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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Roots of Polynomials (FP1.2)

International A Level Mathematics · FP1: Further Pure Mathematics 1 (WFM01)

11 questions · 99 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

11 questions · 99 marks · 1 sub-topic

SUB-TOPIC FP1.2a

ROOTS OF QUADRATIC EQUATIONS (·, ·)

Sub-topic FP1.2a · 11 questions · 99 marks · lowest tariff first



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 7 marks)

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4. The equation $2x^2 + 5x + 7 = 0$ has roots α and β

Without solving the equation

- (a) determine the exact value of $\alpha^3 + \beta^3$

(3)

- (b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta} \text{ and } \frac{\beta^2}{\alpha}$$

(5)

Question 4 continued

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Question 4 continued

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Question 4 continued

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Q4

(Total 8 marks)



6. The quadratic equation

$$Ax^2 + 5x - 12 = 0$$

where A is a constant, has roots α and β

(a) Write down an expression in terms of A for

(i) $\alpha + \beta$

$$(ii) \ \alpha\beta$$

(2)

The equation

$$4x^2 - 5x + B = 0$$

where B is a constant, has roots $\alpha - \frac{3}{\beta}$ and $\beta - \frac{3}{\alpha}$

(b) Determine the value of A

(3)

(c) Determine the value of B

(3)

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Question 6 continued

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Question 6 continued

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(Total 8 marks)

Q6



Question 2 continued

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Question 2 continued

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Lined area for writing the answer to Question 2 continued.



Question 2 continued

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Q2

(Total 9 marks)



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Question 3 continued

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Question 3 continued

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(Total 9 marks)

Q3



Question 5 continued



Question 5 continued

(Total for Question 5 is 9 marks)



5. The quadratic equation

$$2x^2 - 3x + 7 = 0$$

has roots α and β

Without solving the equation,

- (a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

- (b) determine the value of $\alpha^2 + \beta^2$

(2)

- (c) find a quadratic equation which has roots

$$\left(\alpha - \frac{1}{\beta^2}\right) \text{ and } \left(\beta - \frac{1}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p , q and r are integers to be determined.

(6)



Question 5 continued

Lined area for writing the answer to Question 5 continued.





Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 9 marks)



5. The equation $5x^2 - 4x + 2 = 0$ has roots $\frac{1}{p}$ and $\frac{1}{q}$

(a) Without solving the equation,

(i) show that $pq = \frac{5}{2}$

(ii) determine the value of $p + q$

(4)

(b) Hence, without finding the values of p and q , determine a quadratic equation with roots

$$\frac{p}{p^2 + 1} \text{ and } \frac{q}{q^2 + 1}$$

giving your answer in the form $ax^2 + bx + c = 0$ where a , b and c are integers.

(5)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 9 marks)



5. The quadratic equation

$$2x^2 - 3x + 5 = 0$$

has roots α and β

Without solving the equation,

- (a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

- (b) determine the value of

- (i) $\alpha^2 + \beta^2$

- (ii) $\alpha^3 + \beta^3$

(4)

- (c) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(5)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 10 marks)



$$f(x) = x^2 - 6x + 3$$

Without solving the equation,

$$(\alpha^2 + 1)(\beta^2 + 1)$$

(b) find a quadratic equation which has roots

$$\frac{\alpha}{(\alpha^2 + 1)} \text{ and } \frac{\beta}{(\beta^2 + 1)}$$

(6)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

(Total for Question 5 is 10 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the mark scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to $x = \dots$ (may be unsimplified).

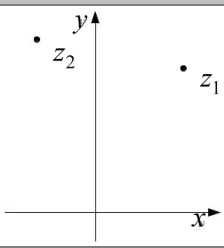
3. Completing the square (where a = 1; if a ≠ 1 must divide by a first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Question Number	Scheme	Notes	Marks
1	$\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix}$		
(a)	{det P=} p(p-1)-(-3)(p+1) Award for any correct expression for det P. If expanded immediately allow one sign error only in one term of p ² -p+3p+3 which can be implied by p ² ±4p+3 provided this has not come from ad+bc		M1
	p ² +2p+3	Correct 3TQ. Terms in any order. Ignore “=0”	A1
			(2)
(b)	If (a) is incorrect, only the M mark is available in (b) and this mark requires working with a 3TQ [from (a) or reattempted] which ≠ 0 for all real p		
	The M mark requires doing enough work so that only an explanation and conclusion is then needed, e.g., correct numerical expression (or value) for determinant, correct use of formula, correct completion of the square, correct vertex (stated or via graph or differentiation) The A mark always requires an appropriate explanation and non-singular/not singular/isn't/can't be singular or "has inverse". Do not accept just "shown" etc.		
	1. Discriminant: d or b ² -4ac = 2 ² -4(1)(3) {=4-12=-8} Negative or d<0 or b ² <4ac, no (real) solution or det P ≠ 0, so non-singular M1: Correct numerical expression (or value) for the discriminant. Allow comparison of b ² with 4ac A1: Explanation and conclusion. 2. Using formula: $p = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)} \left\{ = \frac{-2 \pm \sqrt{-8}}{2} = -1 \pm \sqrt{2}i \right\}$ {√-8} not real/possible or complex (accept imaginary) or no solution or det P ≠ 0, so non-singular M1: Correct numerical expression for p A1: Explanation and conclusion. 3. Using completing the square: p ² +2p+3 = (p+1) ² +2 ≠ 0 or > 0 or □ 2 (not > 2) or min. value = 2 so non-singular Might see (p+1) ² = -2 or p+1 = {±}√-2 or {±}√2 i or p = -1{±}√2 i followed by not real/possible/complex roots, so non-singular M1: p ² +2p+3 = (p+1) ² +2 A1: Explanation and conclusion.		M1 A1

Cases continue overleaf

Question Number	Scheme	Marks
2		
(a)	$\frac{3}{8} - \frac{\sqrt{71}}{8}i$	B1 (1)
(b)	$\left(x - \frac{3}{8} - \frac{\sqrt{71}}{8}i\right)\left(x - \frac{3}{8} + \frac{\sqrt{71}}{8}i\right) ((x-4)=0)$ $\left(x^2 - \frac{3}{4}x + \frac{5}{4}\right) ((x-4)=0)$ $x^3 - \frac{19}{4}x^2 + \frac{17}{4}x - 5 (=0)$ $4x^3 - 19x^2 + 17x - 20 (=0) \quad p=17, q=-20$	M1A1 dM1 A1 (4)
(a) B1 (b) M1 A1 dM1 A1	Correct answer only Attempt the multiplication of the 2 brackets with the complex terms. Allow $(x \pm \text{root})$ for the brackets. Allow “invisible” brackets. Correct quadratic obtained - may have multiplied by the 4 (or other constant factor) and this is fine. (Need not be fully simplified but must have real terms) Attempt to multiply their quadratic by $(x-4)$ or may divide their quadratic into the cubic or other full method leading to at least one of p or q. Correct values. Values of p and q need not be shown explicitly but may be seen in a cubic, provided the cubic starts $4x^3 - 19x^2$ (isw after a correct cubic)	[5]
	Note if a candidate uses a hybrid method, mark under main scheme unless an Alt scores more marks.	
Alt 1 (b)	$-\frac{q}{4} = 4 \times \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) \times \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) = \dots \rightarrow q = \dots \quad \text{or}$ $\frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $\Rightarrow q = -16 \times \left(\frac{9}{64} + \frac{71}{64}\right) = -20 \quad \text{or} \quad p = 17$ E.g. $f(4) = 0 \Rightarrow 4(4)^3 - 19(4)^2 + 4p - 20 = 0 \Rightarrow p = \dots$ $\text{or } \frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $p = 17, q = -20$	M1 A1 dM1 A1 (4)

Question	Scheme	Marks
2(a)		M1 A1
		(2)
(b) (i)	$ z_1 = \sqrt{3^2 + 5^2} = \sqrt{34}$	B1
		(1)
(ii)	$\frac{z_1}{z_2} = \frac{3+5i}{-2+6i} \times \frac{-2-6i}{-2-6i} = \dots$	M1
	$= \frac{-6-18i-10i+30}{40}$	A1
	$= \frac{3}{5} - \frac{7}{10}i$	A1
		(3)
(c)	$\arg \frac{z_1}{z_2} = \arctan \frac{-7/10}{3/5} = \arctan \frac{-7}{6} = \dots$ but allow $\arctan \frac{7}{6}$ for M1	M1
	$= -0.86$ or 5.42 (awrt)	A1
		(2)
(8 marks)		
Notes:		
(a) M1: Points in correct quadrants - z_1 in quadrant 1 and z_2 in quadrant 2. Must be clearly labelled either eg z_1 or $3 + 5i$ or correct numbers on the axes. (Accept with vector arrows.) A1: Correct diagram, z_1 in first quadrant further away from real axis than imaginary and z_2 in second quadrant, closer to imaginary axis but above z_1 OR correct nos on their axes (imag axis may include i), but not dashes w/o any indication of scale. Allow M1A0 for points unlabelled but diagram otherwise correct. (b)(i) B1: Correct modulus. Must be evaluated to $\sqrt{34}$ Question says “without using your calculator” so decimal answers can be ignored (isw) but exact answer must be seen somewhere. (ii) M1: Multiplies numerator and denominator by the conjugate of their denominator. A1: Correct unsimplified (or simplified) numerator, with the i^2 correctly dealt with, and correct denominator. A1: Correct answer. Allow as shown, $\frac{6}{10} - \frac{7}{10}i$, or $0.6 - 0.7i$.		

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1(a)	$\mathbf{A}^{-1} = \frac{1}{3 \times -2 - a \times -2} \begin{pmatrix} -2 & -a \\ 2 & 3 \end{pmatrix}$	Complete method for the inverse. Allow slips in the determinant and at most one error in the adjoint matrix.	M1
	$= \frac{1}{2a-6} \begin{pmatrix} -2 & -a \\ 2 & 3 \end{pmatrix}$	Correct inverse	A1
			(2)
(b)	$\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I} \Rightarrow \begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix} + \frac{1}{2a-6} \begin{pmatrix} -2 & -a \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Sets up the correct matrix equation with their $\mathbf{A}^{-1}$. The identity may be left as $\mathbf{I}$ as long as at least one equation is processed correctly and no equation implies an incorrect identity matrix.	M1
	$3 + \frac{2}{6-2a} = 1, a + \frac{a}{6-2a} = 0, -2 + \frac{2}{2a-6} = 0, -2 + \frac{3}{2a-6} = 1 \Rightarrow a = \dots$ Uses one of the elements to set up a suitable equation and solves for a . Allow a sign slip in the $6-2a$ but have correct coefficient of $\mathbf{I}$		dM1
	$a = \frac{7}{2} \text{ oe}$	Correct value and no others	A1
			(3)
			Total 5

Question Number	Scheme	Notes	Marks
2(a)	$f(x) = 7\sqrt{x} - \frac{1}{2}x^3 - \frac{5}{3x} \quad x > 0$		
	$f(2.8) = 0.1420022...$ $f(2.9) = -0.8486421...$	Attempts both $f(2.8)$ and $f(2.9)$ with at least one correct to 1 s.f.	M1
	Sign change (positive, negative) and $f(x)$ is continuous therefore (a root) α is between $x = 2.8$ and $x = 2.9$	Both $f(2.8) = \text{awrt } 0.1$ (or truncated) and $f(2.9) = \text{awrt } -0.8$ (or truncated), sign change, continuous and minimal conclusion.	A1
			(2)
(b)(i)	$f'(x) = \frac{7}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^2 + \frac{5}{3x^2}$	$x^n \rightarrow x^{n-1}$ at least once	M1
		Correct derivative	A1
(b)(ii)	$x_1 = 2.8 - \frac{f(2.8)}{f'(2.8)} = 2.8 - \frac{0.142002276...}{-9.4557649...}$	Correct application of Newton-Raphson. If no substitution/values see accept a correct statement followed by a value for the attempt.	M1
	$= 2.815$	cao following a correct derivative.	A1
			(4)
(c)	$\frac{2.9 - \alpha}{0.8486421875...} = \frac{\alpha - 2.8}{0.1420022762...}$	Any correct or implied linear interpolation statement.	B1
	$\alpha = \frac{2.8 \times 0.8486421875... + 2.9 \times 0.1420022762...}{0.8486421875... + 0.1420022762...} = ...$ Rearranges an equation suitable form (e.g. allow sign errors in interpolation statement) to give $\alpha = ...$		M1
	$= 2.814$	cao	A1
			(3)
Alt (c)	$\frac{x}{0.1420022762...} = \frac{0.1}{0.1420022762... + 0.8486421875...}$	Any correct or implied linear interpolation statement for x distance.	B1
	$\alpha = 2.8 + x = 2.8 + \frac{0.4 \times 0.1420022762...}{0.8486421875... + 0.1420022762...} = ...$ Rearranges and adds 2.8 to give $\alpha = ...$		M1
	$= 2.814$	cao	A1
			(3)
			Total 9



Question Number	Scheme	Notes	Marks
3	$2x^2 - 5x + 7 = 0$		
(a)	$\alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{7}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{3}{4}$	cso – must have scored the B1	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(\frac{5}{2}\right) = -\frac{85}{8}$	cso – must have scored the B1	A1
			(4)
(c)	$\text{Sum} = \frac{1}{\alpha^2 + \beta} + \frac{1}{\beta^2 + \alpha} = \frac{\alpha^2 + \beta + \beta^2 + \alpha}{(\alpha^2 + \beta)(\beta^2 + \alpha)}$ $= \frac{\alpha^2 + \beta^2 + \alpha + \beta}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{-\frac{3}{4} + \frac{5}{2}}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{14}{41} \right)$ Attempts sum – substitutes their into a correct numerator must but allow slips in the denominator as long as 4 terms are produced from the expansion.		M1
	$\text{Product} = \frac{1}{\alpha^2 + \beta} \times \frac{1}{\beta^2 + \alpha} = \frac{1}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{1}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{8}{41} \right)$ Attempts product – must be correct expansion of denominator with their values.		M1
	$x^2 - \frac{14}{41}x + \frac{8}{41} (= 0)$	Applies $x^2 - (\text{their sum})x + \text{their product} (= 0)$ Depends on at least one previous M awarded.	dM1
	$41x^2 - 14x + 8 = 0$	Allow any integer multiple. Must include “=0”	A1
			(4)
			Total 9

Question	Scheme	Marks
5(a)	$\alpha + \beta = -\frac{3}{4}$	B1
	$\alpha\beta = \frac{k}{4}$	B1
		(2)
(b)	$\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}$	B1
	$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^2}; = \frac{\left(-\frac{3}{4}\right)^3 - 3\left(\frac{k}{4}\right)\left(-\frac{3}{4}\right)}{\left(\frac{k}{4}\right)^2} = \dots$	M1; M1
	$= \frac{36k - 27}{4k^2} = \frac{9}{k} - \frac{27}{4k^2}$	A1
		(4)
(c)	Product of roots is $\frac{\alpha\beta}{\alpha^2\beta^2} = \frac{1}{\alpha\beta} = \frac{4}{k}$	B1ft
	Equation is $x^2 - \left(\frac{36k - 27}{4k^2}\right)x + \frac{4}{k} = 0$	M1
	$4k^2x^2 - (36k - 27)x + 16k = 0$	A1
		(3)
(9 marks)		
Notes:		
(a) B1: Correct expression for $\alpha + \beta$ B1: Correct expression for $\alpha\beta$ (b) B1: Combines the fractions correctly. M1: For a correct identity for the sum of cubes. M1: Substitutes their values for $\alpha + \beta$ and $\alpha\beta$ into their equation for sum of $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ (not dependent, so there may be a slip in the identity used for $\alpha^3 + \beta^3$). A1: Correct expression in terms of k in a simplified form - e.g. either form as shown in scheme. (c) B1ft: Correct product of roots in terms of k, or follow through $\frac{1}{\text{their } \alpha\beta}$ from part (a).		

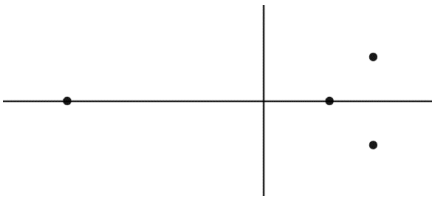
M1: Applies $x^2 - (\text{their sum of roots})x + \text{their product of roots} (= 0)$. Allow without the “=0” for this mark.

A1: Correct equation, as shown or an integer multiple thereof. Accept equivalents for the x term (e.g. $4k^2x^2 + (27 - 36k)x + 16k = 0$). Must include the “=0”.



Question	Scheme	Marks
5.(a)	$2x^2 - 3x + 7 = 0$	
	$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = \frac{7}{2}$	B1
		(1)
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{19}{4}$	A1
		(2)
(c)	$\text{Sum} = \alpha - \frac{1}{\beta^2} + \beta - \frac{1}{\alpha^2} = \alpha + \beta - \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{3}{2} - \frac{-\frac{19}{4}}{\left(\frac{7}{2}\right)^2} = \frac{185}{98}$	M1A1
	$\text{Prod} = \left(\alpha - \frac{1}{\beta^2}\right)\left(\beta - \frac{1}{\alpha^2}\right) = \alpha\beta - \frac{\alpha + \beta}{\alpha\beta} + \frac{1}{\alpha^2\beta^2} = \frac{7}{2} - \frac{3}{7} + \frac{4}{49} = \frac{309}{98}$	M1A1
	$x^2 - \frac{185}{98}x + \frac{309}{98} (= 0)$	M1
	$98x^2 - 185x + 309 = 0$	A1
		(6)
(Total 9 marks)		
Notes		
(a) B1: Both values correct. (b) M1: Attempts to use a correct identity to find the sum of square of roots. A1: Correct value. Note do not allow recovery from $\alpha + \beta = -\frac{3}{2}$ for this mark. (c) M1: Attempts sum for the new roots using their values from (a) and (b). They must be substituting into a correct identity for this mark. If substitution not seen allow for any value appearing after a suitable combined identity is seen. A1: Correct value. M1: Attempts product for the new roots using their values from (a). Must be substituting into an expression of the correct form, but allow if a sign slip occurs when expanding. If substitution not seen allow for any value appearing after a suitable expanded identity is seen. A1: Correct value. M1: Applies $x^2 - (\text{their sum})x + \text{their prod} (= 0)$. May be implied by suitable values for p, q and r stated if no quadratic seen. A1: Allow any integer multiple. Must include the “= 0”, and must be an equation, not just values for p, q and r. Note: Answers from solving the quadratic will gain no credit for (a) and (b) and only score in (c) if the method marks as described are earned.		

Question Number	Scheme	Notes	Marks
2	$f(z) = z^3 - 13z^2 + 59z + p$		
(a)	$\begin{array}{r} [f(3)] = 3^3 - 13(3)^2 + 59(3) + p \\ \text{or e.g., } 27 - 117 + 177 + p \text{ or} \\ z^2 - 10z + 29 \\ z - 3 \overline{) z^3 - 13z^2 + 59z + p} \\ \underline{z^3 - 3z^2} \\ -10z^2 + 59z \\ \underline{-10z^2 + 30z} \\ 29z + p \\ \underline{29z - 87} \\ 0 \end{array}$	Attempts $f(3)$. Must see more than just $87 + p$ Allow one slip (e.g., a miscopy of one coefficient, or one incorrect value/sign if expression just given as $27 - 117 + 177 + p$) Alternatively long divides by $z - 3$ obtaining a 3TQ with two terms of $z^2 - 10z + 29$ correct. Could use synthetic division. An attempt at equating coefficients/factorising requires 2 correct values for the a, b and c of $az^2 + bz + c$	M1
	$f(3) = 0 \Rightarrow p = -87 *$	Obtains "$p = -87$" only with no errors but condone work in x "=0" must have been seen before $p = -87$ if $f(3)$ attempted but allow just $p = -87$ following a full and correct attempt via division/equating coefficients etc with no errors.	A1* (shown as B1 on ePen)
(2)			
(b)	Allow equivalent work in x. Allow use of a calculator to solve a quadratic. Solutions that just follow $z^3 - 13z^2 + 59z - 87 = 0$ score no marks. There are no marks if $z^2 - 10z + 29$ has clearly been produced by using $(z - (5 + 2i))(z - (5 - 2i))$		
	$(z^3 - 13z^2 + 59z - 87) \div (z - 3)$ $= \dots [z^2 - 10z + 29]$	M1: Uses $z \pm 3$ with $f(z)$ (not their $f(z)$) to obtain a 3TQ expression with evidence of any appropriate method including inspection (must be evidence of use of $z \pm 3$) or equating coefficients. Ignore any remainder if long division is used and may see $z^2 - 16z + 107$ ($r(-408)$) if $z + 3$ used. Must be seen or referred to in (b) A1: Correct quadratic	M1 A1
	$z = \frac{-(-10) \pm \sqrt{(-10)^2 - (4)(1)(29)}}{2(1)}$ or $(z - 5)^2 - 25 + 29 = 0 \Rightarrow z = 5 \pm \sqrt{-4}$	Solves their 3TQ arising from using $(z - 3)$ only as a factor (usual rules but allow if one correct root if calculator used on their quadratic) If a sum/product of roots method is used on their 3TQ (i.e., $2a = -(-10)$, $a^2 + b^2 = "29"$) it must be complete and condone only sign errors. Do not allow just $5 \pm 2i$ following an incorrect quadratic Requires previous M mark.	dM1
	$\left(z = \frac{10 \pm \sqrt{-16}}{2} = \right) 5 \pm 2i$	$5 \pm 2i$ or $5 + 2i, 5 - 2i$ only. Not $5 \pm 2\sqrt{-1}$ Accept $\pm 2i + 5$	A1
(4)			

Question Number	Scheme	Notes	Marks
2(c)	 Look for this arrangement if correct but note potential ft	Correct diagram ft their $a \pm bi$ ($a, b \neq 0$) Diagram should be roughly symmetrical in the real axis. The point on the negative x-axis should be further from the origin than the point on the positive x-axis but ignore any other scaling issues – just look for the $a \pm bi$ points to be placed in the correct quadrants, roughly aligned vertically and placed correctly relative to the given point that is on the same side of the y-axis. Points/axes may be unlabelled or mislabelled. If vectors/lines are used the end points must satisfy the conditions above.	B1ft
			(1)
(d)	$2\left(\sqrt{("5"-(-9))^2 + "2"^2} + \sqrt{("5"-3)^2 + "2"^2}\right)$	A correct numerical expression for the perimeter ft their $a \neq 0$ or 3 or -9 and $b \neq 0$ This mark requires working with points that would form a convex or concave kite where the x-axis is a line of symmetry. Working must be seen if $a \pm bi$ incorrect but allow just $4\sqrt{5} + 4\sqrt{17}$ oe from using $-5 \pm 2i$	M1
	$\left[= 2\left(\sqrt{14^2 + 2^2} + \sqrt{2^2 + 2^2}\right) = 2\left(\sqrt{200} + \sqrt{8}\right) \right]$ $= 24\sqrt{2}$	$24\sqrt{2}$ or any simplified equivalent e.g., $12\sqrt{8}$ or $2\sqrt{288}$ but not $\sqrt{1152}$. Correct answer scores both marks and allow M1 A0 for just $\sqrt{1152}$	A1
			(2)
			Total 9



Question Number	Scheme	Notes	Marks
3	$f(x) = x^3 - 5\sqrt{x} - 4x + 7$		
(a)	$f(0.25)=3.515625, \frac{225}{64}, 3\frac{33}{64} f(1) = -1$	Attempts both $f(0.25)$ and $f(1)$ with one correct allowing awrt 3.52 for $f(0.25)$	M1
	examples: "1" refers to "-1" with sign corrected $\frac{\alpha - 0.25}{\text{"3.515625"}} = \frac{1 - \alpha}{\text{"1"}} \Rightarrow \alpha = \dots$ $\frac{\alpha - 0.25}{\text{"}\frac{225}{64}\text{"}} = \frac{1 - \alpha}{\text{"1"}} \Rightarrow \alpha = \dots$ $\frac{\alpha - 0.25}{\text{"3.515625"}} = \frac{1 - 0.25}{\text{"3.515625" + "1"}} \Rightarrow \alpha = \dots$ $\frac{1 - \alpha}{\text{"1"}} = \frac{1 - 0.25}{\text{"3.515625" + "1"}} \Rightarrow \alpha = \dots$ $[\alpha - 0.25 = 3.515625 - 3.515625\alpha$ $4.515625\alpha = 3.765625]$	Forms an equation in α that is correct for their values and solves for α. Can use x etc. Allow e.g., "$f(0.25)$" and "$-f(1)$" in this equation provided values for these are seen. Any modulus signs must be applied and $f(0.25)$ and $f(1)$ must have had different signs. Can be implied by just awrt 0.83 or $\frac{241}{289}$ but otherwise a correct equation for their $f(0.25)$ and $f(1)$ must be seen but allow use of $\frac{af(b)-bf(a)}{f(b)-f(a)} \Rightarrow \frac{1(\text{"3.515625"})-0.25(\text{"-1"})}{\text{"3.515625"}-(\text{"-1"})}$ or a correct partially processed equivalent and only allow formula followed by value if values for $a, b, f(a)$ and $f(b)$ are seen If e.g., A is used for $\alpha - 0.25$ then must see $A + 0.25$ later. Note that sight of 1.2981... or $\frac{209}{161}$ usually indicates a sign error.	M1
	$\alpha = 0.834$	awrt 0.834 (0.8339100346...) Must be decimal. Ignore labelling and just look for this value. [Note: actual root is 0.767843...]	A1
(3)			
Alt for last 2 marks (straight line equation)	e.g., $y = \frac{\text{"3.515625"} - \text{"(-1)"}}{0.25 - 1}x + c$ $(1, \text{"-1"}) \Rightarrow -1 = -6.0208\dot{3} + c$ $\Rightarrow c = 5.0208\dot{3}$ $y = 0 \Rightarrow \alpha = \frac{-5.0208\dot{3}}{-6.0208\dot{3}} = 0.834$	M1: Any full method to find the equation of the line between $(0.25, \text{"3.515625"})$ and $(1, \text{"-1"})$ and then uses $y = 0$ to find a value for α. Condone errors finding c and α but the initial equation should be correct for their $f(0.25)$ and $f(1)$ and the x and y coordinates should always be correctly placed. A1: awrt 0.834	M1 A1
(b)	$[f'(x)] = 3x^2 - \frac{5}{2}x^{-\frac{1}{2}} - 4$	M1: 2 correctly differentiated terms (this includes $7 \rightarrow 0$) Allow unsimplified e.g., $3 \times x^{3-1}$ A1: Fully correct simplified derivative	M1 A1
(2)			
(c)	$x_1 = 1.75 - \frac{1.75^3 - 5\sqrt{1.75} - 4(1.75) + 7}{\text{"}3(1.75)^2 - 2.5(1.75)^{-0.5} - 4\text{"}}$ $\left[= 1.75 - \frac{-1.255003278\dots}{3.297677635\dots} = 1.75 + 0.38057\dots \right]$	Uses a correct Newton-Raphson formula with $x_0 = 1.75$ and their $f'(x)$ to obtain a numerical expression for x_1 but implied by awrt 2.13 (2.13057185). Working must be seen if x_1 is wrong – allow "$x_0 = 1.75, x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \dots$" or $1.75 - \frac{f(1.75)}{f'(1.75)} = \dots$"	M1
	$x_1 = 2.13057185\dots \Rightarrow \beta = 2.131$	awrt 2.131 (ignore labelling and just look for this value). Ignore further iterations. [Note: actual root is 2.011276...]	A1
(2)			
Total 7			

Question Number	Scheme	Notes	Marks
4	If z is restated incorrectly, e.g., " $z = 3 + 4i$ " is seen allow a maximum of M1dM1A0 B0M1A1M1A0		
(a)	$z^2 - 3 = (-3 + 4i)(-3 + 4i) - 3$ $= 9 - 24i - 16 - 3$ $= -10 - 24i$	Substitutes $z = -3 + 4i$ into $z^2 - 3$, expands and reaches $a + bi$ ($a, b \neq 0$) Implied by $-10 - 24i$ seen and condone misapplication of the modulus e.g., using $a + bi$ from $ -a - bi $	M1
	$ z^2 - 3 = \sqrt{10^2 + 24^2}$	Correct expression for modulus of their $a + bi$ ($a, b \neq 0$) Allow with no working for the modulus provided answer correct for their $a + bi$ Requires previous M mark.	dM1
	26	26 only from correct work. e.g., $ -10 + 24i = 26$ is A0 Answer only or without $-10 - 24i$ is no marks.	A1
(3)			
(b)	$(z = -3 + 4i \Rightarrow) \quad z^* = -3 - 4i$	Correct conjugate. Can be implied	B1
	$\frac{50}{z^*} = \frac{50}{-3 - 4i} \times \frac{-3 + 4i}{-3 + 4i} \left[= 50 \times \frac{-3 + 4i}{25} \right]$ or $\frac{1}{z^*} = \frac{1}{-3 - 4i} \times \frac{-3 + 4i}{-3 + 4i} \left[= \frac{-3 + 4i}{25} \right]$	A correct multiplier seen that would make the denominator real for $\frac{50}{z^*}$ or $\frac{1}{z^*}$ where $z^* = \pm 3 \pm 4i$ (except $-3 + 4i$). If the multiplier is not seen must see something better than $50 \times \frac{-3 + 4i}{25}$ or $\frac{-3 + 4i}{25}$ or $-6 + 8i$ e.g., $\frac{50}{z^*} = \frac{50(-3 + 4i)}{9 + 16}$	M1
	$\frac{50}{z^*} = 2(-3 + 4i) \quad \text{or} \quad 2z$	Obtains $2(-3 + 4i)$ or $2z$ Just $-6 + 8i$ is insufficient Allow $k = 2$ provided " kz " or " $k(-3 + 4i)$ " is seen	A1
(3)			
Using Result	May see: $\frac{50}{-3 - 4i} = k(-3 + 4i) \Rightarrow \frac{50}{9 + 16} = k \Rightarrow k = 2$ B1: Correct z^* M1: $\frac{50}{9 + 16} = k$ or better after multiplication A1*: $k = 2$		
Alt Using $\frac{1}{z^*} = \frac{z}{ z ^2}$	$\frac{1}{z^*} = \frac{z}{ z ^2}$ oe e.g., $z^* z = z ^2$	States or uses $\frac{1}{z^*} = \frac{z}{ z ^2}$ oe	B1
	$\frac{c}{z^*} = \frac{cz}{ z ^2}, \quad z = \sqrt{3^2 + 4^2} = \dots$	Expresses $\frac{c}{z^*}$ as $\frac{cz}{ z ^2}$ and attempts $ z $ or $ z ^2$ where $c = 1$ or 50	M1
	$\frac{50}{z^*} = \frac{50z}{25} = 2z$	Correctly finds $2z$ Allow $k = 2$ provided " kz " or " $k(-3 + 4i)$ " is seen	A1

Question Number	Scheme	Notes	Marks
4(c)	$\arctan\left(\pm\frac{4}{3}\right) = \pm 0.927... (53.13^\circ)$ or $\arctan\left(\pm\frac{3}{4}\right) = \pm 0.643... (36.86^\circ)$ May see equivalent trig in which case the hypotenuse should be correct	Finds a relevant angle which could be in degrees correct to 2sf so accept awrt $\pm 0.93 (53^\circ)$ or $\pm 0.64 (37^\circ)$ If neither value is seen allow implication from the work May see e.g., $\tan^{-1}\left(\pm\frac{8}{6}\right) = ...$ M0 if $\arg 2z$ replaced with $2 \arg z$	M1
	$\left[\theta = \pi - 0.927295... \quad \theta = \frac{\pi}{2} + 0.643501... \right]$ $\theta = 2.21$	Final answer of awrt 2.21 – do not isw. (n.b. $\theta = 2.214297436...$) Final answer of e.g., "$\pi - 0.927$" is A0 Answer only scores both marks. Answer only in degrees (awrt 127°) is M1A0	A1
Note: allow access to both marks even if k in part (b) was incorrect			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$5x^2 - 4x + 2 = 0$		
	Solutions that rely on solving the given quadratic/finding values for p and q are likely to score a maximum of 0010 11010 if the relevant work is seen		
(a)(i)	$\frac{1}{p} \times \frac{1}{q}$ or $\frac{1}{pq} = \frac{2}{5} \Rightarrow pq = \frac{5}{2}^*$	Shows product of roots = $\frac{2}{5}$ followed by $pq = \frac{5}{2}$ Minimum as shown. Allow e.g., $qp = 2.5$ Note that $\frac{1}{pq} = \frac{1}{\frac{2}{5}} \Rightarrow pq = \frac{5}{2}$ is B0 No clearly incorrect work/statements.	B1*
	May see: $\left(x - \frac{1}{p}\right)\left(x - \frac{1}{q}\right) = x^2 - \left(\frac{1}{p} + \frac{1}{q}\right)x + \frac{1}{pq} = x^2 - \frac{4}{5}x + \frac{2}{5} \Rightarrow \frac{1}{pq} = \frac{2}{5} \Rightarrow pq = \frac{5}{2}^*$ Must not be any clearly incorrect work/statements.		
	Assuming result: $pq = \frac{5}{2} \Rightarrow \frac{1}{p} \times \frac{1}{q} = \frac{2}{5}$ requires conclusion e.g., "Hence true"		
(a)(ii) May use work from (i)	$\frac{1}{p} + \frac{1}{q} = -\frac{(-4)}{5}$	Uses sum of roots to achieve a correct equation in p and q	M1
	$\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq}$	States or uses $\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq}$	M1
	$\frac{p+q}{pq} = \frac{p+q}{\frac{5}{2}} = \frac{4}{5} \Rightarrow p+q = \frac{4}{5} \times \frac{5}{2} = 2$	" $p+q=2$ " from correct work. Allow " $2=q+p$ "	A1
(4)			
Alt $x \rightarrow \frac{1}{z}$	$x \rightarrow \frac{1}{z} \Rightarrow 5\left(\frac{1}{z}\right)^2 - 4\left(\frac{1}{z}\right) + 2 = 0$	Correctly replaces x with e.g., $\frac{1}{z}$ and allow $\frac{1}{x}$	1st M1
	$2z^2 - 4z + 5 = 0$	Obtains a 3TQ in "z", "w" etc.	2nd M1
	$pq = \frac{5}{2}$	States $pq = \frac{5}{2}$ following correct work	B1* 1st mark
	$p+q=2$	" $p+q=2$ " from correct work	A1

Question Number	Scheme	Notes	Marks
5(b)	$\frac{p}{p^2+1} + \frac{q}{q^2+1} = \frac{pq^2 + p + p^2q + q}{p^2q^2 + p^2 + q^2 + 1}$ $\frac{p}{p^2+1} \times \frac{q}{q^2+1} = \frac{pq}{p^2q^2 + p^2 + q^2 + 1}$ M1: For $p(q^2+1) + q(p^2+1) \rightarrow pq^2 + p + p^2q + q$ or $(p^2+1)(q^2+1) \rightarrow p^2q^2 + p^2 + q^2 + 1$ Allow equivalents e.g., $pq(p+q) + p + q$ provided the initial expansion has been carried out A1: Both correct (expression for denominator seen correctly once) Do not accept pq^2 for $(pq)^2$ unless it is clearly recovered		M1 A1
	$\text{sum} = \frac{pq(p+q) + p + q}{(pq)^2 + (p+q)^2 - 2pq + 1} = \frac{\frac{5}{2} \times 2 + 2}{(\frac{5}{2})^2 + 2^2 - 2 \times \frac{5}{2} + 1} = \frac{7}{\frac{25}{4}} = \dots \left(\frac{28}{25} \text{ or } 1.12 \right)$ $\text{product} = \frac{pq}{(pq)^2 + (p+q)^2 - 2pq + 1} = \frac{\frac{5}{2}}{(\frac{5}{2})^2 + 2^2 - 2(\frac{5}{2}) + 1} = \frac{\frac{5}{2}}{\frac{25}{4}} = \dots \left(\frac{2}{5} \text{ or } 0.4 \right)$ Obtains a value for either the new sum or new product using $pq = \frac{5}{2}$ and a value for $p+q$ which could be their answer from part (a)(ii) and may have been stated as e.g., $\frac{1}{p} + \frac{1}{q}$ or it could be inconsistent with their answer to (a)(ii). May be slips. At least one of their expressions must have included both pq and $p+q$ and have been completely in terms of pq and $p+q$ including at least one use of $p^2 + q^2 = (p+q)^2 - 2pq$. Accept just sum = $\frac{28}{25}$ or product = $\frac{2}{5}$ if there is no clearly incorrect work otherwise some evidence of all of the above conditions and not just values must be seen. Requires previous M mark.		dM1
	Note that for the numerator of the sum it is possible to use $pq^2 + p + p^2q + q = p+q + (p+q)(p^2+q^2) - (p^3+q^3) = p+q + (p+q)((p+q)^2 - 2pq) - ((p+q)^3 - 3pq(p+q))$ in which case both $p^2 + q^2 = (p+q)^2 - 2pq$ and $p^3 + q^3 = (p+q)^3 - 3pq(p+q)$ must be used		
	The above work may be embedded within $x^2 \pm (\text{sum})x \pm \text{product}$		
	$x^2 - \frac{28}{25}x + \frac{2}{5}$	Applies $x^2 - (\text{sum})x + \text{product}$ correctly for their stated values for new sum and product. Not dependent.	M1
	$25x^2 - 28x + 10 = 0$	Correct quadratic (or integer multiple) with “= 0” Allow a different variable e.g., z for x Allow e.g., $a = 25, b = -28, c = 10$ provided $ax^2 + bx + c = 0$ is seen otherwise score M1A0	A1
			(5)
			Total 9

Question Number	Scheme	Notes	Marks
3(a)	$\mathbf{M}^{-1} = \frac{1}{5k-3k} \begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix}$	Attempts $\mathbf{M}^{-1} = \frac{1}{\det \mathbf{M}} \times \text{adj}(\mathbf{M})$ Either part correct but $\text{adj}(\mathbf{M}) = \mathbf{M}$ scores M0	M1
	$= \frac{1}{2k} \begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix} \text{ or } \begin{pmatrix} \frac{5}{2k} & -\frac{1}{2} \\ -\frac{3}{2k} & \frac{1}{2} \end{pmatrix}$	Correct matrix $2k$ must be seen for this mark	A1
			(2)
(b)	$(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1} = \frac{1}{2k} \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix}$	Applies $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$	M1
	$= \frac{1}{2k} \begin{pmatrix} 2k & 0 \\ 23 & -5k \end{pmatrix} \text{ or e.g. } \begin{pmatrix} 1 & 0 \\ \frac{23}{2k} & -\frac{5}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
ALT (b)	Find $\mathbf{N}$ (ie inverse of $\mathbf{N}^{-1}$) Find $\mathbf{MN} = -\frac{1}{5k} \begin{pmatrix} -5k & 0 \\ -23 & 2k \end{pmatrix}$ Find $(\mathbf{MN})^{-1}$ $= \frac{1}{2k} \begin{pmatrix} 2k & 0 \\ 23 & -5k \end{pmatrix} \text{ or e.g. } \begin{pmatrix} 1 & 0 \\ \frac{23}{2k} & -\frac{5}{2} \end{pmatrix}$	Complete method needed Correct matrix	M1 A1
			(2)
			Total 4

Question Number	Scheme	Notes	Marks
4	$f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$		
(a)	$(z =) 5 + i$	Correct complex number	B1
			(1)
	Mark (b) and (c) together – ignore any labelling seen. Award marks in the order given for their choice of method		
(b)/(c) With (b) first	$z = 5 \pm i \Rightarrow (z - (5 + i))(z - (5 - i)) = \dots$ Or e.g. Sum of roots = 10 Product of roots = 26	Correct strategy to find the quadratic factor using the conjugate pair	M1
	$z^2 - 10z + 26$	Correct quadratic	A1
	$f(z) = (z^2 - 10z + 26)(2z^2 + \dots z + k)$	Attempts to find the other quadratic. May use inspection (apply rules for quadratic factorisation ie " $26 \mid k \mid = 156$ ") or e.g. long division with quotient $2z^2 + \dots z + \dots$	M1
	NB long division gives quotient $2z^2 + z + (A - 42)$ and remainder $(10A + B - 446)z + 936 - 26A$		
	$2z^2 + z - 6$	Correct quadratic	A1
	$\Rightarrow z = \frac{3}{2}, -2, (5 \pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(5)
	$f(z) = (z^2 - 10z + 26)(2z^2 + z - 6) = \dots$	Multiplies out both quadratics or extracts the terms needed	M1
	$A = 36, B = 86$	Correct values (can be seen in the quartic equation)	A1
			(2)
			Total 8
(b)/(c) With (c) first	$952 + 960i - 2090 - 24A + 10Ai + 5Bi - 156 = 0$	Substitute $(5 + i)$ into the quartic (by calculator) and equate real and imag parts (can be done with $(5 - i)$)	M1
	$-1294 + 24A + 5B = 0$ $-446 + 10A + B = 0$	Correct equations	A1
	$A = 36 \quad B = 86$	M1 Solve simultaneously A1 One correct A1 Both correct	M1 A1A1
			(5)
	$2z^4 - 19z^3 + 36z^2 + 86z - 156 = 0$ $z = \dots$	Solve the equation by long division, inspection or by calculator	M1
	$\Rightarrow z = \frac{3}{2}, -2, (5 \pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(2)
			Total 8

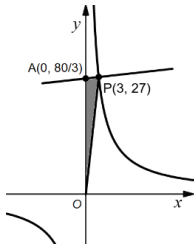


Question Number	Scheme	Notes	Marks
4	$\mathbf{A} = \begin{pmatrix} -3 & 8 \\ -3 & k \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix} \quad \mathbf{BC} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix}$		
(i)	$\det \mathbf{A} = -3k - 8(-3) \{ = -3k + 24 \}$ Could be implied	Attempts $\det \mathbf{A}$ and obtains $\pm 3k \pm 8(\pm 3)$ or $\pm 3k \pm 24$	M1
	$-3k + 24 = 3$ or $-3k + 24 = -3$ $\Rightarrow k = \dots$ May see $(-3k + 24)^2 = +9 \Rightarrow 9k^2 - 144k + 567 = 0 \Rightarrow \dots$	Equates their $\det \mathbf{A}$ of form $ak + b$, $a, b \neq 0$ to 3 or -3 or equivalent work and solves for k (usual rules if quadratic and must use +9)	M1
	$k = 7, k = 9$ 1st A1: Either correct value of k from correct work. Allow e.g., $\frac{-21}{-3}$ or $\frac{-27}{3}$ 2nd A1: Both correct values of k from correct work. 7 and 9 only. No extra		A1 A1
			(4)
(ii)	$\det \mathbf{B} = 1 \times 3a - (-4) \times 2 \{ = 3a + 8 \}$	Correct unsimplified expression for $\det \mathbf{B}$. Could be implied	B1
	$\mathbf{B}^{-1} = \frac{1}{3a+8} \begin{pmatrix} 3 & 4 \\ -2 & a \end{pmatrix}$	Correct $\mathbf{B}^{-1}$ with their $\det \mathbf{B}$. $\text{Adj}(\mathbf{B})$ to be correct but allow elements to have their $\det \mathbf{B}$ as denominators if incorporated.	M1
	$\mathbf{C} = \mathbf{B}^{-1}\mathbf{BC} = \frac{1}{3a+8} \begin{pmatrix} 3 & 4 \\ -2 & a \end{pmatrix} \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix} = \dots$ Access to this mark is allowed if there is no determinant or if $\mathbf{B}^{-1} = \det \mathbf{B} \times \text{Adj}(\mathbf{B})$ used	Multiplies $\mathbf{BC}$ by their $\mathbf{B}^{-1}$ (changed – and not just by incorporation of their determinant) the correct way round . Expect four correct elements for their matrices if the method is unclear. The incorrect order scores M0 even if the correct result is obtained.	M1
	$\mathbf{C} = \frac{1}{3a+8} \begin{pmatrix} 10 & 31 & 11 \\ a-4 & 4a-10 & 2a-2 \end{pmatrix}$ Ignore any reference to inapplicable values of a ($a \neq -\frac{8}{3}$)	Correct $\mathbf{C}$ or equivalent with like terms collected and single fractions if necessary. e.g., $\begin{pmatrix} \frac{10}{3a+8} & \frac{31}{3a+8} & \frac{11}{3a+8} \\ \frac{a-4}{3a+8} & \frac{2(2a-5)}{3a+8} & \frac{2(a-1)}{3a+8} \end{pmatrix}$	A1
			(4)
Alt Sim. equations	$\begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} p & q & r \\ s & t & u \end{pmatrix} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix} \Rightarrow \begin{matrix} ap-4s=2 & aq-4t=5 & ar-4u=1 \\ 2p+3s=1 & 2q+3t=4 & 2r+3u=2 \end{matrix}$ Multiplies in the correct order to obtain at least three correct equations		B1
	$(3a+8)p=10 \quad (3a+8)q=31 \quad (3a+8)r=11$ $p = \frac{10}{3a+8} \quad q = \frac{31}{3a+8} \quad r = \frac{11}{3a+8}$ $s = \frac{1}{3} \left(1 - \frac{20}{3a+8} \right) \quad t = \frac{1}{3} \left(4 - \frac{62}{3a+8} \right) \quad u = \frac{1}{3} \left(2 - \frac{22}{3a+8} \right) \Rightarrow \begin{pmatrix} \frac{10}{3a+8} & \frac{31}{3a+8} & \frac{11}{3a+8} \\ \frac{a-4}{3a+8} & \frac{4a-10}{3a+8} & \frac{2a-2}{3a+8} \end{pmatrix}$ $s = \frac{a-4}{3a+8} \quad t = \frac{4a-10}{3a+8} \quad u = \frac{2a-2}{3a+8}$ M1: Solves their equations to find expressions in terms of a for three elements M1: Finds expressions in terms of a for all six elements A1: Correct matrix – like terms collected and single fractions		M1 M1 A1
			Total 8



Question Number	Scheme	Notes	Marks
3(ii)(a)	$g(x) = 3^{\sin x} - 3 \cos x$ $g(4) = 2.396...$, $g(5) = -0.502...$	Attempts both $g(4)$ and $g(5)$ with at least one correct (using radians): $g(4) = \text{awrt } 2.4$ (or 2.3 truncated), $g(5) = \text{awrt } -0.5$	M1
	Sign change oe and $g(x)$ is continuous therefore a root oe e.g., β is between $x = 4$ and $x = 5$	Both $g(4) = \text{awrt } 2.4$ (or 2.3 truncated) and $g(5) = \text{awrt } -0.5$, sign change (accept equivalents e.g., $g(4) > 0$ & $g(5) < 0$ or $g(4)g(5) < 0$ or "positive, negative"), continuous and conclusion all given. Be generous with attempt at "continuous" and condone e.g., "x is continuous". Minimum in bold. Condone a wrong interval following e.g., "There is a solution in..." May use f for g.	A1
(2)			
(ii)(b)	E.g. $\frac{\beta - 4}{2.396...} = \frac{5 - \beta}{-0.502...} \Rightarrow \beta = ...$ $\frac{5 - \beta}{\beta - 4} = \frac{-0.502...}{2.396...} \Rightarrow \beta = ...$ $\frac{\beta - 4}{5 - 4} = \frac{2.396...}{2.396... - 0.502...} \Rightarrow \beta = ...$ $\frac{4(-0.502...) - 5(2.396...)}{-0.502... - 2.396...} = ...$ or $m = \frac{-0.502... - 2.396...}{5 - 4} \{ = -2.8986... \}$ $\Rightarrow \text{e.g., } y - 2.396... = m(x - 4), y = 0$ $\Rightarrow x \text{ or } \beta = ...$ If only unsubstituted "$\frac{ag(b) - bg(a)}{g(b) - g(a)}$" oe is seen followed by value it must round to 4.83 unless $a, b, g(a) \& g(b)$ are identified. May use f for g.	Uses a correct interpolation equation/expression for their values (allow for any value <u>provided $g(4)$ positive, $g(5)$ negative</u> and condone clear miscopying) and finds a value for β or x etc. Accept any correct statement for their values followed by any attempt to solve. If not fully substituted, values for $g(4)$ & $g(5)$ may be seen separately here or in part (a). May use f for g. Alternatively finds the gradient and then the line joining the endpoints and sets $y = 0$ to get β. Straight line equation must be correct for their values but allow a correct unsimplified gradient seen which is miscalculated later and allow errors finding c from a consistent equation with a correctly substituted point. Ignore how the value is labelled (may be unlabelled or x or α used). If their variable denotes e.g., the distance between $(4, 0)$ and $(\beta, 0)$ then 4 must be added later. Implied by awrt 4.83 provided no evidence of incorrect equation but must not clearly be using the actual root of 4.8245... from solving by calculator. Note that failure to change the sign of $g(5)$ leads to 5.265...	M1
	= 4.827 (4 s.f.)	awrt 4.827 Accept answer only. Ignore further interpolations.	A1
	(2)		
			Total 7

Question Number	Scheme	Notes	Marks
4(a)	For the first two marks if any ambiguous fractions within fractions are seen e.g., $\frac{-9}{t^2}$ then marks must be confirmed by appropriate later processing.		
	$xy = 81, P\left(9t, \frac{9}{t}\right) \Rightarrow \frac{dy}{dx} = -\frac{81}{x^2}$ or $\frac{dy}{dx} = \frac{-9}{t^2}$ or $x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ Any correct equation involving $\frac{dy}{dx}$ (or $\frac{dx}{dy}$). Accept just $\frac{dy}{dx}$ or $m_T = -\frac{1}{t^2}$. May see e.g. $x = \frac{81}{y} \Rightarrow \frac{dx}{dy} = -\frac{81}{y^2}$ or $-\frac{dx}{dy} = \frac{81}{y^2}$		B1
	e.g., $-\frac{81}{x^2} \rightarrow \frac{x^2}{81}, -\frac{81}{(9t)^2} \rightarrow \frac{(9t)^2}{81}, \frac{-9}{t^2} \rightarrow \frac{9}{t^2}, -\frac{y}{x} \rightarrow \frac{x}{y}, -\frac{1}{t^2} \rightarrow -\frac{1}{t^2} \Rightarrow t^2$ Applies correct perpendicular gradient rule to obtain a gradient for the normal. Allow in terms of t, x, y , or x and y . $\frac{dy}{dx}$ may be incorrect - just look for clear use of $-\frac{1}{m_T}$. Could just change the sign of $\frac{dx}{dy}$. If starts with just $m_N = t^2$ then 0110 max		M1
	$y - \frac{9}{t} = t^2(x - 9t)$ or $\frac{9}{t} = t^2 \times 9t + c \Rightarrow c = \dots \left\{ \frac{9}{t} - 9t^3 \right\}$	Correct straight line method with any changed gradient in terms of t . "Changed" gradient may just be the negative or reciprocal instead of negative reciprocal. Condone (for all marks) late substitution if gradient not initially in terms of t (but no "x"s or "y"s can be combined before this substitution)	M1
	$\Rightarrow ty - 9 = t^3x - 9t^4$ or $y = t^2x + \frac{9}{t} - 9t^3$ $\Rightarrow ty = t^3x + 9(1 - t^4)^*$	Correct equation reached with an intermediate step and no errors seen. Allow recovery of poor bracketing provided it is before the final answer. Allow e.g., $yt = 9(1 - t^4) + xt^3$ (yt must be on its own on one side, +9 must have been factorised out)	A1*
(4)			
(b)	$\{x = 0 \Rightarrow\} ty = 9(1 - t^4) \Rightarrow y = \dots$	Sets $x = 0$ in given equation of normal and obtains an expression in t for y (which may be incorrect)	M1
	$A\left(0, \frac{9}{t} \times (1 - t^4)\right)$ or oe e.g., $\left(0, \frac{9}{t} - 9t^3\right), \left(0, \frac{9 - 9t^4}{t}\right), \left(0, 9\left(\frac{1}{t} - t^3\right)\right)$	Correct answer - coordinates or $x = 0, y = \dots$ stated separately. Brackets may be missing. The $x = 0$ must be seen at some point in this part. Isw once a correct y is seen.	A1
(2)			

Question Number	Scheme	Notes	Marks	
4(c)	 $A: \left(0, \frac{9}{\frac{1}{3}} \left(1 - \frac{1}{3^4}\right)\right)$ or $\left(0, \frac{80}{3}\right)$ $\Rightarrow \{\text{Area } OPA =\}$ $\frac{1}{2} \times 9 \times \frac{1}{3} \times \frac{9}{\frac{1}{3}} \left(1 - \frac{1}{3^4}\right)$ or $\frac{1}{2} \times 9 \times \frac{1}{3} \times \frac{80}{3}$ " or $\frac{1}{2} \times 3 \times \frac{80}{3}$ " May see alternatives e.g., $27 \times 3 - \frac{1}{2} \times \left(27 - \frac{80}{3}\right) - \frac{1}{2} \times 3 \times 27$	Uses $t = \frac{1}{3}$ to obtain a positive value/expression for the y coordinate of A using their result from (b) provided it is a function of t and obtains a consistent exact numerical expression or value for the area of OPA. If there is no work calculating y_A then the value must be correct and exact. Note that subsequent work could recover an exact expression. The coordinate/coordinates of P that are used must be correct. Do not allow if any length is negative. May use "shoelace" algorithm - award once multiplications are set up e.g., $\frac{1}{2} \left 3 \times \frac{80}{3} - 0 \right$. If modulus is missing and expression within is negative, it must be corrected to positive. M0 if the x-axis intercept (-240) of the normal is used. Allow if correct method for area in terms of t followed by substitution to obtain a numerical expression or value e.g., $\left[\frac{1}{2} \times 9t \times \frac{9}{t} \left(1 - t^4\right) \right]_{t=\frac{1}{3}} \Rightarrow \dots$	M1	
	= 40	40 only. No equivalents. Allow if e.g., the y-coordinate of P is incorrect but not used	A1	
	If other alternatives for finding the area of the triangle are used work must be exact (or lead to exactly 40 - exactness can be recovered) to score any marks			
			(2)	
		Total 8		