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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Complex Numbers (FP1.1)

International A Level Mathematics · FP1: Further Pure Mathematics 1 (WFM01)

20 questions · 161 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

20 questions · 161 marks · 1 sub-topic

SUB-TOPIC FP1.1a

COMPLEX NUMBER ARITHMETIC & EQUATIONS

Sub-topic FP1.1a · 20 questions · 161 marks · lowest tariff first



Question 2 continued

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Q2

(Total 5 marks)



1. $z_1 = 3 + 3i$ $z_2 = p + qi$ $p, q \in \mathbb{R}$

Given that $|z_1 z_2| = 15\sqrt{2}$

(a) determine $|z_2|$ (2)

Given also that $p = -4$

(b) determine the possible values of q (2)

(c) Show z_1 and the possible positions for z_2 on the same Argand diagram. (2)

Question 1 continued

Lined area for writing the answer to Question 1 continued.

Question 1 continued

Lined area for writing the answer to Question 1 continued.



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)



9. Given that

$$\frac{3z-1}{2} = \frac{\lambda+5i}{\lambda-4i}$$

where λ is a real constant,

- (a) determine z , giving your answer in the form $x + yi$, where x and y are real and in terms of λ .

(4)

Given also that $\arg z = \frac{\pi}{4}$

- (b) find the possible values of λ .

(2)



Question 9 continued

(Total for Question 9 is 6 marks)



2. The complex numbers z_1 , z_2 and z_3 are given by

$$z_1 = 2 - i \quad z_2 = p - i \quad z_3 = p + i$$

where p is a real number.

(a) Find $\frac{z_2 z_3}{z_1}$ in the form $a + bi$ where a and b are real. Give your answer in its simplest form in terms of p .

(3)

Given that $\left| \frac{z_2 z_3}{z_1} \right| = 2\sqrt{5}$

(b) find the possible values of p .

(4)

Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total 7 marks)

Q2



Question 5 continued

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Question 5 continued

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Question 5 continued

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Q5

(Total 7 marks)



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4.

$$f(z) = 2z^3 - z^2 + az + b$$

where a and b are integers.

The complex number $-1 - 3i$ is a root of the equation $f(z) = 0$

- (a) Write down another complex root of this equation.

(1)

- (b) Determine the value of a and the value of b .

(4)

- (c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

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Question 4 continued

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Question 4 continued

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(Total 7 marks)

Q4



2.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that $x = 2 + 3i$ is a root of the equation

$$2x^4 - 8x^3 + 29x^2 - 12x + 39 = 0$$

- (a) write down another complex root of this equation.

(1)

- (b) Use algebra to determine the other 2 roots of the equation.

(4)

- (c) Show all 4 roots on a single Argand diagram.

(2)

Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



Question 2 continued

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Question 2 continued

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Q2

(Total 8 marks)



Question 4 continued

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Question 4 continued

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Q4

4.

$$f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$$

where A and B are constants.

The complex number $5 - i$ is a root of the equation $f(z) = 0$

- (a) Write down another complex root of this equation.

(1)

- (b) Solve the equation $f(z) = 0$ completely.

(5)

- (c) Determine the value of A and the value of B .

(2)



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)



$$f(z) = 6z^3 + Az^2 + Bz + C$$

Given that $z = \frac{2}{3} + \frac{\sqrt{17}}{3}i$ is a root of the equation $f(z) = 0$

(1)

(5)

(2)



Question 4 continued

Lined area for writing the answer to Question 4 continued.





Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)



Question 5 continued

Lined area for writing the answer to Question 5 continued.





Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 8 marks)



$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

The complex numbers $3 + i$ and $-1 - 2i$ are roots of the equation $f(z) = 0$

- (a) Write down the other roots of this equation. (2)
- (b) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram. (2)
- (c) Determine the values of a , b , c and d . (5)



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Q3





Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 9 marks)



Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 9 marks)

Question 3 continued



Question 3 continued

(Total for Question 3 is 10 marks)





Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)

Question 7 continued

Lined area for writing the answer to Question 7 continued.





Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 10 marks)



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 11 marks)

Q6



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



A1: Correct quadratic factor. May be implied if expansion to a quartic achieves the correct answer without intermediate factorisation shown.		
A1: Correct expression		
Question	Scheme	Marks
9.(a)	$\frac{3z-1}{2} = \frac{\lambda+5i}{\lambda-4i} \times \frac{\lambda+4i}{\lambda+4i}$	M1
	$= \frac{\lambda^2+9\lambda i-20}{\lambda^2+16}$	M1
	$\frac{3z-1}{2} = \frac{\lambda^2+9\lambda i-20}{\lambda^2+16} \Rightarrow z = \frac{2\left(\frac{\lambda^2+9\lambda i-20}{\lambda^2+16}\right)+1}{3}$	ddM1
	$= \frac{\lambda^2-8}{\lambda^2+16} + \frac{6\lambda}{\lambda^2+16}i$	A1
		(4)
(a) Way 2	$\frac{3z-1}{2} = \frac{\lambda+5i}{\lambda-4i} \Rightarrow 3z = \frac{2\lambda+10i}{\lambda-4i} + 1 = \frac{3\lambda+6i}{\lambda-4i}$ $\Rightarrow 3z = \frac{3\lambda+6i}{\lambda-4i} \times \frac{\lambda+4i}{\lambda+4i} \text{ or } z = \frac{\lambda+2i}{\lambda-4i} \times \frac{\lambda+4i}{\lambda+4i}$	M1
	$3z = \frac{3\lambda^2+18\lambda i-24}{\lambda^2+16} \text{ or } z = \frac{\lambda^2+6\lambda i-8}{\lambda^2+16}$	M1
	$3z = \frac{3\lambda^2+18\lambda i-24}{\lambda^2+16} \Rightarrow z = \dots$	ddM1
	$= \frac{\lambda^2-8}{\lambda^2+16} + \frac{6\lambda}{\lambda^2+16}i$	A1
(a) Way 3	$\frac{3z-1}{2} = \frac{\lambda+5i}{\lambda-4i} \Rightarrow (3x+3yi-1)(\lambda-4i) = 2\lambda+10i$ $\Rightarrow 3\lambda x - \lambda + 12y + (4+3\lambda y - 12x)i = 2\lambda+10i$	M1
	$\Rightarrow 3\lambda x + 12y = 3\lambda, \quad 3\lambda y - 12x = 6$	M1
	$\Rightarrow x = \dots, y = \dots$	ddM1
	$z = \frac{\lambda^2-8}{\lambda^2+16} + \frac{6\lambda}{\lambda^2+16}i$	A1
(b)	$\arg z = \frac{\pi}{4} \Rightarrow \operatorname{Re} z = \operatorname{Im} z (>0) \Rightarrow \lambda^2 - 6\lambda - 8 = 0 \Rightarrow \lambda = \dots \text{ or }$ $\arg z = \frac{\pi}{4} \Rightarrow \frac{6\lambda}{\lambda^2-8} = \tan \frac{\pi}{4} = 1 \Rightarrow \lambda^2 - 6\lambda - 8 = 0 \Rightarrow \lambda = \dots$	M1
	(Also need $\operatorname{Re}(z), \operatorname{Im}(z) > 0$, so $\lambda > 0$) $\lambda = 3 + \sqrt{17}$	A1



		(2)
(Total 6 marks)		
Notes		
(a) M1: Multiplies rhs by $\frac{\lambda + 4i}{\lambda + 4i}$ M1: Applies $i^2 = -1$ in both numerator and denominator and obtains a real number in the denominator. ddM1: Rearranges to $z = \dots$ A1: Correct and in the required form, but accept $\frac{\lambda^2 - 8 + 6\lambda i}{\lambda^2 + 16}$. Need not be fully simplified. Accept e.g. $\frac{3\lambda^2 - 24}{3\lambda^2 + 48} + \frac{18\lambda}{3\lambda^2 + 48}i$ (a) Way 2 M1: Rearranges to $3z = \dots$ (or $z = \dots$) and multiplies numerator and denominator by the complex conjugate of their denominator. M1: Applies $i^2 = -1$ in both numerator and denominator and obtains a real number in the denominator. ddM1: Rearranges to $z = \dots$ if not already done so. If rearranged to z initially M1ddM1 will be scored together. A1: As per main scheme. There may be variations on the rearrangement, but the key steps will remain the same. (a) Way 3 M1: Cross multiplies, applies $z = x + iy$, expands and applies $i^2 = -1$ to achieve Cartesian form terms. M1: Equates real and imaginary parts to form two equations with real coefficients. ddM1: Solves the equations simultaneously to find x and y in terms of λ. A1: As per main scheme. (b) M1: Sets the imaginary part of z equal to their real part of z, or divides these and sets equal to 1, and forms and solves the resulting quadratic in λ. (Need not be real roots for the M.) Watch for answers to (a) with a negative imaginary component that do not consider the minus sign, as these should score M0 as they have not set real and imaginary parts equal. A1: Correct exact answer only. The negative solution must have been rejected. Allow if both numerators were correct in (a) if there was a slip in the denominator only (e.g. $\lambda^2 + 4$ or $\lambda + 16$) or if they were only out by a positive scale factor (e.g. lost the 3).		

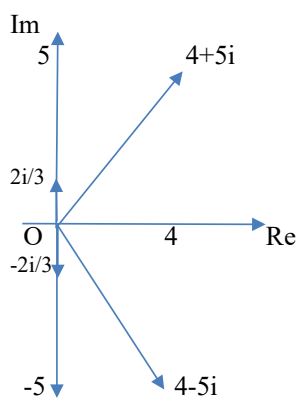
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Question Number	Scheme	Notes	Marks
1	$f(x) = x^3 + 4x - 6$		
1(i)(a)	$f(1) = -1$ $f(1.5) = 3.375 \left(= \frac{27}{8} \right)$	Attempts to evaluate at both end points. If substitution not seen accept $f(1) = -1$ or $f(1.5) = 3.375$ as evidence, with any value for the other end.	M1
	Sign change and $f(x)$ continuous therefore α is between $x = 1$ and $x = 1.5$	$f(1) = -1$ and $f(1.5) = 3.375$ <u>both correct</u> and mentions/indicates <u>sign change</u> , <u>continuous</u> and <u>conclusion</u> .	A1
			(2)
1(i)(b)	$f'(x) = 3x^2 + 4$	Correct derivative. Can be implied by a correct expression seen later such as $3(1.5)^2 + 4$	B1
	$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{3.375}{10.75}$	Attempt Newton-Raphson using the correct formula.	M1
	$x_2 = 1.186...$	Accurate first application either awrt 1.186, $\frac{51}{43}$ or a correct numerical expression.	A1
	$x_3 = \left(1.186... - \frac{0.412...}{8.220...} \right) = 1.1358...$		
	$\alpha \approx 1.136$	cao	A1
			(4)
1(ii)	$g(1.4) = 3.442116... \text{ or } g(1.5) = -3.601419...$	Evidence of at least one value correct to 3 d.p. or better. May be implied by correct answer if never seen.	B1
	$\frac{1.5 - \beta}{\beta - 1.4} = \frac{0 - g(1.5)}{g(1.4) - 0} \text{ or}$ $\frac{\beta - 1.4}{0 - g(1.4)} = \frac{1.5 - 1.4}{g(1.5) - g(1.4)} \text{ oe}$	A correct linear interpolation statement as shown oe (with correct signs). May omit the zeroes or use evaluated values (e.g 0.1 instead of $1.5 - 1.4$). Other forms are possible.	B1
	$\Rightarrow \beta \approx 1.448869...$	Evaluates β from an attempt at a linear interpolation statement, allow if signs are incorrect. $\beta \approx$ awrt 1.449	M1 A1
			(4)
			Total 10

Qn No	Scheme	Notes	Marks
2. (a)	$\frac{z_2 z_3}{z_1} = \frac{(p-i)(p+i)(2+i)}{(2-i)(2+i)}$	Multiply top and bottom by complex conjugate of their denominator. (The two M's may be scored if the given numbers are wrongly placed.)	M1
	$= \frac{(p^2+1)(2+i)}{5}$	Simplifies numerator with evidence that $i^2 = -1$ and denominator real. Accept any equivalent form in the numerator as long as there are not i^2 terms if expanded.	M1
	$= \frac{2(p^2+1)}{5} + \frac{(p^2+1)}{5}i$	Correct real +imaginary form with i factored out. Accept as single fraction with numerator in correct form. Accept 'a =' and 'b ='.	A1
			(3)
	ALT $\frac{z_2 z_3}{z_1} = \frac{(p-i)(p+i)}{(2-i)} = a+bi$ $p^2+1 = (a+bi)(2-i)$ $\left. \begin{aligned} 2a+b &= p^2+1 \\ 2b-a &= 0 \end{aligned} \right\}$	Cross multiplies by $2-i$ (or their denominator), expands and equates real and imaginary parts. (The two M's may be scored if the given numbers are wrongly placed.)	M1
	$\left. \begin{aligned} 2a+b &= p^2+1 \\ 2b-a &= 0 \end{aligned} \right\} \Rightarrow a=..., b=...$	Attempt to solve their equations.	M1
	$a+bi = \frac{2(p^2+1)}{5} + \frac{(p^2+1)}{5}i$	Correct real +imaginary form with i factored out. Accept as single fraction with numerator in correct form. Accept 'a =' and 'b ='.	A1
			(3)
	2(b) $\left \frac{z_2 z_3}{z_1} \right ^2 = \frac{4(p^2+1)^2}{25} + \frac{(p^2+1)^2}{25}$	Correct attempt at the modulus or modulus squared. Accept with their answers to part (a). Any erroneous i or i^2 is M0.	M1
	$\frac{4(p^2+1)^2}{25} + \frac{(p^2+1)^2}{25} = (2\sqrt{5})^2$	Their $\left \frac{z_2 z_3}{z_1} \right ^2 = (2\sqrt{5})^2$	dM1
2(b)	$(p^2+1)^2 = 100 \Rightarrow p = \pm 3$	Attempt to solve and achieves $p = ...$ (may be scored from use of $.. ^2 = 2\sqrt{5}$) $p = \pm 3$	M1 A1
			(4)
	ALT 1 $\left \frac{z_2 z_3}{z_1} \right = 2\sqrt{5} \Rightarrow z_2 z_3 = \sqrt{4+1} \times 2\sqrt{5}$	Cross multiplies and attempts $ z_1 $	M1
	$\Rightarrow z_2 ^2 = \sqrt{4+1} \times 2\sqrt{5} \Rightarrow p^2+1 = ...$	Attempts $ z_2 z_3 $ either directly or using $ z_2 z_2^* = z_2 ^2$ to get an equation in p.	dM1
	$(p^2+1) = 10 \Rightarrow p = \pm 3$	Attempt to solve and achieves $p = ...$ $p = \pm 3$	M1 A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \\ -2 & -2 & 0 \end{pmatrix}$	Attempt to multiply in the correct order with at least four correct elements. May be done as three separate calculations, so look for at least 4 correct values.	M1
	(1,-2), (3,-2) and (1,0)	Accept as individual column vectors but not as a single 2x3 matrix.	A1
			(2)
3(b)	Rotation	Accept rotate or turn oe.	B1
	270° (anticlockwise) about the origin	Accept -90° (anticlockwise) or 90° clockwise (must be stated) and (0,0) or O. <i>Assume anticlockwise unless otherwise stated.</i>	B1
			(2)
3(c)	$Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	One correct, both correct	B1,B1
			(2)
3(d)	$RQ = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Multiplication in correct order for their matrices and at least 1 row or 1 column correct.	M1
	$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Correct matrix	A1
			(2)
3(e)	Reflection	Correct type identified.	B1
	in (the line) $y = x$	Correct line of reflection specified, accepting equivalent forms (e.g. line at angle 45° (anticlockwise) to the (positive) x-axis).	B1
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
4(a)	$y = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2}$ $xy = 25 \Rightarrow y + x \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = -\frac{5}{t^2} \cdot \frac{1}{5}$	Attempts a derivative expression, such as $\frac{dy}{dx} = kx^{-2}$ or $x \frac{dy}{dx} = ky$ or $\frac{dy}{dx} = \frac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	M1
	$\frac{dy}{dx} = -25x^{-2}$ or $\frac{-y}{x}$ or $\frac{-1}{t^2}$	Correct derivative.	A1
	$y - \frac{5}{t} = -\frac{1}{t^2}(x - 5t)$	Uses $y - \frac{5}{t} = (\text{their gradient}) \times (x - 5t)$ or $y = (\text{their gradient})x + c$ using $x = 5t, y = \frac{5}{t}$ in an attempt to find c . Their gradient must be a function of t for marks to be awarded.	M1
	$t^2y + x = 10t$ *	cso	A1
			(4)
Alt 4(a)	Intersect when $(10t - t^2y)y = 25$	Substitutes given line equation into the curve equation to find intersections	M1
	$\Rightarrow t^2y^2 - 10ty + 25 = 0$	Correct equation	A1
	$\Rightarrow (ty - 5)^2 = 0 \Rightarrow y = \frac{5}{t}$ is single root.	Shows the equation has a single root.	M1
	Single root when $y = \frac{5}{t}$ means the line is the tangent to H at P .	All work correct, with correct conclusion.	A1
			(4)
4(b)	$t^2(-5) + 15 = 10t$	Substitute $(15, -5)$ into equation of tangent	M1
	$(t + 3)(t - 1) = 0 \Rightarrow t = ..$	Solves via any valid means	M1
	$\Rightarrow t = -3, t = 1$ (or one correct point)	Both values of t or one correct coordinate pair.	A1
	$\left(-15, -\frac{5}{3}\right)$ and $(5, 5)$	Both correct sets of coordinates.	A1
			(4)
ALT	$t_1^2y + x = 10t_1$ $t_2^2y + x = 10t_2$	Form equations with their t_1, t_2 and attempt to solve for x and y	M1
	$(x =) \frac{10t_1t_2}{(t_1 + t_2)} = 15, (y =) \frac{10}{(t_1 + t_2)} = -5$	Equates to coordinate and attempt to solve for t_1, t_2	M1
	$t_1 = -3, t_2 = 1$ (or one correct point)	Both values of t or one correct coordinate pair.	A1
	$\left(-15, -\frac{5}{3}\right)$ and $(5, 5)$	Both correct sets of coordinates.	A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
	$f(x) = (9x^2 + d)(x^2 - 8x + (10d + 1))$		
5(a)	$9x^2 + d = 0 \Rightarrow x = \pm \sqrt{-\frac{d}{9}} \text{ or } \pm \frac{i\sqrt{d}}{3}$	or exact equivalents	B1
	or $x = \frac{8 \pm \sqrt{64 - 4(10d + 1)}}{2}$ or $(x - 4)^2 - 16 + 10d + 1 = 0 \Rightarrow x = \dots$	Solve $x^2 - 8x + (10d + 1) = 0$ by formula or completing the square. Must have complete constant term.	M1
	$x = 4 + \sqrt{15 - 10d}$ and $x = 4 - \sqrt{15 - 10d}$	oe with discriminant simplified. Mark final answer, do not isw.	A1
			(3)
5(b)	$x = \pm \frac{2i}{3}$ or f.t. their roots	Correct roots, or f.t. their answer for the $9x^2 + d$	B1ft
	$x = 4 \pm 5i$ or f.t. their roots	Correct roots for the given quadratic, or f.t. their 3TQ	B1ft
	SC Award B1ftB0 if only one of each pair is given.		
			(2)
5(c)		Two roots on imaginary axis the same distance from O . Follow through their <i>imaginary</i> roots from (b). B0 if real roots found.	B1ft
		Their two <i>complex</i> roots with real and imaginary parts, one the conjugate of the other, so reflected in the real axis. Must be correct relative scale compared with the imaginary roots if the first B1 in (c) has been awarded (ie clearly further from O if correct, or f.t. their answers). But if first B0 has been given, ignore scales. Accept points or vectors. Complex numbers must be labelled in some way e.g. via scales or coordinates or vectors.	B1ft
			(2)
			Total 7

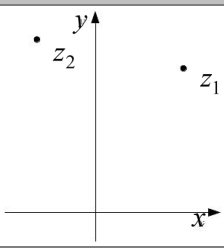
Question Number	Scheme	Notes	Marks
3	$2x^2 - 5x + 7 = 0$		
(a)	$\alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{7}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{3}{4}$	cso – must have scored the B1	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(\frac{5}{2}\right) = -\frac{85}{8}$	cso – must have scored the B1	A1
			(4)
(c)	$\text{Sum} = \frac{1}{\alpha^2 + \beta} + \frac{1}{\beta^2 + \alpha} = \frac{\alpha^2 + \beta + \beta^2 + \alpha}{(\alpha^2 + \beta)(\beta^2 + \alpha)}$ $= \frac{\alpha^2 + \beta^2 + \alpha + \beta}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{-\frac{3}{4} + \frac{5}{2}}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{14}{41} \right)$ Attempts sum – substitutes their into a correct numerator must but allow slips in the denominator as long as 4 terms are produced from the expansion.		M1
	$\text{Product} = \frac{1}{\alpha^2 + \beta} \times \frac{1}{\beta^2 + \alpha} = \frac{1}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{1}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{8}{41} \right)$ Attempts product – must be correct expansion of denominator with their values.		M1
	$x^2 - \frac{14}{41}x + \frac{8}{41} (= 0)$	Applies $x^2 - (\text{their sum})x + \text{their product} (= 0)$ Depends on at least one previous M awarded.	dM1
	$41x^2 - 14x + 8 = 0$	Allow any integer multiple. Must include “=0”	A1
			(4)
			Total 9

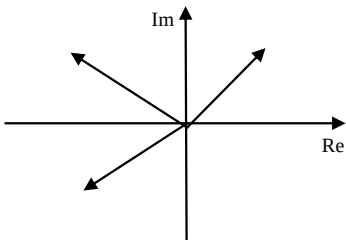
Question Number	Scheme	Notes	Marks
1	$\sum_{r=1}^n r^2 (r+2) = \sum_{r=1}^n r^3 + 2 \sum_{r=1}^n r^2$ or $\sum_{r=1}^n r^3 + \sum_{r=1}^n 2r^2$	Correct split with 2 summations. Could be implied by correct work. Condone missing or incorrect summation limits.	B1
	$= \frac{1}{4} n^2 (n+1)^2 + 2 \times \frac{1}{6} n(n+1)(2n+1)$	Attempts to use both standard results and obtains an expression of the form $pn^2(n+1)^2 + qn(n+1)(2n+1)$ $p, q \neq 0$ Could be implied by immediate expansion	M1
	$= \frac{1}{12} n(n+1)[3n(n+1) + 4(2n+1)]$ $= \frac{1}{12} n(n+1)(3n^2 + 11n + 4)$	dM1: Attempts factorisation to obtain $\frac{1}{12} n(n+1)(an^2 + bn + c)$ $a, b, c \neq 0$. Condone poor algebra. Could follow cubic or quartic. Allow a consistent $a = \dots, b = \dots, c = \dots$ if quadratic never seen simplified Requires previous M mark. A1: Correct expression or $a = 3, b = 11, c = 4$ Allow e.g., $\frac{1}{12} n(n+1)$ written as $\frac{n}{12}(n+1)$	dM1 A1
	Note: $n(n+1)(3n^2 + 11n + 4) = 3n^4 + 14n^3 + 15n^2 + 4n$		Total 4

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Question	Scheme	Marks
1.	$\det \mathbf{M} = 3x \times (2 - x) - (4x + 1) \times 7 = \dots$	M1
	$= -3x^2 - 22x - 7$ or $3x^2 + 22x + 7$	A1
	$-3x^2 - 22x - 7 = 0 \Rightarrow (-3x - 1)(x + 7) = 0 \Rightarrow x = \dots$	M1
	$-3x^2 - 22x - 7 > 0 \Rightarrow "-7" < x < "-\frac{1}{3}"$	M1
	So range is $-7 < x < -\frac{1}{3}$ or $(x \in) \left(-7, -\frac{1}{3}\right)$	A1
		(5)
(5 marks)		
Notes:		
M1: Attempts to expand the determinant of M. Allow with + between the 2 products. A1: Correct simplified quadratic with = or an inequality sign or neither M1: Attempts to solve their three term quadratic, any valid means (usual rules – see front pages). Correct answers seen implies correct method. Can be awarded even if the roots are complex. M1: Chooses the inside region for their roots, accept with strict or loose inequalities. A1: Correct answer. Accept $x > -7 \cap x < -\frac{1}{3}$		



Question	Scheme	Marks
2(a)		M1 A1
		(2)
(b) (i)	$ z_1 = \sqrt{3^2 + 5^2} = \sqrt{34}$	B1
		(1)
(ii)	$\frac{z_1}{z_2} = \frac{3+5i}{-2+6i} \times \frac{-2-6i}{-2-6i} = \dots$	M1
	$= \frac{-6-18i-10i+30}{40}$	A1
	$= \frac{3}{5} - \frac{7}{10}i$	A1
		(3)
(c)	$\arg \frac{z_1}{z_2} = \arctan \frac{-7/10}{3/5} = \arctan \frac{-7}{6} = \dots$ but allow $\arctan \frac{7}{6}$ for M1	M1
	$= -0.86$ or 5.42 (awrt)	A1
		(2)
(8 marks)		
Notes:		
(a) M1: Points in correct quadrants - z_1 in quadrant 1 and z_2 in quadrant 2. Must be clearly labelled either eg z_1 or $3 + 5i$ or correct numbers on the axes. (Accept with vector arrows.) A1: Correct diagram, z_1 in first quadrant further away from real axis than imaginary and z_2 in second quadrant, closer to imaginary axis but above z_1 OR correct nos on their axes (imag axis may include i), but not dashes w/o any indication of scale. Allow M1A0 for points unlabelled but diagram otherwise correct. (b)(i) B1: Correct modulus. Must be evaluated to $\sqrt{34}$ Question says “without using your calculator” so decimal answers can be ignored (isw) but exact answer must be seen somewhere. (ii) M1: Multiplies numerator and denominator by the conjugate of their denominator. A1: Correct unsimplified (or simplified) numerator, with the i^2 correctly dealt with, and correct denominator. A1: Correct answer. Allow as shown, $\frac{6}{10} - \frac{7}{10}i$, or $0.6 - 0.7i$.		

Question Number	Scheme	Notes	Marks
1(a)	$z_1 = 3 + 3i$ $z_2 = p + qi$ $p, q \in \mathbb{R}$		
	$ z_1 = \sqrt{3^2 + 3^2}$ $ z_1 z_2 = z_1 z_2 \Rightarrow z_2 \sqrt{18} = 15\sqrt{2} \Rightarrow z_2 = \dots$	Attempts $ z_1 $ using Pythagoras and uses $ z_1 z_2 = z_1 z_2 $ to find $ z_2 $	M1
	$ z_2 = 5$	Cao	A1
			(2)
ALT	$ z_1 z_2 = 15\sqrt{2}$ $ (3p - 3q) + i(3p + 3q) = 15\sqrt{2}$ $\sqrt{18p^2 + 18q^2} = 15\sqrt{2}$ $p^2 + q^2 = 25$ $ z_2 = \sqrt{p^2 + q^2} = 5$	Uses $ z_1 z_2 = z_1 z_2 $ to reach $p^2 + q^2 = \dots$	M1
			A1 (2)
(b)	$ z_2 = 5 \Rightarrow p^2 + q^2 = 25$ $\Rightarrow (-4)^2 + q^2 = 25 \Rightarrow q = \dots$	Uses $p^2 + q^2 = 25$ with $p = \pm 4$ leading to a value for q .	M1
	$q = \pm 3$	Both values. Must be clear $p = 4$ has not been used	A1
			(2)
(c)	 Points to be in the correct quadrants and either with correct numbers on the axes or labelled correctly	3 + 3i plotted correctly and labelled	B1
		Vectors/ lines not needed; point(s) alone are sufficient	
		A conjugate pair plotted correctly following through their q .	B1ft
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2	$f(x) = 10 - 2x - \frac{1}{2\sqrt{x}} - \frac{1}{x^3} \quad x > 0$		
(a)	$f(0.4) = -7.21\dots, f(0.5) = 0.292\dots$	Attempts both $f(0.4)$ and $f(0.5)$	M1
	Sign change (positive, negative) and $f(x)$ is continuous therefore (a root) α is between $x = 0.4$ and $x = 0.5$	Both $f(0.4) = \text{awrt } -7$ and $f(0.5) = \text{awrt } 0.3$, sign change and conclusion. Must mention continuity . Can have $f(0.4) \times f(0.5) < 0$ instead of “sign change”	A1
			(2)
(b)	$f'(x) = -2 + \frac{1}{4}x^{-\frac{3}{2}} + 3x^{-4}$	$x^n \rightarrow x^{n-1}$ in at least 1 term other than 10	M1
		2 of the 3 terms shown correct	A1
		All correct	A1
			(3)
(c)	$x_1 = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{0.29289321\dots}{46.70710678\dots}$	Correct application of Newton-Raphson	M1
	$= 0.494$	Correct value 3dp. A correct derivative must have been used	A1
			(2)
(d)	$\frac{4.9 - \beta}{ f(4.9) } = \frac{\beta - 4.8}{ f(4.8) } \Rightarrow \beta = \dots$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta = 4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 1	$\beta = \frac{a f(b) + b f(a) }{ f(a) + f(b) }$ $\beta = \frac{4.8 \times 0.0344 + 4.9 \times 0.1627}{0.0344 + 0.1627} = \dots$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta = 4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 2	Gradient $= \frac{-0.0344 - 0.1627}{4.9 - 4.8} = -1.971$ Equation of line: $y - 0.1627 = -1.971(x - 4.8)$ or $y = -1.971x + 9.6235$ Substitute $y = 0 \quad x = \dots$	Complete method for line equation followed by substitution to obtain a value for x	M1
	$\beta = 4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
2	$f(z) = z^3 - 13z^2 + 59z + p$		
(a)	$\begin{array}{r} [f(3)] = 3^3 - 13(3)^2 + 59(3) + p \\ \text{or e.g., } 27 - 117 + 177 + p \text{ or} \\ z^2 - 10z + 29 \\ z - 3 \overline{) z^3 - 13z^2 + 59z + p} \\ \underline{z^3 - 3z^2} \\ -10z^2 + 59z \\ \underline{-10z^2 + 30z} \\ 29z + p \\ \underline{29z - 87} \\ 0 \end{array}$	Attempts $f(3)$. Must see more than just $87 + p$ Allow one slip (e.g., a miscopy of one coefficient, or one incorrect value/sign if expression just given as $27 - 117 + 177 + p$) Alternatively long divides by $z - 3$ obtaining a 3TQ with two terms of $z^2 - 10z + 29$ correct. Could use synthetic division. An attempt at equating coefficients/factorising requires 2 correct values for the a, b and c of $az^2 + bz + c$	M1
	$f(3) = 0 \Rightarrow p = -87^*$	Obtains "$p = -87$" only with no errors but condone work in x "=0" must have been seen before $p = -87$ if $f(3)$ attempted but allow just $p = -87$ following a full and correct attempt via division/equating coefficients etc with no errors.	A1* (shown as B1 on ePen)
(2)			
(b)	Allow equivalent work in x. Allow use of a calculator to solve a quadratic. Solutions that just follow $z^3 - 13z^2 + 59z - 87 = 0$ score no marks. There are no marks if $z^2 - 10z + 29$ has clearly been produced by using $(z - (5 + 2i))(z - (5 - 2i))$		
	$(z^3 - 13z^2 + 59z - 87) \div (z - 3)$ $= \dots [z^2 - 10z + 29]$	M1: Uses $z \pm 3$ with $f(z)$ (not their $f(z)$) to obtain a 3TQ expression with evidence of any appropriate method including inspection (must be evidence of use of $z \pm 3$) or equating coefficients. Ignore any remainder if long division is used and may see $z^2 - 16z + 107$ ($r(-408)$) if $z + 3$ used. Must be seen or referred to in (b) A1: Correct quadratic	M1 A1
	$z = \frac{-(-10) \pm \sqrt{(-10)^2 - (4)(1)(29)}}{2(1)}$ or $(z - 5)^2 - 25 + 29 = 0 \Rightarrow z = 5 \pm \sqrt{-4}$	Solves their 3TQ arising from using $(z - 3)$ only as a factor (usual rules but allow if one correct root if calculator used on their quadratic) If a sum/product of roots method is used on their 3TQ (i.e., $2a = -(-10)$, $a^2 + b^2 = "29"$) it must be complete and condone only sign errors. Do not allow just $5 \pm 2i$ following an incorrect quadratic Requires previous M mark.	dM1
	$\left(z = \frac{10 \pm \sqrt{-16}}{2} = \right) 5 \pm 2i$	$5 \pm 2i$ or $5 + 2i$, $5 - 2i$ only. Not $5 \pm 2\sqrt{-1}$ Accept $\pm 2i + 5$	A1
(4)			

Question Number	Scheme	Notes	Marks
1	$\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix}$		
(a)	$\{\det \mathbf{P} =\} p(p-1) - (-3)(p+1)$ Award for any correct expression for $\det \mathbf{P}$. If expanded immediately allow one sign error only in one term of $p^2 - p + 3p + 3$ which can be implied by $p^2 \pm 4p + 3$ provided this has not come from $ad + bc$		M1
	$p^2 + 2p + 3$	Correct 3TQ. Terms in any order. Ignore “=0”	A1
			(2)
(b)	If (a) is incorrect, only the M mark is available in (b) and this mark requires working with a 3TQ [from (a) or reattempted] which $\neq 0$ for all real p		
	The M mark requires doing enough work so that only an explanation and conclusion is then needed, e.g., correct numerical expression (or value) for determinant, correct use of formula, correct completion of the square, correct vertex (stated or via graph or differentiation) The A mark always requires an appropriate explanation and non-singular/not singular/isn't/can't be singular or "has inverse". Do not accept just "shown" etc.		
	1. Discriminant: d or $b^2 - 4ac = 2^2 - 4(1)(3) \{= 4 - 12 = -8\}$ Negative or $d < 0$ or $b^2 < 4ac$, no (real) solution or $\det \mathbf{P} \neq 0$, so non-singular M1: Correct numerical expression (or value) for the discriminant. Allow comparison of b^2 with $4ac$ A1: Explanation and conclusion. 2. Using formula: $p = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)} \left\{ = \frac{-2 \pm \sqrt{-8}}{2} = -1 \pm \sqrt{2}i \right\}$ $\{\sqrt{-8}\}$ not real/possible or complex (accept imaginary) or no solution or $\det \mathbf{P} \neq 0$, so non-singular M1: Correct numerical expression for p A1: Explanation and conclusion. 3. Using completing the square: $p^2 + 2p + 3 = (p+1)^2 + 2$ $\neq 0$ or > 0 or $\square 2$ (not > 2) or min. value = 2 so non-singular Might see $(p+1)^2 = -2$ or $p+1 = \{\pm\}\sqrt{-2}$ or $\{\pm\}\sqrt{2}i$ or $p = -1\{\pm\}\sqrt{2}i$ followed by not real/possible/complex roots, so non-singular M1: $p^2 + 2p + 3 = (p+1)^2 + 2$ A1: Explanation and conclusion.		M1 A1
Cases continue overleaf			



Question Number	Scheme	Notes	Marks
2(e)	$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$		
	For either mark there must be no clear evidence of use of an incorrect method or matrix in this part e.g., using their matrix R for the reflection in part (c), i.e, sight of their $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ even if this is labelled as P. Do not allow confusion between their $A(-3, -4)$ and the given point $(4, 3)$ and there are no marks if a matrix or point has been miscopied/misread even if it is restated in this part. This also applies if there is an incorrect A in (d) but it is miscopied into the correct $(-3, -4)$. Both marks can be scored from minimal or no working.		
	$\left(\frac{4}{5}, \dots\right) \text{ or } (0.8, \dots) \text{ or } \left(\dots, -\frac{3}{5}\right) \text{ or } (\dots, -0.6) \text{ or } \left(-\frac{4}{5}, \frac{3}{5}\right)$ For one correct coordinate or ft non-zero coordinate on their $A(-3, -4)$ but must not use $(4, 3)$ even if that is their answer for A from part (d). Examples of follow throughs are: $(-4, -3) \rightarrow \left(\frac{3}{5}, -\frac{4}{5}\right)$ $(-4, 3) \rightarrow \left(-\frac{3}{5}, -\frac{4}{5}\right)$ $(4, -3) \rightarrow \left(\frac{3}{5}, \frac{4}{5}\right)$ $(-3, 4) \rightarrow \left(-\frac{4}{5}, -\frac{3}{5}\right)$ $(3, -4) \rightarrow \left(\frac{4}{5}, \frac{3}{5}\right)$ $(3, 4) \rightarrow \left(-\frac{4}{5}, \frac{3}{5}\right)$ May be stated as $x = \dots / y = \dots$ and condone if given without brackets or as a vector but do not condone a correct value seen within e.g., a 2×2 matrix		M1
	$\{B:\}\left(\frac{4}{5}, -\frac{3}{5}\right) \text{ or } (0.8, -0.6) \text{ only}$ Both correct coordinates. Not ft. May be stated as $x = \dots, y = \dots$ and condone if given without brackets or as a vector. Isw if necessary.		A1
	Examples of working: Do not allow confusion between their $A(-3, -4)$ and the given point $(4, 3)$ Using matrices (Note that the matrices can be applied in either order with these two transformations): $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} y \\ -x \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, -5x = -4 \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 5x \\ 5y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, -5x = -4 \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ Using inverse: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \frac{1}{25} \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & -\frac{1}{5} \\ \frac{1}{5} & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ Using the actual transformations: $(-3, -4) \Rightarrow \left(-\frac{3}{5}, -\frac{4}{5}\right) \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right) \text{ or } (-3, -4) \Rightarrow (4, -3) \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$		(2)
			Total 8

Question Number	Scheme	Notes	Marks
3(i)	There is no credit if only signs instead of values are used		
	$f(x) = x^2 + 5 - 8^{5x}$ $f(0) = 4, f(0.5) = -175.769..., [f(1) = -32762]$	Obtains a value for $f(0.5)$ and at least one of $f(0)$ and $f(1)$ with at least one correct: $f(0) = 4, f(0.5) = \text{awrt } -180, [f(1) = \text{awrt } -33000]$	M1
	$\{f(0) > 0, f(0.5) < 0 \text{ so root in } [0, 0.5]\} \Rightarrow f(0.25) = \dots$ Having obtained different signs of values for $f(0.5)$ and $f(0)$, obtains a value for $f(0.25)$. If by error the sign change occurs with $f(0.5)$ and $f(1)$, allow for attempting $f(0.75)$ $\{-2429.93... \approx -2400\}$. Accept using a full list/table of values e.g., $f(0) = 4, f(0.25) = -8.39184..., f(0.5) = -175.769..., f(0.75) = -2429.93..., f(1) = -32762$ $f(0) = 4, f(0.25) = \text{awrt } -8.4, f(0.5) = \text{awrt } -180, f(0.75) = \text{awrt } -2400, f(1) = \text{awrt } -33000$ Allow the values to be calculated in any order. Previous mark required.		dM1
	$f(0.25) = -8.39184... \Rightarrow (\alpha \in) [0, 0.25]$ $f(0.25) = \text{awrt } -8.4$ and all other values used correct to 2 sf (ignore values from further bisections e.g., $f(0.125), f(0.375), f(0.625), f(0.875)$) and concludes required interval is $[0, 0.25]$. Accept e.g., $(0, 0.25)$. Condone e.g., $0 \leq x \leq 0.25, 0 < \beta < 0.25$ but do not allow e.g., $(0.25, 0)$. Must give an interval so e.g., "It's between 0 & 0.25" is A0. Ignore further bisections provided correct interval has been seen. Allow 2 sf truncated values.		A1
	(3)		

Question Number	Scheme	Notes	Marks
3(ii)(a)	$g(x) = 3^{\sin x} - 3 \cos x$ $g(4) = 2.396\dots, g(5) = -0.502\dots$	Attempts both $g(4)$ and $g(5)$ with at least one correct (using radians): $g(4) = \text{awrt } 2.4$ (or 2.3 truncated), $g(5) = \text{awrt } -0.5$	M1
	Sign change oe and $g(x)$ is continuous therefore a root oe e.g., β is between $x = 4$ and $x = 5$	Both $g(4) = \text{awrt } 2.4$ (or 2.3 truncated) and $g(5) = \text{awrt } -0.5$, sign change (accept equivalents e.g., $g(4) > 0$ & $g(5) < 0$ or $g(4)g(5) < 0$ or "positive, negative"), continuous and conclusion all given. Be generous with attempt at "continuous" and condone e.g., "x is continuous". Minimum in bold. Condone a wrong interval following e.g., "There is a solution in..." May use f for g.	A1
(2)			
(ii)(b)	E.g. $\frac{\beta - 4}{2.396\dots} = \frac{5 - \beta}{-0.502\dots} \Rightarrow \beta = \dots$ $\frac{5 - \beta}{\beta - 4} = \frac{-0.502\dots}{2.396\dots} \Rightarrow \beta = \dots$ $\frac{\beta - 4}{5 - 4} = \frac{2.396\dots}{2.396\dots - 0.502\dots} \Rightarrow \beta = \dots$ $\frac{4(-0.502\dots) - 5(2.396\dots)}{-0.502\dots - 2.396\dots} = \dots$ or $m = \frac{-0.502\dots - 2.396\dots}{5 - 4} \{ = -2.8986\dots \}$ $\Rightarrow \text{e.g., } y - 2.396\dots = m(x - 4), y = 0$ $\Rightarrow x \text{ or } \beta = \dots$ If only unsubstituted " $\frac{ag(b) - bg(a)}{g(b) - g(a)}$ " oe is seen followed by value it must round to 4.83 unless $a, b, g(a) \& g(b)$ are identified. May use f for g.	Uses a correct interpolation equation/expression for their values (allow for any value <u>provided $g(4)$ positive, $g(5)$ negative</u> and condone clear miscopying) and finds a value for β or x etc. Accept any correct statement for their values followed by any attempt to solve. If not fully substituted, values for $g(4)$ & $g(5)$ may be seen separately here or in part (a). May use f for g. Alternatively finds the gradient and then the line joining the endpoints and sets $y = 0$ to get β . Straight line equation must be correct for their values but allow a correct unsimplified gradient seen which is miscalculated later and allow errors finding c from a consistent equation with a correctly substituted point. Ignore how the value is labelled (may be unlabelled or x or α used). If their variable denotes e.g., the distance between $(4, 0)$ and $(\beta, 0)$ then 4 must be added later. Implied by awrt 4.83 provided no evidence of incorrect equation but must not clearly be using the actual root of 4.8245... from solving by calculator. Note that failure to change the sign of $g(5)$ leads to 5.265...	M1
	= 4.827 (4 s.f.)	awrt 4.827 Accept answer only. Ignore further interpolations.	A1
	(2)		
			Total 7

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

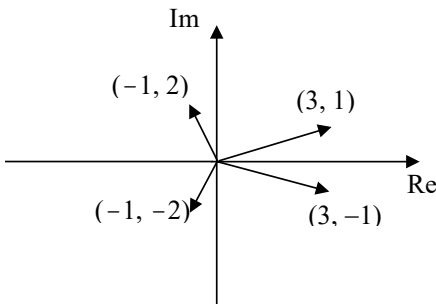
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Notes	Marks
1(a)	$f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \quad x > 0$		
	$f(1.4) = -0.435673...$ $f(1.5) = 0.598356...$	Attempts both $f(1.4)$ and $f(1.5)$	M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a root) α is between $x = 1.4$ and $x = 1.5$	Both $f(1.4) = \text{awrt } -0.4$ and $f(1.5) = \text{awrt } 0.6$, sign change and conclusion. For 'sign change' indication that $f(1.4) < 0$ and $f(1.5) > 0$ is sufficient. Also $f(1.4)f(1.5) < 0$ is sufficient. 'Therefore root' (without mention of the interval) is a sufficient conclusion. Mention of 'continuous' is not required.	A1
			(2)
(b)	$\left(f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} = x^3 - 10x^{-\frac{3}{2}} + 4x^{-1} \right)$		
	$f'(x) = 3x^2 + 15x^{-\frac{5}{2}} - 4x^{-2}$ (or equivalent, see below)	$x^n \rightarrow x^{n-1}$ for one term	M1
		2 correct terms simplified or unsimplified	A1
		All correct simplified or unsimplified	A1
			(3)
(c)	$(x_1) = 1.4 - \frac{f(1.4)}{f'(1.4)}$ $\left(= 1.4 - \frac{-0.43567...}{10.30720...} \right) = ...$	Correct application of N-R leading to an answer. <u>Values</u> of $f(1.4)$ and $f'(1.4)$ need not be seen before their final answer.	M1
	$= 1.442$	cao (must be corrected to 3 d.p.) isw if x_2 , etc. have been found, but the answer for 'one use of N-R' must be seen as 1.442 to score this mark.	A1
			(2)
			Total 7
(b)	Equivalent unsimplified versions are acceptable, e.g. (using quotient rule); $3x^2 - \frac{(5x^{\frac{3}{2}} - 4x^2) - 20x^{\frac{3}{2}} + 8x^2}{x^4}$	The 'two correct terms' still applies for the first A1. Here a 'term' would be, for example, the $x^{-\frac{5}{2}}$ terms in unsimplified form. Isw after a correct unsimplified form.	
(b)(c)	A common error in (b) is to have $+4x^{-2}$ instead of $-4x^{-2}$, giving 1.430 in (c). This, if otherwise correct, would score (b) 110 and (c) 10		

Question Number	Scheme	Notes	Marks
2	$5x^2 - 2x + 3 = 0$		
(a)	$\alpha + \beta = \frac{2}{5}, \quad \alpha\beta = \frac{3}{5}$	Both correct	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Uses a correct identity	M1
	$= \left(\frac{2}{5}\right)^2 - 2\left(\frac{3}{5}\right) = -\frac{26}{25}$	Correct value (allow -1.04), even after $\alpha + \beta = -\frac{2}{5}$ in (a)	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$	Uses a correct identity	M1
	$= \left(\frac{2}{5}\right)^3 - 3\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) = -\frac{82}{125}$	Correct value (allow -0.656)	A1
			(4)
(c)	Sum = $\alpha + \beta + \alpha^2 + \beta^2 = \frac{2}{5} - \frac{26}{25} \left(= -\frac{16}{25} \right)$	Attempts value of sum	M1
	Product = $\alpha\beta + \alpha^3 + \beta^3 + (\alpha\beta)^2 = \frac{3}{5} - \frac{82}{125} + \left(\frac{3}{5}\right)^2 \left(= \frac{38}{125} \right)$	Attempts value of product, using the <u>correct</u> expansion of $(\alpha + \beta^2)(\beta + \alpha^2)$	M1
	$x^2 + \frac{16}{25}x + \frac{38}{125} (= 0)$	Applies $x^2 - (\text{their sum})x + \text{their product}$ Accept unsimplified versions. The '=' is not required	M1
	$125x^2 + 80x + 38 = 0$	Allow any integer multiple. Must be a fully correct equation, including the '=' Not just $p = 125, q = 80, r = 38$	A1
			(4)
			Total 9

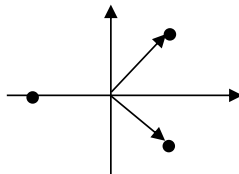
Question Number	Scheme	Notes	Marks
3	$f(z) = z^4 + az^3 + bz^2 + cz + d$		
(a)	$(z =) 3 - i$ or $(z =) -1 + 2i$		B1
	$(z =) 3 - i$ and $(z =) -1 + 2i$		B1
			(2)
(b)		$3 \pm i$ correctly plotted with vectors or dots or crosses etc. or $-1 \pm 2i$ correctly plotted with vectors or dots or crosses etc.	B1
		All 4 correct roots correctly plotted with scaling approximately correct (e.g. $(-1, 2)$ higher than $(3, 1)$, etc.) There should be approximate symmetry about the real axis, but be generous	B1
			(2)
(c)	$z = 3 \pm i \Rightarrow (z - (3 + i))(z - (3 - i)) = \dots$ or $z = -1 \pm 2i \Rightarrow (z - (-1 + 2i))(z - (-1 - 2i)) = \dots$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2 - 6z + 10$ or $z^2 + 2z + 5$	One correct quadratic	A1
	$z^2 - 6z + 10$ and $z^2 + 2z + 5$	Both correct	A1
	$(z^2 - 6z + 10)(z^2 + 2z + 5) = \dots$	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a = -4, b = 3, c = -10, d = 50$ or $f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1
			(5)
			Total 9

Question	Scheme	Marks
2. (a)	$f(z) = 2z^3 + pz^2 + qz - 41$	
	$(z =) 5 + 4i$	B1
		(1)
(b)	$z = 5 \pm 4i \Rightarrow (z - (5 + 4i))(z - (5 - 4i)) = \dots$ Or e.g. Sum of roots = 10, Product of roots = 41	M1
	$z^2 - 10z + 41$	A1
	$f(z) = (z^2 - 10z + 41)(2z + \dots)$	M1
	$\Rightarrow z = \frac{1}{2}, (5 \pm 4i)$	A1
		(4)
(c)	$f(z) = (z^2 - 10z + 41)(2z - 1) = \dots$	M1
	$p = -21, q = 92$	A1
		(2)
(d)	$\text{Area} = \frac{1}{2} \times 8 \times \left(5 - \frac{1}{2}\right)$	M1
	$= 18$	A1ft
		(2)
(Total 9 marks)		
Notes Mark (b) and (c) together		
(a) B1: Correct complex number (b) M1: Correct strategy to find the quadratic factor using the conjugate pair. A1: Correct quadratic factor. M1: Attempts to find the linear factor. Look for $2z + k$ where k is number (or allow if k is in terms of p as long as a value of p is also found at some stage). May arise from attempts at long division. A1: Correct real root (condone if labelled x). The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together). (c) M1: Multiplies out to obtain values for p and q. May have been found as part of a long division process in (b). A1: Correct values. May be seen embedded in the cubic. (d) M1: For $\frac{1}{2} \times 8 \times \left 5 - \frac{1}{2}\right$ where their real root is non-zero. A1ft: Correct area (follow through their non-zero real root).		

Alt 1 (b)	Product of complex roots = 41, Product of all roots = $\frac{\pm 41}{2}$	M1
	Product of complex roots = 41, Product of all roots = $\frac{41}{2}$	A1
	$z = \frac{\text{Product of roots of } f(z)}{\text{Product of complex roots}} = \frac{41}{2} \div 41$	M1
	$\Rightarrow z = \frac{1}{2}, (5 \pm 4i)$	A1
		(4)
(c)	$\frac{p}{2} = -\sum \alpha_i, \frac{q}{2} = \sum \alpha_i \alpha_j \Rightarrow p = \dots, q = \dots$	M1
	$p = -21, q = 92$	A1
		(2)
Notes		
Alt 1 (b) M1: Identifies the product of roots of $f(z)$ up to sign error, and the product of complex roots. A1: Correct products seen or implied. M1: Attempts to find the third root of $f(z)$. A1: Correct real root. The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together) (c) M1: Applies sum and pair sum properties, or multiplies out to obtain values for p and q. A1: Correct values. May be seen embedded in the cubic.		
Alt 2 (b)	$f(5 \pm 4i) = -230 \pm 472i + p(9 \pm 40i) + q(5 \pm 4i) - 41 = 0$ $\Rightarrow -271 + 9p + 5q = 0, 472 + 40p + 4q = 0$	M1 A1
	$\Rightarrow p = \dots, q = \dots \Rightarrow f(z) = 2z^3 + "p"z^2 + "q"z - 41 \Rightarrow z = \dots$	M1
	$\Rightarrow z = \frac{1}{2}, (5 \pm 4i)$	A1
		(4)
(c)	$\Rightarrow p = \dots, q = \dots$	M1
	$p = -21, q = 92$	A1
		(2)
Notes Alt (b)+(c) together		
Alt 2 (b) M1: Attempts factor theorem with one of the complex roots and equates real and imaginary terms to produce simultaneous equations. A1: Correct equations. M1: Uses their p and q from solving the simultaneous equations in $f(z)$ and solves the cubic (may just see roots). A1: Correct real root. The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together) (c) M1: Awarded before previous M. Solves their simultaneous equations to obtain values for p and q. A1: Correct values. May be seen embedded in the cubic.		

Question Number	Scheme	Notes	Marks
1(i)	$\mathbf{A} = \begin{pmatrix} 3k & 4k-1 \\ 2 & 6 \end{pmatrix}$		
(a)	$3k \times 6 - 2(4k-1) = 0 \Rightarrow k = \dots$ Forms $\det \mathbf{A} = 0$ and solves for k . The “= 0” can be implied by a solution for k . Award for $3k \times 6 - 2(4k-1) = 0 \Rightarrow k = \dots$ If LHS is only seen expanded 2 terms of $18k - 8k + 2$ must be correct (implied by $10k$) May use $ad = bc$ and condone $\det \mathbf{A} = bc - ad = 0$ but clear use of $ad + bc$ is M0		M1
	$(10k + 2 = 0 \Rightarrow k = -\frac{1}{5} \text{ or } -0.2$	A1: Correct value. Accept $-\frac{2}{10}$	A1
(2)			
(b)	$(\mathbf{A}^{-1} =) \frac{1}{10k+2} \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix} \text{ or } \begin{pmatrix} \frac{6}{10k+2} & \frac{1-4k}{10k+2} \\ \frac{-2}{10k+2} & \frac{3k}{10k+2} \end{pmatrix} \text{ or e.g., } \begin{pmatrix} \frac{3}{5k+1} & \frac{1-4k}{10k+2} \\ \frac{-1}{5k+1} & \frac{3k}{10k+2} \end{pmatrix}$ M1: for $\dots \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix}$ Ignore any multiplier and accept without one and condone if this matrix is labelled as $\mathbf{A}^{-1}$. Allow unsimplified e.g., $\dots \begin{pmatrix} 6 & -(4k-1) \\ -2 & 3k \end{pmatrix}$ Allow if determinant incorporated provided it is clear that the elements of $\text{Adj}(\mathbf{A})$ are correct A1ft: $\frac{1}{"10k+2"} \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix}$ Fully correct inverse ft their determinant in form $ak + b$ $a, b \neq 0$ and simplified but if determinant incorporated there is no requirement to write e.g., $\frac{6}{10k+2}$ as $\frac{3}{5k+1}$. Allow different brackets e.g., $[\dots]$, $\{\dots\}$ but $ \dots $ is M0 if followed by an attempt at $\det(\text{Adj}(\mathbf{A}))$. Allow if “ $\times$ ” is between fraction and matrix and allow fraction to appear on the right of the matrix. Isw when a correct answer is seen but this mark is not available if they substitute a value of k into the determinant and/or matrix.		M1 A1ft
(2)			
(ii)(a)	$p = q = -2 \text{ or } (\mathbf{B} =) \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	Both values identified or correct matrix (any or no bracket). Allow “Both are -2” or “-2, -2”	B1
(b)	$p = -1 \quad q = 1 \text{ or } (\mathbf{B} =) \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Both values identified or correct matrix (any or no bracket). Allow “-1, +1” (Mark in order presented). No trig expressions.	B1
(2)			
Total 6			



Question	Scheme	Marks	
	$f(z) = 4z^3 + pz^2 - 24z + 108, -3$ a root.		
3(a)	$f(-3) = 0 \Rightarrow 4(-3)^3 + p(-3)^2 - 24(-3) + 108 = 0 \Rightarrow p = \dots$	M1	
	$p = -8$	A1	
		(2)	
(b)	$4z^3 - 8z^2 - 24z + 108 = (z + 3)(4z^2 + \dots z + 36)$	M1	
	$= (z + 3)(4z^2 - 20z + 36)$	A1	
	$4z^2 - 20z + 36 = 0 \Rightarrow z = \frac{20 \pm \sqrt{400 - 4 \times 4 \times 36}}{8} = \dots$	dM1	
	Roots are $-3, \frac{5 \pm i\sqrt{11}}{2}$	A1	
		(4)	
(c)	e.g. Product of complex roots is $\frac{36}{4} = 9$, so modulus is $\sqrt{9}$ or	M1	
	Modulus is $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{\sqrt{11}}{2}\right)^2}$		
	Hence modulus is 3	A1	
		(2)	
(d)		Complex conjugate pair in correct quadrant for their roots	M1
		All three roots correctly positioned.	A1
			(2)
(10 marks)			

Notes:

Mark the question as a whole - do not be concerned part labelling.

(a)

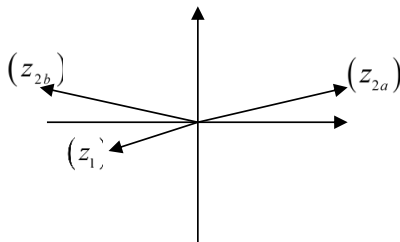
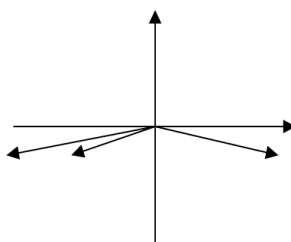
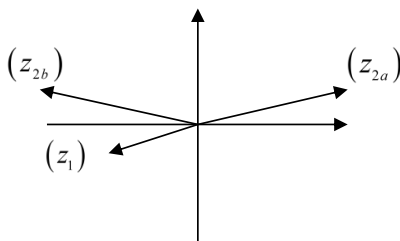
 M1: A complete method to find the value of p . Use of the factor theorem is most direct, look for setting $f(-3) = 0$ and solving for p . May attempt to factor out $(z + 3)$ and compare coefficients, e.g.

Question Number	Scheme	Notes	Marks
5	Solutions that rely entirely on solving the equation are generally unlikely to score but there may be attempts which include some of the work below which can receive appropriate credit.		
(a)	$\alpha + \beta = 6 \quad \alpha\beta = 3$	Correct sum and product. Could be implied. Allow $\frac{6}{1}$ and $\frac{3}{1}$	B1
	$(\alpha^2 + 1)(\beta^2 + 1) = \alpha^2\beta^2 + \alpha^2 + \beta^2 + 1$	Multiplies $(\alpha^2 + 1)(\beta^2 + 1)$ to obtain 3 or 4 terms with 3 correct. Do not condone $\alpha\beta^2$ for $(\alpha\beta)^2$ unless implied later	M1
	$= \alpha^2\beta^2 + (\alpha + \beta)^2 - 2\alpha\beta + 1$	Uses $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	$\{ = 3^2 + 6^2 - 2 \times 3 + 1 \}$ $= 40$	Correct answer from correct work. Use of e.g., $\alpha + \beta = -6$ is A0	A1
			(4)
(b)	Allow use of their $(\alpha^2 + 1)(\beta^2 + 1)$ which could be from (a) or a first or reattempt in (b). Numerator must be correct		
	$\frac{\alpha}{(\alpha^2 + 1)} + \frac{\beta}{(\beta^2 + 1)} = \frac{\alpha(\beta^2 + 1) + \beta(\alpha^2 + 1)}{(\alpha^2 + 1)(\beta^2 + 1)}$	Any correct expression with their $(\alpha^2 + 1)(\beta^2 + 1)$ for the new sum as a single fraction (or two fractions both with the common denominator)	B1
	$= \frac{\alpha\beta(\beta + \alpha) + (\alpha + \beta)}{(\alpha^2 + 1)(\beta^2 + 1)} = \frac{"3" \times "6" + "6"}{"40"} = \dots$	Uses a correct expression with their $(\alpha^2 + 1)(\beta^2 + 1)$ for the new sum to obtain a correct numerical expression with their denominator, $\alpha + \beta$ & $\alpha\beta$ and achieves a value.	M1
	$\frac{\alpha\beta}{(\alpha^2 + 1)(\beta^2 + 1)} = \frac{"3"}{"40"}$	Uses a correct expression with their $(\alpha^2 + 1)(\beta^2 + 1)$ for the new product to obtain a correct value with their denominator and $\alpha\beta$	M1
	new sum = $\frac{24}{40} \left\{ = \frac{3}{5} \right\}$ or new product = $\frac{3}{40}$	One value for new sum or new product correct. Any equivalent fractions. Not ft. Requires appropriate previous M mark.	A1
	$x^2 - \frac{24}{40}x + \frac{3}{40} \{ = 0 \}$	Correctly uses $x^2 - (\text{sum of roots})x + (\text{product of roots})$ or equivalent work with their new sum and product. Condone use of a different variable. Allow appropriate values for p, q and r	M1
	$40x^2 - 24x + 3 = 0$	Any correct equation with integer coefficients and " $= 0$ ". Condone use of a different variable. Allow e.g., $p = 40$, $q = -24$, $r = 3$. Requires all marks.	A1
			(6)
	Note that although $(\alpha^2 + 1)(\beta^2 + 1)$ may be attempted or reattempted in (b) there is no credit for work in (a) that is only seen in (b)		Total 10

Question Number	Scheme	Notes	Marks
6(a)	$ z_1 + z_2 = \{ 3 + 2i + 2 + 3i = 5 + 5i \} = \sqrt{5^2 + 5^2}$	Attempts the sum (allow one slip) and uses Pythagoras correctly	M1
	$\sqrt{50}$ or $5\sqrt{2}$	Either correct exact answer	A1
	Answer only is no marks but working can be minimal e.g., $ 5 + 5i = 5\sqrt{2}$		(2)
(b)	$\frac{z_2 z_3}{z_1} = \frac{(2 + 3i)(a + bi)}{(3 + 2i)} = \frac{(2 + 3i)(a + bi)}{(3 + 2i)} \times \frac{(3 - 2i)}{(3 - 2i)}$ or $\frac{z_2}{z_1} = \frac{2 + 3i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$ or $\frac{z_3}{z_1} = \frac{a + bi}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$	Substitutes complex numbers and correct multiplier to rationalise the denominator seen or implied. See note below Could use $\times \frac{-3 + 2i}{-3 + 2i}$	M1
	$(3 + 2i)(3 - 2i) = 13$	13 <u>obtained from</u> $(3 + 2i)(3 - 2i)$ Could be implied.	B1
	$\frac{z_2 z_3}{z_1} = \frac{12a - 5b}{13} + \frac{5a + 12b}{13}i$ or $\frac{1}{13}(12a - 5b) + \frac{i}{13}(5a + 12b)$ or $\frac{12}{13}a - \frac{5}{13}b + i\left(\frac{5}{13}a + \frac{12}{13}b\right)$ etc. Condone $\frac{(12a - 5b) + (5a + 12b)i}{13}$	dM1: Attempts to simplify the numerator and collects terms to obtain $pa + qb + rai + sbi$ with at least three of p, q, r and s non-zero. Requires previous M mark. A1: Correct answer in any form with a single "i". Correct bracketing where needed. Allow $x = \dots, y = \dots$	dM1 A1
	Note: The following marks are accessible if complex numbers are substituted in the wrong places: z_2 as denominator max 1010, z_3 as denominator max 1000		(4)
(c)	$\frac{12a - 5b}{13} = \frac{4}{13}, \frac{5a + 12b}{13} = \frac{58}{13} \Rightarrow a = \dots, b = \dots$	Equates their x to $\frac{4}{13}$ and their y to $\frac{58}{13}$ to obtain 2 linear equations in both a and b and solves to obtain values for both a and b .	M1
	No need to check values but must be some working between equations and values. „ $\frac{12a - 5b}{13} = \frac{4}{13}, \frac{5a + 12b}{13} = \frac{58}{13}$ $12a - 5b = 4, 5a + 12b = 58$ $a = 2, b = 4$ “ is M0A0 Values can immediately follow if equations are produced with coefficients of a or b of the same magnitude		
	$a = 2$ and $b = 4$	Correct values for a and b from correct equations with working.	A1
	SC: Allow access to both marks for the exact $a = -\frac{242}{169}$ and $b = \frac{716}{169}$ from using $w = \frac{z_1 z_3}{z_2} = \frac{12a + 5b}{13} + \frac{12b - 5a}{13}i$ There are no marks in (c) if z_3 was used as the denominator in (b) [leads to $a = b = 0$]		(2)
(d)	$\arctan\left(\frac{\frac{58}{13}}{\frac{4}{13}}\right) = \{1.5019\dots \text{ or } 86.05\dots^\circ\}$ or $\arctan\left(\frac{\frac{4}{13}}{\frac{58}{13}}\right) = \{0.068856\dots \text{ or } 3.945\dots^\circ\}$	Either correct arctan or $\tan^{-1}$ seen or implied by a correct 2sf value (awrt 1.5, 86, 0.069/0.068, 3.9) Could use equivalent trig. Note: $\tan \frac{58}{4} = -2.634$ or 0.258	M1
	1.502	1.502 only (not awrt) Mark final answer if 1.502 is followed by e.g., $\frac{\pi}{2} - 1.502 = 0.06880$	A1
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
5	$z_1 = r \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right), \quad z_1 z_2 = 15 \quad z_2 = 5 \quad \left\{ z_1 = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i = 3e^{i\frac{7\pi}{6}} \right\}$		
(a)	$r = 3$ Allow $z_1 = 3 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$	Correct value. No others and not ± 3 . Accept just "3" but $ z_1 = 3$ is insufficient unless later work implies $r = 3$	B1
(1)			
(b)	$z_2 = a + bi, z_1 + z_2 = c + 0i \Rightarrow r \sin \frac{7\pi}{6} + b = 0$ $\Rightarrow b = -3 \sin \frac{7\pi}{6} \quad \left\{ = -3 \times -\frac{1}{2} = \frac{3}{2} \right\}$	Obtains " b " = $\pm 3 \sin \frac{7\pi}{6}$ or $\pm 3 \times \pm \frac{1}{2}$ with their real "3" from (a) which may have been negative or $\pm$. The " b " may be implied by e.g., $z_2 = x + \frac{3}{2}i$ or later work	M1
	$a^2 + b^2 = 25$ or e.g., $\sqrt{a^2 + \left(\frac{3}{2}\right)^2} = 5$ Uses the modulus of z_2 to form a correct equation linking real and imaginary parts. Allow even if equation has no real solution. See SC below if $r = 5$ Using $ z_1 z_2 = 15$ leads to $\sqrt{\left(-\frac{3\sqrt{3}}{2}a + \frac{3}{2}b\right)^2 + \left(-\frac{3}{2}a - \frac{3\sqrt{3}}{2}b\right)^2} = 15$ oe Must see a correct equation ft their r and value for b if necessary.		M1
	$a = \sqrt{25 - \left(\frac{3}{2}\right)^2} \quad \left\{ = \dots \pm \frac{\sqrt{91}}{2} \right\}$ Allow $a = (\pm) \sqrt{25 - \left(3 \sin \frac{7\pi}{6}\right)^2}$	Substitutes b into a correct equation and finds at least one value or expression for a . Pythagoras must be used correctly and expression must be real.	M1
	SC: If $r = 5$ in (a) we will allow access to the M marks (and the M only in (c)): " b " = $\pm 5 \sin \frac{7\pi}{6}$ or $\pm 5 \times \pm \frac{1}{2} \Rightarrow a^2 + b^2 = 9$ or e.g., $\sqrt{a^2 + \left(\frac{5}{2}\right)^2} = 3 \Rightarrow a = \sqrt{9 - \left(\frac{5}{2}\right)^2} \quad \left\{ = \dots \pm \frac{\sqrt{11}}{2} \right\}$		
	$\{z_{2a}\} = \frac{\sqrt{91}}{2} + \frac{3}{2}i, \quad \{z_{2b}\} = -\frac{\sqrt{91}}{2} + \frac{3}{2}i$ or exact equivalents e.g., $\pm \sqrt{\frac{91}{4}} + \frac{3}{2}i$		
	A1: One correct answer: $(\pm \text{awrt } 4.8) + 1.5i$ and allow $-3 \sin \frac{7\pi}{6}$ for 1.5 (but final A0 and a must be a value for any marks)		A1
	A1: Both correct exact answers. Accept e.g., $\frac{\pm \sqrt{91} + 3i}{2}$ Allow both marks for e.g., $a = \pm \frac{\sqrt{91}}{2}, b = \frac{3}{2}$ (if $a + bi$ not seen there should be no subsequent evidence of a or b wrongly defined) or e.g., $z_2 = x + \frac{3}{2}i$ seen and $x = \pm \frac{\sqrt{91}}{2}$ No additional answers. Ignore any labelling of the answers. Isw if necessary.		A1
			(5)
	A possible variation is: $z_1 = 3 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right), z_2 = 5(\cos \theta + i \sin \theta), z_1 + z_2 = c + 0i \Rightarrow 3 \sin \frac{7\pi}{6} + 5 \sin \theta = 0$ (M1) $\Rightarrow \sin \theta = \frac{3}{10} \Rightarrow \left(\frac{3}{10}\right)^2 + \cos^2 \theta = 1$ (M1) $\Rightarrow \cos \theta = \pm \frac{\sqrt{91}}{10}$ (M1) $\Rightarrow z_2 = 5 \left(\pm \frac{\sqrt{91}}{10} + \frac{3}{10}i \right)$ or $\pm \frac{\sqrt{91}}{2} + \frac{3}{2}i$ (A1A1)		



Question Number	Scheme	Notes	Marks
5(c)	M1 Requires answers of form $(\pm p) + qi$ $p > 0, q > 0$ or $(\pm p) - qi$ $p > 0, q > 0$ from (b) which could be inexact. Allow if still have trig expressions, e.g., $\pm \sqrt{25 - \left(3\sin \frac{7\pi}{6}\right)^2} - 3\sin \frac{7\pi}{6}i$ <u>Ignore relative positions</u> and complex number/axis labelling. May use points/lines. More than 3 complex numbers indicated is M0.		M1
	For $(\pm p) + qi$: Three complex numbers: 1 in each of quadrant 1, Q2 and Q3 (not on axes). e.g., 	For $(\pm p) - qi$: Three complex numbers: 2 in Q3 and one in Q4 (not on axes). e.g., 	
	A1 Requires correct answers from (b) but condone inexact equivalents $(\pm \text{awrt } 4.8) + 1.5i$ and allow if still have trig expressions, e.g., $\pm \sqrt{25 - \left(3\sin \frac{7\pi}{6}\right)^2} - 3\sin \frac{7\pi}{6}i$ and ignore attempts to write z_2 in trig form AND $r = 3$ from (a) but and ignore miscalculation of z_1 $\left\{ \text{i.e., if } \neq -\frac{3\sqrt{3}}{2} - \frac{3}{2}i \right\}$ provided complex number is placed in Q3 and is closest to O		A1
		Correct sketch with complex number in Q3 clearly closest of the three to the origin, <u>otherwise ignore relative positions</u> . Condone e.g., asymmetry of z_{2a} and z_{2b} May use points/lines. If real and imaginary axes have been labelled the wrong way round then A0 but ignore all other labelling	
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$f(x) = 3x^2 + kx - 5 \Rightarrow \{\alpha\beta\} = -\frac{5}{3}$	Correct value for $\alpha\beta$. Accept $-1.\dot{6}$ Allow if e.g., "(a)" omitted and seen later	B1
	(1)		
(b)	$\alpha + \beta = 9\alpha\beta \Rightarrow -\frac{k}{3} = 9 \times -\frac{5}{3} \Rightarrow k = \dots$	Uses $\pm \frac{k}{3}$ for the sum of roots, sets equal to 9 times their product of roots and solves for k	M1
	$\{k\} = 45$	Correct value (no equivalents) from correct work . Answer only is M1A1	A1
	(2)		
(c)	$\{(\alpha + \beta)^3\} = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	Correct expansion seen. Terms may be uncollected e.g., $\alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3$	B1
	$\Rightarrow \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)^*$	Achieves given answer via an intermediate step following expansion that is not just collecting terms on RHS of $(\alpha + \beta)^3 = \dots$ No errors seen. Both sides must be seen but allow correct use of LHS=RHS=. Previous mark required.	dB1*
	Working backwards: $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 - 3\alpha^2\beta - 3\alpha\beta^2 = \alpha^3 + \beta^3$ B1: $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ or e.g., $\alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3$ dB1* : Correct proof with $-3\alpha\beta(\alpha + \beta)$ seen expanded. Both sides must be seen but allow correct use of LHS=RHS=. Previous mark required.		
	(2)		

Question Number	Scheme	Notes	Marks
6(d)	Note that the work for the first four marks might be seen embedded in a quadratic expression/equation. May also see use of $(x - \alpha^2 - \beta)(x - \alpha - \beta^2)$ If the work clearly relies on using the solutions to $3x^2 + 45x - 5 = 0$ $\left(\frac{-45 \pm \sqrt{2085}}{6} \text{ or } 0.1103... \text{ \& } -15.1103...\right)$ then allow a max of 101010		
	$\alpha^2 + \beta + \alpha + \beta^2 = \alpha + \beta^2 + \alpha^2 + \beta$ $= \alpha + \beta + (\alpha + \beta)^2 - 2\alpha\beta$	Evidence of a correct algebraic expression in terms of $\alpha + \beta$ and $\alpha\beta$ only for the new sum of roots. If not seen in its entirety it could be implied by e.g. a numerical expression/value.	1st M1 (Sum)
	$= -15 + (-15)^2 - 2\left(-\frac{5}{3}\right)$ $= -15 + 225 + \frac{10}{3} = \frac{640}{3}$	Correct value for new sum. If inexact allow awrt 213 from a correct calculation. Allow if exact values are recovered later. Must have used a correct expression and $\alpha\beta = -\frac{5}{3}$, $\alpha + \beta = -\frac{45}{3}$ but allow slip in algebra if it is clearly recovered by e.g., an appropriate calculation	A1
	$(\alpha^2 + \beta)(\alpha + \beta^2) = \alpha^3 + \beta^3 + \alpha\beta + (\alpha\beta)^2$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) + \alpha\beta + (\alpha\beta)^2$ Allow $\alpha^2\beta^2$ for $(\alpha\beta)^2$ but $\alpha\beta^2$ must be recovered by later work	Evidence of a correct algebraic expression for the new product of roots in terms of $\alpha + \beta$ and $\alpha\beta$ only. If not seen in its entirety it could be implied by e.g. a numerical expression/value.	2nd M1 (Product)
	$= (-15)^3 - 3\left(-\frac{5}{3}\right)(-15) - \frac{5}{3} + \left(-\frac{5}{3}\right)^2$ $= -3375 - 75 - \frac{5}{3} + \frac{25}{9} = -\frac{31040}{9}$	Correct value for new product. If inexact allow awrt -3450 from a correct calculation. Must have used a correct expression and $\alpha\beta = -\frac{5}{3}$, $\alpha + \beta = -\frac{45}{3}$ but allow slip in algebra if it is clearly recovered by e.g., an appropriate calculation	A1
	$x^2 - \left(\frac{640}{3}\right)x + \left(-\frac{31040}{9}\right) \{= 0\}$	Applies $x^2 - (\text{their newsum of roots})x + \text{their new product of roots}$ correctly (e.g., no missing "x") with non-zero values (which could be inexact). Allow without the "=0" for this mark. Not dependent. If just see e.g., $a = \dots$, $b = \dots$, $c = \dots$ then must see e.g., $ax^2 + bx + c$	M1
	$9x^2 - 1920x - 31040 = 0$	Correct equation as shown or an integer multiple. Could recover inexact values. Must include the "=0". Could use e.g., z consistently for x . Requires all previous marks. If just see e.g., $a = \dots$, $b = \dots$, $c = \dots$ then must see e.g., $ax^2 + bx + c = 0$	A1
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
7	$f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68, \quad z = 3 + 5i$ Condone $\times$ used consistently for z throughout		
(a)	3 - 5i	Correct second root	B1
(1)			
(b)	Either $(z - 3 - 5i)(z - 3 + 5i) = z^2 \pm 6z \pm m, \quad m \in \square$ or $\alpha_1 + \alpha_2 = 6, \alpha_1 \alpha_2 = M \Rightarrow z^2 \pm 6z \pm M, \quad M \in \square$ For completing a correct strategy to find a 3 term quadratic factor. May either attempt to expand with correct starting point achieving $z^2 \pm 6z \pm m, \quad m \in \square$, or attempts the sum and product of roots and reaches $z^2 \pm 6z \pm$ their (real) product		M1
	$z^2 - 6z + 34$	Correct factor. Accept answer only & "=0"	A1
(2)			
(c)	Alternatives for (c) are shown overleaf		
	$(z^2 - 6z + 34)(\text{"a"}z^2 + \text{"b"}z + \text{"c"}) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$		
	$34c = 68 \Rightarrow c = 2$		B1
	"c" = 2 seen or implied. Allow $c = -2$ from $z^2 - 6z - 34$ but no other follow throughs.		(ft on -34)
	$az^4 + (b - 6a)z^3 + (c - 6b + 34a)z^2 + (34b - 6c)z + 34c$ $\Rightarrow b - 6a = -36, \quad c - 6b + 34a = Q, \quad -6c + 34b = 192$ $\{a = P, \quad 34c = 68\}$ With $c = 2$ substituted: $\Rightarrow b - 6a = -36, \quad 2 - 6b + 34a = Q, \quad -12 + 34b = 192$	Expands $(z^2 - 6z + 34)(az^2 + bz + c)$ [which could be implied] and compares coefficients for at least two of the z^3, z^2 and z terms obtaining at least 2 equations (could be implied) with real coefficients (may include the variable e.g., $34bz - 12z = 192z$). Must use at least 2 terms from the expansion per equation. Their 3TQ must have real coeffs.	M1
	e.g., $\Rightarrow 34b - 12 = 192 \Rightarrow b = 6, \quad b - 6a = -36 \Rightarrow a = 7 \Rightarrow P = 7$ Solves sufficient equations of correct form to find a real non-zero value for P (allow "a") or Q . No need to check algebra and accept a value following equations. Note that only 2 equations from comparing z^3 and z coefficients are needed to find P . It is possible to find Q first although it is not common e.g. $b = 6$ in $Q = 2 - 6b + 34(\frac{b}{6} + 6) \Rightarrow Q = 204$. Previous mark required.		dm1
	e.g., $Q = c - b + 34a = 2 - 36 + 238 = 204$ Solves sufficient equations of correct form to find real non-zero values for both P (allow "a") and Q . No need to check algebra and accept values following equations. 2 previous marks required.		ddM1
	$P = 7$ and $Q = 204$ only Allow $a = 7$ if $P = a$ (only) seen	$P = 7$ (not "a") and $Q = 204$ No other answers. May be embedded in $f(z)$	A1
(5)			
(d)	$"7z^2 + 6z + 2" = 0$ (Allow with their other 3TQ factor - must have real coefficients) $\Rightarrow z = \frac{-6 \pm \sqrt{6^2 - 4 \times 7 \times 2}}{2 \times 7}$ or $\frac{-6 \pm \sqrt{-20}}{14}$ or $\frac{-6 \pm \sqrt{20}i}{14}$ or $z^2 + \frac{6}{7}z + \frac{2}{7} = \left(z + \frac{3}{7}\right)^2 - \frac{9}{49} + \frac{2}{7} = 0 \Rightarrow z = \dots \left\{ -\frac{3}{7} \pm \frac{\sqrt{-5}}{7} \right\}$ Either uses correct formula correctly or completes the square - usual rules - shown for solving their other three term quadratic factor (which must have complex roots). Do not accept factorisation or just writing down simplified roots from calculator. 1 root is sufficient. If forms equations e.g., $z_1 + z_2 = -\frac{6}{7}, \quad z_1 z_2 = \frac{2}{7}$ (allow sign errors only) a full algebraic method for obtaining 1 root must be seen.		M1
	$\frac{-3 \pm i\sqrt{5}}{7}$ or $-\frac{3 \pm i\sqrt{5}}{7}$ or $-\frac{3}{7} \pm \frac{\sqrt{5}}{7}i$	Correct simplified other roots from correct factor. Must see "i". Ignore the presence of $3 \pm 5i$ if also listed. Ignore labelling. Does not require 5/5 in (c)	A1
(2)			
			Total 10

Question Number	Scheme/Notes	Marks
7(c)	If long division is not completed/equations not seen allow access to the marks if the correct values for P and/or Q are deduced provided there is no clearly inappropriate work. For example it is possible to deduce that $b = 6$ as well as $c = 2$ and compare z^3 coeffs. from long division. If their quadratic factor is incorrect then equations must be seen for the 1st M1	
Alt 1a Full Long Division Use Alt 1b if they have a value for c	Allow for equivalent work using e.g., a multiplication grid $Pz^2 + (6P - 36)z + (Q + 2P - 216)$ $z^2 - 6z + 34 \overline{)Pz^4 - 36z^3 + Qz^2 + 192z + 68}$ $\underline{Pz^4 - 6Pz^3 + 34Pz^2}$ $(6P - 36)z^3 + (Q - 34P)z^2 + 192z$ $\underline{(6P - 36)z^3 - 6(6P - 36)z^2 + 34(6P - 36)z}$ $(Q + 2P - 216)z^2 + (1416 - 204P)z + 68$ $\underline{(Q + 2P - 216)z^2 - 6(Q + 2P - 216)z + 34(Q + 2P - 216)}$ $(120 - 192P + 6Q)z - 68P - 34Q + 7412$ $\Rightarrow 120 - 192P + 6Q = 0, -68P - 34Q + 7412 = 0$ B1: Attempts long division using a 3TQ with real coefficients and proceeds correctly to the first z^3 coefficient after subtraction (double underlined) - ignore subsequent errors. M1: Carries out sufficient long division and sets remainder = 0 (could be implied by equations if remainder not seen explicitly) to form a pair of linear simultaneous equations with real coefficients. Must have come from comparing coefficients oe for z terms and constant terms. Both equations must include P and Q .	B1 M1
	$\{32P - Q = 20, 2P + Q = 218\}$ $\Rightarrow P = \dots$ or $Q = \dots$	dM1
	$\Rightarrow P = \dots$ and $Q = \dots$	ddM1
	$P = 7$ and $Q = 204$ only	A1
		(5)
Alt 1b Long Division: using a value for c	$Pz^2 + (6P - 36)z + 2$ $z^2 - 6z + 34 \overline{)Pz^4 - 36z^3 + Qz^2 + 192z + 68}$ $\underline{Pz^4 - 6Pz^3 + 34Pz^2}$ $(6P - 36)z^3 + (Q - 34P)z^2 + 192z$ $\underline{(6P - 36)z^3 - 6(6P - 36)z^2 + 34(6P - 36)z}$ $(Q + 2P - 216)z^2 + (1416 - 204P)z + 68$ $\underline{ 2z^2 - 12z + 68}$ $(2P + Q - 218)z^2 - (204P - 1428)z$ $\Rightarrow 2P + Q - 218 = 0, -204P + 1428 = 0$ B1: $c = 2$ implied M1: Carries out sufficient long division and sets remainder = 0 (which could be implied) to form at least an equation with real coefficients in just P . Note that a remainder may not be seen explicitly. Must have come from comparing coefficients (oe) of z^2 terms and z terms. dM1: Solves an equation in P to find a real non-zero value for P or solves two equations (one in just P and one in both P and Q) to find a real non-zero value for Q . However, it is very unlikely that Q will be seen with no answer for P given. ddM1: Solves two equations to find real non-zero values for both P and Q . Must have had one equation in P only and one equation in both P and Q . A1: $P = 7$ and $Q = 204$ only. May be embedded in $f(z)$	

Question Number	Scheme/Notes		Marks
7(c) Alt 2 Substitution	$f(3 \pm 5i) = P(3 \pm 5i)^4 - 36(3 \pm 5i)^3 + Q(3 \pm 5i)^2 + 192(3 \pm 5i) + 68$ $= P(-644 \mp 960i) + Q(-16 \pm 30i) + 7772 \pm 600i$ $\text{or } -644P - 16Q + 7772 + (\mp 960P \pm 30Q \pm 600)i$ Correct six term expression for $f(3+5i)$ or $f(3-5i)$. May use calculator Implied by correct equations		B1
	$f(3 \pm 5i) = 0 \Rightarrow P(-644 \mp 960i) + Q(-16 \pm 30i) + 7772 \pm 600i = 0$ $\Rightarrow -644P - 16Q + 7772 = 0, \mp 960P \pm 30Q \pm 600 = 0$ Attempts $f(3+5i)$ or $f(3-5i)$ and sets equal to 0 and equates real and imaginary parts to form a pair of linear simultaneous equations with real coefficients. Both equations must have no imaginary terms and both must include P and Q. Ignore extra equations		M1
	$\{161P + 4Q = 1943, 32P - Q = 20\}$ $\Rightarrow P = \dots \text{ or } Q = \dots$	Solves the equations to find a real non-zero value for either P or Q . No requirement to check algebra and accept a value following equations. Previous mark required.	dM1
	$\Rightarrow P = \dots \text{ and } Q = \dots$	Solves the equations to find real non-zero values for both P and Q . No requirement to check algebra and accept values following equations. 2 previous marks required.	ddM1
	$P = 7 \text{ and } Q = 204 \text{ only}$	$P = 7 \text{ and } Q = 204. \text{ No other answers.}$ Maybe embedded in $f(z)$	A1
			(5)

Question Number	Scheme	Marks
2		
(a)	$\frac{3}{8} - \frac{\sqrt{71}}{8}i$	B1 (1)
(b)	$\left(x - \frac{3}{8} - \frac{\sqrt{71}}{8}i\right)\left(x - \frac{3}{8} + \frac{\sqrt{71}}{8}i\right) ((x-4)=0)$ $\left(x^2 - \frac{3}{4}x + \frac{5}{4}\right) ((x-4)=0)$ $x^3 - \frac{19}{4}x^2 + \frac{17}{4}x - 5 (=0)$ $4x^3 - 19x^2 + 17x - 20 (=0) \quad p=17, q=-20$	M1A1 dM1 A1 (4)
(a) B1 (b) M1 A1 dM1 A1	Correct answer only Attempt the multiplication of the 2 brackets with the complex terms. Allow $(x \pm \text{root})$ for the brackets. Allow “invisible” brackets. Correct quadratic obtained - may have multiplied by the 4 (or other constant factor) and this is fine. (Need not be fully simplified but must have real terms) Attempt to multiply their quadratic by $(x-4)$ or may divide their quadratic into the cubic or other full method leading to at least one of p or q . Correct values. Values of p and q need not be shown explicitly but may be seen in a cubic, provided the cubic starts $4x^3 - 19x^2$ (isw after a correct cubic)	[5]
	Note if a candidate uses a hybrid method, mark under main scheme unless an Alt scores more marks.	
Alt 1 (b)	$-\frac{q}{4} = 4 \times \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) \times \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) = \dots \rightarrow q = \dots$ or $\frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $\Rightarrow q = -16 \times \left(\frac{9}{64} + \frac{71}{64}\right) = -20$ or $p = 17$ E.g. $f(4) = 0 \Rightarrow 4(4)^3 - 19(4)^2 + 4p - 20 = 0 \Rightarrow p = \dots$ or $\frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $p = 17, q = -20$	M1 A1 dM1 A1 (4)

Question Number	Scheme	Marks
M1 A1 dM1 A1	A correct attempt to use product of roots is $-\frac{q}{4}$ to find a value for q or pair sum is $\frac{p}{4}$ to find a value of p . Correct value for p or q Correct full method to find both p and q . Correct values for both	
Alt 2	Attempts at using the factor theorem are possible but unlikely to succeed. Score as follows: M1: Uses the factor theorem to generate two equations in the two unknowns (note they will need to use a complex root to achieve this and equate real and imaginary parts.). A1: Correct equations. dM1: Solves their two equations to find values for p and q . A1: Correct values Send to review if unsure.	
Alt 3 (b)	$ \begin{array}{r} 4x^2 - 3x + p - 12 \\ x - 4 \overline{) 4x^3 - 19x^2 + px + q} \\ \underline{4x^3 - 16x^2} \\ -3x^2 + px + q \\ \underline{-3x^2 + 12x} \\ (p - 12)x + q \\ \underline{(p - 12)x - 4(p - 12)} \\ \Rightarrow q + 4(p - 12) = 0 \quad \& \quad \frac{p - 12}{4} = \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i \right) \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i \right) = \frac{5}{4} \\ P = 17, q = -20 \end{array} $	M1 A1 dM1 A1 (4)
(b) M1 A1 dM1 A1	Divides $x - 4$ into the cubic to achieve a 3TQ quotient and a remainder Correct quotient and remainder Correct full method to find p or q Correct values	

Question Number	Scheme	Marks
3(a)	$k(k+5) - 6 = 0$ $k^2 + 5k - 6 = 0$ $((k-1)(k+6) = 0 \Rightarrow) \quad k = 1, -6$	M1 A1 (2)
(b)	$\frac{1}{k^2 + 5k - 6} \begin{pmatrix} k & 2 \\ 3 & k+5 \end{pmatrix}$	M1A1 (2) [4]
(a) M1 A1	Attempts determinant and sets equal to zero (or equivalent method) to obtain an unsimplified quadratic equation Correct values for k (may solve the quadratic by any valid means)	
(b) M1 A1	Forms the matrix of signed minors (must have at least three correct elements) divided or multiplied by an attempt at the determinant Fully correct inverse	