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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Numerical Solution of Equations (FP1.3)

International A Level Mathematics · FP1: Further Pure Mathematics 1 (WFM01)

12 questions · 100 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

12 questions · 100 marks · 1 sub-topic

SUB-TOPIC FP1.3a

INTERVAL BISECTION, LINEAR INTERPOLATION & NEWTON-RAPHSON

Sub-topic FP1.3a · 12 questions · 100 marks · lowest tariff first



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1. (a) Show that the equation $4x - 2\sin x - 1 = 0$, where x is in radians, has a root α in the interval $[0.2, 0.6]$
(2)
- (b) Starting with the interval $[0.2, 0.6]$, use interval bisection twice to find an interval of width 0.1 in which α lies.
(3)

Question 1 continued

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(Total 5 marks)

Q1



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1. $f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \quad x > 0$

- (a) Show that the equation $f(x) = 0$ has a root α in the interval $[1.4, 1.5]$ (2)
- (b) Determine $f'(x)$. (3)
- (c) Using $x_0 = 1.4$ as a first approximation to α , apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α , giving your answer to 3 decimal places. (2)

Question 1 continued

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Q1

(Total 7 marks)



(Total for Question 3 is 7 marks)





Question 3 continued



Question 3 continued

(Total for Question 3 is 7 marks)



Question 4 continued



Question 4 continued



Question 4 continued

(Total for Question 4 is 8 marks)







(Total for Question 6 is 8 marks)



Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total 9 marks)

Q2







Question 2 continued

(Total for Question 2 is 9 marks)







Question 2 continued

(Total for Question 2 is 9 marks)



Question 1 continued

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Question 1 continued

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Question 1 continued

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(Total 10 marks)

Q1



Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 10 marks)







Question 7 continued

(Total for Question 7 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Question Number	Scheme	Notes	Marks
1(i)	$\mathbf{A} = \begin{pmatrix} 3k & 4k-1 \\ 2 & 6 \end{pmatrix}$		
(a)	$3k \times 6 - 2(4k-1) = 0 \Rightarrow k = \dots$ Forms $\det \mathbf{A} = 0$ and solves for k . The “= 0” can be implied by a solution for k . Award for $3k \times 6 - 2(4k-1) = 0 \Rightarrow k = \dots$ If LHS is only seen expanded 2 terms of $18k - 8k + 2$ must be correct (implied by $10k$) May use $ad = bc$ and condone $\det \mathbf{A} = bc - ad = 0$ but clear use of $ad + bc$ is M0		M1
	$(10k + 2 = 0 \Rightarrow k = -\frac{1}{5} \text{ or } -0.2$	A1: Correct value. Accept $-\frac{2}{10}$	A1
(2)			
(b)	$(\mathbf{A}^{-1} =) \frac{1}{10k+2} \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix} \text{ or } \begin{pmatrix} \frac{6}{10k+2} & \frac{1-4k}{10k+2} \\ \frac{-2}{10k+2} & \frac{3k}{10k+2} \end{pmatrix} \text{ or e.g., } \begin{pmatrix} \frac{3}{5k+1} & \frac{1-4k}{10k+2} \\ \frac{-1}{5k+1} & \frac{3k}{10k+2} \end{pmatrix}$ M1: for $\dots \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix}$ Ignore any multiplier and accept without one and condone if this matrix is labelled as $\mathbf{A}^{-1}$. Allow unsimplified e.g., $\dots \begin{pmatrix} 6 & -(4k-1) \\ -2 & 3k \end{pmatrix}$ Allow if determinant incorporated provided it is clear that the elements of $\text{Adj}(\mathbf{A})$ are correct A1ft: $\frac{1}{"10k+2"} \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix}$ Fully correct inverse ft their determinant in form $ak + b$ $a, b \neq 0$ and simplified but if determinant incorporated there is no requirement to write e.g., $\frac{6}{10k+2}$ as $\frac{3}{5k+1}$. Allow different brackets e.g., $[\dots]$, $\{\dots\}$ but $ \dots $ is M0 if followed by an attempt at $\det(\text{Adj}(\mathbf{A}))$. Allow if “ $\times$ ” is between fraction and matrix and allow fraction to appear on the right of the matrix. Isw when a correct answer is seen but this mark is not available if they substitute a value of k into the determinant and/or matrix.		M1 A1ft
(2)			
(ii)(a)	$p = q = -2 \text{ or } (\mathbf{B} =) \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	Both values identified or correct matrix (any or no bracket). Allow “Both are -2” or “-2, -2”	B1
(b)	$p = -1 \quad q = 1 \text{ or } (\mathbf{B} =) \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Both values identified or correct matrix (any or no bracket). Allow “-1, +1” (Mark in order presented). No trig expressions.	B1
(2)			
Total 6			

Question Number	Scheme	Notes	Marks
2	$f(z) = z^3 - 13z^2 + 59z + p$		
(a)	$\begin{array}{r} [f(3) =] 3^3 - 13(3)^2 + 59(3) + p \\ \text{or e.g., } 27 - 117 + 177 + p \text{ or} \\ \phantom{27 - 117 + 177 + p \text{ or}} \underline{z^2 - 10z + 29} \\ z - 3 \overline{) z^3 - 13z^2 + 59z + p} \\ \phantom{z - 3 \overline{) z^3 - 13z^2 + 59z + p}} \underline{z^3 - 3z^2} \\ \phantom{z - 3 \overline{) z^3 - 13z^2 + 59z + p}} -10z^2 + 59z \\ \phantom{z - 3 \overline{) z^3 - 13z^2 + 59z + p}} \underline{-10z^2 + 30z} \\ \phantom{z - 3 \overline{) z^3 - 13z^2 + 59z + p}} 29z + p \\ \phantom{z - 3 \overline{) z^3 - 13z^2 + 59z + p}} \underline{29z - 87} \\ \phantom{z - 3 \overline{) z^3 - 13z^2 + 59z + p}} 0 \end{array}$	Attempts $f(3)$. Must see more than just $87 + p$ Allow one slip (e.g., a miscopy of one coefficient, or one incorrect value/sign if expression just given as $27 - 117 + 177 + p$) Alternatively long divides by $z - 3$ obtaining a 3TQ with two terms of $z^2 - 10z + 29$ correct. Could use synthetic division. An attempt at equating coefficients/factorising requires 2 correct values for the a, b and c of $az^2 + bz + c$	M1
	$f(3) = 0 \Rightarrow p = -87 *$	Obtains "$p = -87$" only with no errors but condone work in x "=0" must have been seen before $p = -87$ if $f(3)$ attempted but allow just $p = -87$ following a full and correct attempt via division/equating coefficients etc with no errors.	A1* (shown as B1 on ePen)
(2)			
(b)	Allow equivalent work in x. Allow use of a calculator to solve a quadratic. Solutions that just follow $z^3 - 13z^2 + 59z - 87 = 0$ score no marks. There are no marks if $z^2 - 10z + 29$ has clearly been produced by using $(z - (5 + 2i))(z - (5 - 2i))$		
	$(z^3 - 13z^2 + 59z - 87) \div (z - 3)$ $= \dots [z^2 - 10z + 29]$	M1: Uses $z \pm 3$ with $f(z)$ (not their $f(z)$) to obtain a 3TQ expression with evidence of any appropriate method including inspection (must be evidence of use of $z \pm 3$) or equating coefficients. Ignore any remainder if long division is used and may see $z^2 - 16z + 107$ ($r(-408)$) if $z + 3$ used. Must be seen or referred to in (b) A1: Correct quadratic	M1 A1
	$z = \frac{-(-10) \pm \sqrt{(-10)^2 - (4)(1)(29)}}{2(1)}$ or $(z - 5)^2 - 25 + 29 = 0 \Rightarrow z = 5 \pm \sqrt{-4}$	Solves their 3TQ arising from using $(z - 3)$ only as a factor (usual rules but allow if one correct root if calculator used on their quadratic) If a sum/product of roots method is used on their 3TQ (i.e., $2a = -(-10)$, $a^2 + b^2 = "29"$) it must be complete and condone only sign errors. Do not allow just $5 \pm 2i$ following an incorrect quadratic Requires previous M mark.	dM1
	$\left(z = \frac{10 \pm \sqrt{-16}}{2} = \right) 5 \pm 2i$	$5 \pm 2i$ or $5 + 2i$, $5 - 2i$ only. Not $5 \pm 2\sqrt{-1}$ Accept $\pm 2i + 5$	A1
(4)			

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the mark scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation but refer to scheme.

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ or}$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$

both leading to at least one solution $x = \dots$

2. Formula

Correctly use the correct formula with values for a, b and c to obtain at least one solution $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$; if $a \neq 1$ must divide by a first but allow equivalent work e.g., if a is a perfect square)

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$$

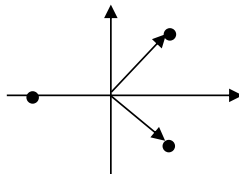
leading to at least one solution $x = \dots$



Question Number	Scheme	Notes	Marks
1	$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} \quad \mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$ Condone any brackets (or missing brackets) for matrices throughout. Allow clear miscopying slips (e.g., $-5-3a$ becomes $-3-5a$) to access M marks.		
(a)	$\{\det \mathbf{M}\} = -5-3a$	Correct determinant. Allow unsimplified	B1
	$\{\mathbf{M}^{-1}\} = \frac{1}{-5-3a} \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}$ or $\begin{pmatrix} \frac{-5}{-5-3a} & \frac{-a}{-5-3a} \\ \frac{-3}{-5-3a} & \frac{1}{-5-3a} \end{pmatrix}$ May see e.g., $\frac{-1}{5+3a} \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}, \frac{1}{5+3a} \begin{pmatrix} 5 & a \\ 3 & -1 \end{pmatrix}$	M1: For $\frac{1}{\pm 5 \pm 3a} \times$ a changed matrix or a correct $\text{Adj}(\mathbf{M})$ seen i.e., $\dots \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}$ A1: Any correct inverse	M1 A1
	(3)		
(b)	$\left\{ \frac{1}{-5-3a} \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix} \right\} = 2 \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} + 8 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ 6 & -10 \end{pmatrix} + \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 10 & 2a \\ 6 & -2 \end{pmatrix}$ Substitutes $\mathbf{M}$ into the RHS of the equation and combines matrices appropriately obtaining one correct element which may be unsimplified e.g., $2+8$. Apply BOD if only see e.g., $-5 = 2(-5-3a) + 8$ but next mark is M0. Allow equivalent work if writes as e.g., $\mathbf{M}^{-1} - 2\mathbf{M} = +8\mathbf{I}$ so $\frac{-5}{-5-3a} - 2 = 8$ implies the $2+8$. Condone use of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ for $\mathbf{I}$. If $\mathbf{I} = \mathbf{0}$ then M0. Note that it is fine to just use 1 element and proceed directly to an equation.		M1
	e.g., $\Rightarrow -5 = 10(-5-3a)$ or $-5 = -50-30a$ or $5 = 10(5+3a) \Rightarrow a = \dots \left\{ -\frac{3}{2} \right\}$ Obtains a non-zero value for a from a consistent "extracted" equation (with no fraction). If the fraction is not dealt with (going straight to a value for a) it must be consistent. Equation must come from $\frac{1}{\pm 5 \pm 3a} \times$ a changed matrix = credible attempt at $2\mathbf{M} + 8\mathbf{I}$ M0 for equating to a zero element of their $2\mathbf{M} + 8\mathbf{I}$. Accept a value following an appropriate equation. If no equation must correctly get $-\frac{3}{2}$ oe. Ignore usual rules if equation is quadratic. If RHS is only seen multiplied out it must be consistent with the work. Condone use of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. If $\mathbf{I} = \mathbf{0}$ then M0. May see: $-a = 2a(-5-3a), -3 = 6(-5-3a), 1 = -2(-5-3a) \Rightarrow a = \dots$ $a = 2a(5+3a), 3 = 6(5+3a), -1 = -2(5+3a) \Rightarrow a = \dots$		M1
	A1: $-\frac{3}{2}$ or $-1\frac{1}{2}$ or -1.5 and allow equivalent fractions e.g., $-\frac{45}{30}$ Requires both previous marks and no sight of incorrect $\mathbf{I}$. Must come from correct work for the element used. Ignore any extra equations or checks if they are wrong but unused. But do not isw if any extra incorrect unrejected solutions are offered e.g., $a = 0$ from $6a^2 + 9a = 0$		A1
	(3)		
	Some alternatives for (b) are shown overleaf		Total 6

1(b)	$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} \quad \mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$	
Alt 1	$\mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I} \Rightarrow \mathbf{I} = 2\mathbf{M}^2 + 8\mathbf{M} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 6a+10 & 0 \\ 0 & 6a+10 \end{pmatrix} \Rightarrow 6a+10=1 \Rightarrow a = -\frac{3}{2}$ M1: Achieves <u>$pa+q$</u> M1: Solves $pa+q=1$ A1: Correct value (no others or incorrect I)	
Alt 2	$\det \mathbf{M}^{-1} = \det(2\mathbf{M} + 8\mathbf{I}) \Rightarrow \frac{1}{-5-3a} = \frac{-20-12a}{-5-3a} \Rightarrow 36a^2 + 120a + 99 = 12a^2 + 40a + 33 = (2a+3)(6a+11) = 0 \Rightarrow a = -\frac{3}{2}$ M1: Achieves <u>$pa+q$</u> M1: Solves $\frac{1}{\pm 5 \pm 3a} = pa+q$ A1: Correct value and no others	
Others	Determinants may also be used to form the equation in Alt 1 i.e., $1 = (6a+10)^2$ Another (unlikely) possibility is to equate the traces of the matrices: $\frac{1}{-5-3a}(-5+1) = 10+(-2) \Rightarrow -4 = -40-24a \Rightarrow a = -\frac{3}{2}$	

Question Number	Scheme	Notes	Marks
2	$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ Condone any brackets (or missing brackets) for matrices throughout		
(a)	Rotation...	Identifies the transformation as a rotation. Allow any reasonable attempt at this word e.g., "rotate". M0 for a combination of transformations or if any different types of transformation are given as alternatives. Note that giving an angle does not imply "rotation".	M1
	...(anti-clockwise) of $\frac{3\pi}{2}$ (270°) or clockwise of $\frac{\pi}{2}$ (90°) about/around/from (etc.) O Correct full description for the rotation, including angle & direction (if no direction is stated then assume anticlockwise is meant) and any mention of origin or O or (0, 0). Condone "original" for "origin". Accept in degrees (symbol not required) or exact radians. {Note also that $-\frac{\pi}{2}$ (-90°) or $-\frac{3\pi}{2}$ (-270°) clockwise are acceptable}		A1
	(2)		
(b)	Enlargement...	Identifies the transformation as an enlargement. Allow any reasonable attempt at this word e.g., "inlarge", "large". Nothing else so e.g. "stretch" is M0. M0 for a combination of transformations or if any different types of transformation are given as alternatives.	M1
	...of scale factor/factor/scale/size 5, centre/from (etc.) O	Correct full description for the enlargement including scale factor and centre. Allow any mention of "5" and any mention of origin, O or (0, 0) Condone "original" for "origin".	A1
	(2)		
(c)	$\{\mathbf{R}=\}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	Correct matrix	B1
	(1)		
(d)	$\left\{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix}=\right\}(-3, -4)$	Correct coordinates for A. Brackets may be missing. May be stated as $x = \dots, y = \dots$ Condone answer given as a vector. Isw if necessary. Score B0 if an incorrect R has clearly been used in this part. Allow if correct answer comes from multiplying the wrong way round. No ft.	B1
	(1)		

Question	Scheme	Marks	
	$f(z) = 4z^3 + pz^2 - 24z + 108, -3$ a root.		
3(a)	$f(-3) = 0 \Rightarrow 4(-3)^3 + p(-3)^2 - 24(-3) + 108 = 0 \Rightarrow p = \dots$	M1	
	$p = -8$	A1	
		(2)	
(b)	$4z^3 - 8z^2 - 24z + 108 = (z + 3)(4z^2 + \dots z + 36)$	M1	
	$= (z + 3)(4z^2 - 20z + 36)$	A1	
	$4z^2 - 20z + 36 = 0 \Rightarrow z = \frac{20 \pm \sqrt{400 - 4 \times 4 \times 36}}{8} = \dots$	dM1	
	Roots are $-3, \frac{5 \pm i\sqrt{11}}{2}$	A1	
		(4)	
(c)	e.g. Product of complex roots is $\frac{36}{4} = 9$, so modulus is $\sqrt{9}$ or Modulus is $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{\sqrt{11}}{2}\right)^2}$	M1	
	Hence modulus is 3	A1	
		(2)	
(d)		Complex conjugate pair in correct quadrant for their roots	M1
		All three roots correctly positioned.	A1
			(2)
(10 marks)			

Notes:

Mark the question as a whole - do not be concerned part labelling.

(a)

 M1: A complete method to find the value of p . Use of the factor theorem is most direct, look for setting $f(-3) = 0$ and solving for p . May attempt to factor out $(z + 3)$ and compare coefficients, e.g.

$$f(z) = 4z^3 + pz^2 - 24z + 108 = (z + 3)(4z^2 + bz + 36) \Rightarrow 3b + 36 = -24, 12 + b = p \Rightarrow b = \dots, p = \dots \text{ or}$$

may attempt long division and set remainder equal to zero to find p or variations on these.

A1: For $p = -8$

(b)

Note: Allow marks in (b) for work seen in (a) e.g. via attempts in (a) by long division.

M1: Correct strategy to find a quadratic factor. If factorising, look for correct first and last terms. May use long division, in which case look for the correct first term and attempt to use it - may have been seen in (a).

Question instructs use of algebra so an algebraic method must be seen.

A1: Correct quadratic factor - may have been seen in (a).

dM1: Uses the quadratic formula or completing the square or calculator to find the roots of their quadratic factor (allow for attempts at a quadratic factor via long division which had non-zero remainder). If a calculator is used (no method shown), there must be at least one correct **complex** root for their equation. Factorisation is M0.

A1: Correct roots in simplest form. All three should be included at some point in the solution in (b).

(c)

M1: Any correct method to find the modulus of the complex roots. Most likely to see Pythagoras, but some may deduce from product of roots. They must have complex roots to score the marks in (c).

A1: Modulus 3 only. If -3 is also given as a modulus then score A0.

(d)

Note: Allow the marks in (d) if the i 's were missing in their roots in (b) but they clearly mean the correct complex roots on their diagram.

M1: Plots the complex roots as a conjugate pair in the correct quadrants for their roots.

A1: Fully correct diagram with one root on the negative real axis, and the other as a complex pair of roughly the same length in quadrants 1 and 4.

Question	Scheme	Marks
4(a)	(i) $f'(x) = \underline{Ax^{-5}} + \underline{Bx^{\frac{9}{2}}}$ oe for at least one power	M1
	$f'(x) = -\frac{4x^{-5}}{8} + \frac{2 \times \frac{7}{2} x^{\frac{9}{2}}}{7} = \frac{1}{2x^5} - \frac{1}{x^{\frac{9}{2}}}$ oe	A1
	(ii) Since $f'(0.25) = 512 - 512 = 0$ the process cannot be carried out as it would require division by zero.	B1
	(iii) $\alpha = 0.15 - \frac{f(0.15)}{f'(0.15)} = 0.15 - \frac{-27.332...}{1484.137...} = ...$	M1
	$= 0.168$ to 3 d.p.	A1cso
		(5)
(b)	e.g. $\frac{f(0.25) - f(0.15)}{0.25 - 0.15} = \frac{f(0.15) - 0}{0.15 - \alpha}$ or $\frac{\alpha - 0.25}{0 - f(0.25)} = \frac{\alpha - 0.15}{0 - f(0.15)}$ etc	M1
	$\Rightarrow \alpha = 0.15 - \frac{0.1 \times f(0.15)}{f(0.25) - f(0.15)} = ...$ or $\alpha = \frac{0.25f(0.15) - 0.15f(0.25)}{(f(0.15) - f(0.25))} = ...$	M1
	etc	
	$= 0.15 - \frac{0.1 \times -27.332...}{5.571... - (-27.332)} = 0.23306... = \text{awrt } 0.233$ (3 d.p.)	A1
		(3)
(8 marks)		

Notes:**(a)(i)****M1:** Attempts to differentiate $f(x)$, obtaining the correct power for at least one term.**A1:** Correct differentiation, need not be simplified.**(ii)****B1:** Correct reason given, accept e.g. “as $f'(0.25) = 0$ ” as a minimum and isw after a correct reason is given. Just stating $f'(0.25) = 0$ is not sufficient, there must be an indication this is the reason why the process cannot be used but accept any indication (such as “not valid”) following this.**(iii)****M1:** Correct Newton-Raphson process attempted using their derivative or implied by the correct answer from use of a calculator.**A1cso:** Correct answer from correct work (derivative must have been correct). Must be 3dp.**(b)****M1:** Correct interpolation strategy. Accept any correct statement such as the one shown. They may use e.g. x for $\alpha - 0.15$, in which case the method will be gained once the correct overall strategy is clear.**M1:** Proceeds from a recognisable attempt at interpolation to find a value for α . Not dependent, but they must have attempted to set up a suitable equation in α . If no working is shown accept any value for the root following the suitable statement.

A1: Accept awrt 0.233 following correct working.



Question	Scheme	Marks
6.(i)	$f(x) = x - 4 - \cos(5\sqrt{x}) \quad x > 0$	
(a)	$f(2.5) = -1.44\dots, f(3.5) = 0.497\dots$	M1
	Sign change (negative, positive) and $f(x)$ is continuous therefore (a root) α is between $x = 2.5$ and $x = 3.5$	A1
		(2)
(b)	E.g. $\frac{\alpha - 2.5}{ f(2.5) } = \frac{3.5 - \alpha}{f(3.5)} \Rightarrow \alpha = \dots$ or $\frac{\alpha - 2.5}{0 - f(2.5)} = \frac{3.5 - 2.5}{f(3.5) - f(2.5)} \Rightarrow \alpha = \dots$	M1
	$\alpha = \text{awrt } 3.24$	A1
		(2)
(ii)	$g(x) = \frac{1}{10}x^2 - \frac{1}{2x^2} + x - 11 \quad x > 0$	
(a)	$g'(x) = \frac{1}{5}x + \frac{1}{x^3} + 1$	M1
		A1
		(2)
(b)	$x_1 = 6 - \frac{g(6)}{g'(6)} = 6 - \frac{-1.41388\dots}{2.20462\dots}$	M1
	$= 6.641$	A1cao
		(2)
(Total 8 marks)		

Notes

(i)(a)

M1: Attempts both $f(2.5)$ and $f(3.5)$ with at least one correct in either radians or degrees. Note that in degrees $f(2.5) = -2.49\dots$ and $f(3.5) = -1.4867\dots$

A1: Both $f(2.5) = \text{awrt } -1$ and $f(3.5) = \text{awrt } 0.5$, sign change (accept $f(2.5)f(3.5) < 0$), continuous and conclusion all given but be forgiving with exact language. Use of degrees will be A0 as there is no change in sign.

(b)

M1: Uses a correct interpolation method to find a value for α . There are other alternative versions but look for a correct full process. E.g. may attempt the equation of the line through the two end points, then substitute $y = 0$ to find x . Allow if using degrees so long as a correct interpolation statement is clear.

A1: Correct value, accept awrt 3.24.

(ii)(a)

M1: $x^n \rightarrow x^{n-1}$ in at least two of the first 3 terms.

A1: All correct simplified or unsimplified.

(b)

M1: Correct application of Newton-Raphson. If no expression is seen, the method may be implied by a correct answer. (Look for the process rather than labelling if they write f but use g .)

A1cao: Correct value. Must be to 3d.p.. ISW if they try a second application of N-R.

Note: If correct answers for (b) appear after an incorrect derivative then please send to review.



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the mark scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$; if $a \neq 1$ must divide by a first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Question	Scheme	Marks
4(a)	$4 - 3i$	B1
		(1)
(b)	$(x - (4 + 3i))(x - (4 - 3i)) = \dots$	M1
	$x^2 - 8x + 25$	A1
		(2)
(c)	E.g. Product of roots is 225, so product of real roots is $\frac{225}{25} = 9$ Or $x^4 + Ax^3 + Bx^2 + Cx + 225 = (x^2 - 8x + 25)(x^2 + \dots + 9)$	M1
	Hence (as root is positive) repeated real root is 3	A1
		(2)
(d)	$(x^2 - 8x + 25)(x^2 - 6x + 9)$ $= x^4 - 6x^3 + 9x^2 - 8x^3 + 48x^2 - 72x + 25x^2 - 150x + 225$	M1
	$= x^4 - 14x^3 + 82x^2 - 222x + 225$ Two correct middle term coefficients	A1
	So $A = -14$, $B = 82$ and $C = -222$ (or accept in the quartic)	A1
		(3)

(8 marks)**Notes:****(a)****B1:** For $4 - 3i$ **(b)****M1:** Correct strategy to find a quadratic factor. May expand as shown in scheme, or may look for sum of roots and product of roots first and then write down the factor.**A1:** Correct quadratic factor. Can be written down – give M1A1 if correct, M0A0 if incorrect.

Ignore “= 0” with their quadratic factor.

Alt for (b):**M1:** Product of complex roots is 25, so product of real roots is $\frac{225}{25} = 9$, so the (positive) real root is**A1:** $x^2 - 6x + 9$ or $(x - 3)^2$ **(c)****M1:** A complete strategy to deduce the real root or its square. May consider product of roots, as in scheme, or may first attempt to factorise/long division to find the other quadratic factor – award at the point the quadratic factor with real roots is found. May have been seen in (b)**A1:** Real root is 3. (No need to see rejection of the negative possibility.)

Not a “show that” so award M1A1 if correct root is written down with no working.



Question Number	Scheme	Notes	Marks
7(a)	$f(x) = x^{\frac{3}{2}} + x - 3$ $f(1) = 1 + 1 - 3 = -1 \quad f(2) = \sqrt{8} + 2 - 3 = 1.828...$	Calculates values for both $f(1)$ and $f(2)$ with one correct. Allow e.g., $f(2) = 2\sqrt{2} - 1$ or awrt 2	M1
	f is continuous and changes sign , so root or α in $[1, 2]$. Correct interval $[1, 2]$ if given. Sign change can be implied by “negative, positive”, “ $f(1) < 0, f(2) > 0$ ” or “ $f(1)f(2) < 0$ ”	Correct values and sight of continuous, sign change and e.g., root/shown/QED/true/proven/✓	A1
			(2)
(b) Work may be seen in a table	$f(1.5) = 1.5^{\frac{3}{2}} + 1.5 - 3 = \dots \{ \dots 0.3371 \dots \}$	Obtains a <u>numerical expression or value</u> for $f(1.5)$	M1
	$f(1.25) = 1.25^{\frac{3}{2}} + 1.25 - 3 = \dots \{ -0.3524 \dots \}$	Obtains a <u>value</u> for $f(1.25)$. Requires previous M mark.	dM1
	$\Rightarrow$ root/ α /it's in/on/ $\in [1.25, 1.5]$ or “in $[1.25, 1.5]$ ” or $1.25 \leq \text{root}/\alpha/x \leq 1.5$	Correct values (awrt 0.3 and -0.3 or -0.4) and suitable conclusion. Allow “between $\frac{5}{4}$ and $\frac{3}{2}$ inclusive ”	A1
	Do not accept $[1.5, 1.25]$. Just “ $f(1.25) = \dots$ followed by $f(1.5) = \dots$ so...” is 100 if no evidence of interval bisection. There are no marks if it is a clear attempt at interpolation.		(3)
(c)(i)	$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 1$	Correct differentiation. Any correct equivalent e.g., $1.5\sqrt{x} + 1$	B1
(ii)	$\alpha \approx 1.375 - \frac{1.375^{\frac{3}{2}} + 1.375 - 3}{\frac{3}{2} \times 1.375^{\frac{1}{2}} + 1} = \dots$ $\left\{ = 1.375 - \frac{-0.01266958256\dots}{2.75890591\dots} = 1.375 + 0.004592248875\dots \right\}$ $= 1.379592249\dots$ $\left\{ \text{exact values: } \frac{11}{8} - \frac{11\sqrt{22} - 52}{32} \div \frac{8 + 3\sqrt{22}}{8} \right\}$	Correctly applies the Newton-Raphson formula with 1.375 & their $f'(x)$ and obtains a value. Some working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow “ $\dots = 1.375 - \frac{f(1.375)}{f'(1.375)}$ ” followed by value but formula must be fully substituted if just followed by value unless “ x_0 ” defined	M1
	awrt 1.380 or “1.38” (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		(3)
(d)	e.g., $\frac{\alpha - 1.25}{1.5 - \alpha} = \frac{0.3524575141\dots}{0.3371173071\dots}$ or e.g., $\frac{1.5 - \alpha}{0.337\dots} = \frac{1.5 - 1.25}{0.337\dots + 0.352\dots}$	Forms an equation in e.g., α with their $f(1.25)$ and $f(1.5)$ allowing for sign errors only but must be using differences. Allow use of “ $f(1.25)$ ” and “ $f(1.5)$ ” - could recover sign error	M1
	$\alpha = 1.377780737\dots = 1.378$	dM1: Solves $\Rightarrow$ value Requires previous M mark. A1: awrt 1.378	dM1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		(3)
Alt (Equation of line methods)	or $y - (-0.3524\dots[\text{or } 0.3371\dots]) = \frac{0.3371\dots - (-0.3524\dots)}{1.5 - 1.25}(x - 1.25[\text{or } 1.5])$ or $-0.3524\dots[\text{or } 0.3371\dots] = \frac{0.3371\dots - (-0.3524\dots)}{1.5 - 1.25}(1.25[\text{or } 1.5]) + c \Rightarrow c = \dots$		M1
	A full method to determine the equation of the line using their $f(1.25)$ and $f(1.5)$ allowing for sign errors only (but allow subsequent errors finding c if $y = mx + c$ used)		
	$\{\Rightarrow y = 2.758\dots x - 3.800\dots\}$ $\alpha = 1.377780737\dots = 1.378$	dM1: Puts $y = 0$ and solves $\Rightarrow$ value Requires previous M mark. A1: awrt 1.378	dM1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		(3)
			Total 11