$ A1ft: ft their values of a and b , $a > 0$ and $b > 0$ $G_Y(t) = \frac{t^2}{64}("3" + "5"t^6)^2$ or $G_Y(t) = \frac{t^2}{64}("9" + "30"t^6 + "25"t^{12})$ or $G_Y(t) = \frac{1}{64}("9t^2" + "30"t^8 + "25"t^{14})$		

Qu	Scheme	Mark	AO
6			
(a)	$[G_X(t) = (4 - 3t)^{-\frac{1}{2}} \Rightarrow] \quad G'_X(t) = \frac{3}{2}(4 - 3t)^{-\frac{3}{2}}; \quad [\text{So } E(X) =] \quad G'_X(t) = \frac{3}{2}$ $G''_X(t) = \frac{27}{4}(4 - 3t)^{-\frac{5}{2}}; \text{ so } G''_X(1) = \frac{27}{4}$ $\text{Var}(X) = \frac{27}{4} + \frac{3}{2} - \left(\frac{3}{2}\right)^2; = \underline{6}$	M1; A1 M1; A1ft M1; A1 (6)	2.1; 1.1b 2.1 1.1b 1.1b 1.1b
(b)	Using Maclaurin: $\frac{G''_X(0)}{2!} = \left\{\frac{1}{2}\right\} \times \frac{27}{4} \times \frac{1}{32}$ $[P(X=2)] = \frac{1}{2} \times \frac{27}{4} \times \frac{1}{32} \left[= \frac{27}{256} \right]$ $[P(X=0) + P(X=1)] = \frac{3}{2} \times \frac{1}{8} + \frac{1}{2}$ $P(X \leq 2) = \frac{203}{256}$ Using Binomial: $[G_X(t)] = \frac{1}{2} \left(1 - \frac{3}{4}t\right)^{-\frac{1}{2}}$ $\frac{1}{2} \left(\dots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(-\frac{3}{4}t\right)^2 + \dots \right) = \left[\dots + \frac{27}{256}t^2 + \dots \right]$ $\frac{1}{2} \left(1 + \frac{3}{8}t + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(-\frac{3}{4}t\right)^2 + \dots \right)$ $P(X \leq 2) = \frac{203}{256}$	M1 A1 M1 A1 (4)	2.1 1.1b 2.1 1.1b
(c)	$[G_W(t) = G_{X_1}(t) \times G_{X_2}(t) \times t =] \frac{1}{\sqrt{4-3t}} \times \frac{1}{\sqrt{4-3t}} \times t; \quad G_Y(t) = \frac{t}{4-3t}$ $G_Y(t) = \frac{\frac{1}{4}}{1 - \frac{3}{4}t} \text{ or } G_Y(t) = \frac{\frac{1}{4}t}{1 - \frac{3}{4}t}; \quad Y \sim \text{Geo}\left(\frac{1}{4}\right)$	M1; A1 M1 A1 (4)	3.1a/2.1 1.1b 2.1 3.2a/2.2a
(d)	$P(Y > 6) = \left(1 - \frac{1}{4}\right)^6; \quad = \frac{729}{4096} = 0.177978... \quad \text{awrt } \underline{0.178}$	M1 A1 (2)	3.4, 1.1b
Notes		(16 marks)	
(a)	1 st M1 for attempt to differentiate leading to $k(4 - 3t)^{-1.5}$; 1 st A1 for $E(X) = 1.5$ or exact equivalent 2 nd M1 for attempting to differentiate again leading to $m(4 - 3t)^{-2.5}$ 2 nd A1ft for $\frac{27}{4}$ or correct ft from their k provided both Ms are scored 3 rd M1 for a correct method for finding $\text{Var}(X)$; can ft their $\frac{3}{2}$ and their $\frac{27}{4}$ 3 rd A1 for 6		
(b)	1 st M1 for Maclaurin to find $P(X = 2)$ condone $\frac{27}{4} \times \frac{1}{32} \left[= \frac{27}{128} \text{ or } 0.2109375 \right]$ or putting in the form $a(1 - 0.75t)^{-0.5}$ 1 st A1 for a correct unsimplified prob for $P(X = 2)$ (may be in binomial expansion) Allow 0.105(468...) 2 nd M1 for use of pgf to find $P(X = 1)$ and $P(X = 0)$ <u>or</u> attempt 1 st 3 terms of bin expansion 2 nd A1 for $\frac{203}{256}$ or exact equivalent.		
(c)	1 st M1 for using product of pgf or multiplication by t 1 st A1 for correct unsimplified form of pgf 2 nd dm1 (dep. on 1 st M1) for attempting to convert pgf to form given in the formula book or for stating Geometric alongside a correct PGF for Y (may be unsimplified) 2 nd A1 for correctly deducing the distribution of Y as geometric with $p = 0.25$ [may be seen in (d)] NB: Final A1 dependent on previous marks being awarded.		
(d)	M1 for attempting to use geometric formula or their pdf (using correct coefficients) : A1 for awrt 0.178		