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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Continuous Probability Distributions (2)

Further Statistics 2 · Topic 2

10 questions · 105 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
2.2 PDFS AND CDFS		4 Qs · 40 marks	3
1. Q4	Option 4B: Further Statistics 2 June2019	8	4
2. Q3	Option 4B: Further Statistics 2 November2021	10	8
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2.3 MEAN AND VARIANCE		2 Qs · 20 marks	17
5. Q5	Option 4B: Further Statistics 2 June2024	10	18
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2.4 THE CONTINUOUS UNIFORM DISTRIBUTION		3 Qs · 30 marks	26
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QUESTIONS

10 questions · 105 marks · 4 sub-topics



SUB-TOPIC 2.2

PDFS AND CDFS

Sub-topic 2.2 · 4 questions · 40 marks · lowest tariff first





Question 4 continued



(Total for Question 4 is 8 marks)







(Total for Question 2 is 16 marks)



(Total for Question 6 is 10 marks)





SUB-TOPIC 2.3

MEAN AND VARIANCE

Sub-topic 2.3 · 2 questions · 20 marks · lowest tariff first







Question 5 continued

(Total for Question 5 is 10 marks)







SUB-TOPIC 2.4

THE CONTINUOUS UNIFORM DISTRIBUTION

Sub-topic 2.4 · 3 questions · 30 marks · lowest tariff first



7. The random variable R has a continuous uniform distribution over the interval $[2, 10]$

(a) Write down the probability density function $f(r)$ of R

(1)

A sphere of radius R cm is formed.

The surface area of the sphere, $S \text{ cm}^2$, is given by $S = 4\pi R^2$

(b) Show that $E(S) = \frac{496\pi}{3}$

(4)

The volume of the sphere, $V \text{ cm}^3$, is given by $V = \frac{4}{3}\pi R^3$

(c) Find, using algebraic integration, the expected value of V

(4)

Question 7 continued

(Total for Question 7 is 9 marks)



5. The continuous random variable X is uniformly distributed over the interval $[0, 4\beta]$, where β is an unknown constant.

Three independent observations, X_1, X_2 and X_3 , are taken of X and the following estimators for β are proposed

$$A = \frac{X_1 + X_2}{2}$$

$$B = \frac{X_1 + 2X_2 + 3X_3}{8}$$

$$C = \frac{X_1 + 2X_2 - X_3}{8}$$

- (a) Calculate the bias of A , the bias of B and the bias of C

(5)

- (b) By calculating the variances, explain which of B or C is the better estimator for β

(4)

- (c) Find an unbiased estimator for β

(1)





Question 5 continued

(Total for Question 5 is 10 marks)



Question 5 continued



Question 5 continued



SUB-TOPIC 5.1

ESTIMATORS, BIAS AND STANDARD ERROR

Sub-topic 5.1 · 1 question · 15 marks · lowest tariff first



6. Korhan and Louise challenge each other to find an estimator for the mean, μ , of the continuous random variable X which has variance σ^2

$X_1, X_2, X_3, \dots, X_n$ are n independent observations taken from X

Korhan's estimator is given by

$$K = \frac{2}{n(n+1)} \sum_{r=1}^n rX_r$$

Louise's estimator is given by

$$L = \frac{X_1 + X_2}{3} + \frac{X_3 + X_4 + \dots + X_n}{3(n-2)}$$

- (a) Show that K and L are both unbiased estimators of μ

(5)

- (b) (i) Find $\text{Var}(K)$

- (ii) Find $\text{Var}(L)$

(7)

The winner of the challenge is the person who finds the better estimator.

- (c) Determine the winner of the challenge for large values of n . Give reasons for your answer.

(3)



Question 6 continued

Lined area for writing the answer to Question 6 continued.

Question 6 continued

(Total for Question 6 is 15 marks)

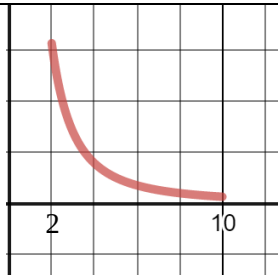


MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
4(a)	$k\left(2^3 - \frac{3}{8}2^4\right) = 1$	B1*	1.1b
	$2k = 1$		
	$k = \frac{1}{2}^*$		
	Or $\frac{1}{2}\left(2^3 - \frac{3}{8}2^4\right) = 1 \therefore k = \frac{1}{2}^*$	(B1*)	
		(1)	
(b)	$f(x) = k\left[3x^2 - \frac{3}{2}x^3\right]$	M1	2.1
	(i) $\int_0^2 xf(x) dx = k \int_0^2 \left(3x^3 - \frac{3}{2}x^4\right) dx$	M1d	1.1b
	$= \left[\frac{3x^4}{8} - \frac{3x^5}{20}\right]_0^2$		
	$= \frac{6}{5}$ or 1.2	A1	1.1b
	(ii) $3x - \frac{9x^2}{4} = 0$	M1d	3.1a
	$x\left(3 - \frac{9x}{4}\right) = 0$	M1d	1.1b
	$x = 0$ or $\frac{4}{3} \therefore \text{mode} = \frac{4}{3}$	A1	1.1b
		(6)	
(c)	Mode > mean implies it is negative skew	B1ft	2.4
		(1)	
(8 marks)			
Notes:			
(a)	B1*	substituting $x = 2$ into $F(x)$ and equating to 1 leading to $k = \frac{1}{2}$ with no errors. Minimum subst seen is $k(8 - 6) = 1$ or $0.5(8 - 6) = 1$	
(b)	M1	Realising they need to find the pdf and attempting to differentiate $k\left[x^3 - \frac{3}{8}x^4\right]$ at least 1 correct term	
(i)	M1d	dep on 1 st M1 Attempting to find $\int_0^2 x(\text{their } f(x))dx$ At least one correct term ft their pdf	
	A1	$\frac{6}{5}$ or 1.2 oe NB 1.2 with no working gains M0M0A0	
(ii)	M1d	dep on 1 st M1 for realising they need to differentiate their pdf. At least one correct term but ft their pdf	
	M1d	Dep on 3 rd M1. correct method for solving their differential of their pdf = 0 pdf must be of the form $ax^2 + bx$	
	A1	$\therefore \text{mode} = \frac{4}{3}$ only. They must eliminate 0	
(c)	B1ft	ft their mode and mean or a correct sketch.	

Question	Scheme	Marks	AOs
3(a)	$F(5) + (1 - F(8)) \quad \left[\frac{3}{4} + \left(1 - \frac{15}{16} \right) \right]$	M1	2.1
	$= \frac{13}{16}$	A1	1.1b
		(2)	
(b)	$F(m) = 0.5 \quad \left[1.25 - \frac{2.5}{m} = 0.5 \right]$	M1	1.1b
	$m = \frac{10}{3}$	A1	1.1b
		(2)	
(c)	$f(x) [= \frac{d}{dx}(F(x))] = 2.5x^{-2}$	M1	2.1
	$E(X^2) [= \int_2^{10} x^2 f(x) dx] = \int_2^{10} 2.5 dx$	M1	1.1b
	$= 20$	A1	1.1b
		(3)	
(d)(i)		M1	1.1b
(ii)	2 and 10 correctly labelled on horizontal axis	A1	2.1
	Therefore positive skew.	A1	2.2b
		(3)	
(10 marks)			
Notes			
(a)	M1: Equivalent correct probability statement, e.g. $[1 - (F(8) - F(5))]$ A1: $\frac{13}{16}$ oe		
(b)	M1: Use of $F(m) = 0.5$ A1: $\frac{10}{3}$ oe		
(c)	M1: Realising that $f(x)$ is required and attempting to differentiate $F(x)$ M1: Use of $\int_2^{10} x^2 f(x) dx$ A1: 20 cao		
(d)(i) and (ii)	M1: Correct shape A1: Correct labels A1: Positive skew provided M1 scored		

Question	Scheme	Marks	AOs
6(a)	$F(2) = \frac{2}{3}$ and $F(4) = 1$		
	$k(2 - 4a) = \frac{2}{3}$ $k(4 - 16a) = 1$	M1	2.1
	$\frac{2}{3(2 - 4a)} = \frac{1}{(4 - 16a)} \rightarrow a = \frac{1}{10}, k = \frac{5}{12}$	M1A1	1.1b 1.1b
	$\frac{5}{12}\left(m - \frac{1}{10}m^2\right) = 0.5 \rightarrow m^2 - 10m + 12 = 0$	M1M1	3.1a 1.1b
	$m = 5 - \sqrt{13}$ ($m = 5 + \sqrt{13}$ reject)	A1	1.1b
		(6)	
(b)	$f(x) = \frac{d}{dx}(F(x))$	M1	1.1b
	$f(x) = \begin{cases} \frac{5}{12}(1 - 0.2x) & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$	A1ft	1.1b
		(2)	
(c)	Mode is $X = 0 \dots$	B1	2.2a
	...since $f(x)$ is linear with negative gradient. or ... $f(x)$ is a decreasing function	dB1	2.4
		(2)	
(10 marks)			
Notes			
(a)	M1: using $F(2) = \frac{2}{3}$ and $F(4) = 1$ to form two correct expressions in a and k M1: solving simultaneously to find a value of a or k A1: both $a = \frac{1}{10}$ and $k = \frac{5}{12}$, may be implied by correct equation or median M1: setting up $F(m) = 0.5$ to form a 3TQ M1: using an appropriate method to solve quadratic, resulting in a value of m A1: selecting the correct value for m		
(b)	M1: differentiating $F(x)$, at least one term correct for their a and k A1ft: correct ft expression with correct limits		
(c)	B1: deducing that the mode is $X = 0$ dB1: correct reasoning May be given via a diagram of a linear function $0 \leq x \leq 4$ with: - a negative gradient - that intersects the y-axis - is never negative in y		

Question	Scheme	Marks	AOs
8(a)	$1.5m - 0.25m^2 - 1.25 = 0.5 \quad (\rightarrow 0.25m^2 - 1.5m + 1.75 = 0)$	M1	1.1b
	$m = 3 - \sqrt{2} \quad (\text{reject } m = 3 + \sqrt{2})$	A1	2.2a
		(2)	
(b)	$P(X < 1.6 \mid X > 1.2) = \frac{P(1.2 < X < 1.6)}{P(X > 1.2)}$	M1	3.1a
	$\frac{F(1.6) - F(1.2)}{1 - F(1.2)}$	M1	1.1b
	$= \frac{32}{81}$	A1	1.1b
		(3)	
(c)	$P(Y \leq y) = P\left(\frac{1}{X} \leq y\right)$		
	$= P\left(X \leq \frac{1}{y}\right) = 1 - F\left(\frac{1}{y}\right)$	M1	3.1a
	$= 1 - \left(\frac{1.5}{y} - 0.25\left(\frac{1}{y}\right)^2 - 1.25\right)$	M1	1.1b
	$F(y) = \begin{cases} 0 & y < \frac{1}{3} \\ 2.25 - \frac{1.5}{y} + 0.25\left(\frac{1}{y}\right)^2 & \frac{1}{3} \leq y \leq 1 \\ 1 & y > 1 \end{cases}$	A1 A1	2.1 1.1b
		(4)	
(d)	$f(y) = \frac{d}{dy}(F(y)) = 1.5y^{-2} - 0.5y^{-3} \rightarrow f'(y) = -3y^{-3} + 1.5y^{-4}$	M1	3.1a
	$f'(y) = -3y^{-3} + 1.5y^{-4} = 0$	depM1	1.1b
	$1.5y^{-4} = 3y^{-3} \rightarrow \frac{1.5}{y^4} = \frac{3}{y^3}$		
	[Mode of Y] 0.5 (since $f''(0.5) = -48 < 0$)	A1	1.1b
		(3)	
(12 marks)			
Notes			
(a)	M1: Correct equation for m A1: $m = 3 - \sqrt{2}$ only (isw if exact answer is given then rounded)		
(b)	M1: Determining the two probabilities required to find the probability M1: Correct ratio of probabilities A1: allow awrt 0.395		
(c)	M1: Realising that $P\left(X \leq \frac{1}{y}\right)$ is necessary M1: Correct use of $F(x)$ A1: Determining correct limits (allow $\leq$ for $<$ etc.) A1: All lines of cdf correct (ignoring limits) must be in terms of y SC: If 0 scored, then $[F(y) =] \frac{1.5}{y} - 0.25\left(\frac{1}{y}\right)^2 - 1.25 \quad \frac{1}{3} \leq y \leq 1$ scores M0M0A1A0		
(d)	M1: Realising that the cdf must be differentiated twice depM1: (dep on previous M1) Equating their $f'(y) = 0$ with attempt to solve A1: 0.5 cao		

Question	Scheme		Marks	AOs
5(a)	$\int ax^{-2} - bx^{-3} dx = -\frac{a}{x} + \frac{b}{2x^2}$		M1	1.1b
	$\left[-\frac{a}{x} + \frac{b}{2x^2}\right]_2^\infty = 0 - \left(-\frac{a}{2} + \frac{b}{8}\right) \quad \left[= \frac{a}{2} - \frac{b}{8} \right]$ or $\left[-\frac{a}{x} + \frac{b}{2x^2}\right]_2^4 = \left(-\frac{a}{4} + \frac{b}{32}\right) - \left(-\frac{a}{2} + \frac{b}{8}\right) \quad \left[= \frac{a}{4} - \frac{3b}{32} \right]$ or $\left[-\frac{a}{x} + \frac{b}{2x^2}\right]_4^\infty = 0 - \left(-\frac{a}{4} + \frac{b}{32}\right) \quad \left[= \frac{a}{4} - \frac{b}{32} \right]$		M1 M1	1.1b 1.1b
	$\frac{a}{2} - \frac{b}{8} = 1$ or $4a - b = 8$ oe $\frac{a}{4} - \frac{3b}{32} = \frac{3}{8}$ or $8a - 3b = 12$ oe $\frac{a}{4} - \frac{b}{32} = \frac{5}{8}$ or $8a - b = 20$ oe		dM1 A1	1.1b 1.1b
	$\therefore a = 3^*$		A1*cs0	2.1
			(6)	
(b)	$b = 4$		B1	1.1b
	$\left[-\frac{3}{x} + \frac{4}{2x^2}\right]_2^m = 0.5$ or e.g. $-3x^{-1} + 2x^{-2} + 1 = 0.5$		M1	1.2
	$m^2 - 6m + 4 = 0$ oe		A1	1.1b
	$(m =) 3 + \sqrt{5}$		A1	2.2a
			(4)	
(10 marks)				
Notes:				
(a)	M1	Attempt to integrate one term correct. Look for the power increasing by 1		
	M1	Integrating both terms and substitute limits the correct way round [any one of $(2, \infty)$ or $(2, 4)$ or $(4, \infty)$] to form one expression where c is a non-zero constant. Alternatively, allow this mark for: Allow this mark for $F(x) = \left(-\frac{a}{x} + \frac{b}{2x^2}\right) - \left(-\frac{a}{2} + \frac{b}{8}\right)$ or one of $F(4)$ or $F(\infty)$		
	M1	Integrating and substitute limits the correct way round [any one of $(2, \infty)$ or $(2, 4)$ or $(4, \infty)$] to form a second expression. Alternatively, for $F(4)$ and $F(\infty)$		
	dM1	Dependent on the 2 nd M. For one of the expressions equal to correct value from 1, $\frac{3}{8}$ or $\frac{5}{8}$		
	A1	Translating a problem in mathematical context into two correct equations with one a term, one b term and one number		
	A1*cs0	Fully correct solution, achieving $a = 3$		

Alt (a)	M1	As in main scheme
	M1	substitutes two of 2, 4 or “ ∞ ” into their integral which must have a constant of integration eg $-\frac{a}{2} + \frac{b}{8} + c (=0)$, $-\frac{a}{4} + \frac{b}{32} + c \left(=\frac{3}{8} \right)$ or $-\frac{a}{\text{"}\infty\text{"}} + \frac{b}{\text{"}\infty\text{"}} + c (=1)$ oe
	M1	substitutes all three of 2, 4 or “ ∞ ”
	dM1	forms at least two equations involving c eg $-\frac{a}{2} + \frac{b}{8} + c = 0$, $-\frac{a}{4} + \frac{b}{32} + c = \frac{3}{8}$ or $-\frac{a}{\text{"}\infty\text{"}} + \frac{b}{\text{"}\infty\text{"}} + c = 1$ (may just state $c = 1$)
	A1A1	As in main scheme
(b)	B1	Writing or using $b = 4$ may be seen in (a)
	M1	Equating their integral with b , limits 2 and m substituted and equated to 0.5 Allow their $F(x) = 0.5$. It must be of the form $\alpha x^{-1} + \beta x^{-2} + \gamma = 0.5$ oe May be in terms of eg x instead of m
	A1	A correct 3 term quadratic $= 0$ Terms do not need to be collected on the same side. May be implied by ans. May be in terms of eg x
	A1	$3 + \sqrt{5}$ and any other solutions should be eliminated

Qu	Scheme	Marks	AO
5(a)	$\int (1 - \cos x) \, dx = [x - \sin x]$	M1	1.1b
	Use of correct limits and $\int f(x) \, dx = 1 \Rightarrow 2\pi - 0 - 0 = 1$	M1	1.1b
	so $k = \frac{1}{2\pi} \text{ (*)}$	A1*cs0	1.1b
		(3)	
	(b) $E(X) = \pi$ (symmetry) so $\mu = \pi$ so $f(\mu) = \frac{1}{2\pi}(1 - \cos \pi) = \frac{1}{\pi}$	B1	2.2a
	$\frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{\pi} \text{ ; so } \sigma = \sqrt{\frac{\pi}{2}}$	M1; A1	1.1b 1.1b
		(3)	
	(c) $P\left(\frac{\pi}{2} < X < \frac{3\pi}{2}\right) = \frac{1}{2\pi} [x - \sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \frac{1}{2\pi} \left[\left(\frac{3\pi}{2} - -1\right) - \left(\frac{\pi}{2} - 1\right) \right]$	M1	3.4
	$= \frac{2 + \pi}{2\pi} (= 0.81830\dots)$	A1	1.1b
	$P\left(\frac{\pi}{2} < Y < \frac{3\pi}{2}\right) = 0.7899\dots$	B1	1.1b
	So error is $0.81830\dots - 0.7899\dots = 0.0284$	A1	1.1b
	(4)		
		(10 marks)	
Notes			
(a)	1 st M1 attempt to integrate $(1 - \cos x)$ – one correct term 2 nd M1 for use of correct limits and correct method for k A1* cs0 use of $\int f(x) \, dx = 1$ seen and no incorrect working seen		
(b)	B1 for correctly deducing the value of $f(\mu)$ M1 for a correct equation for σ – ft their value for $f(\mu)$ [condone for sight of correct $g(\mu)$] A1 for $\sqrt{\frac{\pi}{2}}$ or exact equivalent		
(c)	M1 for a correct attempt to find prob – some correct integration and use of limits 1 st A1 for a correct answer (exact or 0.818.. or better) B1 for a correct probability from their calculator i.e. 0.7899 or better accept 0.79 2 nd A1 for 0.0284 or better		

Question	Scheme		Marks	AOs
7(a)	$f(r) = \frac{1}{8}$ for $2 \leq r \leq 10$ [0 otherwise]		B1	1.1b
			(1)	
(b)	$E(4\pi R^2) = 4\pi E(R^2)$	$\int_2^{10} 4\pi r^2 \left(\frac{1}{8}\right) dr$	M1	1.1b
	$E(R) = 6$	$= \left[\frac{1}{2} \pi \left(\frac{r^3}{3} \right) \right]_2^{10}$	M1	1.1b
	$E(4\pi R^2)$ $= 4\pi(\text{Var}(R) + [E(R)]^2)$ $= 4\pi\left(\frac{16}{3} + 6^2\right)$	$= \frac{1}{2} \pi \left(\frac{10^3}{3} - \frac{2^3}{3} \right)$	M1	2.1
	$= \frac{496\pi}{3}$ *		A1*	2.2a
			(4)	
(c)	$E\left(\frac{4}{3}\pi R^3\right) = \int_2^{10} \frac{4}{3}\pi r^3 \left(\frac{1}{8}\right) dr$		B1ft	1.1b
	$= \left[\frac{1}{6} \pi \left(\frac{r^4}{4} \right) \right]_2^{10}$		M1	2.1
	$= \frac{1}{6} \pi \left(\frac{10^4}{4} - \frac{2^4}{4} \right)$		dM1	1.1b
	$= 416\pi$		A1	2.2a
			(4)	
(9 marks)				
Notes				
(a)	B1: correct probability density function with limits Allow any letter but must be consistent			
(b)	M1: use of $E(aR) = aE(R)$ / correct integral with their $f(r)$ (limits not needed) M1: $E(R) = 6$ seen or implied / use of integration to find $E(SA)$ M1: use of $E(R^2) = \text{Var}(R) + [E(R)]^2$ / correct integration of their $f(r)$, limits shown A1*: exact value from correct working			
	follow through their pdf from (a) for first 3 marks of (c) B1ft: correct integral for $E(V)$ including limits, M1: integration of their pdf from (a) to find $E(V)$ dM1: (dep on previous M1) use of correct limits, may be implied A1: 416π (allow awrt 1310 to 3 sf)			



Question	Scheme	Marks	AOs
5(a)	$[E(X) = 2\beta]$ $E(A) = E\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{2}[E(X) + E(X)]$ $E(B) = E\left(\frac{X_1 + 2X_2 + 3X_3}{8}\right) = \frac{1}{8}[E(X) + 2E(X) + 3E(X)]$ $E(C) = E\left(\frac{X_1 + 2X_2 - X_3}{8}\right) = \frac{1}{8}[E(X) + 2E(X) - E(X)]$	M1	3.1a
	Bias for $A = E(A) - \beta = 2\beta - \beta = \beta$ Bias for $B = E(B) - \beta = 1.5\beta - \beta = 0.5\beta$ Bias for $C = E(C) - \beta = 0.5\beta - \beta = -0.5\beta$	M1 A1 A1 A1	2.1 1.1b 1.1b 1.1b
		(5)	
	$[\text{Var}(X) = \frac{4}{3}\beta^2]$ Better estimator would have the smallest bias and the least variance. B and C have equal bias, so we select the estimator with the smallest variance $\text{Var}(B) = \text{Var}\left(\frac{X_1 + 2X_2 + 3X_3}{8}\right)$ $= \frac{1}{64}[\text{Var}(X) + 4\text{Var}(X) + 9\text{Var}(X)]$ $\text{Var}(C) = \text{Var}\left(\frac{X_1 + 2X_2 - X_3}{8}\right)$ $= \frac{1}{64}[\text{Var}(X) + 4\text{Var}(X) + \text{Var}(X)]$	M1	2.1
	$\text{Var}(B) = \frac{7}{32}\text{Var}(X) [= \frac{7}{24}\beta^2]$ $\text{Var}(C) = \frac{3}{32}\text{Var}(X) [= \frac{1}{8}\beta^2]$	A1 A1	1.1b 1.1b
(b)	(Since both have same bias and) $\text{Var}(C) < \text{Var}(B)$ therefore C is the better estimator.	B1ft	2.2a
		(4)	
	Any unbiased estimator, e.g. $\frac{X_1 + X_2 + X_3}{6}$	B1	3.5c
		(1)	
(10 marks)			
Notes			
(a)	M1: Using independence to calculate the $E(A)$, $E(B)$ or $E(C)$ M1: Use of bias = $E(X) - \beta$ A1: Correct bias for A A1: Correct bias for B A1: Correct bias for C [allow $+0.5\beta$]		
(b)	M1: Realising that variances need to be compared and attempt at linear combination of variances for B or C A1: Correct $\text{Var}(B)$ A1: Correct $\text{Var}(C)$ A1ft: Correct comparison and deduction that C a better estimator than A and B .		
(c)	R1: Allow any unbiased estimator, e.g. $\frac{X_1}{1}$		

Qu	Scheme	Mark	AO
5 (a)	(i) $P(1.8 < X < 3.2) = \left[\frac{3.2-1.8}{4-1} \right] = \frac{7}{15}$ oe	B1	3.4
	(ii) $P(X > 3.2 X > 1.8) = \frac{P(X > 3.2)}{P(X > 1.8)}$ or $\frac{\frac{4}{15}}{\frac{11}{15}}$ or $\frac{0.8}{2.2}$ oe	M1	2.1
	$= \frac{4}{11}$	A1	1.1b
		(3)	
	(b) $[1 - F(3.75)] = 1 - \left(\frac{3.75-1}{3} \right)^{10} = 0.581096$ awrt 0.581	B1	3.4
(c)	(i) $f(y) = \frac{dF(y)}{dy} = \frac{10}{3} \times \left(\frac{y-1}{3} \right)^9$ or sketch of correct shape Correct sketch showing $y = 1$ and 4 and $f(1) = 0$	M1	1.1b
		A1	1.1b
	(ii) [From sketch mode of M is] 4	B1	2.2a
		(3)	
	(d) $E(M) = k \int y \times 10(y-1)^9 dy = k \int y d(y-1)^{10}$ or $K \int (3u+1)u^9 du$ $= k \left[y(y-1)^{10} \right]_1^4 - k \int_1^4 (y-1)^{10} dy$ or $K \left\{ \int_0^1 (3u^{10} + u^9) du = \left[\frac{3u^{11}}{11} + \dots \right] \right\}$ $= \left[y \left(\frac{y-1}{3} \right)^{10} \right]_1^4 - \frac{3}{11} \left[\left(\frac{y-1}{3} \right)^{11} \right]_1^4$ oe or $10 \left[\frac{3u^{11}}{11} + \frac{u^{10}}{10} \right]_0^1$ oe $= \left[4 - \frac{3}{11} \right] = \frac{41}{11}$	M1	2.1
		M1	1.1b
		M1	1.1b
		A1	1.1b
		(4)	
(11 marks)			
Notes			
(a)(i)	B1	for $\frac{7}{15}$ or exact equivalent isw	
(ii)	M1	for a correct ratio of prob expressions (must be $P(X > 3.2)$ on num) or values	
	A1	or awrt 0.364	
	A1	for $\frac{4}{11}$ or exact equivalent isw	
(b)	B1	for awrt 0.581	
(c)(i)	M1	for correct expression or a sketch of correct shape with positive increasing gradient	
	A1	for a fully correct sketch with 1 and 4 correctly indicated (dashed line not needed). Condone curves which appear almost linear provided this was not the intention. Ignore any labelling of the axes or values indicated on the vertical axis.	
(ii)	B1	for 4	
(d)	1 st M1	for a correct expression for $E(M)$ and an attempt to start to integrate. May be implied by further work. May use substitution e.g. $3u = y - 1$ so forms the integral expression in u . Allow any constant k (or K)	
	2 nd M1	for a correct first step of integration. Allow any k (or K) and still ignore limits.	
	3 rd M1	for a correctly integrated expression including limits (need not be substituted in)	
	A1	dep on all previous method marks for $\frac{41}{11}$ or exact equivalent	

Question	Scheme	Marks	AOs
6(a)	$E(K) = \frac{2E(X)}{n(n+1)} + \frac{4E(X)}{n(n+1)} + \frac{6E(X)}{n(n+1)} + \dots + \frac{2nE(X)}{n(n+1)}$ oe	M1	3.1a
	$= \frac{1}{n(n+1)} \left(\sum_{r=1}^n 2r \right) E(X) = \frac{1}{n(n+1)} \left(\frac{(2+2n)n}{2} \right) E(X) (= E(X))$	M1	2.1
	$= \mu$ (therefore K is an unbiased estimator of μ)	A1cso	1.1b
	$E(L) = \frac{2E(X)}{3} + \frac{(n-2)E(X)}{3(n-2)} = \frac{2E(X)}{3} + \frac{1E(X)}{3} (= E(X))$	M1	1.1b
	$= \mu$ (therefore L is an unbiased estimator of μ)	A1cso	1.1b
		(5)	
(b)(i)	$\text{Var}(K) = \text{Var}\left(\frac{2X}{n(n+1)}\right) + \text{Var}\left(\frac{4X}{n(n+1)}\right) + \dots + \text{Var}\left(\frac{2n(X)}{n(n+1)}\right)$	M1	3.1a
	$= \frac{2^2}{n^2(n+1)^2} \text{Var}(X) + \frac{4^2}{n^2(n+1)^2} \text{Var}(X) + \dots + \frac{(2n)^2}{n^2(n+1)^2} \text{Var}(X)$	M1	1.1b
	$= \frac{\sum_{r=1}^n (2r)^2}{n^2(n+1)^2} \text{Var}(X) = \frac{4 \sum_{r=1}^n r^2}{n^2(n+1)^2} \text{Var}(X) = \frac{\frac{4}{6}(n)(n+1)(2n+1)}{n^2(n+1)^2} \sigma^2$	M1	2.1
	$= \frac{2(2n+1)}{3n(n+1)} \sigma^2$	A1	1.1b
	(ii) $\text{Var}(L) = \text{Var}\left(\frac{X_1 + X_2}{3}\right) + \text{Var}\left(\frac{X_3 + X_4 + \dots + X_n}{3(n-2)}\right)$	M1	1.1b
	$= \frac{2}{9} \text{Var}(X) + \frac{(n-2)}{9(n-2)^2} \text{Var}(X)$	M1	2.1
	$= \frac{2n-3}{9(n-2)} \sigma^2$	A1	1.1b
		(7)	
	(c) For large values of n $\text{Var}(K) \rightarrow 0$ $\text{Var}(L) \rightarrow \frac{2}{9}(\sigma^2)$	M1	2.1
	(Since both are unbiased,) the better estimator is the one with the smaller variance or $0 < \frac{2}{9}(\sigma^2)$	M1	2.4
	Therefore K is the better estimator and Korhan wins the challenge.	A1	2.2a
		(3)	
(15 marks)			
Notes			
(a)	M1: Using independence to set up expression for $E(K)$		
	M1: Use of $\sum_{r=1}^n r \left(= \frac{n(n+1)}{2} \right)$		
	A1cso: Correct conclusion from correct working		
(b)(i)	M1: Using independence to set up expression for $E(L)$		
	A1cso: Correct conclusion from correct working		
	M1: Using independence to set up expression for $\text{Var}(K)$		
	M1: Use of $\text{Var}(aX) = a^2 \text{Var}(X)$ Implied by $\frac{2^2}{n^2(n+1)^2}$		

(ii)	M1: Use of $\sum_{r=1}^n r^2$ A1: Correct equivalent expression for $\text{Var}(K)$ oe M1: Using independence to set up expression for $\text{Var}(L)$ M1: Use of $\text{Var}(aX) = a^2\text{Var}(X)$ and understanding $\text{Var}(X_1 + X_2) = 2\text{Var}(X)$ Implied by either correct term A1: Correct equivalent expression for $\text{Var}(L)$ oe
(c)	M1: Finding correct limits as n gets larger for each expression allow ft Note: Solving $\frac{2n-3}{9(n-2)} = \frac{2(2n+1)}{3n(n+1)} \rightarrow n = -0.53, 2.46, 4.57$ so allow comments relating to $n \geq 5$ (4.57) M1: Correct explanation A1: Deducing that Korhan is the winner (dependent upon both M marks)