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EDEXCEL A LEVEL FURTHER MATHEMATICS: QUESTIONS BY TOPIC

Other Hypothesis Tests and Confidence Intervals (6)

Further Statistics 2 · Topic 6

14 questions · 138 marks · Mark schemes included

Pearson Edexcel A Level Further Mathematics (9FM0), June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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5.4 TESTS FOR A DIFFERENCE OF MEANS		2 Qs · 21 marks	3
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QUESTIONS

14 questions · 138 marks · 5 sub-topics



SUB-TOPIC 5.4

TESTS FOR A DIFFERENCE OF MEANS

Sub-topic 5.4 · 2 questions · 21 marks · lowest tariff first



Question 2 continued



Question 2 continued



Question 2 continued

(Total for Question 2 is 9 marks)



Question 2 continued





SUB-TOPIC 6.1

TESTS AND CONFIDENCE INTERVALS FOR A VARIANCE

Sub-topic 6.1 · 5 questions · 50 marks · lowest tariff first



Question 1 continued

(Total for Question 1 is 8 marks)



Question 3 continued

(Total for Question 3 is 8 marks)



- 6 A company manufactures bolts. The diameter of the bolts follows a normal distribution with a mean diameter of 5 mm.

Stan believes that the mean diameter of the bolts is less than 5 mm. He takes a random sample of 10 bolts and measures their diameters. He calculates some statistics but spills ink on his work before completing them. The only information he has left is as follows

4.5 4.5 5.5 4.8 4.9 4.7 5
 $X \sim N(5)$
 $\sum x = 48.4$
 $\bar{x} =$
 99% confidence interval for the variance is = (0.01712, 0.23280)

Question 6 continued

(Total for Question 6 is 9 marks)



- 6** A new employee, Kim, joins an existing employee, Jiang, to work in the quality control department of a company producing steel rods. Each day a random sample of rods is taken, their lengths measured and a 95% confidence interval for the mean length of the rods, in metres, is calculated. It is assumed that the lengths of the rods produced are normally distributed.
- Kim took a random sample of 25 rods and used the t distribution to obtain a 95% confidence interval of (1.193, 1.367) for the mean length of the rods. Jiang commented that this interval was a little wider than usual and explained that they usually assume that the standard deviation does not change and can be taken as 0.175 metres.
- (a) Test, at the 10% level of significance, whether or not Kim's sample suggests that the standard deviation is different from 0.175 metres. State your hypotheses clearly.

Using Kim's sample and the normal distribution with a standard deviation of 0.175 metres,

- (b) find a 95% confidence interval for the mean length of the rods.



Question 6 continued

(Total for Question 6 is 12 marks)



Question 4 continued



Question 4 continued

SUB-TOPIC 6.2

F-TESTS FOR EQUAL VARIANCES

Sub-topic 6.2 · 5 questions · 49 marks · lowest tariff first



(Total for Question 3 is 8 marks)

(Total for Question 5 is 8 marks)





Question 2 continued

(Total for Question 2 is 12 marks)







(Total for Question 6 is 15 marks)



SUB-TOPIC 7.1

T-TESTS AND CONFIDENCE INTERVALS FOR A MEAN

Sub-topic 7.1 · 1 question · 6 marks · lowest tariff first



Question 2 continued

(Total for Question 2 is 6 marks)



SUB-TOPIC 7.3

T-TESTS FOR A DIFFERENCE OF MEANS

Sub-topic 7.3 · 1 question · 12 marks · lowest tariff first



6. A researcher set up a trial to assess the effect that a food supplement has on the increase in weight of Herdwick lambs. The researcher randomly selected 8 sets of twin lambs. One of each set of twins was given the food supplement and the other had no food supplement. The gain in weight, in kg, of each lamb over the period of the trial was recorded.

	Set of twin lambs	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
Weight gain (kg)	With food supplement	4.1	5.3	6.0	3.6	5.9	4.2	7.1	6.4
	No food supplement	5.0	4.8	5.2	3.4	5.1	3.9	7.0	6.5

- State why a two sample t -test is not suitable for use with these data. (1)
- Suggest 2 other factors about the lambs that the researcher may need to control when selecting the sample. (2)
- State one assumption, in context, that needs to be made for a paired t -test to be valid. (1)

For a pair of twin lambs, the random variable W represents the weight gain of the lamb given the food supplement minus the weight gain of the lamb not given the food supplement.

- (d) Using the data in the table, calculate a 98% confidence interval for the mean of W . Show your working clearly. (5)

The researcher believes that the mean of W is greater than 200 g

- (e) Stating your hypotheses clearly, use your confidence interval to explain whether or not there is evidence to support the researcher's belief. (3)

Question 6 continued





Question 6 continued

(Total for Question 6 is 12 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
2(a)	$H_0: \mu_{b(lue)} = \mu_{w(hite)} + 5 \quad H_1: \mu_{b(lue)} > \mu_{w(hite)} + 5$	B1	2.1
	$s.e. = \sqrt{\frac{2.6^2}{90} + \frac{2.4^2}{80}}$	M1	1.1b
	$z = \frac{39.5 - 33.7 - 5}{\sqrt{\frac{2.6^2}{90} + \frac{2.4^2}{80}}} = 2.085773...$ awrt 2.09	M1 A1	3.1b 1.1b
	CV = 2.3263 [or p -value = 0.01849...]	B1	1.1b
	Not significant, insufficient evidence to support Nissim's claim.	A1	2.2b
		(6)	
(b)	Use the t - test or (Since sample sizes are large,) use s^2 as an approximation to σ^2	B1	2.4
		(1)	
(c)	Since sample size is large, by the Central Limit Theorem, sample means will be (approximately) normally distributed ...so no effect as the calculations in part (a) can still be carried out.	B1	2.4
		dB1	3.2b
		(2)	
(9 marks)			
Notes			
(a)	B1: Both hypotheses (oe) correct with correct notation (if using μ_x and μ_y these must be defined). M1: Calculation of standard error M1: Standardising using normal distribution test statistic for difference of two means with known variance A1: awrt 2.09 B1: Correct critical value 2.3263 or better A1: Drawing a correct inference in context		
(b)	B1: Correct explanation		
(c)	B1: Understanding that the assumptions required for the hypothesis test, by CLT sample means follow a normal distribution dB1: (dep on previous B1) Correct evaluation		

Question	Scheme	Marks	AOs
2(a)	$z = 2.3263$	B1	1.1b
	98% CI for μ is: $128 \pm 2.3263 \times \frac{3.5}{\sqrt{80}}$	M1	2.1
	$= (127.0896..., 128.9103...) = \text{awrt } (127.09, 128.91)$	A1	1.1b
		(3)	
(b)	$H_0: \mu_{g(\text{reen})} - \mu_{r(\text{ed})} = 10 \quad H_1: \mu_{g(\text{reen})} - \mu_{r(\text{ed})} > 10$	B1	2.5
		(1)	
(c)	Normal model: $\bar{G} - \bar{R} \sim N\left(10, \frac{3.5^2}{80} + \frac{4^2}{n}\right)$	M1 A1	3.3 1.1b
	$z = \frac{(128 - 117) - 10}{\sqrt{\frac{3.5^2}{80} + \frac{4^2}{n}}} > 1.6449$	M1 B1	3.1b 1.1b
	$\frac{1}{1.6449} > \sqrt{\frac{3.5^2}{80} + \frac{4^2}{n}} \rightarrow \left(\frac{1}{1.6449}\right)^2 - \frac{12.25}{80} > \frac{16}{n} \rightarrow n > 73.9...$	M1	1.1b
	$n = 74$	A1	1.1b
		(6)	
(d)	<u>Sample sizes are large</u> so CLT means even though we do not know the distributions of G and R , the <u>means</u> $\bar{G}$ and $\bar{G} - \bar{R}$ will be (approximately) <u>normally distributed</u> .	B1	2.4
		(1)	
(e)	[Since $85 > 73.9...$] there is significant evidence to <u>support Camilo's belief/the mean weight of green apples is more than 10 grams greater than red apples</u> is supported.	B1ft	2.2b
		(1)	
(12 marks)			
Notes			
(a)	B1: 2.3263 or better		
	M1: for $128 \pm \sqrt{\frac{3.5^2}{80}} \times \text{"z value"}$. $ z > 1$ May be implied by a correct CI		
	A1: awrt 127.09 and awrt 128.91		
(b)	B1: Both hypotheses (oe) correct with correct notation (if using μ_x and μ_y these must be defined)		
(c)	M1: setting up normal model with correct mean or variance May be shown in standardisation with correct mean or variance and $ z > 1$		
	A1: correct variance and mean, may be seen in standardisation		
	M1: setting up an equation or inequality by standardising with their mean and their standard deviation and $ z > 1$		
	B1: Correct critical value 1.6449 or better, allow 1.64 or better		
	M1: rearranging to solve for n (may be implied by 73.9...)		
	A1: $n = 74$ cao		
(d)	B1: correct explanation mentioning <u>large sample size</u> and the <u>means</u> being <u>distributed normally</u>		
(e)	B1ft: drawing a correct inference in context, must be consistent with their value of n		

1(a)	Mean = 504	B1	1.1b
	1.96	B1	3.3
	$504 \pm \frac{5.4}{\sqrt{8}} \times "1.96"$	M1	2.1
	(500.258, 507.742)	A1	1.1b
		(4)	
(b)	505 is in the confidence interval therefore there is evidence that the machine is working properly	B1ft	2.2b
		(1)	
(c)	5% oe	B1	1.1b
		(1)	
(d)	s needs to be used instead of σ and a t-value instead of the z value	B1	3.3
	since the sample is small therefore you can't use the normal distribution	B1	3.5b
		(2)	
(8 marks)			
Notes:			
(a)	B1	504 may be seen in part(b)	
	B1	For realising a normal distribution must be used as a model and finding the correct value 1.96	
	M1	For $504 \pm \frac{5.4}{\sqrt{8}} \times "z \text{ value}"$. $ z > 1$ May be implied by a correct CI	
	A1	awrt 500.26 and 507.74 NB using t gives 500.29 and 507.71	
(b)	B1ft	Drawing a correct inference (ft) using their answer to part (a) and the 505 from the question. Reason must be given. Ignore incorrect non – contextual	
(c)	B1	5%	
(d)	B1	create new model by using s and t. Allow if state use CI $\mu \pm \frac{s}{\sqrt{n}} \times "t"$ or use s = 4.44 and t = 2.365	
	B1	For recognising that the sample is small	

Question	Scheme		Marks	AOs
3(a)	1.6449		B1	3.3
	$30.03 \pm \frac{0.868}{\sqrt{15}} \times "1.6449"$		M1	2.1
	(29.6613..., 30.3986...)		A1	1.1b
			(3)	
(b)	$\chi^2_{14}(0.1) \text{ CV} = 21.064$		B1	3.3
	(Reject H_0 if) $\frac{14S^2}{0.868^2} > 21.06$		M1	2.1
	Critical region is $S^2 > 1.133...$		A1	1.1b
			(3)	
(c)	Insufficient evidence that the machine is not working properly as 30.06 is within the confidence interval		M1	2.2b
	and 1.02^2 (1.0404) is not in the CR oe		A1ft	2.4
			(2)	
Total 8				
Notes				
(a)	B1	For realising a normal distribution must be used as a model and finding the correct value 1.6449 or better.		
	M1	For $30.03 \pm \frac{0.868}{\sqrt{15}} \times "z \text{ value}"$. May be implied by a correct CI		
	A1	awrt 29.7 and 30.4 from correct working (B0M1A1 is possible)		
(b)	B1	For realising a chi squared distribution must be used as a model and $\text{CV} = \text{awrt } 21.06$		
	M1	Correct method comparing $\frac{14S^2}{0.868^2}$ to $19 < \chi^2_{14} < 24$ (condone equals instead of $>$) Ignore any additional calculations outside of this range.		
	A1	Correct CR allow awrt 1.13 only		
(c)	M1	Dependent on an answer to part (a) or part (b) in order to draw an inference. Drawing a correct inference that there is insufficient evidence to suggest that the machine is not working properly following through one of their CR or CI with one correct comparison. Must mention the <u>machine</u> at least once. If they have either 30.06 outside their CI or 1.02^2 in their CR then allow an inference that the machine is not working properly for this mark following a correct comparison and no contradictory statements relating to that comparison. This mark can still be scored if there is a second comparison which is incorrect.		
	A1ft	Dependent on 30.06 in their confidence interval and 1.02^2 not in the CR. For drawing a correct inference following through their CR or CI with both correct comparisons and no contradictory statements. Must mention the <u>machine</u> at least once. Do not accept a comparison of 30.06 with just one of the limits for the CI.		



Question	Scheme	Marks	AOs
6	99% confidence interval for Var uses χ^2 values of 1.735 or 23.589	B1	3.3
	$\frac{9s^2}{1.735} = 0.2328$ or $\frac{9s^2}{23.589} = 0.01712$	M1	2.1
	$s^2 = \frac{0.2328 \times "1.735"}{9}$ or $\frac{0.01712 \times "23.589"}{9}$ [= 0.04487...]	dM1	1.1b
	$\bar{x} = 4.84$	B1	1.1b
	$H_0: \mu = 5$ $H_1: \mu < 5$	B1	2.5
	CV $t_9 = -1.833$	B1	1.1b
	$t = \pm \frac{"4.84" - 5}{\sqrt{"0.0449"/10}}$	M1	1.1b
	= awrt -2.39	A1	1.1b
	Stan's belief is supported or there is evidence that the mean diameter of the bolts is less than 5mm	A1ft	2.2b
		(9)	
(9 marks)			
Notes:			
B1: For realising a χ^2 distribution must be used as a model and finding a correct value			
M1: For realising the need to set $\frac{9s^2}{\text{"smallest } \chi^2"} = 0.2328$ or $\frac{9s^2}{\text{"largest } \chi^2"} = 0.01712$			
dM1: correct method used to solve equation to find s^2			
B1: awrt 4.84			
B1: Both hypotheses correct using the notation μ			
B1: ± 1.833			
M1: For us of correct formula ie $\pm \frac{"their 4.84" - 5}{\sqrt{"their 0.0449"/10}}$ If "4.84" not shown it must be correct here			
A1: -2.39			
A1ft: Drawing a correct inference following through their CV and test statistic (must have matching signs)			
NB if chi squared values not shown $s^2 = 0.045$ or 0.0449 award B0 M1M1 for awrt 0.04487 award B1 M1 A1			
Use of $2(2.5758) \frac{\sigma}{\sqrt{10}} = 0.21568$ gives $\sigma = \sqrt{0.0175}$ could get B0M0M0B1B1B1M0A0A0			
Unless continue to get $s^2 = \frac{10}{9} 0.0175 = 0.0194...$			
Use of $2(1.833) \frac{s}{\sqrt{10}} = 0.21568$ gives $s = 0.1860$ could get B0M0M0B1B1B1M1A0A1			

Qu	Scheme	Marks	AO
6(a)	From CI $\bar{x} = \frac{1.193+1.367}{2} = 1.28$	B1	1.1b
	$\text{or width} = 1.367 - 1.193 = 0.174$		
	$1.367 = "1.28" \pm 2.064 \times \frac{s}{\sqrt{25}} \text{ or } "0.174" = 2 \times 2.064 \times \frac{s}{\sqrt{25}}$	M1;A1	3.4 1.1b
	$\Rightarrow s = 0.210755...$	A1	1.1b
	$H_0: \sigma = 0.175 \quad H_1: \sigma \neq 0.175$	B1	2.5
	$\chi_{24}^2 = \frac{24s^2}{\sigma^2} =, \quad 34.8092...$ awrt 34.8	M1, A1	3.3 1.1b
	$\chi_{24}^2 (10\%)$ 2-tail CR is $\chi_{24}^2 < \underline{13.848}$ or $\chi_{24}^2 > \underline{36.415}$	B1	2.1
	34.8 is not significant so insufficient evidence that $\sigma \neq 0.175$	A1	2.2b
		(9)	
	(b) $"1.28" \pm z \times \frac{0.175}{\sqrt{25}}$	M1	3.3
	$= (1.211..., 1.349...) = \text{awrt } \underline{(1.21, 1.35)}$	$z = 1.96$ B1 A1	1.1b 1.1b
		(3)	
		(12 marks)	
Notes			
(a)	1 st B1 for finding mean from CI or calculation of width of CI 1 st M1 for using the given t model to form an equation in s . (Allow t for 2.064 where $2 < t < 3$) 1 st A1 for correct use of $t_{24} = 2.064$ 2 nd A1 for $s = 0.21$ or better 2 nd B1 for correct hypotheses in terms of σ . 2 nd M1 for selecting the appropriate model for this test 3 rd A1 for test statistic awrt 34.8 3 rd B1 for at least one correct critical value 4 th A1 for a correct conclusion confirming that assuming st. dev = 0.175 is OK		
(b)	M1 for use of correct formula with $1.6 < z < 2$ (ft $\bar{x}$ if found in (a)) B1 for $z = 1.96$ or better used A1 for an interval awrt (1.21, 1.35)		

Qu	Scheme	Marks	AO
7(a)	$X = L_1 + L_2 + L_3 \sim N\left(594, \sqrt{75}^2\right)$	M1	3.3
		A1	1.1b
	$Y = S_1 + \dots + S_8 \sim N\left(592, \sqrt{72}^2\right)$	A1	1.1b
	$P(X > Y) = P(D > 0)$ where $D \sim N\left(2, \sqrt{147}^2\right)$	M1	2.1
		A1ft	1.1b
	$= 0.56551\dots$ awrt 0.566	A1	3.4
		(6)	
	(b) $W = L - \frac{8}{3}S \Rightarrow W \sim N\left(\frac{2}{3}, 25 + \frac{64}{9} \times 9\right) = N\left(\frac{2}{3}, \sqrt{89}^2\right)$	M1	3.3
		M1,A1	2.1,1.1b
	$P(W > 0) = 0.528168\dots$ awrt 0.528	M1A1	3.4,1.1b
		(5)	
	(c) $F = L_1 + \dots + L_5 \sim N\left(990, \sqrt{125}^2\right)$	M1	3.1b
		A1	1.1b
	$P(F < 1000) = 0.814455\dots$ (o.e.)	A1	3.4
	$E(\text{cost of Rosa's plan}) = 430 \times "0.814\dots" + 400 \times (1 - "0.814\dots")$	M1	2.1
	$= \text{£ } 424.43$	A1	1.1b
	Buying 14 small panels cost $14 \times 30 = \text{£}420$	A1	3.2a
	So Rosa's plan is likely to be <u>more expensive</u>	(6)	
		(17 marks)	
Notes			
(a)	1 st M1	for an attempt at X or Y – expression or implied by one correct distribution	
	1 st A1	for a correct distribution for X <u>or</u> implied by $E(D) = 2$	
	2 nd A1	for a correct distribution for Y <u>or</u> implied by $\text{Var}(D) = 147$	
	2 nd M1	for a correct strategy – attempt $X - Y$ and $P(D > 0)$ statement	
	3 rd A1ft	for a correct distribution for D ft their X and Y	
	4 th A1	for awrt 0.566	
(b)	1 st M1	for attempt at a correct model (normal and mean)	
	2 nd M1	for correct expression for variance of their model provided of the form $L - kS$ <u>or</u> $kL - S$	
	1 st A1	for a fully correct distribution	
	3 rd M1	for a correct probability statement using their distribution	
	2 nd A1	for awrt 0.528	
(c)	1 st M1	for a correct start to solve the problem attempt at F and correct mean	
	1 st A1	for a correct distribution	
	2 nd A1	for using this model to find $P(F < 1000) = \text{awrt } 0.814$ or $P(F > 1000) = \text{awrt } 0.186$	
	2 nd M1	for a correct strategy to solve the problem i.e. attempt at expected cost ft their prob	
	3 rd A1	for awrt £424	
	4 th A1	for a correct conclusion must have comparison with £420 and reject Rosa's plan	
[(c) is an extended problem and a 3.1, 3.2 question]			

Qu	Scheme	Mark	AO
4. (a)	(Prob =) 0.68268... = awrt 0.683 or awrt 68.3% (o.e.)	B1	1.1b
		(1)	
(b)	$H_0 : \sigma^2 = 6^2 \quad H_1 : \sigma^2 < 6^2$ $s^2 = \frac{422\,862 - 25 \times \left(\frac{3250}{25}\right)^2}{24} = (15.0833...)$ Test statistic = $\frac{24 \times "15.0833..."}{6^2} =, 10.0555... \text{ awrt } \underline{\underline{10.1}}$ 5% critical value (lower tail) $\chi_{24}^2(5\%) = 13.848$ (Significant result so reject H_0) there is evidence that the <u>treatment works</u> oe or <u>variance of weights is lower</u> oe	B1 M1 M1,A1 B1 A1	2.5 2.1 1.1b,1.1b 3.4 2.2b
		(6)	
(c)	$\bar{x} \pm t_{24} \frac{s}{\sqrt{25}} =, 130 \pm 2.064 \times \frac{"3.88..."}{\sqrt{25}}$ = (128.4..., 131.6...) = awrt (128, 132)	M1A1ft A1	3.3, 3.4 1.1b
		(3)	
(d)(i)	Yes (or treatment worked) since 131 is in CI (and σ reduced)	B1	2.4
(ii)	Assume $\sigma = "3.9"$ (or better) and suitable μ e.g. 131 Proportion in range (79% ~ 88%)	M1 A1 (3)	3.5a 1.1b
(13 marks)			
Notes			
(a)	B1 for awrt 0.683 or awrt 68.3% o.e. Do not accept a fraction		
(b)	1 st B1 for correct hypotheses in terms of σ or σ^2 Do not accept in words. 1 st M1 for a correct expression for s or s^2 Implied by $\frac{181}{12}$ or 15.0833... 2 nd M1 for a correct expression for ts (ft their 15.0833...) May be implied by awrt 10.1 1 st A1 for awrt 10.1 or may see $\frac{181}{18}$ 2 nd B1 for the correct cv of 13.848 (accept 13.8 or better or allow 13.85) 2 nd A1 for a correct conclusion in context (independent of hypotheses but dep on M2 and their ts < cv where $12 < cv < 15$). Incorrect comparison or contradictory statement is A0		
(c)	M1 for an attempt at a correct formula with 130 or their 3.88... and $\sqrt{25}$ and $t > 2$ 1 st A1ft for a correct expression using $t = 2.064$ (or better) can ft their 3.88... = their s 2 nd A1 for awrt (128, 132) provided M1 clearly scored.		
(d)(i)	B1 dep on a CI in (c) which includes 131 and concluding variance lower in (b). For stating treatment was successful (o.e. condone e.g. yes) and mentioning 131 (or referred to as the mean) is inside CI. Do not allow reference to critical region for CI.		
(ii)	M1 for evidence of a suitable σ used (ft their s) and a value of μ from their CI. Must see their values for μ and σ to score. A1 dep on $\sigma =$ awrt 3.9 and $\mu = [\text{awrt } 128, \text{awrt } 132]$ for an answer in the range 0.79 to 0.88 inclusive (decimal or %)		
NB	$\sigma = 3.9, \mu = 128.4$ gives 79.5%; $\sigma = 3.9, \mu = 131.6$ gives 87.2% $\sigma = 3.9, \mu = 131$ gives 87.6%		



Question	Scheme	Marks	AOs
3(a)	$[\bar{x} = \frac{2268}{9} = 252] \quad s_A^2 = \frac{571700 - 9 \times 252^2}{8} [= 20.5]$	M1	2.1
	$\chi_{8,0.025}^2 = 17.535 \quad \chi_{8,0.975}^2 = 2.180$	B1	3.3
	$\frac{8 \times 20.5}{17.535} < \sigma_A^2 < \frac{8 \times 20.5}{2.180}$	M1	1.1b
	$9.3527... < \sigma_A^2 < 75.2293...$	A1	1.1b
		(4)	
(b)	$H_0 : \sigma_A^2 = \sigma_B^2, H_1 : \sigma_A^2 \neq \sigma_B^2$	B1	2.5
	$\frac{s_A^2}{s_B^2} = \frac{20.5}{12.7} = 1.61...$	M1	3.3
	$F_{8,10}(0.05) = 3.07$	B1	1.1b
	There is insufficient evidence to suggest that the variances are different.	A1	2.2b
		(4)	
(8 marks)			
Notes			
(a)	M1: correct expression for s_A^2 B1: for selecting χ^2 with $v = 8$. May indicated in notation or either of 17.535, 2.180 M1: setting up 95% CI with their sample variance, chi-squared values, and 8 A1: correct CI with awrt 9.35 and awrt 75.2		
(b)	B1: both hypotheses correct using σ or σ^2 M1: using the F -distribution as the model eg $\frac{s_A^2}{s_B^2}$ B1: awrt 3.07 A1: Drawing a correct inference following through their CV and value for Allow $\sigma_B^2 = \sigma_A^2$ Allow standard deviation instead of variance NB: Allow candidates to use their $\frac{s_B^2}{s_A^2}$ with $\frac{1}{3.35} =$ awrt 0.299 for the final M1B1A1 in (b)		

Question	Scheme	Marks	AOs
5(a)	The concentration of <u>air pollutant</u> (for each site) follows a normal distribution	B1	2.4
		(1)	
(b)	$F_{12,8}(0.01) = 5.67$ or $F_{8,12}(0.01) = 4.5(0)$	B1	3.3
	$\frac{6.39}{s_B^2} > '5.67'$ $\frac{s_B^2}{6.39} > '4.50'$	M1M1	1.1b 2.1
	$(0 <) s_B^2 < 1.12698... , s_B^2 > 28.755$	A1	1.1b
		(4)	
(c)	$\chi_{12,0.005}^2 = 28.3$ or $\chi_{12,0.995}^2 = 3.074$	M1	3.3
	$\frac{12 \times 6.39}{"28.3"} < \sigma_A^2 < \frac{12 \times 6.39}{"3.074"}$	M1	1.1b
	$2.7095... < \sigma_A^2 < 24.94469...$	A1	1.1b
		(3)	
(8 marks)			
Notes			
(a)	B1: Correct modelling assumption. Allow <u>air pollutant</u> samples are independent. Samples of <u>air pollutant</u> are normally distributed is B0		
(b)	B1: Setting up either F statistic M1: Either correct inequality (condone equation) M1: Understanding both correct inequalities are needed (condone equation) A1: Correct range of values with awrt 1.13 and awrt 28.8 $s_B^2 < 1.12698...$ and $s_B^2 > 28.755$ is A0		
(c)	M1: For realising a χ^2 distribution must be used as a model with at least one value correct . Allow $\chi_{12,0.01}^2 = 26.217$ or $\chi_{12,0.99}^2 = 3.571$ for this mark. M1: Setting up 99% CI A1: Correct CI with awrt 2.71 and awrt 24.9		

Question	Scheme	Marks	AOs
2(a)	1.96	B1	3.3
	$57.2 \pm 1.96 \times \frac{1.2}{\sqrt{120}}$	M1	2.1
	(56.985..., 57.414...) awrt (57.0, 57.4)	A1	1.1b
		(3)	
(b)(i) (ii)	$\bar{Y} - \bar{W} \sim N(\dots, \dots)$ $\left(0, \frac{1.2^2}{120} + \frac{0.9^2}{140} \right)$	M1 A1A1 (3)	3.3 1.1b1.1b
	Central limit theorem applies so we do not need to know the distributions of Y and W/ Allows us to assume that sample means ($\bar{Y}$ and $\bar{W}$) are normally distributed.	B1	2.4
		(1)	
(c)	$H_0: \mu_{y(\text{ellow})} = \mu_{w(\text{hite})} \quad H_1: \mu_{y(\text{ellow})} > \mu_{w(\text{hite})}$	B1	2.5
	$z = \frac{57.2 - 56.9}{\sqrt{\frac{1.2^2}{120} + \frac{0.9^2}{140}}} = 2.24949\dots$	M1 A1	3.1b 1.1b
	CV = 1.6449 [or p-value = 0.01224...] $p =$ awrt 0.01	B1	1.1b
	[Reject H_0] Significant evidence to support Jamie's claim/mean weight of the population of yellow tennis balls is greater than mean weight of the population of white tennis balls .	A1	2.2b
		(5)	
(12 marks)			
Notes			
(a)	B1: Understanding that sample mean can be modelled using a Normal distribution with $z = 1.96$ Allow 1.959...or better from calculator		
	M1: Setting up confidence interval $57.2 \pm z \times \frac{1.2}{\sqrt{120}}$		
	A1: awrt (57.0, 57.4) Must come from correct z value = 1.96 (or better).		
(b)(i)(ii)	M1: Translating context into a Normal distribution model		
	A1: Correct mean		
	A1: Correct variance (allow awrt 0.0178 or exact fraction $\frac{249}{14000}$)		
(c)	B1: Correct explanation about the distributions of Y and W or $\bar{Y}$ and $\bar{W}$		
	B1: Both hypotheses (oe) correct with correct notation (if using μ_x and μ_y these must be defined).		
	M1: Standardising using normal distribution test statistic for difference of two means with known variance		
	A1: awrt 2.25		
	B1: Correct critical value 1.6449 or better, [or $p = 0.01$ or better from correct working]		
	A1: Drawing a correct inference in context. Do not allow contradictory statements, e.g. 'Do not reject H_0 , so Jamie's claim is supported'		



Question	Scheme	Marks	AOs
6(a)	$H_0: \sigma_r^2 = \sigma_{mb}^2 \quad H_1: \sigma_r^2 \neq \sigma_{mb}^2$	B1	2.5
	$s_r^2 = \frac{1}{7} \left(21032 - 8 \times \left(\frac{410}{8} \right)^2 \right) = 2.7857...$	M1 A1	2.1 1.1b
	$s_{mb}^2 = \frac{1}{5} \left(14426 - 6 \times \left(\frac{294}{6} \right)^2 \right) = 4$	A1	1.1b
	$\frac{s_{mb}^2}{s_r^2} = 1.4358...$	M1	3.4
	CV $F_{5,7} = 7.46$	B1	1.1b
	$(1.4358... < 7.46 \text{ so there is})$ insufficient evidence to suggest the variances of the wingspans are different.	A1	2.2b
		(7)	
(b)	$\chi_{5,\alpha}^2 = \frac{5 \times '4'}{1.194} \quad \text{or} \quad \chi_{5,\alpha}^2 = \frac{5 \times '4'}{48.54}$	M1	3.1b
	$\chi_{5,\alpha}^2 = 16.75 \rightarrow \alpha = 0.005 \quad \text{or} \quad \chi_{5,\alpha}^2 = 0.412 \rightarrow \alpha = 0.995$	M1	1.1b
	$k = 99$	A1	1.1b
		(3)	
(c)	$s_p^2 = \frac{7 \times '2.7857...' + 5 \times '4'}{8 + 6 - 2} [= 3.29...]$	M1	3.1b
	$t_{12} = 2.179$	B1	1.1b
	$\left(\frac{410}{8} - \frac{294}{6} \right) \pm '2.179' \times \sqrt{'3.29'} \sqrt{\frac{1}{8} + \frac{1}{6}}$	M1	3.4
	(awrt 0.115, awrt 4.39)	A1 A1	1.1b 1.1b
		(5)	
(15 marks)			
Notes			
(a)	B1: For both hypotheses in terms of σ M1: Correct method for s_r^2 or s_{mb}^2 A1: awrt 2.79 (allow $\frac{39}{14}$) A1: 4 cao M1: Using correct model for test statistic with correct ratio B1: Correct CV A1ft: Drawing a correct inference in context using their CV and their test statistic (dep on both M marks)		
(b)	M1: Either correct attempt at $\chi_{5,\alpha}^2$ with $\nu = 5$ M1: Using tables to find appropriate probability A1: 99 cao		
(c)	M1: Correct expression for s_p^2 B1: Correct 95% t -value M1: CI in the correct form (may be implied by either A mark) A1: awrt 0.115 A1: awrt 4.39		
Question	Scheme	Marks	AOs



Qu	Scheme	Mark	AO
3(a)	$H_0 : \sigma_1^2 = \sigma_2^2 \quad H_1 : \sigma_1^2 \neq \sigma_2^2$	B1	2.5
	$F = \frac{143.8^2}{64.0^2} = (5.0484...)$	M1	3.4
	Critical value (5% one-tail) $F_{5,4} = 6.26$	B1	1.1b
	(Not significant) insufficient evidence of a difference in variances	A1	2.2b
		(4)	
(b)(i)	Need to assume variances are the same to carry out the test for means (the test showed that the variances could be assumed to be equal)	B1	2.4
(ii)	There is evidence it <u>increases mean yield</u> (but does not effect variance)	B1	2.2b
		(2)	
(6 marks)			
Notes			
(a)	B1 for both hypotheses correct in terms of σ or σ^2 May use labelling of e.g. 1 and 2 or e.g. “with” and “without”. Condone x and y. M1 for a correct numerical expression for F (must have squares but does not need to be evaluated) May be implied by awrt 5.05 Could be reciprocal. B1 for 6.26 (or better) If reciprocal (0.198...) is used as test statistic then need $\frac{1}{5.19}$ or awrt 0.193 A1 dep on a critical value such that awrt $5.05 < cv < 7$ for a correct test statistic awrt 5.05, and correct conclusion. Condone e.g. variances are the same. Do not accept conclusions which do not refer to the variances. If an incorrect comparison or statement is made A0		
(b)(i)	B1 for recognising that the test (for difference of means) requires the populations to have the same variance oe. Must indicate that it is a requirement (not just that you can assume the variances are equal). Must be in words. Allow explanations suggesting that it enables a pooled estimate for σ^2 or a better estimate of σ^2 to be found (so that a t -test can be carried out on the difference between means of two independent distributions with unknown variances) Do not allow references to the normal distribution being used.		
(ii)	B1 for a correct conclusion in context mentioning increase in mean yield . Does not require the comment about variances.		

Qu	Scheme	Marks	AO
1	$H_0: \sigma_a^2 = \sigma_b^2 \quad H_1: \sigma_a^2 > \sigma_b^2$	B1	2.5
	$s_a^2 = \frac{145\,496}{6} = [24\,249.33...] \quad s_b^2 = \frac{56\,364.4}{9} = [6262.711...]$	M1	1.1b
	$F_{6,9} = \frac{"24\,249.33..."}{"6262.711..."}$	M1	2.1
	$= 3.872...$	A1	1.1b
	$F_{6,9}(5\% \text{ one-tail}) \text{ c.v.} = 3.37$	B1	1.1b
	Significant, there is evidence to support Gina's belief	A1	2.2b
		(6)	
		(6 marks)	
Notes			
1 st B1	for both hypotheses in terms of σ .		
1 st M1	for at least one of the s^2 calculations correctly attempted NB $s_a = 155.72...$ accept awrt 156 and $s_b = 79.137...$ accept awrt 79.1		
2 nd M1	for a correct calculation of the test statistic (ft their s^2)		
1 st A1	for awrt 3.87		
2 nd B1	for correct cv awrt 3.37		
2 nd A1	for a correct conclusion mentioning Gina's belief (o.e.)		

Qu	Scheme	Marks	Grade	AO
2(a)	$\bar{x} = 444$	M1	Low	2.1
	$s_x^2 = \frac{1577\,314 - 8 \times 444^2}{7} = \frac{226}{7} = 32.2857..$	A1	Low	1.1b
	$t_7(5\%)$ 2-tail cv = 2.365	B1	Low	1.1b
	95% CI for μ is: $444 \pm 2.365 \times \sqrt{\frac{32.2857...}{8}}$	M1	Med	2.1
	$= (439.248..., 448.75...)$ = awrt (439, 449)	A1	Med	1.1b
(b)	440 is in CI so the average contents statement is OK	B1	High	2.2b
		(5)		
		(1)		
		(6 marks)		
Notes				
(a)	1 st M1 for finding mean and attempting s^2 1 st A1 for correct mean and s^2 (accept awrt 3sf) B1 for a correct cv of 2.365 or better 2 nd M1 for use of correct formula, ft their mean, s_x and cv for t (use of 1.96 is M0) 2 nd A1 for awrt (439,449)			
(b)	1 st B1 for correct statement about 440 and interval and conclusion			

Question	Scheme	Marks	AOs
6(a)	The samples are not independent	B1	3.5b
		(1)	
(b)	They should consider the birth weight, gender, or whether or not the lambs are premature. oe	B1	2.4
		B1	2.4
		(2)	
(c)	Need the assumption that the underlying distribution of the difference between the weight gains must be normally distributed .	B1	2.4
		(1)	
(d)	Difference -0.9 0.5 0.8 0.2 0.8 0.3 0.1 -0.1	M1	3.1b
	$\bar{w} = 0.2125$ $s^2 = 0.3041...$ ($s = 0.551...$)	M1	1.1b
	Confidence interval : $0.2125 \pm t \times \sqrt{\frac{0.3041}{8}}$	M1	2.1
	$0.2125 \pm 2.998 \times \sqrt{\frac{0.3041}{8}}$	A1ft	1.1b
	$= (-0.37202..., 0.79702...) \text{ oe}$	A1	1.1b
		(5)	
(e)	$H_0 : \mu_w = 0.2$ $H_1 : \mu_w > 0.2$	B1	2.5
	200g = 0.2 kg is in the interval	M1	2.1
	There is no evidence that μ_w is greater than 0.2 oe	A1ft	2.2b
		(3)	
Total 12			
Notes			
(a)	B1	The idea that samples are not independent. Condone other irrelevant comments provided they do not contradict this.	
(b)	B1	For one suitable comment on twins being identical relating to selecting the sample. Condone start weight. Do not accept age / diets of the lambs	
	B1	For a second suitable comment.	
(c)	B1	Need the emboldened words.	
(d)	M1	attempting differences (at least 4 correct) implied by awrt 0.304 or 0.551 but not 0.2125	
	M1	attempt to find $\bar{w}$ and s or s^2 for their differences implied by 0.2125 and either awrt 0.304 or 0.551	
	M1	For using the correct formula their $\bar{w} \pm t \times \sqrt{\frac{\text{their } s^2}{8}}$ where $ t > 2$ all values need to be substituted in.	
	A1ft	for their $\bar{w} \pm \text{awrt } 2.998 \times \sqrt{\frac{\text{their } s^2}{8}}$ all values need to be substituted in	
	A1	dependent on all previous method marks (awrt -0.372, awrt 0.797)	
	SC	If they have carried out a CI for two independent samples allow 3 rd M for using their difference of their means and a pooled variance and A1ft using correct formula with awrt 2.624	
(e)	B1	For both hypotheses correct in terms of μ or μ_w Condone 200	
	M1	For changing 200 g to 0.2 kg (ignore units for this mark) and comparing to their CI	
	A1ft	Independent of hypotheses. Drawing a correct inference following through on their CI provided 0.2 is within their confidence interval, with no contradictory statements. Does not need to be in context. Accept “insufficient evidence to support the (researcher’s) belief”.	

