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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Coordinate Systems: Parabolas & Hyperbolas (FP1.4)

International A Level Mathematics · FP1: Further Pure Mathematics 1 (WFM01)

21 questions · 195 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

21 questions · 195 marks · 1 sub-topic

SUB-TOPIC FP1.4a

PARABOLAS & RECTANGULAR HYPERBOLAS

Sub-topic FP1.4a · 21 questions · 195 marks · lowest tariff first



Question 9 continued

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Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS



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3. The parabola C has equation $y^2 = 18x$

The point S is the focus of C

- (a) Write down the coordinates of S

(1)

The point P , with $y > 0$, lies on C

The shortest distance from P to the directrix of C is 9 units.

- (b) Determine the exact perimeter of the triangle OPS , where O is the origin.

Give your answer in simplest form.

(4)

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Question 3 continued

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Question 3 continued

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(Total 5 marks)

Q3



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 6 marks)



- The point P with coordinates $\left(3t, \frac{3}{t}\right)$, where $t \neq 0$, lies on H

- Give your answer in the form $ty - t^3x = f(t)$

(b) determine the coordinates of the point where the normal meets H again.

Give your answer in simplest form.

(3)



Question 3 continued

(Total for Question 3 is 7 marks)



7. The parabola C has equation $y^2 = \frac{4}{3}x$

The point $P\left(\frac{1}{3}t^2, \frac{2}{3}t\right)$, where $t \neq 0$, lies on C .

(a) Use calculus to show that the normal to C at P has equation

$$3tx + 3y = t^3 + 2t \quad (3)$$

The normal to C at the point where $t = 9$ meets C again at the point Q .

(b) Determine the exact coordinates of Q . (4)

Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 7 marks)



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4. A rectangular hyperbola H has equation $xy = 25$

The point $P\left(5t, \frac{5}{t}\right)$, $t \neq 0$, is a general point on H .

- (a) Show that the equation of the tangent to H at P is $t^2y + x = 10t$

(4)

The distinct points Q and R lie on H . The tangent to H at the point Q and the tangent to H at the point R meet at the point $(15, -5)$.

- (b) Find the coordinates of the points Q and R .

(4)

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Question 4 continued

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Q4

(Total 8 marks)



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 8 marks)

Q6







Question 9 continued

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Question 9 continued

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(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



- The point $P\left(9t, \frac{9}{t}\right)$, $t \neq 0$, lies on C .

- $$ty = t^3x + 9(1 - t^4) \quad (4)$$

(b) Determine the exact coordinates of A in terms of t . (2)

Given that $t = \frac{1}{3}$

- (c) find the exact area of triangle OPA , where O is the origin. (2)



Question 4 continued

Lined area for writing the answer to Question 4.



Question 4 continued

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(Total for Question 4 is 6 marks)



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Question 5 continued

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Q5

(Total 9 marks)



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total 9 marks)

Q7





Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 9 marks)



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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The line l with equation $3x - 4y + 48 = 0$ is a tangent to C at the point P .

- (a) Show that $a = 9$ (4)

- (b) Hence determine the coordinates of P . (2)

Given that the point S is the focus of C and that the line l crosses the directrix of C at the point A ,

- (c) determine the exact area of triangle PSA . (4)

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Question 7 continued

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Q7

(Total 10 marks)



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Question 8 continued

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Question 8 continued

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Q8

(Total 10 marks)





Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)







8. The point $P(2p^2, 4p)$ lies on the parabola with equation $y^2 = 8x$

(a) Show that the point $Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$, where $p \neq 0$, lies on the parabola.

(1)

(b) Show that the chord PQ passes through the focus of the parabola.

(4)

The tangent to the parabola at P and the tangent to the parabola at Q meet at the point R

(c) Determine, in simplest form, the coordinates of R

(8)

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 13 marks)

Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9 continued.





Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

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Question 8 continued

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(Total 14 marks)

Q8



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Question 6 continued

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Question 6 continued

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(Total 16 marks)

Q6



MARK SCHEMES

official mark schemes, in the same order as the questions



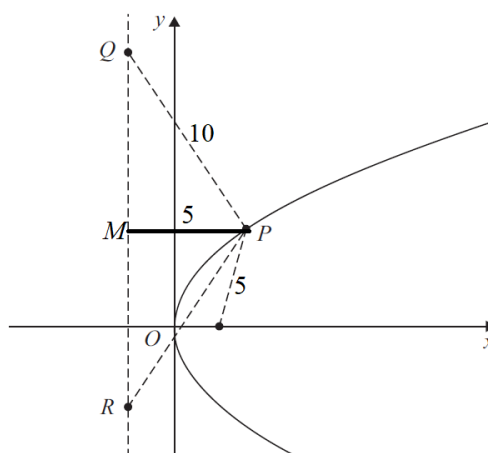
Question Number	Scheme	Notes	Marks
8(b)	$\sum_{r=1}^1 r(3r-2)^2$ or LHS = $1(3 \times 1 - 2)^2 = 1$ and $\frac{n^2(n+1)(9n-7)}{4}$ or RHS = $\frac{1^2 \times 2 \times 2}{4} = 1$ Achieves 1 from two numerical expressions (both not just "1"). No requirement here to say "true" etc. Allow as minimum, e.g., $1 \times 1 = \frac{4}{4} = 1$. "(When $n = 1$), both =1" is B0 but final A1 available.		B1
	{Assume true for $n = k$, then} $\sum_{r=1}^{k+1} r(3r-2)^2 = \sum_{r=1}^k r(3r-2)^2 + (k+1)(3(k+1)-2)^2$ $= \frac{k^2(k+1)(9k-7)}{4} + (k+1)(3k+1)^2$	Adds an attempt at the $(k+1)$ th term to an attempt at the sum to k terms. Allow clear copying slips (e.g., losing the squared from the k^2) but must be a recognisable attempt at forming $\frac{k^2(k+1)(9k-7)}{4} + (k+1)(3(k+1)-2)^2$ So e.g., ... + $k(3k-2)^2$ is M0	M1
	$= \frac{(k+1)}{4}(9k^3 - 7k^2 + 4(9k^2 + 6k + 1)) = \frac{(k+1)}{4}(9k^3 + 29k^2 + 24k + 4)$ $\left\{ \text{or } = (k+1) \left(\frac{9}{4}k^3 + \frac{29}{4}k^2 + 6k + 1 \right) \right\}$ OR $= \frac{1}{4}(9k^4 + 38k^3 + 53k^2 + 28k + 4) \left\{ \text{or } = \frac{9}{4}k^4 + \frac{19}{2}k^3 + \frac{53}{4}k^2 + 7k + 1 \right\}$ Reaches $\frac{(k+1)}{4}$ (4 term cubic in k) or $(k+1)$ (4 term cubic in k) OR expands to a 5 term quartic. Must collect terms. Condone poor algebra. Previous mark required		dM1
	$= \frac{1}{4}(k+1)((k+1)(9k^2 + 20k + 4))$ or $\frac{1}{4}(k+1)^2(k+2)(9k+2) \Rightarrow \frac{(k+1)^2((k+1)+1)(9(k+1)-7)}{4}$ Correctly reaches the result completely in terms of $k+1$ [but allow $k+2$ or $k+1+1$ for $((k+1)+1)$] with an intermediate step . Condone poor notation and allow the odd recovered algebraic slip/poor bracketing provided they reach the correct $\frac{k+1}{4}$ (4 term cubic in k) or $(k+1)$ (4 term cubic in k) or 5 term quartic and there are no subsequent errors. Factorisation may be achieved via calculator use. Allow a final answer of $\frac{1}{4}(k+1)^2(k+2)(9(k+1)-7)$ and with brackets in any order. Meet in the middle approaches must clearly join and the final expression in $k+1$ must be seen in the work		A1
	<u>True for $n = 1$</u> and <u>if true for $n = k$ then</u> it is also <u>true for $n = k + 1$</u> so by induction it is <u>true for (all) n</u> Acceptable proof (i.e., must have scored the previous A) and narrative/conclusion. Requires previous three marks and can only follow B0 if B0 was given for insufficient evidence of substitution. There must be no errors if one substitution was attempted and must have reached both sides = 1. "Assume (true) for $n = k$ " or "If true for $n = k$ " in narrative followed by "true for $n = k + 1$ " is sufficient for the "if...then". Allow suitable surrogates for "true". Allow stating the result from the question paper or " P_n is true" <u>with added reference to n</u> . Condone $n \in \square$ but not $n \in \square$. Allow work in n rather than k throughout. There is no credit for attempts using summation formulae instead of induction unless there is work that can score via the scheme above		A1

(5)



Question Number	Scheme	Notes	Marks
8(c)	$8 \sum_{r=1}^n r(3r-2)^2 = 15 \sum_{r=1}^{2n} (2r^2-1) \Rightarrow$ $8 \times \frac{n^2(n+1)(9n-7)}{4} = 15 \times \frac{4}{3} n(n+1)(4n-1)$ Attempts to form the given equation, obtaining an equation in n only with their answer to (a) which must be cubic but not necessarily in the right form. (Note - part (a) could have been reattempted in this part but do not allow attempts that seek to recreate an equivalent for the answer to (b) via summation formulae). Allow if 8 and 15 are swapped or if one is missing (but other correctly placed) provided there are no other errors. If 8 and 15 correctly placed, condone one minor copying error but must have quartic = cubic unless n instead of n^2 on LHS is the only error. Note that if they attempt to replace the 'n's in their answer to (a) with '2n's then award M0 unless they have used n throughout part (a) in which case all marks are potentially available. If e.g., $15 \times \frac{4}{3}$ is only seen evaluated it must be correct. Allow with "a" and "b" or invented values used but no further marks.		M1
	$\Rightarrow 2n(9n-7) = 20(4n-1)$ $\Rightarrow 18n^2 - 94n + 20 = 0$ $\Rightarrow 9n^2 - 47n + 10 = 0$ $\Rightarrow (9n-2)(n-5) = 0$ $\Rightarrow n = \dots$ May see, e.g. $18n^3 - 94n^2 + 20n = 0$ $9n^3 - 47n^2 + 10n = 0$	Simplifies to a 3TQ or 3 term cubic with no constant (see below if 4TC or quartic) and solves to find a value for n. If working is shown, apply usual rules (in this case solution does not have to be a positive integer). However, if answer/s are just written down one real positive integer root must be achieved and be correct for their 3TQ/3TC with no constant. If there is no 3TQ/3TC with no constant and $n = 5$ is just written down score 100. Do not allow solutions directly from just a quartic e.g., $9n^4 - 38n^3 - 37n^2 + 10n = 0$ or a cubic with constant e.g., $9n^3 - 38n^2 - 37n + 10 = 0$ unless there is a clear full method to factorise (e.g., factor theorem, long division, multiplication grid). Do not allow immediate factorisation. Previous mark required.	dM1
	$\{n=\}5$ Correct answer and no other unrejected solutions e.g., $n = \frac{2}{9}, 0, -1$ (Ignore any rejected incorrect values of n provided quadratic oe was correct and dM1 was scored) Answer may not be labelled and accept e.g., $x = 5$		A1
			Total 13

Question Number	Scheme	Notes	Marks
9	Let M be point where perpendicular to directrix from P meets the directrix, then $PM = 5$	States, uses or implies the horizontal distance from P to the directrix is 5. If indicated on Figure 1 or on their own diagram it must be clearly the horizontal distance from P to the directrix. Could be implied by a correct QM or QR . Do not be concerned about any preceding algebra. There is no credit for just forming expressions/equations in e.g., a and/or t until a numerical distance is found.	B1
	$QM = \sqrt{10^2 - 5^2} (= \sqrt{75} = 5\sqrt{3} \approx 8.66...)$ May see: $QM = 10 \sin \frac{\pi}{3}, 5 \tan \frac{\pi}{3}, 10 \cos \frac{\pi}{6}, \frac{5}{\tan \frac{\pi}{6}}$ $QM = 10 \sin 60^\circ, 5 \tan 60^\circ, 10 \cos 30^\circ, \frac{5}{\tan 30^\circ}$	Obtains a correct numerical expression for QM (not QM^2) with their $PM = k$ where $0 < k < 10$. Pythagoras must be fully correctly applied. Implied by QR . This mark is not available for arbitrarily choosing a value for an angle but apply BOD and potentially full marks if a correct angle is used without working.	M1
	$QR = 2 \times \sqrt{10^2 - 5^2} (= 2 \times 5\sqrt{3} \approx 17.3...)$ May see: $QR = \sqrt{10^2 + 10^2 - 2 \times 10 \times 10 \times \cos \frac{2\pi}{3}}$	A correct numerical expression for QR (not QR^2) with their $PM = k$ where $0 < k < 10$. Previous mark required.	dM1
	$\{QR =\} 10\sqrt{3}$	Correct answer. Allow any exact equivalent e.g., $2\sqrt{75}, \sqrt{300}$	A1
	Correct answer only or correct answer following no or minimal work scores 4/4 If trigonometry is used the scheme applies as above so all work must be correct for their k where $0 < k < 10$ but allow premature rounding.		
	(4)		

Total 4

Correct angles: $\angle MPQ = \angle MPR = \frac{\pi}{3} = 60^\circ$, $\angle MQP = \angle MRP = \frac{\pi}{6} = 30^\circ$, $\angle QPR = \frac{2\pi}{3} = 120^\circ$



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
3.(a)	$\frac{dy}{dx} = -\frac{1}{t^2}$	B1
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$	M1
	$\Rightarrow x = \dots, y = \dots$	M1
	$A(2ct, 0) \text{ and } B\left(0, \frac{2c}{t}\right)$	A1
		(4)
(b)	$\frac{1}{2} \times 2ct \times \frac{2c}{t} = 90 \Rightarrow c = \dots$	M1
	$c^2 = 45 \Rightarrow c = 3\sqrt{5}$	A1cso
		(2)
(Total 6 marks)		
Notes		
(a)		
B1: Correct $\frac{dy}{dx}$ in terms of t . May be implied at the point of substitution into the equation if only found in terms of x and/or y initially. Allow if just stated, no working needed (may have been memorised). You may see attempts where this is derived but they must get to or imply the correct derivative in terms of t .		
M1: Correctly forms the equation of the tangent. If no working for the gradient is shown accept $m = \pm \frac{1}{t^2}$ for this mark. If using $y = mx + c$ they must proceed at least as far as finding c .		
M1: Uses their “tangent” (which must be a straight line equation) to find the non-zero coordinates of A and B . Both must be attempted.		
A1: Both correct. Accept $A, x = 2ct, B, y = \frac{2c}{t}$ or list as separate coordinate as long as it is clear, but just $\left(2ct, \frac{2c}{t}\right)$ is A0. If coordinates are labelled the wrong way award A0 but allow both marks in (b).		
(b)		
M1: Uses their coordinates and the “90” correctly to form and solve an equation for c .		
A1cso: Correct value for c (must be simplified surd so $3\sqrt{5}$). Must have come from correct coordinates. A0 if the negative root is also given (and not rejected).		

Question Number	Scheme	Notes	Marks
1	$\sum_{r=1}^n r^2 (r+2) = \sum_{r=1}^n r^3 + 2 \sum_{r=1}^n r^2$ or $\sum_{r=1}^n r^3 + \sum_{r=1}^n 2r^2$	Correct split with 2 summations. Could be implied by correct work. Condone missing or incorrect summation limits.	B1
	$= \frac{1}{4} n^2 (n+1)^2 + 2 \times \frac{1}{6} n(n+1)(2n+1)$	Attempts to use both standard results and obtains an expression of the form $pn^2(n+1)^2 + qn(n+1)(2n+1)$ $p, q \neq 0$ Could be implied by immediate expansion	M1
	$= \frac{1}{12} n(n+1)[3n(n+1) + 4(2n+1)]$ $= \frac{1}{12} n(n+1)(3n^2 + 11n + 4)$	dM1: Attempts factorisation to obtain $\frac{1}{12} n(n+1)(an^2 + bn + c)$ $a, b, c \neq 0$. Condone poor algebra. Could follow cubic or quartic. Allow a consistent $a = \dots, b = \dots, c = \dots$ if quadratic never seen simplified Requires previous M mark. A1: Correct expression or $a = 3, b = 11, c = 4$ Allow e.g., $\frac{1}{12} n(n+1)$ written as $\frac{n}{12}(n+1)$	dM1 A1
	Note: $n(n+1)(3n^2 + 11n + 4) = 3n^4 + 14n^3 + 15n^2 + 4n$		Total 4

Question	Scheme	Marks
7.(a)	$x = \frac{1}{3}t^2, y = \frac{2}{3}t \Rightarrow \frac{dy}{dx} = \frac{2}{3} \div \frac{2}{3}t = \frac{1}{t}$ or $y^2 = \frac{4}{3}x \Rightarrow 2y \frac{dy}{dx} = \frac{4}{3} \Rightarrow \frac{dy}{dx} = \frac{2}{3y} = \frac{1}{t}$ or $y^2 = \frac{4}{3}x \Rightarrow y = \frac{2}{\sqrt{3}}\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{3}\sqrt{x}} = \frac{1}{t}$	B1
	$y - \frac{2}{3}t = -t \left(x - \frac{1}{3}t^2 \right)$	M1
	$3tx + 3y = t^3 + 2t^*$	A1*
		(3)
(b)	$t = 9 \Rightarrow 27x + 3y = 747$	B1
	$y^2 = \frac{4}{3}x \Rightarrow x = \frac{3y^2}{4} \Rightarrow 3y + 3 \times 9 \times \frac{3y^2}{4} = 729 + 18$ or $y^2 = \frac{4}{3}x \Rightarrow \frac{1}{9}(747 - 27x)^2 = \frac{4}{3}x \Rightarrow 729x^2 - 40350x + 558009 = 0$	M1
	$27y^2 + 4y - 996 = 0 \Rightarrow y = \dots$ or $729x^2 - 40350x + 558009 = 0 \Rightarrow x = \dots$	M1
	$y = -\frac{166}{27}, x = \frac{6889}{243}$	A1
		(4)
(b) ALT	$t = 9 \Rightarrow 27x + 3y = 747$	B1
	$9x + y = 249 \Rightarrow 3t^2 + \frac{2}{3}t = 249$	M1
	$9t^2 + 2t - 747 = 0 \Rightarrow t = \dots \left(-\frac{83}{9} \right)$	M1
	$y = -\frac{166}{27}, x = \frac{6889}{243}$	A1

(Total 7 marks)

Notes

(a)

B1: Correct $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of t from a calculus method. Must have seen a derivative used.**M1:** Correct straight line method for the normal – must be using $\frac{-1}{\text{their } m_t}$ (or other correct approach). If using $y = mx + c$ they must proceed at least as far as finding c .**A1:** cso – must have seen the evidence of use of calculus.

(b) + Alt (b)

B1: Correct equation for the normal at $t = 9$ **M1:** Solves normal and equation of C simultaneously to obtain a quadratic equation in x or y or substitutes the parametric form to obtain a quadratic in t .**M1:** Solves 3TQ in y to obtain a value (other than 6) or in x to obtain a value (other than 27) or in t to obtain a value (other than 9)

Question Number	Scheme	Notes	Marks
1	$f(x) = x^3 + 4x - 6$		
1(i)(a)	$f(1) = -1$ $f(1.5) = 3.375 \left(= \frac{27}{8} \right)$	Attempts to evaluate at both end points. If substitution not seen accept $f(1) = -1$ or $f(1.5) = 3.375$ as evidence, with any value for the other end.	M1
	Sign change and $f(x)$ continuous therefore α is between $x = 1$ and $x = 1.5$	$f(1) = -1$ and $f(1.5) = 3.375$ <u>both correct</u> and mentions/indicates <u>sign change</u> , <u>continuous</u> and <u>conclusion</u> .	A1
			(2)
1(i)(b)	$f'(x) = 3x^2 + 4$	Correct derivative. Can be implied by a correct expression seen later such as $3(1.5)^2 + 4$	B1
	$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{3.375}{10.75}$	Attempt Newton-Raphson using the correct formula.	M1
	$x_2 = 1.186...$	Accurate first application either awrt 1.186, $\frac{51}{43}$ or a correct numerical expression.	A1
	$x_3 = \left(1.186... - \frac{0.412...}{8.220...} \right) = 1.1358...$		
	$\alpha \approx 1.136$	cao	A1
			(4)
1(ii)	$g(1.4) = 3.442116... \text{ or } g(1.5) = -3.601419...$	Evidence of at least one value correct to 3 d.p. or better. May be implied by correct answer if never seen.	B1
	$\frac{1.5 - \beta}{\beta - 1.4} = \frac{0 - g(1.5)}{g(1.4) - 0} \text{ or }$ $\frac{\beta - 1.4}{0 - g(1.4)} = \frac{1.5 - 1.4}{g(1.5) - g(1.4)} \text{ oe}$	A correct linear interpolation statement as shown oe (with correct signs). May omit the zeroes or use evaluated values (e.g 0.1 instead of $1.5 - 1.4$). Other forms are possible.	B1
	$\Rightarrow \beta \approx 1.448869...$	Evaluates β from an attempt at a linear interpolation statement, allow if signs are incorrect. $\beta \approx$ awrt 1.449	M1 A1
			(4)
			Total 10

Question Number	Scheme	Notes	Marks
6(a)	$\left(\frac{a}{\sqrt{k}}, a\sqrt{k} \right)$	Correct coordinates – need not be simplified, so accept any equivalents.	B1
	$xy = a^2 \Rightarrow y = a^2 x^{-1}$ $\Rightarrow \frac{dy}{dx} = -a^2 x^{-2} = -a^2 \left(\frac{a}{\sqrt{k}} \right)^{-2} (= -k)$	Correct method for the gradient of the tangent at P . Must have substituted for x (and y) in their derivative.	M1
	$y - a\sqrt{k} = -k \left(x - \frac{a}{\sqrt{k}} \right)$ oe or $y = -kx + c \Rightarrow c = a\sqrt{k} + \frac{ka}{\sqrt{k}};$ $\Rightarrow y = -kx + 2a\sqrt{k}$ oe	M1: Correct straight line method for the tangent at P A1: Correct equation. Need not be fully simplified but do not accept $\sqrt{a^2}$ terms left unsimplified. ISW after a suitable correct equation seen.	M1A1
			(4)
(b)	$x = 0 \Rightarrow y = \dots \quad y = 0 \Rightarrow x = \dots$	Uses $x = 0$ and $y = 0$ to find A and B	M1
	$A \left(\frac{2a}{\sqrt{k}}, 0 \right) \quad B \left(0, 2a\sqrt{k} \right)$	Correct coordinates with same criteria as in (a).	A1
			(2)
(c)	Area = $\frac{1}{2} \times 2a\sqrt{k} \times \frac{2a}{\sqrt{k}} = \dots$	Fully correct strategy for the area	M1
	$= 2a^2$ Which is independent of k	All correct with conclusion	A1
			(2)
			Total 8

Question Number	Scheme/Notes	Marks
8	Condone work in n instead of k throughout	
8(i)	$n = 1 \left\{ \text{in} \begin{pmatrix} 1-3n & 9n \\ -n & 3n+1 \end{pmatrix} \right\} \Rightarrow \begin{pmatrix} 1-3 & 9 \\ -1 & 3+1 \end{pmatrix} = \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}$ Obtains $\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}$ or $\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^1$ with (minimal) substitution seen. One of $1-3$ or $1-3(1) \rightarrow -2$ or $9(1) \rightarrow 9$ or $-(1) \rightarrow -1$ or $3+1$ or $3(1)+1 \rightarrow 4$ is sufficient. No requirement to say "true" (oe) yet. Ignore further verifications for $n = 2$ etc.	B1
	$\{\text{Assume true for } n = k:\} \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^k = \begin{pmatrix} 1-3k & 9k \\ -k & 3k+1 \end{pmatrix}$	
	$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^{k+1} = \begin{pmatrix} 1-3k & 9k \\ -k & 3k+1 \end{pmatrix} \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix} \text{ or } \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1-3k & 9k \\ -k & 3k+1 \end{pmatrix}$ Completes an attempt to form $\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^{k+1}$ in terms of k	M1
	$= \begin{pmatrix} -2+6k-9k & 9-27k+36k \\ 2k-3k-1 & -9k+12k+4 \end{pmatrix} \text{ or } \begin{pmatrix} -2+6k-9k & -18k+27k+9 \\ -1+3k-4k & -9k+12k+4 \end{pmatrix} \text{ or } \begin{pmatrix} -2-3k & 9+9k \\ -k-1 & 3k+4 \end{pmatrix}$ Correct unsimplified or simplified matrix with no unexpanded expressions	A1
	$\left\{ = \begin{pmatrix} -2-3k & 9+9k \\ -k-1 & 3k+4 \end{pmatrix} \right\} = \begin{pmatrix} 1-3(k+1) & 9(k+1) \\ -(k+1) & 3(k+1)+1 \end{pmatrix}$ Reaches a correct matrix fully in terms of $k+1$ (terms in any order and allow for any $k+1$ to be written as $1+k$) with no errors. Meet in the middle approaches must be convincing. Requires previous two marks.	A1
	If the result is true for $n = k$ then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for (all) n. Correct conclusion/narrative. All the elements in bold should be satisfied. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. "Assume $n = k$" in the narrative followed by "true for $n = k+1$" in the conclusion plus "true for $n = 1$" and "true for (all) n" is sufficient. For the last statement allow "true for n", "true for $\square^+$", "true for $\square$" and condone "true for $\square$", "true for integers", "true for integers after 1" or similar but do not allow "true for all $n \in \square$" or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If the base case work is omitted or wrong in any other way then 01110 is the maximum available.	A1
		(5)
	Note that is valid to e.g., assume true for $n = k+1$ and show true for $n = k+2$: $\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^{k+2} = \begin{pmatrix} 1-3(k+1) & 9(k+1) \\ -(k+1) & 3(k+1)+1 \end{pmatrix} \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} -3k-2 & 9k+9 \\ -k-1 & 3k+4 \end{pmatrix} \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}$ $= \begin{pmatrix} 6k+4-9k-9 & -27k-18+36k+36 \\ 2k+2-3k-4 & -9k-9+12k+16 \end{pmatrix} = \begin{pmatrix} -3k-5 & 9k+18 \\ -k-2 & 3k+7 \end{pmatrix} = \begin{pmatrix} 1-3(k+2) & 9(k+2) \\ -(k+2) & 3(k+2)+1 \end{pmatrix}$ Use Review for any such approaches you are not sure about.	

Question Number	Scheme/Notes	Marks
8(ii)	$u_1 = 1 \quad u_2 = 4 \quad u_{n+2} = 6u_{n+1} - 9u_n \quad n \geq 1 \Rightarrow u_n = 3^{n-2}(n+2)$	
	$n = 1 \Rightarrow u_1 = 3^{-1}(2+1) = 1, \quad n = 2 \Rightarrow u_2 = 3^0(2+2) = 4$ Obtains $u_1 = 1$ and $u_2 = 4$ from $u_n = 3^{n-2}(n+2)$ with some substitution seen for both cases although it may be minimal. Look for any numerical expressions that give 1 and 4. No requirement to say "true" (oe) yet. Ignore work for u_3 and beyond	B1
	$\{ \text{Assume true for } n = k \text{ and } n = k + 1: \}$ $u_k = 3^{k-2}(k+2) \text{ and } u_{k+1} = 3^{k-1}(k+3)$	
	$u_{k+2} = 6u_{k+1} - 9u_k = 6 \times 3^{k-1}(k+3) - 9 \times 3^{k-2}(k+2)$ Attempts u_k and u_{k+1} using $u_n = 3^{n-2}(n+2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+2} in terms of k	M1
	$= 6 \times 3^k + 2k \times 3^k - 2 \times 3^k - k \times 3^k$ or e.g., $2 \times 3^k(k+3) - 3^k(k+2)$ Obtains an expression where all terms are multiples of 3^k . Requires previous mark	dM1
	$\{ = 4 \times 3^k + k \times 3^k = 3^k(k+4) \} = 3^{(k+2)-2}((k+2)+2) \text{ or } 3^{k+2-2}(k+2+2)$ Reaches a correct expression in terms of $k+2$ with no errors. Meet in the middle approaches must be convincing.	A1
	If the result is true for $n = k$ and $n = k + 1$ then shown true for $n = k + 2$. As the result has been shown to be true for $n = 1$ and $n = 2$, then result is true for (all) n . Correct conclusion/narrative. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. All the elements in bold should be satisfied. "Assume $n = k$ and $n = k + 1$ " in the narrative followed by "true for $n = k + 2$ " in the conclusion plus "true for $n = 1$ and $n = 2$ " and "true for (all) n " or "true for $n \in \square^+$ " is sufficient. For the last statement allow "true for n ", "true for $\square^+$ ", "true for $\square$ " and condone "true for $\square$ ", "true for integers", "true for integers after 1" etc. but do not allow "true for all $n \in \square$ " or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If just " $u_1 = 1, u_2 = 4$ " is seen this is not sufficient evidence of any attempt to substitute and so the maximum score could only be 01110. The same applies if there are any errors in substitution. However e.g., just e.g., "when $n = 1, u_1 = 1$, when $n = 2, u_2 = 4$ " can score 01111 since this implies an attempt to verify the values and no errors are seen.	A1
		(5)
		Total 10
	See overleaf for approaches that assume true for $n = k - 1$ and $n = k$ and show true for $n = k + 1$	

8(ii)	Note that is valid to e.g., assume true for $n = k - 1$ and $n = k$ and show true for $n = k + 1$: $n = 1 \Rightarrow u_1 = 3^{-1}(2 + 1) = 1, \quad n = 2 \Rightarrow u_2 = 3^0(2 + 2) = 4$ $u_{k-1} = 3^{k-3}(k + 1) \quad u_k = 3^{k-2}(k + 2)$ $u_{k+1} = 6u_k - 9u_{k-1} = 6 \times 3^{k-2}(k + 2) - 9 \times 3^{k-3}(k + 1)$ $= 2 \times 3^{k-1}(k + 2) - 3^{k-1}(k + 1) \text{ or e.g., } 2k \times 3^{k-1} + 4 \times 3^{k-1} - k \times 3^{k-1} - 3^{k-1}$ $= k \times 3^{k-1} + 3 \times 3^{k-1} = 3^{k-1}(k + 3) = 3^{(k+1)-2}((k + 1) + 2) \text{ or } 3^{k+1-2}(k + 1 + 2)$ B1: As main scheme M1: Attempts u_{k-1} and u_k using $u_n = 3^{n-2}(n + 2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+1} in terms of k dM1: Obtains an expression where all terms are multiples of 3^{k-1}. Requires previous mark A1: Reaches a correct expression in terms of $k + 1$ with no errors. A1: If result is true for $n = k - 1$ and $n = k$ then shown true for $n = k + 1$. As the result has been shown to be true for $n = 1$ and $n = 2$, then result is true for (all) n. See main scheme for guidance on the last mark. Use Review for any similar approaches you are not sure about.	
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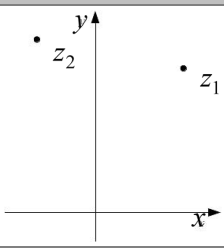
Question Number	Scheme	Notes	Marks
9(a)	$y^2 = \frac{1}{2}x, P\left(\frac{t^2}{8}, \frac{t}{4}\right) \Rightarrow y = \frac{1}{\sqrt{2}}\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{2}\sqrt{x}} = \frac{\sqrt{2}}{4\sqrt{x}} = \frac{\sqrt{2}}{4\sqrt{\frac{t^2}{8}}} = \frac{1}{t}$ or $x = \frac{t^2}{8}, y = \frac{t}{4} \Rightarrow \frac{dy}{dx} = \frac{\frac{1}{4}}{\frac{t}{8}} = \frac{1}{t}$ or $2y \frac{dy}{dx} = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{4y} = \frac{1}{4\left(\frac{t}{4}\right)} = \frac{1}{t}$ or $x = 2y^2 \Rightarrow \frac{dx}{dy} = 4y \Rightarrow \frac{dy}{dx} = \frac{1}{4y} = \frac{1}{4\left(\frac{t}{4}\right)} = \frac{1}{t}$ Correct $\frac{dy}{dx}$ in terms of t . Could be unsimplified. Accept just " $\frac{dy}{dx}$ or $m = \frac{1}{t}$ "		B1
	$y - \frac{t}{4} = \frac{1}{t}\left(x - \frac{t^2}{8}\right)$ or $\frac{t}{4} = \frac{1}{t}\left(\frac{t^2}{8}\right)x + c \Rightarrow c = \dots \left\{\frac{t}{8}\right\}$	Correct straight line method with an unchanged gradient in terms of t . Condone late substitution of x/y into gradient if initial gradient not in terms of t (the first two marks are then accessible and A1* is possible)	M1
	e.g., $8ty - 2t^2 = 8x - t^2$ or $8yt = 8x + t^2$ or $y = \frac{1}{t}x + \frac{t}{8}$ $\Rightarrow 8yt - 8x = t^2$ *	Obtains the answer via intermediate line and no errors. Accept answer with t^2 on one side and $8ty - 8x$ or $8(yt - x)$ or $8(ty - x)$ on the other (these 2 terms in either order). Requires both previous marks.	A1*
			(3)
(b)	$x = 0 \Rightarrow 8yt = t^2 \Rightarrow y = \frac{t}{8} \quad \left\{Q_y = \frac{t}{16}\right\}$	Correct y coordinate of A Could be unsimplified.	B1 (M1 on ePen)
	{Midpoint of AP :} $\left(\frac{0 + \frac{t^2}{8}}{2}, \frac{\frac{t}{8} + \frac{t}{4}}{2}\right) \left\{ = \left(\frac{t^2}{16}, \frac{3t}{16}\right) \right\}$	Finds midpoint of AP using a fully correct method for their A which is of the form $(0, f(t))$. May be given as $x = \dots, y = \dots$	M1
	{equation of l_2 :} $y - \frac{3t}{16} = -t\left(x - \frac{t^2}{16}\right)$ or $y = -tx + c \Rightarrow \frac{3t}{16} = -t\left(\frac{t^2}{16}\right) + c \Rightarrow c = \dots \left\{\frac{3t + t^3}{16}\right\}$	Forms the equation of the perpendicular bisector of AP correct for their midpoint of AP and with gradient $-t$ (oe). Not dependent but the coordinates of their midpoint must both be functions of t	M1
	$y = \frac{t}{16} \Rightarrow \frac{t}{16} - \frac{3t}{16} = -tx + \frac{t^3}{16} \Rightarrow x = \frac{2 + t^2}{16} \Rightarrow 2 + 256y^2 = 16x$ or e.g., $\frac{t}{16} = -tx + \frac{3t + t^3}{16} \Rightarrow 1 = -16x + 3 + t^2 \Rightarrow -2 = -16x + 256y^2$ $y - 3y = -16y\left(x - \frac{256y^2}{16}\right) \Rightarrow -2 = -16\left(x - \frac{256y^2}{16}\right) \quad \{\Rightarrow -2 = -16x + 256y^2\}$ Any correct 3 term equation (may be factorised) with t eliminated		A1
	$y^2 = \frac{1}{16}x - \frac{1}{128}$ Correct equation in the correct form with y^2 on its own on one side. Allow $y^2 = 0.0625x - 0.0078125$ or equivalent fractions for α and β in $y^2 = \alpha x + \beta$ e.g., $-\frac{2}{256} + \frac{16}{256}x = y^2$ and isw if subsequent factorisation/multiplication etc.		A1
			(5)
			Total 8

Question Number	Scheme	Notes	Marks
2	$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ Condone any brackets (or missing brackets) for matrices throughout		
(a)	Rotation...	Identifies the transformation as a rotation. Allow any reasonable attempt at this word e.g., "rotate". M0 for a combination of transformations or if any different types of transformation are given as alternatives. Note that giving an angle does not imply "rotation".	M1
	...(anti-clockwise) of $\frac{3\pi}{2}$ (270°) or clockwise of $\frac{\pi}{2}$ (90°) about/around/from (etc.) O Correct full description for the rotation, including angle & direction (if no direction is stated then assume anticlockwise is meant) and any mention of origin or O or (0, 0). Condone "original" for "origin". Accept in degrees (symbol not required) or exact radians. {Note also that $-\frac{\pi}{2}$ (-90°) or $-\frac{3\pi}{2}$ (-270°) clockwise are acceptable}		A1
	(2)		
(b)	Enlargement...	Identifies the transformation as an enlargement. Allow any reasonable attempt at this word e.g., "inlarge", "large". Nothing else so e.g. "stretch" is M0. M0 for a combination of transformations or if any different types of transformation are given as alternatives.	M1
	...of scale factor/factor/scale/size 5, centre/from (etc.) O	Correct full description for the enlargement including scale factor and centre. Allow any mention of "5" and any mention of origin, O or (0, 0) Condone "original" for "origin".	A1
	(2)		
(c)	$\{\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\}$	Correct matrix	B1
	(1)		
(d)	$\left\{ \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \right\} (-3, -4)$	Correct coordinates for A. Brackets may be missing. May be stated as $x = \dots, y = \dots$ Condone answer given as a vector. Isw if necessary. Score B0 if an incorrect R has clearly been used in this part. Allow if correct answer comes from multiplying the wrong way round. No ft.	B1
	(1)		

Question Number	Scheme	Notes	Marks
2(e)	$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$		
	For either mark there must be no clear evidence of use of an incorrect method or matrix in this part e.g., using their matrix $\mathbf{R}$ for the reflection in part (c), i.e, sight of their $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ even if this is labelled as $\mathbf{P}$. Do not allow confusion between their $A(-3, -4)$ and the given point $(4, 3)$ and there are no marks if a matrix or point has been miscopied/misread even if it is restated in this part. This also applies if there is an incorrect A in (d) but it is miscopied into the correct $(-3, -4)$. Both marks can be scored from minimal or no working.		
	$\left(\frac{4}{5}, \dots\right) \text{ or } (0.8, \dots) \text{ or } \left(\dots, -\frac{3}{5}\right) \text{ or } (\dots, -0.6) \text{ or } \left(-\frac{4}{5}, \frac{3}{5}\right)$ For one correct coordinate or ft non-zero coordinate on their $A(-3, -4)$ but must not use $(4, 3)$ even if that is their answer for A from part (d). Examples of follow throughs are: $(-4, -3) \rightarrow \left(\frac{3}{5}, -\frac{4}{5}\right)$ $(-4, 3) \rightarrow \left(-\frac{3}{5}, -\frac{4}{5}\right)$ $(4, -3) \rightarrow \left(\frac{3}{5}, \frac{4}{5}\right)$ $(-3, 4) \rightarrow \left(-\frac{4}{5}, -\frac{3}{5}\right)$ $(3, -4) \rightarrow \left(\frac{4}{5}, \frac{3}{5}\right)$ $(3, 4) \rightarrow \left(-\frac{4}{5}, \frac{3}{5}\right)$ May be stated as $x = \dots / y = \dots$ and condone if given without brackets or as a vector but do not condone a correct value seen within e.g., a 2×2 matrix		M1
	$\{B:\}\left(\frac{4}{5}, -\frac{3}{5}\right) \text{ or } (0.8, -0.6) \text{ only}$ Both correct coordinates. Not ft. May be stated as $x = \dots, y = \dots$ and condone if given without brackets or as a vector. Isw if necessary.		A1
	Examples of working: Do not allow confusion between their $A(-3, -4)$ and the given point $(4, 3)$ Using matrices (Note that the matrices can be applied in either order with these two transformations): $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} y \\ -x \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, -5x = -4 \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 5x \\ 5y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, -5x = -4 \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ Using inverse: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \frac{1}{25} \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & -\frac{1}{5} \\ \frac{1}{5} & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ Using the actual transformations: $(-3, -4) \Rightarrow \left(-\frac{3}{5}, -\frac{4}{5}\right) \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right) \text{ or } (-3, -4) \Rightarrow (4, -3) \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$		(2)
			Total 8

Question Number	Scheme	Notes	Marks
5(a)	$xy = 64 \Rightarrow y = 64x^{-1} \Rightarrow \frac{dy}{dx} = -64x^{-2}$ or $xy = 64 \Rightarrow x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ or $x = 8p, y = \frac{8}{p} \Rightarrow \frac{dy}{dx} = \frac{-8p^{-2}}{8}$	Correct $\frac{dy}{dx}$ This can be in any form, simplified or unsimplified. The parameter could be a different variable, e.g. t	B1
	$m_T = -\frac{64}{64p^2} \Rightarrow m_N = p^2$ or $m_T = -\frac{8/p}{8p} \Rightarrow m_N = p^2$ or $m_T = -p^{-2} \Rightarrow m_N = p^2$	Correct use of the perpendicular gradient rule and the point P to obtain the normal gradient Correct normal gradient of p^2	M1 A1
	$y - \frac{8}{p} = p^2(x - 8p)$ or $y = p^2x + c, \frac{8}{p} = p^2 \times 8p + c \Rightarrow c = \dots$	Correct straight line method for normal	M1
	$p^3x - py = 8(p^4 - 1)^*$	cso. (No errors, but possibly direct from the version in line 1 above)	A1*
			(5)
(b)	$p^3x - py = 8(p^4 - 1), xy = 64 \Rightarrow$ $p^3x - p \frac{64}{x} = 8(p^4 - 1)$ or $p^3 \frac{64}{y} - py = 8(p^4 - 1)$	Uses both equations to obtain an equation in one variable	M1
	$p^3x^2 + 8(1 - p^4)x - 64p = 0$ or $py^2 + 8(p^4 - 1)y - 64p^3 = 0$	Correct quadratic. Must have the x^2 or y^2 term, but the x or y terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions, e.g. $p^2x^2 + \frac{8x}{p} - 8p^3x = 64$	A1
	$(x - 8p)(p^3x + 8) = 0 \Rightarrow x = \dots$ or $(py - 8)(y + 8p^3) = 0 \Rightarrow y = \dots$	Solves their 3TQ (usual rules) to obtain the <u>other</u> value of x or y . The <u>other</u> value must be picked out as a solution. This could be done by algebraic division... (see below)	dM1
	$x = -\frac{8}{p^3} \quad y = -8p^3 \quad \text{or} \quad \left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)
			Total 9

5(b)	Rather than solving the 3TQ for the dM1, algebraic division can be used. To score the mark the division should follow the usual rules for solution by factorisation, so in the first case, e.g. if the quadratic is correct, the quotient should be $\pm p^3x \pm 8$, then this must lead to the other value $x_2 = \dots$		
5(b)	Note that another way to find the other value for the dM1 is to use the ‘sum of roots’ $= -\frac{b}{a}$, e.g. $8p + x_2 = \frac{-8(1 - p^4)}{p^3} \quad x_2 = \dots$		
(b) Way 2	$p^3x - py = 8(p^4 - 1), \left(8q, \frac{8}{q}\right) \Rightarrow$ $p^38q - p\frac{8}{q} = 8(p^4 - 1)$	Uses the given normal equation and the parametric form for Q to form an equation in p and q	M1
	$p^3q^2 - p = qp^4 - q$	Correct quadratic. Must have the q^2 term, but the q terms need not be combined. The terms do not need to be ‘all on one side’, and the coefficients could involve fractions.	A1
	$(p - q)(p^3q + 1) = 0 \Rightarrow q = \dots$	Solves their 3TQ (usual rules) to obtain the value of q . This could be done by algebraic division (condition as for main scheme)	dM1
	$q = -\frac{1}{p^3} \Rightarrow x = -\frac{8}{p^3} \quad y = -8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)

Question	Scheme	Marks
2(a)		M1 A1
		(2)
(b) (i)	$ z_1 = \sqrt{3^2 + 5^2} = \sqrt{34}$	B1
		(1)
(ii)	$\frac{z_1}{z_2} = \frac{3+5i}{-2+6i} \times \frac{-2-6i}{-2-6i} = \dots$	M1
	$= \frac{-6-18i-10i+30}{40}$	A1
	$= \frac{3}{5} - \frac{7}{10}i$	A1
		(3)
(c)	$\arg \frac{z_1}{z_2} = \arctan \frac{-7/10}{3/5} = \arctan \frac{-7}{6} = \dots$ but allow $\arctan \frac{7}{6}$ for M1	M1
	$= -0.86$ or 5.42 (awrt)	A1
		(2)
(8 marks)		
Notes:		
(a) M1: Points in correct quadrants - z_1 in quadrant 1 and z_2 in quadrant 2. Must be clearly labelled either eg z_1 or $3 + 5i$ or correct numbers on the axes. (Accept with vector arrows.) A1: Correct diagram, z_1 in first quadrant further away from real axis than imaginary and z_2 in second quadrant, closer to imaginary axis but above z_1 OR correct nos on their axes (imag axis may include i), but not dashes w/o any indication of scale. Allow M1A0 for points unlabelled but diagram otherwise correct. (b)(i) B1: Correct modulus. Must be evaluated to $\sqrt{34}$ Question says “without using your calculator” so decimal answers can be ignored (isw) but exact answer must be seen somewhere. (ii) M1: Multiplies numerator and denominator by the conjugate of their denominator. A1: Correct unsimplified (or simplified) numerator, with the i^2 correctly dealt with, and correct denominator. A1: Correct answer. Allow as shown, $\frac{6}{10} - \frac{7}{10}i$, or $0.6 - 0.7i$.		

(c)

M1: For $\arctan\left(\pm\frac{7}{6}\right)$ (not necessarily simplified to this) or $\tan\alpha = \pm\frac{7}{6}$ $\alpha = \dots$ This mark is available if answer is given in degrees. Can use $\arctan\left(\pm\frac{6}{7}\right)$ provided a complete method to reach the correct arg is seen.

A1: For awrt -0.86 or awrt 5.42 Must be radians.

ALT for (c):

M1: Use $\arg z_1 - \arg z_2$ correctly

A1: Correct answer

Question	Scheme	Marks
3(a)	$\left(\frac{9}{2}, 0\right)$	B1
		(1)
(b)	$PS = 9$	B1
	$x_p = -\frac{9}{2} + 9 = \frac{9}{2} \Rightarrow OP = \sqrt{\left(\frac{9}{2}\right)^2 + \left(18 \times \frac{9}{2}\right)} = \dots$	M1
	So perimeter = " $\frac{9}{2}$ " + "9" + " $\frac{9\sqrt{5}}{2}$ "	dM1
	$= \frac{27 + 9\sqrt{5}}{2}$ oe	A1
		(4)
(5 marks)		
Notes:		
(a) B1: Correct coordinates. (b) B1: Deduces $PS = 9$ from the focus directrix property (may be implied by seeing it embedded in an expression for the perimeter). May find coordinates of P first and then attempt Pythagoras theorem – must be correct. May be seen on the diagram. Allow even if incorrect value used later. M1: Uses distance from directrix to find x coordinate of P and goes on to find OP by Pythagoras (with a plus sign). dM1: Sums their three side lengths. Extras – including 0 – score M0. Depends on the previous M mark. A1: Correct answer. Equivalent must be in simplified surd form.		

Question	Scheme	Marks
4(a)	$4 - 3i$	B1
		(1)
(b)	$(x - (4 + 3i))(x - (4 - 3i)) = \dots$	M1
	$x^2 - 8x + 25$	A1
		(2)
(c)	E.g. Product of roots is 225, so product of real roots is $\frac{225}{25} = 9$ Or $x^4 + Ax^3 + Bx^2 + Cx + 225 = (x^2 - 8x + 25)(x^2 + \dots + 9)$	M1
	Hence (as root is positive) repeated real root is 3	A1
		(2)
(d)	$(x^2 - 8x + 25)(x^2 - 6x + 9)$ $= x^4 - 6x^3 + 9x^2 - 8x^3 + 48x^2 - 72x + 25x^2 - 150x + 225$	M1
	$= x^4 - 14x^3 + 82x^2 - 222x + 225$ Two correct middle term coefficients	A1
	So $A = -14$, $B = 82$ and $C = -222$ (or accept in the quartic)	A1
		(3)

(8 marks)**Notes:****(a)****B1:** For $4 - 3i$ **(b)****M1:** Correct strategy to find a quadratic factor. May expand as shown in scheme, or may look for sum of roots and product of roots first and then write down the factor.**A1:** Correct quadratic factor. Can be written down – give M1A1 if correct, M0A0 if incorrect.

Ignore “= 0” with their quadratic factor.

Alt for (b):**M1:** Product of complex roots is 25, so product of real roots is $\frac{225}{25} = 9$, so the (positive) real root is**A1:** $x^2 - 6x + 9$ or $(x - 3)^2$ **(c)****M1:** A complete strategy to deduce the real root or its square. May consider product of roots, as in scheme, or may first attempt to factorise/long division to find the other quadratic factor – award at the point the quadratic factor with real roots is found. May have been seen in (b)**A1:** Real root is 3. (No need to see rejection of the negative possibility.)

Not a “show that” so award M1A1 if correct root is written down with no working.



Question	Scheme	Marks
6(a)	$a = 5$	B1
		(1)
(b)	$\frac{dy}{dx} = -\frac{20}{x^2}$ or $\frac{dy}{dx} = -\frac{2\sqrt{5}}{t^2} \div 2\sqrt{5} = -\frac{1}{t^2}$ or $x\frac{dy}{dx} + y = 0$ oe	B1
	Gradient of normal is $\frac{-1}{"-1/t^2"} = t^2$	M1
	Normal is $y - \frac{2\sqrt{5}}{t} = t^2(x - 2t\sqrt{5})$	M1
	$\Rightarrow ty - 2\sqrt{5} = t^3x - 2t^4\sqrt{5} \Rightarrow ty - t^3x - 2\sqrt{5}(1 - t^4) = 0^*$	A1*
		(4)
(c)	$cy - c^3x - 2\sqrt{5}(1 - c^4) = 0$ passes through $\left(-\frac{\sqrt{5}}{c}, -4c\sqrt{5}\right)$	M1
	$\Rightarrow -4c^2\sqrt{5} + c^2\sqrt{5} - 2\sqrt{5}(1 - c^4) = 0$	
	$\Rightarrow 2c^4 - 3c^2 - 2 = 0$ (oe)	A1
	$\Rightarrow c^2 = \frac{3 \pm \sqrt{9 - 4 \times 2 \times -2}}{4} = \dots \left(2, -\frac{1}{2}\right)$	dM1
	$c^2 > 0 \Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	A1
		(4)
(9 marks)		

Notes:
(a)
B1: Correct value stated.

(b)
B1: Correct expression for $\frac{dy}{dx}$, or any equivalent correct expression including it, such as

$$x\frac{dy}{dx} + y = 0 \text{ or } \frac{dy}{dx} = -\frac{1}{t^2}$$

M1: Attempts negative reciprocal gradient at the point P . Allow with a instead of 5 for this mark, so score for e.g. $m_N = \frac{4at^2}{20}$.

M1: Uses their normal (changed from tangent) gradient and P to find the equation of the tangent.

 Look for $y - \frac{2\sqrt{5}}{t} = "m_n"(x - t\sqrt{5})$. If using $y = mx + c$ they must proceed as far as finding c .

A1*: Correct equation achieved from correct working with intermediate step.

(c)

M1: Substitutes the parameter for A into the normal equation and attempts to substitute the coordinates of B to obtain an equation in one variable. Allow if there are slips during substitution. A1: Correct quadratic in c^2 need not be simplified. dM1: Solves their (at least two term) quadratic in c^2 to find a value for at least c^2 A1: Deduces correct values. Both required. Ignore reference to any complex roots.		
Alts		
(c)	$cy - c^3x - 2\sqrt{5}(1 - c^4) = 0$ intersects H again $\Rightarrow \frac{20}{x}c - c^3x - 2\sqrt{5}(1 - c^4) = 0$ or $cy - \frac{20}{y}c^3 - 2\sqrt{5}(1 - c^4) = 0$ $\Rightarrow c^3x^2 + 2\sqrt{5}(1 - c^4)x - 20c = 0$ or $cy^2 - 2\sqrt{5}(1 - c^4)y - 20c^3 = 0$	M1 A1
	$\Rightarrow (c^3x + 2\sqrt{5})(x - 2c\sqrt{5}) = 0$ or $(cy - 2\sqrt{5})(y + 2\sqrt{5}c^3) = 0$ $\left(x = 2c\sqrt{5} \text{ is } A \text{ so}\right)$ for B $x = -\frac{2\sqrt{5}}{c^3} = -\frac{\sqrt{5}}{c} \Rightarrow c = \dots$ $\left(\text{or } y = \frac{2\sqrt{5}}{c} \text{ is } A \text{ so}\right)$ for B $y = -2\sqrt{5}c^3 = -4c\sqrt{5} \Rightarrow c = \dots$	M1
	$\Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	A1
		(4)
	Notes:	
(c) M1: Substitutes parameters for A and equation for H into normal to obtain a quadratic in x . A1: Correct quadratic in x or y M1: Solves the quadratic in x or y , identifies correct solution and equates to the relevant coordinate of B and solves for c A1: Deduces correct values. Both required.		
(c)	$m_{AB} = \frac{\frac{2\sqrt{5}}{c} - (-4c\sqrt{5})}{2\sqrt{5}c - \left(-\frac{\sqrt{5}}{c}\right)}$	M1
	$m_{AB} = \frac{2 + 4c^2}{2c^2 + 1} = 2$	A1
	From (b), normal at A has gradient $(t^2 =)c^2 \Rightarrow c^2 = 2$	M1
	$\Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	A1
		(4)
Notes:		

(c)

M1: Attempts the gradient of AB .

A1: Correct gradient need not be simplified.

M1: Finds/deduces the gradient of the normal at A and sets equal to their gradient of AB .

A1: Deduces correct values. Both required.



Question Number	Scheme	Notes	Marks
4	$f(z) = 6z^3 + Az^2 + Bz + C$ Condone work in x throughout		
(a)	$(z =) \frac{2}{3} - \frac{\sqrt{17}}{3}i$	Correct conjugate. Must be seen in (a)	B1
			(1)
(b)	$\left(z - \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right) = \dots \left\{z^2 - \frac{4}{3}z + \frac{7}{3}\right\} \quad \text{or e.g.}$$\alpha + \beta = \frac{2}{3} + \frac{\sqrt{17}}{3}i + \frac{2}{3} - \frac{\sqrt{17}}{3}i = \dots \left\{\frac{4}{3}\right\}, \quad \alpha\beta = \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right) = \dots \left\{\frac{4}{9} + \frac{17}{9} = \frac{7}{3}\right\} \Rightarrow \dots \left\{z^2 - \frac{4}{3}z + \frac{7}{3}\right\}$ M1: Completes a correct strategy for finding a quadratic factor and obtains a 3TQ with real coefficients. May be slips with the expansion or calculation/forming quadratic but score M0 if the starting point is clearly $\left(z + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z + \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right)$ A1: Any correct three term quadratic factor. Allow any multiple of $z^2 - \frac{4}{3}z + \frac{7}{3}$ e.g., $3z^2 - 4z + 7$ May see $(3z - (2 + \sqrt{17}i))(3z - (2 - \sqrt{17}i))$ or $(3z - 2)^2 = (\pm\sqrt{17}i)^2 \Rightarrow \dots \{9z^2 - 12z + 21\}$		M1A1
		$6\left(z + \frac{3}{2}\right)\left(z^2 - \frac{4}{3}z + \frac{7}{3}\right) = \dots \text{ or e.g., } (2z + 3)(3z^2 - 4z + 7) = \dots$ Multiplies their 3 term quadratic factor with real coefficients (or multiple) by $z + \frac{3}{2}$ (or multiple) to obtain a 4TC with real coefficients so allow $\left(z + \frac{3}{2}\right)\left(z^2 - \frac{4}{3}z + \frac{7}{3}\right) = \dots \left\{z^3 + \frac{z^2}{6} + \frac{z}{3} + \frac{7}{2}\right\}$	M1
$\{f(z) = \} 6z^3 + z^2 + 2z + 21 \quad \text{or} \quad A = 1, B = 2, C = 21$		A1: Any two correct values for A, B or C (could be embedded) A1: Fully correct expression (ignore an “=0”) or three correct values	A1A1
		Note that if the complex factors are not multiplied first e.g.,$\left(z - \frac{3}{2}\right)\left(z - \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right) = \left(z^2 + \left(\frac{5}{6} - \frac{\sqrt{17}}{3}i\right)z - 1 - \frac{\sqrt{17}}{2}i\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right)$$= z^3 + \frac{1}{6}z^2 + \frac{1}{3}z + \frac{7}{2} \Rightarrow 6z^3 + z^2 + 2z + 21$ Score the first 2 M marks together for obtaining a 4TC with real coefficients. M0M1 is possible e.g., with $\left(z + \frac{3}{2}\right)\left(z + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z + \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right)$ Any correct multiple of the 4TC scores the first A then score as main scheme	
			(5)

Question Number	Scheme	Notes	Marks
6(i)(a)	Stretch scale factor 3 parallel to the y-axis	Stretch (<u>not</u> enlargement)	B1
		Scale factor 3 parallel to the y-axis. Allow, e.g. '3 times y values', 'y increased by 3 factor', or similar. Allow, e.g. 'direction of y', 'along y', 'vertical', or similar. Ignore any mention of the origin. If additional transformations are included, send to Review	B1
			(2)
(b)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix. $\frac{1}{\sqrt{2}}$ may be seen rather than $\frac{\sqrt{2}}{2}$	B1
			(1)
(c)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	Attempt to multiply the right way round, i.e. BA , not AB At least two correct terms (for their matrix B) are needed to indicate a correct multiplication attempt	M1
	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \end{pmatrix}$ or equiv. e.g. $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	Trapezium area = $\frac{1}{2}(5+2)(k+8)$	Correct method for the area of the trapezium	M1
	$\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix} = 5 \times 3 - (-2) \times 1 = 17$	Correct method for the determinant	M1
		17 (Allow ± 17)	A1
	$\frac{1}{2}(5+2)(k+8) \times 17 = 510 \Rightarrow k = \dots$	Multiplies their trapezium area by their determinant, sets equal to 510 and solves for k . Or equivalently: Equates their trapezium area to $(510 \div \text{determinant})$ and solves for k	M1
	$k = \frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			(5)

(ii) Way 2	$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -2 & -2 & 5 & 5 \\ 0 & k & 8 & 0 \end{pmatrix}$	Multiplies correct matrices to find the coordinates for T'	2 nd M
	$= \begin{pmatrix} -10 & -10 + k & 33 & 25 \\ 4 & 4 + 3k & 14 & -10 \end{pmatrix}$	Correct coordinates (can be left in matrix form)	A1
	$\frac{1}{2}[-10(4 + 3k) + 14(-10 + k) - 330 + 100 - 4(-10 + k) - 33(4 + 3k) - 350 - 100]$	Correct method for area of T' ('shoelace rule' with or without a modulus), using their coordinates for T'	1 st M
	$\pm \frac{1}{2}(952 + 119k) = 510, \quad k = \dots$	Sets area of T' equal to 510 and solves for k	M1
	$k = \frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			Total 10

Question Number	Scheme	Notes	Marks
7(a) Way 1	$3x - 4y + 48 = 0 \Rightarrow x = \frac{4y - 48}{3}$ $y^2 = 4ax \Rightarrow y^2 = 4a \left(\frac{4y - 48}{3} \right)$ or $3x - 4y + 48 = 0 \Rightarrow y = \frac{3x + 48}{4}$ $y^2 = 4ax \Rightarrow \left(\frac{3x + 48}{4} \right)^2 = 4ax$ or $x = \frac{y^2}{4a} \Rightarrow \frac{3y^2}{4a} - 4y + 48 = 0$	Uses both equations to obtain an equation in one variable.	M1
	$3y^2 - 16ay + 192a = 0$ or $9x^2 + (288 - 64a)x + 2304 = 0$ or $3x - 8\sqrt{a}\sqrt{x} + 48 = 0$	Correct 3TQ (Coefficients could be ‘fractional’) (This could be a quadratic in $\sqrt{x}$)	A1
	Equal roots: $(16a)^2 = 4 \times 3 \times 192a$ or $(288 - 64a)^2 = 4 \times 9 \times 2304$ $\Rightarrow a = \dots$	Uses “ $b^2 = 4ac$ ” to find a value for a	M1
	$a = 9^*$	cs0	A1*
	Beware the use of the given result $a = 9$, but there may be cases where ‘working backwards’ deserves merit (if in doubt, send to Review).		
			(4)

(a) Way 2	$y^2 = 4ax \Rightarrow 2y \frac{dy}{dx} = 4a$ $3x - 4y + 48 = 0 \Rightarrow \frac{dy}{dx} = \frac{3}{4}$ $\Rightarrow 2y \times \frac{3}{4} = 4a$	Uses differentiation to obtain the gradient of C and substitutes the gradient of l to obtain an equation connecting y and a , or connecting x and a , or an equation in t	M1
	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}} \Rightarrow a^{\frac{1}{2}}x^{-\frac{1}{2}} = \frac{3}{4}$		
	$x = at^2, y = 2at \Rightarrow \frac{dy}{dx} = \frac{1}{t}$ $\frac{1}{t} = \frac{3}{4}$		
	$y = \frac{8a}{3} \text{ or } x = \frac{16a}{9}$	Correct y value, or correct x value (possibly implied in subsequent work, particularly if using the parametric equations)	A1
	$y^2 = 4ax \Rightarrow \frac{64a^2}{9} = 4ax \Rightarrow x = \frac{16a}{9}$ $3 \times \frac{16a}{9} - 4 \times \frac{8a}{3} + 48 = 0 \Rightarrow a = \dots$	Uses $y^2 = 4ax$ or l to find a value for x (or y) and substitutes their x and y into the other equation to find a value for a If using parameter t , substitutes their value for t into $3(at^2) - 4(2at) + 48 = 0$ and solves to find a value for a	M1
	$x = \frac{4y - 48}{3} = \frac{32a}{9} - 16$ $\frac{64a^2}{9} = 4a \left(\frac{32a}{9} - 16 \right) \Rightarrow a = \dots$		
	$y^2 = 4ax \Rightarrow y^2 = \frac{64a^2}{9} \Rightarrow y = \frac{8a}{3}$ $3 \times \frac{16a}{9} - 4 \times \frac{8a}{3} + 48 = 0 \Rightarrow a = \dots$		
	$y = \frac{3x + 48}{4} = \frac{4a}{3} + 12$ $\left(\frac{4a}{3} + 12 \right)^2 = 4a \left(\frac{16a}{9} \right) \Rightarrow a = \dots$		
	$3(at^2) - 4(2at) + 48 = 0$ $3 \left(\frac{16a}{9} \right) - 4 \left(\frac{8a}{3} \right) + 48 = 0 \Rightarrow a = \dots$		
	$a = 9 *$	CSO	A1*

Question Number	Scheme	Notes	Marks
8(a)	$y^2 = 20x \Rightarrow y = \sqrt{20}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{20}}{2}x^{-\frac{1}{2}} = \frac{\sqrt{20}}{2\sqrt{5p^2}}$ or $y^2 = 20x \Rightarrow 2y \frac{dy}{dx} = 20 \Rightarrow \frac{dy}{dx} = \frac{10}{y} = \frac{10}{10p}$ or $x = 5p^2, y = 10p \Rightarrow \frac{dy}{dx} = \frac{10}{10p}$	Correct strategy for finding $\frac{dy}{dx}$ in terms of p	M1
	$y - 10p = \frac{1}{p}(x - 5p^2)$ or $y = \frac{1}{p}x + c \Rightarrow c = 10p - \frac{1}{p} \times 5p^2$	Correct straight line method	M1
	$py - x = 5p^2 *$	Cso	A1*
			(3)
(b)	$(0, 5p)$	Correct coordinates	B1
			(1)
(c)	$(5, 0)$	Correct coordinates	B1
			(1)
(d)	$y = \frac{2}{p}x$	Correct equation for l_2	B1
	E.g. $y = -\frac{5p}{5}(x - 5)$ or $y = -px + c \rightarrow c = 5p$	Correct strategy for the equation of l_1 (providing it has non-zero gradient)	M1
	$y = \frac{2}{p}x \Rightarrow p = \frac{2x}{y} \Rightarrow y = -\frac{2x}{y}(x - 5)$	Eliminates p to obtain an equation connecting x and y	M1
		Correct equation in any form	A1
	$2x^2 + y^2 = 10x *$	Fully correct proof	A1*
			(5)
Alternative for last 3 marks of (d)			
	$y = \frac{2}{p}x, y = -\frac{5p}{5}(x - 5)$ $\Rightarrow x = \dots, y = \dots$	Solves simultaneously to find x and y in terms of p	M1
	$x = \frac{5p^2}{p^2 + 2}, y = \frac{10p}{p^2 + 2}$	Correct coordinates for B	A1
	$2x^2 + y^2 = 2\left(\frac{25p^4}{(p^2 + 2)^2}\right) + \frac{100p^2}{(p^2 + 2)^2} = \frac{50p^4 + 100p^2}{(p^2 + 2)^2} = \frac{50p^2(p^2 + 2)}{(p^2 + 2)^2} = 10x *$ Completes the proof by substituting into the given equation and shows sufficient working to establish the equivalence (as above)		A1*
			Total 10

Question Number	Scheme	Notes	Marks
4	$f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$		
(a)	$(z =) 5 + i$	Correct complex number	B1
			(1)
	Mark (b) and (c) together – ignore any labelling seen. Award marks in the order given for their choice of method		
(b)/(c) With (b) first	$z = 5 \pm i \Rightarrow (z - (5 + i))(z - (5 - i)) = \dots$ Or e.g. Sum of roots = 10 Product of roots = 26	Correct strategy to find the quadratic factor using the conjugate pair	M1
	$z^2 - 10z + 26$	Correct quadratic	A1
	$f(z) = (z^2 - 10z + 26)(2z^2 + \dots z + k)$	Attempts to find the other quadratic. May use inspection (apply rules for quadratic factorisation ie " $26 \mid k = 156$ ") or e.g. long division with quotient $2z^2 + \dots z + \dots$	M1
	NB long division gives quotient $2z^2 + z + (A - 42)$ and remainder $(10A + B - 446)z + 936 - 26A$		
	$2z^2 + z - 6$	Correct quadratic	A1
	$\Rightarrow z = \frac{3}{2}, -2, (5 \pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(5)
	$f(z) = (z^2 - 10z + 26)(2z^2 + z - 6) = \dots$	Multiplies out both quadratics or extracts the terms needed	M1
	$A = 36, B = 86$	Correct values (can be seen in the quartic equation)	A1
			(2)
			Total 8
(b)/(c) With (c) first	$952 + 960i - 2090 - 24A + 10Ai + 5Bi - 156 = 0$	Substitute $(5 + i)$ into the quartic (by calculator) and equate real and imag parts (can be done with $(5 - i)$)	M1
	$-1294 + 24A + 5B = 0$ $-446 + 10A + B = 0$	Correct equations	A1
	$A = 36 \quad B = 86$	M1 Solve simultaneously A1 One correct A1 Both correct	M1 A1A1
			(5)
	$2z^4 - 19z^3 + 36z^2 + 86z - 156 = 0$ $z = \dots$	Solve the equation by long division, inspection or by calculator	M1
	$\Rightarrow z = \frac{3}{2}, -2, (5 \pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$2x^2 - 3x + 5 = 0$		
(a)	$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = \frac{5}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Uses a correct identity	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right) = -\frac{11}{4} (= -2.75)$	Correct value Allow to come from $\alpha + \beta = -\frac{3}{2}$	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Reaches an identity ready for substitution	M1
	$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{2}\right) = -\frac{63}{8} (= -7.875)$	Correct value	A1
			(4)
(c)	Sum = $\alpha^3 + \beta^3 - (\alpha + \beta) = -\frac{63}{8} - \frac{3}{2} (= -\frac{75}{8})$	Attempts sum Allow eg $(\alpha^3 - \beta) + (\beta^3 - \alpha)$ followed by $(\alpha^3 + \beta^3) + (\alpha + \beta) = \dots$	M1
	Prod = $(\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta$ and $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$	Expands $(\alpha^3 - \beta)(\beta^3 - \alpha)$ and uses a correct identity for $\alpha^4 + \beta^4$	M1
	Alt identities: $\alpha^4 + \beta^4 =$ $(\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2; \alpha^4 + \beta^4 = (\alpha^3 + \beta^3)(\alpha + \beta) - \alpha\beta(\alpha^2 + \beta^2)$		
	$(\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta = \left(\frac{5}{2}\right)^3 + \frac{5}{2} - \left(\left(-\frac{11}{4}\right)^2 - 2\left(\frac{5}{2}\right)^2\right) = \frac{369}{16}$		A1
	$x^2 + \frac{75}{8}x + \frac{369}{16} (= 0)$	Applies $x^2 - (\text{their sum})x + \text{their prod} (= 0)$	M1
	$16x^2 + 150x + 369 = 0$	Allow any integer multiple	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
6(a)	$x = 9t^2, y = 18t \Rightarrow \frac{dy}{dx} = \frac{18}{18t}$ or $y^2 = 36x \Rightarrow 2y \frac{dy}{dx} = 36 \Rightarrow \frac{dy}{dx} = \frac{18}{y} = \frac{18}{18t}$ or $y^2 = 36x \Rightarrow y = 6\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{x}} = \frac{3}{3t}$	Correct $\frac{dy}{dx}$ in terms of t There must be evidence of use of calculus ($\frac{dy}{dx} = \frac{1}{t}$ with no working scores B0)	B1
	$m_T = \frac{1}{t} \Rightarrow m_N = -t$	Correct use of the perpendicular gradient rule.	M1
	$y - 18t = -t(x - 9t^2)$	Correct straight line method for the normal. Must use their perpendicular gradient – not dy/dx . (Any complete method – use of $y = mx + c$ requires an attempt at “c”)	dM1
	$y + tx = 9t^3 + 18t^*$	Cso All previous marks must have been earned	A1*
			(4)
(b)	$x = 54, y = 0 \Rightarrow 54t = 9t^3 + 18t$ $\Rightarrow 9t^3 - 36t = 0$	Substitutes $x = 54$ and $y = 0$ into the equation from part (a) and attempts to collect terms.	M1
	$9t^3 - 36t = 0 \Rightarrow 9t(t^2 - 4) = 0$ $\Rightarrow t = \pm 2 \Rightarrow y \pm 2x = 9(\pm 2)^3 + 18(\pm 2)$	Solves to obtain at least one non zero value for t and attempts at least one normal equation	dM1
	$y = -2x + 108$ or $y = 2x - 108$	One correct equation in any equivalent form	A1
	$y = -2x + 108$ and $y = 2x - 108$	Both correct and in the required form	A1
			(4)
(c)	$x = -9 \Rightarrow y = 18 + 108 \text{ or } -18 - 108$	Uses $x = -9$ to find the y coordinate of A or B	M1
	Area = $\frac{1}{2} \times 252 \times 18$	Fully correct strategy for the area Award M0 if their x coord of the focus is not doubled	M1
	= 2268	Cao	A1
			(3)
			Total 11
ALT	Last 2 marks by “shoelace” method: eg $\begin{vmatrix} 1 & -9 & 9 & -9 & -9 \\ 2 & 126 & 0 & -126 & 126 \end{vmatrix}$ $= \frac{1}{2} (9 \times -126 - 9 \times 126 - (-9 \times -126 + 9 \times 126))$ = 2268	Their coordinates with first and last the same $\frac{1}{2}$ must be included Attempt to expand also needed Must be positive	M1 A1

Question	Scheme	Marks
8(a)	$y^2 = 4ax, y = k \Rightarrow P = \left(\frac{k^2}{4a}, k \right)$	B1
	Either $PS = \frac{k^2}{4a} + a = \dots$ or $PS^2 = \left(\frac{k^2}{4a} - a \right)^2 + k^2 = \dots \Rightarrow PS = \dots$	M1
	$PS = \frac{k^2 + 4a^2}{4a} *$	A1*
		(3)
(b)	Gradient of l_2 is $\frac{k}{\frac{k^2 + 4a^2}{4a}} = \frac{4ak}{k^2 + 4a^2}$ oe	B1
	$l_2 : y = \frac{4ak}{k^2 + 4a^2}(x + a) \Rightarrow y = \frac{4ak}{k^2 + 4a^2} \times (0 + a) = \dots$	M1
	$y _{x=0} = \frac{4a^2k}{k^2 + 4a^2} *$	A1*
		(3)
(c)	Area $OSP = \frac{1}{2} \times a \times k$	B1
	Area $BPA = \frac{1}{2} \times \frac{k^2 + 4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2 + 4a^2} \right) \quad \left(= \frac{k^3}{8a} \right)$	M1
	$\frac{\text{Area } BPA}{\text{Area } OSP} = \frac{\frac{k^2 + 4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2 + 4a^2} \right)}{ak} = 4k^2$	M1
	$\Rightarrow k^3 + 4a^2k - 4a^2k = 16a^2k^3 \Rightarrow a = \dots$	dM1
	$a = \frac{1}{4}$	A1
		(5)
(11 marks)		
Notes:		
(a) B1: Correct x coordinate at P found. May be seen on diagram. M1: For a full method to find an expression for PS . Either use of focus-directrix property or may use Pythagoras with their coordinates.		

A1*: Reaches the correct expression with a suitable intermediate step and no errors seen. If using Pythagoras the suitable step must be one with brackets expanded before factorising again.

(b)

B1: Correct expression for the gradient of l_2 given or implied by working. Need not be simplified. If

using a similar triangles approach this may be scored for e.g. $\frac{k^2 + 4a^2}{4a} \div k = \frac{a}{y}$ or $k = \left(\frac{k^2 + 4a^2}{4a} \right) m$

M1: Full method to find the y intercept, e.g. by forming the equation of the line and substituting $x = 0$

May use $y - k = \frac{4ak}{k^2 + 4a^2} \left(x - \frac{k^2}{4a} \right) \Rightarrow y = k + \frac{-k^3}{k^2 + 4a^2}$

A1*: Reaches correct answer with no errors seen.

(c)

NB: If a value is chosen for k (or $k = 2a$) used, all marks are available, score for the relevant correct expressions/methods with their value.

B1: Correct area of OSP stated or implied. Note that if they go direct to ratios, the $\frac{1}{2}$ may not be seen (as it cancels with that in BPA)

M1: Correct method for the area of the triangle BPA . Allow sign slips if the method is clear (e.g.

$\frac{k^2}{4a} - "a" = \frac{k^2}{4a} - a$ if it is clear BP is meant). Allow if the negative of the area is found.

M1: Applies the ratio correctly to the problem.

dM1: Attempts to solve their equation.

A1: Correct answer. Allow if the negative of the area was found and later made positive as recovery.

Question	Scheme	Marks
9	For $n = 1$, $\sum_{r=1}^1 \log(2r-1) = \log(2-1) = \log 1$ and $\log\left(\frac{(2 \times 1)!}{2^1 1!}\right) = \log 1$ So true for $n = 1$	B1
	(Assume the result is true for $n = k$, so $\sum_{r=1}^k \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right)$ Then) $\sum_{r=1}^{k+1} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right) + \log(2(k+1)-1)$	M1
	$= \log\left(\frac{(2k)!}{2^k k!} \times (2k+1)\right)$	M1
	$= \log\left(\frac{(2k+1)!}{2^k k!} \times \frac{2k+2}{2k+2}\right) = \log\left(\frac{(2k+2)!}{2^k \times 2(k+1)!}\right)$	M1
	$= \log\left(\frac{(2k+2)!}{2^{k+1}(k+1)!}\right)$	A1
	Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all $n \in \mathbb{N}$ by induction.	A1
		(6)
(6 marks)		
Notes:		
(a) B1: Checks the result for $n = 1$. Must see both sides (possibly in one line) identified as $\log 1$ or 0 but may not see much more than this. M1: Makes the inductive assumption (may be implied) and applies it to the question by adding the $(k+1)^{\text{th}}$ term to the expression for the sum to k terms. Allow if there are minor slips (e.g. a missing factorial) if the intent is clear. M1: Attempts to combine or split log terms appropriately. Not dependent, so may be scored if the wrong term is added in the previous M as long it is a log term. M1: Introduces the relevant cancelling factors to achieve the $(2k+2)!$ term. The introduction of the factors must shown or implied in an intermediate step. Alternatively, may decompose from the $k+1$ statement to achieve the same intermediate expression. A1: Achieves correct expression from correct work (or correctly shows equivalence). A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying “true for $n = 1$”, “assumed true for $n = k$” and “shown true for $n = k+1$”, and “hence true for all n”. Depends on all except the B mark, though a check for $n = 1$ must have been attempted. NB Allow the M’s and first A if n is used throughout but the steps are correct, but must have used a different variable for the final A.		

Question Number	Scheme	Notes	Marks
7(a)	$f(x) = x^{\frac{3}{2}} + x - 3$ $f(1) = 1 + 1 - 3 = -1 \quad f(2) = \sqrt{8} + 2 - 3 = 1.828...$	Calculates values for both $f(1)$ and $f(2)$ with one correct. Allow e.g., $f(2) = 2\sqrt{2} - 1$ or awrt 2	M1
	f is continuous and changes sign , so root or α in $[1, 2]$. Correct interval $[1, 2]$ if given. Sign change can be implied by “negative, positive”, “ $f(1) < 0, f(2) > 0$ ” or “ $f(1)f(2) < 0$ ”	Correct values and sight of continuous, sign change and e.g., root/shown/QED/true/proven/✓	A1
			(2)
(b) Work may be seen in a table	$f(1.5) = 1.5^{\frac{3}{2}} + 1.5 - 3 \quad \{= \dots 0.3371\dots\}$	Obtains a <u>numerical expression or value</u> for $f(1.5)$	M1
	$f(1.25) = 1.25^{\frac{3}{2}} + 1.25 - 3 = \dots \quad \{-0.3524\dots\}$	Obtains a <u>value</u> for $f(1.25)$. Requires previous M mark.	dM1
	$\Rightarrow$ root/ α /x/it's in/on/ $\in [1.25, 1.5]$ or “in $[1.25, 1.5]$ ” or $1.25 \leq \text{root}/\alpha/x \leq 1.5$	Correct values (awrt 0.3 and -0.3 or -0.4) and suitable conclusion. Allow “between $\frac{5}{4}$ and $\frac{3}{2}$ inclusive ”	A1
	Do not accept $[1.5, 1.25]$. Just “ $f(1.25) = \dots$ followed by $f(1.5) = \dots$ so...” is 100 if no evidence of interval bisection. There are no marks if it is a clear attempt at interpolation.		(3)
(c)(i)	$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 1$	Correct differentiation. Any correct equivalent e.g., $1.5\sqrt{x} + 1$	B1
(ii)	$\alpha \approx 1.375 - \frac{1.375^{\frac{3}{2}} + 1.375 - 3}{\frac{3}{2} \times 1.375^{\frac{1}{2}} + 1} = \dots$ $\left\{ = 1.375 - \frac{-0.01266958256\dots}{2.75890591\dots} = 1.375 + 0.004592248875\dots \right\}$ $= 1.379592249\dots$ $\left\{ \text{exact values: } \frac{11}{8} - \frac{11\sqrt{22} - 52}{32} \div \frac{8 + 3\sqrt{22}}{8} \right\}$	Correctly applies the Newton-Raphson formula with 1.375 & their $f'(x)$ and obtains a value. Some working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow “ $\dots = 1.375 - \frac{f(1.375)}{f'(1.375)}$ ” followed by value but formula must be fully substituted if just followed by value unless “ x_0 ” defined	M1
	awrt 1.380 or “1.38” (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		(3)
(d)	e.g., $\frac{\alpha - 1.25}{1.5 - \alpha} = \frac{0.3524575141\dots}{0.3371173071\dots}$ or e.g., $\frac{1.5 - \alpha}{0.337\dots} = \frac{1.5 - 1.25}{0.337\dots + 0.352\dots}$	Forms an equation in e.g., α with their $f(1.25)$ and $f(1.5)$ allowing for sign errors only but must be using differences. Allow use of “ $f(1.25)$ ” and “ $f(1.5)$ ” - could recover sign error	M1
	$\alpha = 1.377780737\dots = 1.378$	dM1: Solves $\Rightarrow$ value Requires previous M mark. A1: awrt 1.378	dM1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		(3)
Alt (Equation of line methods)	or $y - (-0.3524\dots[\text{or } 0.3371\dots]) = \frac{0.3371\dots - (-0.3524\dots)}{1.5 - 1.25} (x - 1.25[\text{or } 1.5])$ or $-0.3524\dots[\text{or } 0.3371\dots] = \frac{0.3371\dots - (-0.3524\dots)}{1.5 - 1.25} (1.25[\text{or } 1.5]) + c \Rightarrow c = \dots$		M1
	$\{ \Rightarrow y = 2.758\dots x - 3.800\dots \}$ $\alpha = 1.377780737\dots = 1.378$	dM1: Puts $y = 0$ and solves $\Rightarrow$ value Requires previous M mark. A1: awrt 1.378	dM1 A1
			(3)
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		Total 11

Question Number	Scheme	Notes	Marks
8	$y^2 = 8x$ $P(2p^2, 4p)$ $Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$		
	Each part is marked separately. For example there is no credit in (c) for work seen in (b) unless that work is referred to in (c)		
(a) Subs. both x and y into $y^2 = 8x$	$\text{LHS or } y^2 = \left(\frac{-4}{p}\right)^2 = \frac{16}{p^2} \quad \text{RHS or } 8x = 8 \times \frac{2}{p^2} = \frac{16}{p^2}$ So Q lies on the parabola* Allow e.g., $\left(\frac{-4}{p}\right)^2 = 8\left(\frac{2}{p^2}\right) \Rightarrow \frac{16}{p^2} = \frac{16}{p^2} \Rightarrow \text{true}$	Substitutes both coordinates of Q into the parabola equation, obtains e.g., $\frac{16}{p^2}$ twice and makes minimal conclusion - e.g., shown/QED/true/proven/✓ Sight of just "$y^2 = 8x$" is insufficient but allow "$y_Q^2 = 8x_Q$"	B1*
			(1)
Alt Subs. x or y to find y or x	$x = \frac{2}{p^2} \Rightarrow y^2 = 8 \times \frac{2}{p^2} \text{ or } \frac{16}{p^2} \Rightarrow y = \frac{-4}{p} \text{ or } \pm \frac{4}{p}$ or $y = \frac{-4}{p} \Rightarrow \frac{16}{p^2} = 8x \Rightarrow x = \frac{2}{p^2}$ So Q lies on the parabola*	Substitutes one coordinate of Q into the parabola equation to correctly find the other coordinate and makes minimal conclusion - e.g., - e.g., shown/QED/true/proven/✓ Sight of just "$y^2 = 8x$" is insufficient but allow "$y_Q^2 = 8x_Q$"	B1*
			(1)
8(b)	Focus is (2, 0) or $x = 2, y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2, y = 0$ used but award final A0	B1
	$\text{gradient of } PQ = \frac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}} \text{ or } \frac{-\frac{4}{p} - 4p}{\frac{2}{p^2} - 2p^2}$ $\left\{ = \frac{4p^3 + 4p}{2p^4 - 2} = \frac{2p^3 + 2p}{p^4 - 1} = \frac{2p(p^2 + 1)}{p^4 - 1} = \frac{2p}{p^2 - 1} \right\}$	Attempts the gradient of PQ condoning one term of incorrect sign. Allow this mark if they subsequently attempt to convert it to a normal gradient. Note that m may be obtained from $4p = 2mp^2 + c, -\frac{4}{p} = \frac{2m}{p^2} + c \Rightarrow m = \dots$	M1
	$\text{e.g., } y - 4p = \frac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}(x - 2p^2)$	Any correct equation for PQ. May use Q. Allow this mark to be implied if their equation would have been correct but errors were made simplifying a correct gradient.	A1
	If $y = mx + c$ is used, one of the following expressions or for c must be reached following correct gradient seen: $c = 4p - 2p^2(\text{gradient})$ or $c = \frac{-4}{p} - \frac{2}{p^2}(\text{gradient})$		
	Examples with fully simplified gradient (see overleaf for a fuller list): $x = 2 \Rightarrow y - 4p = \frac{2p}{p^2 - 1}(2 - 2p^2) \Rightarrow y = \frac{4p - 4p^3 + 4p^3 - 4p}{p^2 - 1} = 0$ $\text{or } y - 4p = \frac{2p}{p^2 - 1}(2 - 2p^2) \Rightarrow y - 4p = -4p \Rightarrow y = 0$ $y = 0 \Rightarrow -4p = \frac{2p}{p^2 - 1}(x - 2p^2) \Rightarrow x = \frac{-4p^3 + 4p + 4p^3}{2p} = 2$ $(2, 0) \Rightarrow -4p = \frac{2p}{p^2 - 1}(2 - 2p^2) \Rightarrow -4p = -4p$		$\Rightarrow$ So PQ passes through the focus*
			A1*

	Substitutes $x = 2$ and shows $y = 0$ or vice versa or substitutes both values and shows that the equation is true. Must have minimal conclusion e.g., shown/QED/true/proven/✓ and no incorrect work. Condone no conclusion if the mark in (a) was withheld for this reason only. The examples indicate the minimum level of algebra acceptable. With the exception of using $(2, 0)$ with a fully simplified gradient, <u>look for substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "$1=1$" followed by a minimal conclusion</u>		
	Work in "a" can only access the accuracy marks when $a = 2$ is substituted		(4)
Alt 1 Grad PF = Grad QF	Focus is $(2, 0)$ or $x = 2, y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone $(0, 2)$ if $x = 2, y = 0$ used but award final A0	B1
	$\text{gradient } PF = \frac{4p}{2p^2 - 2} \text{ or } \frac{-4p}{2 - 2p^2}$ $\text{and gradient } QF = \frac{\frac{4}{2 - \frac{2}{p^2}}}{\frac{-4}{\frac{2}{p^2} - 2}}$	M1: Obtains expressions for both gradients condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe	M1 A1
	$\text{Grad } QF = \frac{4p}{2p^2 - 2} = \text{Grad } PF$ So PQ passes through the focus*	Shows that the gradients are the same plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).	A1*
	Note: A variation is to show grad PF or grad QF = grad PQ – marked as Alt		(4)
	Alt 2 Follows (similar triangles)		
8(b) Examples of minimum amount of algebra required with different expressions for gradient:			
$y - 4p = \frac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}(x - 2p^2)$			
$x = 2, y = \dots$	$x = 2 \Rightarrow y - 4p = \frac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}(2 - 2p^2) \Rightarrow y = \frac{8p + \frac{8}{p} - 8p^3 - 8p + 8p^3 - \frac{8}{p}}{2p^2 - \frac{2}{p^2}} = 0$		
$y = 0, x = \dots$	$y = 0 \Rightarrow -4p = \frac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}(x - 2p^2) \Rightarrow x = \frac{-8p^3 + \frac{8}{p} + 8p^3 + 8p}{4p + \frac{4}{p}} = 2$		
$(2, 0) \Rightarrow$	$(2, 0) \Rightarrow -4p = \frac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}(2 - 2p^2) \Rightarrow -4p = \frac{8p + \frac{8}{p} - 8p^3 - 8p}{2p^2 - \frac{2}{p^2}} \Rightarrow -4p = -4p$		
$y - 4p = \frac{4p^3 + 4p}{2p^4 - 2}(x - 2p^2)$			
$x = 2, y = \dots$	$x = 2 \Rightarrow y - 4p = \frac{4p^3 + 4p}{2p^4 - 2}(2 - 2p^2) \Rightarrow y = \frac{8p^3 + 8p - 8p^5 - 8p^3 + 8p^5 - 8p}{2p^4 - 2} = 0$ $\text{or } y - 4p = \frac{4p^3 + 4p}{2p^4 - 2}(2 - 2p^2) \Rightarrow y = \frac{-4p^3 - 4p + 4p^3 + 4p}{p^2 + 1} = 0$		
$y = 0, x = \dots$	$y = 0 \Rightarrow -4p = \frac{4p^3 + 4p}{2p^4 - 2}(x - 2p^2) \Rightarrow x = \frac{-8p^5 + 8p + 8p^5 + 8p^3}{4p^3 + 4p} = 2$		

$(2, 0) \Rightarrow$	$(2, 0) \Rightarrow -4p = \frac{4p^3 + 4p}{2p^4 - 2} (2 - 2p^2) \Rightarrow -4p = \frac{8p^3 + 8p - 8p^5 - 8p^3}{2p^4 - 2} \Rightarrow -4p = -4p$		
$y - 4p = \frac{2p}{p^2 - 1} (x - 2p^2)$			
$x = 2, y = \dots$	$x = 2 \Rightarrow y - 4p = \frac{2p}{p^2 - 1} (2 - 2p^2) \Rightarrow y = \frac{4p - 4p^3 + 4p^3 - 4p}{p^2 - 1} = 0$ or $y - 4p = \frac{2p}{p^2 - 1} (2 - 2p^2) \Rightarrow y - 4p = -4p \Rightarrow y = 0$		
$y = 0, x = \dots$	$y = 0 \Rightarrow -4p = \frac{2p}{p^2 - 1} (x - 2p^2) \Rightarrow x = \frac{-4p^3 + 4p + 4p^3}{2p} = 2$		
$(2, 0) \Rightarrow$	$(2, 0) \Rightarrow -4p = \frac{2p}{p^2 - 1} (2 - 2p^2) \Rightarrow -4p = -4p$		
Note that this not an exhaustive list (for example there are all the corresponding $y = mx + c$ approaches or those using Q) and the precise choice of algebra will vary widely but with the exception of the last example above this mark requires <u>substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "$1=1$" followed by a minimal conclusion</u> (unless B0 was given in (a) for that reason).			
8(b) cont.	$y^2 = 8x \quad P(2p^2, 4p) \quad Q\left(\frac{2}{p^2}, \frac{-4}{p}\right) \quad X\left(2, \frac{-4}{p}\right) \quad Y\left(2p^2, \frac{-4}{p}\right)$		
Alt 2 Similar triangles	Focus is $(2, 0)$ or $x = 2, y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone $(0, 2)$ if $x = 2, y = 0$ used but award final A0	B1
	$\frac{XF}{XP} = \frac{\frac{4}{p}}{4p + \frac{4}{p}} \quad \frac{QX}{QY} = \frac{2 - \frac{2}{p^2}}{2p^2 - \frac{2}{p^2}}$	M1: Obtains expressions for two ratios condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe	M1 A1
	$\frac{XF}{XP} = \frac{1}{p^2 + 1} = \frac{QX}{QY}$ So $\Delta s XFQ$ & YPQ are similar $\Rightarrow PQ$ passes through the focus*	Shows that the ratios are the same, makes reference to similarity plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).	A1*
			(4)
8 cont.	$y^2 = 8x \quad P(2p^2, 4p) \quad Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$		
(c)	$y = \sqrt{8x^{\frac{1}{2}}} \Rightarrow \frac{dy}{dx} = \frac{1}{2} \sqrt{8x^{-\frac{1}{2}}}$ or $2y \frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{8}{2y}$ or $x = at^2, y = 2at \Rightarrow \frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a \Rightarrow \frac{dy}{dx} = \frac{2a}{2at}$ (They may use p for t and/or $a = 2$)	Achieves an expression of the correct form (sign/coefficient errors only) for the gradient. There is no requirement for calculus so they may use e.g., $m_t = \frac{1}{t}$. Must go beyond $\frac{dx}{dy}$	M1
	$m_t \text{ at } P = \frac{1}{2} \sqrt{8} \frac{1}{\sqrt{2p^2}} \text{ or } \frac{8}{2 \times 4p} \text{ or } \frac{2 \times 2}{2 \times 2 \times p} \left\{ = \frac{1}{p} \right\}$ or $m_t \text{ at } Q = -\frac{1}{2} \sqrt{8} \frac{1}{\sqrt{\frac{2}{p^2}}} \text{ or } -\frac{8}{2 \times \left(\frac{-4}{p}\right)} \text{ or } x = \frac{2}{p^2}, y = \frac{-4}{p} \Rightarrow \frac{dx}{dp} = \frac{-4}{p^3}, \frac{dy}{dp} = \frac{4}{p^2} \Rightarrow \frac{dy}{dx} = -p$ Any correct unsimplified expression for the tangent gradient in terms of p at either point. If awarding for Q via explicit differentiation must choose the negative root		A1

	Eqn of tgt at P: $y - 4p = \frac{1}{p}\left(x - 2p^2\right)$ oe or Eqn of tgt at Q: $y + \frac{4}{p} = -p\left(x - \frac{2}{p^2}\right)$ oe	M1: Correct straight line method for either point with their tangent gradient in terms of p (but allow if “a” also present) Coordinates correctly placed. If $y = mx + c$ is used must reach $c = \dots$ following correctly placed coordinates A1: Any correct unsimplified equation for either tangent	M1 A1
	Note: $y = mx + c$: At P, $y = \frac{1}{p}x + 2p$ At Q, $y = -px - \frac{2}{p}$		
	$\frac{1}{p}\left(x - 2p^2\right) + 4p = -p\left(x - \frac{2}{p^2}\right) - \frac{4}{p} \Rightarrow x = \dots$ or $\frac{1}{p}x + 2p = -px - \frac{2}{p} \Rightarrow x = \dots$ Eliminates y from their tangent equations and solves for x (See note below if eliminate x). Gradients must be different and no clear evidence of conversion of any line to a normal. Condone poor algebra.		M1
	$x\left(\frac{1}{p} + p\right) = -\frac{2}{p} - 2p \Rightarrow x = \frac{-(2p^2 + 2)}{(p^2 + 1)} = -2$	$x = -2$ only	A1
	$y = \frac{1}{p}\left(-2 - 2p^2\right) + 4p, -p\left(-2 - \frac{2}{p^2}\right) - \frac{4}{p}, \frac{1}{p}(-2) + 2p, -p(-2) - \frac{2}{p}$ dM1: Substitutes their x (a constant or function of p) into one of their two tangent equations to obtain an expression for y . Requires previous M mark.		dM1
	e.g., $y = 2p - \frac{2}{p}, 2\left(p - \frac{1}{p}\right), \frac{2}{p}(p^2 - 1), \frac{2p^2 - 2}{p}, \frac{2(p + 1)(p - 1)}{p}$ A1: Correct y in simplest form – two terms which could be factorised in any correct way and/or written as a single fraction. Note there is no requirement for coordinate notation.		A1
	Note it is obviously possible to eliminate x . In this case, award the last 4 marks in this order: M1: Eliminates x and solves for y A1: Any correct y in simplest form dM1: Substitutes their y (a constant or function of p) into one of their two tangent equations to obtain an expression for x . Requires previous M mark. A1: $x = -2$		(8)
	Working which involves “ a ” where a is never replaced by 2 can score the Ms		Total 13

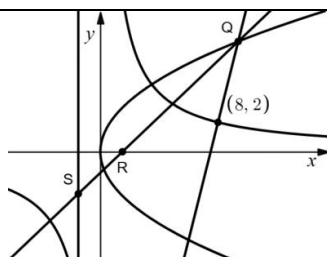


Question Number	Scheme	Notes	Marks
8	$f(k) = 7^{k-1} + 8^{2k+1}$		
	General guidance: Apply the Way that best fits the overall approach. Condone work in e.g., n instead of k. Allow use of -57 but if any different multiples of 57 are involved, e.g., 114, the last A1 additionally requires “114 is a multiple of/divisible by (but not “factor of”) 57” oe for each case. Ignore work re the divisibility of $f(2)$, $f(3)$ etc but starting with e.g., $f(2)$ scores a max of 01110. Final A1: There must be evidence that true for $n = k \Rightarrow$ true for $n = k + 1$ but it could be minimal and be scored in a conclusion or a narrative or via both. So if e.g., “Assume true for $n = k \dots$” is seen in the work followed by “true for $n = k + 1$” in a conclusion this is sufficient. May say “is divisible by 57” for “true”. Condone “for all $n \in \mathbb{Z}$” but not $n \in \mathbb{R}$		
Way 1 $f(k+1)$ $-f(k)$	$n = 1: f(1) = [7^0 + 8^3 =] 513,$ $513 \div 57 = 9$ oe	Obtains 513 for $f(1)$ and shows 513 is divisible by 57. Allow $\frac{1+512}{57} = 9$	B1
	$[f(k+1) =] 7^{(k+1)-1} + 8^{2(k+1)+1} \{ = 7^k + 8^{2k+3} \}$	Attempts $f(k+1)$	M1
	$[f(k+1) - f(k) =]$ $7(7^{k-1}) - 7^{k-1} + 8^2(8^{2k+1}) - 8^{2k+1}$	Obtains expression for $f(k+1) - f(k)$ in 7^{k-1} and 8^{2k+1} only	M1
	$= 6(7^{k-1} + 8^{2k+1}) + 57(8^{2k+1})$ $\Rightarrow f(k+1) = 7f(k) + 57(8^{2k+1})$ or $= 63(7^{k-1} + 8^{2k+1}) - 57(7^{k-1})$ $\Rightarrow f(k+1) = 64f(k) - 57(7^{k-1})$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$. May not see $f(k+1) = \dots$ A1: Correct expression. Must see $f(k+1) = \dots$ Allow if e.g., $7f(k)$ written as $7(7^{k-1} + 8^{2k+1})$ or $7(7^{k-1}) + 7(8^{2k+1})$	M1 A1
	Shown true for $n = 1$ and if true for $n = k$ then true for $n = k + 1$ so true for all $n \in \mathbb{Z}^+$	Makes correct conclusion or narrative with no errors throughout. Minimum in bold . Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	A1
(6)			
Way 2 $f(k+1) =$	$n = 1: f(1) = [7^0 + 8^3 =] 513,$ $513 \div 57 = 9$ oe	Obtains 513 for $f(1)$ and shows 513 is divisible by 57. Allow $\frac{1+512}{57} = 9$	B1
	$[f(k+1) =] 7^{(k+1)-1} + 8^{2(k+1)+1} \{ = 7^k + 8^{2k+3} \}$	Attempts $f(k+1)$	M1
	$[f(k+1) =] 7(7^{k-1}) + 8^2(8^{2k+1})$	Obtains expression for $f(k+1)$ in 7^{k-1} and 8^{2k+1} only	M1
	$= 7(7^{k-1} + 8^{2k+1}) + 57(8^{2k+1})$ $\Rightarrow f(k+1) = 7f(k) + 57(8^{2k+1})$ or $= 64(7^{k-1} + 8^{2k+1}) - 57(7^{k-1})$ $\Rightarrow f(k+1) = 64f(k) - 57(7^{k-1})$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$. May not see $f(k+1) = \dots$ A1: Correct expression. Must see $f(k+1) = \dots$ Allow if e.g., $7f(k)$ written as $7(7^{k-1} + 8^{2k+1})$ or $7(7^{k-1}) + 7(8^{2k+1})$	M1 A1
	Shown true for $n = 1$ and if true for $n = k$ then true for $n = k + 1$ so true for all $n \in \mathbb{Z}^+$	Makes correct conclusion or narrative with no errors throughout. Minimum in bold . Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	A1
(6)			

Question Number	Scheme	Notes	Marks
8 Way 3 $f(k+1) - mf(k)$	$n=1: f(1) = [7^0 + 8^3] = 513,$ $513 \div 57 = 9$ oe	Obtains 513 for $f(1)$ and shows 513 is divisible by 57. Allow $\frac{1+512}{57} = 9$	B1
	$[f(k+1)] = 7^{(k+1)-1} + 8^{2(k+1)+1} \{ = 7^k + 8^{2k+3} \}$	Attempts $f(k+1)$	M1
	$f(k+1) - mf(k)$ $= 7(7^{k-1}) - (7^{k-1})m + 8^2(8^{2k+1}) - (8^{2k+1})m$	Obtains expression for $f(k+1) - mf(k)$ in 7^{k-1} and 8^{2k+1} only	M1
	e.g., $m = 7 \Rightarrow$ $f(k+1) - 7f(k) = 57(8^{2k+1})$ $\Rightarrow f(k+1) = 7f(k) + 57(8^{2k+1})$ e.g., $m = 64 \Rightarrow$ $f(k+1) - 64f(k) = -57(7^{k-1})$ $\Rightarrow f(k+1) = 64f(k) - 57(7^{k-1})$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$ using a value for m . May not see $f(k+1) = \dots$ A1: A correct expression. Must see $f(k+1) = \dots$ Allow if $\beta f(k)$ written as $\beta(7^{k-1} + 8^{2k+1})$ or $\beta(7^{k-1}) + \beta(8^{2k+1})$	M1 A1
	Shown true for $n = 1$ and if true for $n = k$ then true for $n = k + 1$ so true for all $n \in \mathbb{Z}^+$	Makes correct conclusion or narrative with no errors throughout. Minimum in bold . Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	A1
(6)			
Way 4 $f(k) = 57\lambda$	$n=1: f(1) = [7^0 + 8^3] = 513,$ $513 \div 57 = 9$ oe	Obtains 513 for $f(1)$ and shows 513 is divisible by 57. Allow $\frac{1+512}{57} = 9$	B1
	$[f(k+1)] = 7^{(k+1)-1} + 8^{2(k+1)+1} \{ = 7^k + 8^{2k+3} \}$	Attempts $f(k+1)$	M1
	$[f(k+1)] = 7(7^{k-1}) + 8^2(8^{2k+1})$ $= 7(7^{k-1} + 8^{2k+1}) + 57(8^{2k+1})$	Obtains expression for $f(k+1)$ in 7^{k-1} and 8^{2k+1} only	M1
	$f(k) = 57\lambda \Rightarrow f(k+1) = 399\lambda + 57(8^{2k+1})$ or $= 7 \times 57\lambda + 57(8^{2k+1})$ or $= 64(7^{k-1} + 8^{2k+1}) - 57(7^{k-1})$ $f(k) = 57\lambda \Rightarrow f(k+1) = 64 \times 57\lambda - 57(7^{k-1})$ or $= 3648\lambda - 57(7^{k-1})$	M1: Obtains expression for $f(k+1)$ in terms of λ with $f(k) = 57\lambda$ seen. May not see $f(k+1) = \dots$ A1: Correct expression Must see $f(k+1) = \dots$	M1 A1
	Shown true for $n = 1$ and if true for $n = k$ then true for $n = k + 1$ so true for all $n \in \mathbb{Z}^+$	Makes correct conclusion or narrative with no errors throughout. Minimum in bold . Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	A1
(6)			
Total 6			

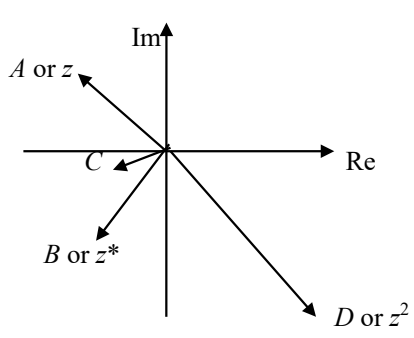
Question Number	Scheme	Notes	Marks
9(a)	$y = c^2 x^{-1}$ $\frac{dy}{dx} = -c^2 x^{-2} = -\frac{c^2}{x^2}$ $\left(ct, \frac{c}{t}\right) \Rightarrow \frac{dy}{dx} = -\frac{c^2}{c^2 t^2}$ Correct expression for $\frac{dy}{dx}$ in terms of c and t (or just t). Award when seen and isw. Allow for a correct $\frac{dx}{dy}$ or $-\frac{dx}{dy}$		$xy = c^2$ $y + x \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{-\frac{c}{t}}{ct}$ $x = ct$ $y = \frac{c}{t}$ $\frac{dx}{dt} = c$ $\frac{dy}{dt} = -ct^{-2}$ $\frac{dy}{dx} = -\frac{ct^{-2}}{c}$ B1
	$m_T = -\frac{1}{t^2} \Rightarrow m_N = t^2$	Correct perpendicular gradient rule for their $\frac{dy}{dx}$ in terms of t (or c and t)	M1
	$y - \frac{c}{t} = "t^2"(x - ct)$ or $y = "t^2"x + b \Rightarrow \frac{c}{t} = "t^2"(ct) + b \Rightarrow b = ...$	Correct straight line method with a changed gradient in terms of t (or c and t) with coordinates correctly placed. Condone the use of $y = mx + c$ instead of e.g. $y = mx + b$	M1
	$ty - c = t^3x - ct^4$ or $y = t^2x + \frac{c}{t} - ct^3$ $\Rightarrow t^3x - ty = c(t^4 - 1)^*$	Fully correct proof with at least one intermediate line before printed answer but allow if equation reversed and/or order altered e.g., $(-1 + t^4)c = -ty + t^3x$	A1*
	Score a maximum of 0110 if they start with just $\frac{dy}{dx} = -\frac{1}{t^2}$ and 0010 if just $m_N = t^2$		
(4)			

Question Number	Scheme	Notes	Marks
9(b)	$(8, 2) \Rightarrow \text{e.g., } c^2 = 16, c = 4;$ $ct = 8 \text{ or } \frac{c}{t} = 2 \Rightarrow t = 2$	Correct values for c and t seen, used or implied (e.g., by correct normal). If $c = \pm 4, t = \pm 2$ then the positive values must be implied by subsequent work	B1
	Note that another way of finding t is by using $c = 4$ and $(8, 2)$ in the normal: $\Rightarrow 8t^3 - 2t = 4(t^4 - 1) \Rightarrow 4t^4 - 8t^3 + 2t - 4 = (t - 2)(4t^3 + 2) = 0 \Rightarrow t = 2$		
	normal : $8x - 2y = 60 \Rightarrow$ $y = 4x - 30 \text{ or } x = \frac{15}{2} + \frac{1}{4}y$ $\Rightarrow (4x - 30)^2 = 6x \text{ or } y^2 = 45 + \frac{3}{2}y$	Uses their values of c and t in the given normal $t^3x - ty = c(t^4 - 1)$ [could repeat the work in (a) with $y = 16x^{-1}$] and substitutes into the parabola to obtain a quadratic equation. Note that appropriate work must be seen for this mark. $4x - 30 = \sqrt{6x}$ must be followed by a credible attempt to square (i.e., a 3TQ on LHS and ... x on the RHS) but see note below	M1
	Note that replacing x with e.g., k^2 in $4x - 30 = \sqrt{6x} \rightarrow$ $4k^2 - 30 = \sqrt{6}k \Rightarrow k = \frac{\sqrt{6} \pm \sqrt{6 - 4(4)(-30)}}{2(4)} = \frac{5\sqrt{6}}{4}, -\sqrt{6} \Rightarrow x = \frac{75}{8}, 6$ Scores the M1 for the quadratic in k and the dM1 for solving via usual rules and also reaching $x = \dots$ by squaring.		
	$16x^2 - 246x + 900 = 0 \Rightarrow 8x^2 - 123x + 450 = 0$ $\Rightarrow (8x - 75)(x - 6) = 0 \Rightarrow x = \dots \text{ or }$ $2y^2 - 3y - 90 = 0 \Rightarrow (2y - 15)(y + 6) = 0 \Rightarrow y = \dots$	Solves 3TQ (usual rules – one correct root if no working). Requires previous method mark.	dM1
	$x = \frac{75}{8}, y = \frac{15}{2} \text{ or e.g., } Q(9.375, 7.5)$	Correct values/coordinates. Allow any equivalent fractions. If a second point is given e.g., $(6, -6)$ or $(6, 6)$ score A0 if it is not rejected in (b).	A1
(4)			
Alt Approaches using parametric coords	$c = 4, t = 2$	Correct values for c and t seen or used	B1
	Let Q have coordinates $(\frac{3}{2}k^2, 3k)$: Substituting into the normal with $c = 4$ and $t = 2$: $8(\frac{3}{2}k^2) - 2(3k) = 4(16 - 1)$ OR Since gradient of normal to hyperbola $= t^2 = 4$, gradient of AQ where A is $(8, 2) = \frac{3k - 2}{\frac{3}{2}k^2 - 8} = 4$ Forms a quadratic equation with their values. The equation in case 2 implies the B1.		M1
	$12k^2 - 6k = 60$ or $3k - 2 = 6k^2 - 32 \Rightarrow 6k^2 - 3k - 30 = 0$ $\Rightarrow 2k^2 - k - 10 = 0 \Rightarrow (2k - 5)(k + 2) = 0 \Rightarrow k = \dots[\frac{5}{2}]$ $\Rightarrow x = \dots \text{ or } y = \dots$	Solves 3TQ (usual rules – one correct root if no working) and proceeds to a value of x or y Requires previous method mark.	dM1
	$x = \frac{75}{8}, y = \frac{15}{2} \text{ or e.g., } Q(9\frac{3}{8}, 7\frac{1}{2})$	Correct values/coordinates. Allow any equivalent fractions. If a second point is given e.g., $(6, -6)$ or $(6, 6)$ score A0 if it is not rejected in (b).	A1
(4)			

Question Number	Scheme	Notes	Marks
9(c)	$R\left(\frac{3}{2}, 0\right)$ Correct coordinates for the focus seen or used. Can score anywhere e.g., written across the question. Condone sight of $\left(0, \frac{3}{2}\right)$ if used correctly e.g. in gradient calculation. If on a diagram, accept $\frac{3}{2}$ appropriately placed. Accept 1.5, $\frac{6}{4}$ etc. for $\frac{3}{2}$. Just " a or $x = \frac{3}{2}$ " or " $R = \frac{3}{2}$ " is insufficient. There must be some recognition of the <u>position</u> of R .		B1
	Allow work with decimals for the 3 M marks.		
	QR : e.g., $y - 0 = \left(\frac{\frac{15}{2} - 0}{\frac{75}{8} - \frac{3}{2}}\right)(x - \frac{3}{2})$ or $y = \left(\frac{\frac{15}{2} - 0}{\frac{75}{8} - \frac{3}{2}}\right)x + c \Rightarrow 0 = \frac{20}{21}\left(\frac{3}{2}\right) + c \Rightarrow c = \dots$ Correctly forms equation of QR for their Q and R . Q could be "made up" or be an incorrect choice from part (b) but must have real coordinates (A, B) , $A > 0$, $B \neq 0$ so allow e.g., $(6, 6)$ and $(6, -6)$. R must be of form $(\alpha, 0)$, $\alpha > 0$ Allow if a correct gradient is seen but wrongly calculated before line equation is given. If using $y = mx + c$ the equation must be formed correctly and " $c = \dots$ " reached following correct placement of $(\alpha, 0)$. For $0 = \frac{3}{2}m + c$, $\frac{15}{2} = \frac{75}{8}m + c \Rightarrow m = \dots$, $c = \dots$ must find both m and c with one correct M0 for a vertical line or if a normal gradient is used		M1
	$y = \frac{20}{21}x - \frac{10}{7}$, $x = -\frac{3}{2}$ $\Rightarrow y = \frac{20}{21}\left(-\frac{3}{2}\right) - \frac{10}{7} = -\frac{10}{7} - \frac{10}{7} = -\frac{20}{7}$	Substitutes $x = -\alpha$, $\alpha > 0$ into their equation to find a value for the y coordinate. Must be using a consistent α Requires previous M mark.	dM1
	$S\left(-\frac{3}{2}, -\frac{20}{7}\right) \Rightarrow$ $QS = \sqrt{\left(\frac{75}{8} - \left(-\frac{3}{2}\right)\right)^2 + \left(\frac{15}{2} - \left(-\frac{20}{7}\right)\right)^2}$	Applies correct distance formula for their $Q(A, B)$, $A > 0$, $B \neq 0$ and $S(-\alpha, \pm \beta)$ $\alpha > 0$ and consistent, $\beta \neq 0$ Implied by 15.017857... otherwise working must be seen. Requires both previous M marks. Note that using $(6, 6)$ or $(6, -6) \rightarrow QS = \frac{25}{2}$	ddM1
	$\left[= \sqrt{\left(\frac{87}{8}\right)^2 + \left(\frac{145}{14}\right)^2} = \sqrt{\frac{7569}{64} + \frac{21025}{196}} = \sqrt{\frac{707281}{3136}} \right] \Rightarrow QS = \frac{841}{56}$ Correct exact distance. Any exact equivalent e.g., $15\frac{1}{56}$ and may not be in simplest form		A1
Alt For the last two marks (QS = QR + RS)	$QS = QR + RS$ but $QR =$ shortest distance of Q to directrix $= \frac{75}{8} + \frac{3}{2} = \frac{87}{8}$ $QS = \sqrt{\left(0 - \left(-\frac{20}{7}\right)\right)^2 + \left(\frac{3}{2} - \left(-\frac{3}{2}\right)\right)^2} + \frac{87}{8} = \frac{29}{7} + \frac{87}{8} = \frac{841}{56}$ M1: A full method correct for their Q and S . Implied only by awrt 15.017857... A1: Correct exact distance (any equivalent)		
			
(5)			
Total 13			
PAPER TOTAL: 75 marks			

Question Number	Scheme	Marks
4		
(a)	$\alpha + \beta = -\frac{5}{2} \quad \alpha\beta = \frac{7}{2}$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \left(-\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(-\frac{5}{2}\right)$ $= \frac{85}{8}$	B1 M1 A1 (3)
(b)	$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \dots$ $= \left(\frac{85}{8}\right) \times \left(\frac{2}{7}\right) = \frac{85}{28}$ $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = \frac{7}{2}$ $x^2 - \frac{85}{28}x + \frac{7}{2} (= 0)$ $28x^2 - 85x + 98 = 0$	M1 A1 B1ft M1 A1 (5) [8]
(a)	Note: if a candidate solves the equation and uses the roots to answer the question, then send to review.	
B1	Both correct. (Seen anywhere in the working)	
M1	Uses their sum and product of roots in a correct expression for $\alpha^3 + \beta^3$.	
A1	Correct value. Must be exact. Accept 10.625	
(b)		
M1	$\alpha^3 + \beta^3 \quad \alpha\beta \quad \frac{\alpha^3 + \beta^3}{\alpha\beta} = \dots$ substitutes their values for $\alpha^3 + \beta^3$ and $\alpha\beta$ into $\frac{\alpha^3 + \beta^3}{\alpha\beta}$ (allow slips in substitution).	
A1	Correct sum as a single fraction (may be seen or implied in their equation)	
B1ft	Correct product or follow through their product	
M1	Use $x^2 - \text{sum of roots} \times x + \text{product of roots}$ with their values for sum and product. “= 0” may be missing.	
A1	A correct final equation as shown or any integer multiple of this. “= 0” must be included.	

Question Number	Scheme	Marks
5(a)	$\sum_{r=1}^n (r+1)(r+5) = \sum_{r=1}^n (r^2 + 6r + 5)$ $= \sum_{r=1}^n r^2 + 6 \sum_{r=1}^n r + 5n$ $= \frac{n}{6}(n+1)(2n+1) + 6 \frac{n}{2}(n+1) + 5n$ $= \frac{n}{6}(2n^2 + 3n + 1 + 18n + 18 + 30)$ $= \frac{n}{6}(2n^2 + 21n + 49) = \frac{n}{6}(n+7)(2n+7) \quad *$	B1 M1A1 dM1 A1 * (5)
(b)	$\sum_{r=n+1}^{2n} = \sum_{r=1}^{2n} - \sum_{r=1}^n = \frac{2n}{6}(2n+7)(4n+7) - \frac{n}{6}(n+7)(2n+7)$ $= \frac{n}{6}(2n+7)\{8n+14 - (n+7)\}$ $= \frac{7n}{6}(2n+7)(n+1)$	M1 A1 (2)
[7]		
(a) B1 M1 A1 dM1 A1* (b) M1 A1	Brackets multiplied out correctly. Summation signs not needed. Use at least two correct formulae from $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n 1 = n$. Fully correct expression. Attempt to remove factor $\frac{n}{6}$ from an expression with common factor n present. (if “5n” is just 5 then this mark will not be scored). Must be seen before the given answer is quoted. No need to simplify the remaining quadratic factor. Obtain the correct 3 term quadratic and factorise. This is a “show that” question, so the 3 TQ must be seen. No errors seen. Use $\sum_{r=n+1}^{2n} = \sum_{r=1}^{2n} - \sum_{r=1}^n$ Simplify to the correct answer.	

Question Number	Scheme	Marks
6(a)	$\lambda = 4$	B1 (1)
(b)	$\arctan \frac{3}{4}$ or $\arctan \frac{-3}{4}$ (Second quadrant so $\arg z = 2.498\dots = 2.5$ (rad))	M1 A1 (2)
(c)(i)	$\frac{z+3i}{2-4i} = \frac{-4+6i}{2-4i} \times \frac{2+4i}{2+4i}$ or $\frac{z+3i}{2-4i} = a+ib \Rightarrow -4+6i = (a+ib)(2-4i)$ $= \frac{-8+12i-16i+24i^2}{4+16} = -\frac{8}{5} - \frac{1}{5}i$ Accept e.g. $\frac{-32-4i}{20}$ Or $2a+4b = -4, 2b-4a = 6 \Rightarrow a = \dots, b = \dots$	M1 dM1A1
(ii)	$z^2 = (-4+3i)^2 = 16-24i+9i^2 = 16-24i-9$ $= 7-24i$	M1 A1ft (5)
(d)		B1 B1ft B1ft (3)
[11]		
(a) B1 (b) M1 A1 (c)(i) M1 dM1 A1 (ii) M1 A1ft (d) B1 B1ft	Correct answer. No working needed. For $\arctan\left(\pm \frac{3}{4}\right)$ with their "4". Can be awarded from $\tan \theta = \pm \frac{3}{4} \Rightarrow \theta = \dots$ or by implication if correct value for either $\arctan$ or correct final answer (rounded or not rounded, may be degrees) is seen. Cao 2.5 Multiplies numerator and denominator by complex conjugate of denominator. Award if denominator of $4+16$ or 20 is seen instead of product. May still have z at this stage, or even allow with $\lambda+3i$ as numerator. Alternatively, sets equal to $a+ib$ and cross multiplies. Using their λ or 4 substitutes correctly for z, fully expands the numerator and uses $i^2 = -1$ Alt, uses $i^2 = -1$, equates real and imaginary terms and solves their equations for a and b Correct answer only, as shown or single fraction accepting equivalent fractions or with exact decimals $(-1.6 - 0.2i)$. Squaring an expression of form $k+3i$ (with a real value for k) to get 3 terms (may be implied) and uses $i^2 = -1$ Correct answer, follow through their $\lambda > 0$ (ie for "$\lambda^2 - 9$" - "6λ" i must be negative i term) NB: Penalise once only (in the first mark due) for mislabelling or failing to label points as long as they look to be placed correctly. Award if lines/arrows not included. Points may be labelled by letter, name or their Cartesian coordinates (which may be given on the axes). Plots z in second quadrant and z^* as mirror image in the Real axis. Both must be labelled. Plot and label C for their solution to (c)(ii) It must be the correct side of B for their	

Question Number	Scheme	Notes	Marks
6(a)	$y = \sqrt{8x^{\frac{1}{2}}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{8x^{-\frac{1}{2}}} = \sqrt{2x^{-\frac{1}{2}}}$ $y^2 = 8x \Rightarrow 2y \frac{dy}{dx} = 8$ or $\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = 4 \cdot \frac{1}{4t}$	Attempts a derivative expression, such as $\frac{dy}{dx} = kx^{-\frac{1}{2}}$ or $ky \frac{dy}{dx} = c$ or their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}} \right)$	M1
	$\frac{dy}{dx} = \frac{1}{2}\sqrt{8x^{-\frac{1}{2}}} \left(= \sqrt{2x^{-\frac{1}{2}}} \right)$ or $2y \frac{dy}{dx} = 8$ or $\frac{dy}{dx} = 4 \cdot \frac{1}{4t} \left(= \frac{1}{t} \right)$	Correct differentiation (need not have substituted for t etc)	A1
	At P , gradient of normal is $m_N = -p$	Correct gradient for the normal.	A1
	$y - 4p = -p(x - 2p^2)$	$y - 4p = (\text{their } m_N) \times (x - 2p^2)$ or $y = (\text{their } m_N)x + c$ using $x = 2p^2, y = 4p$ in an attempt to find c . Their gradient must be a function of p for marks to be awarded. Must use a changed gradient, not tangent gradient.	M1
	$y + px = 2p^3 + 4p^*$	cso	A1
			(5)
6(b)	$y + qx = 2q^3 + 4q$	oe	B1
			(1)
6(c)	$y + px = 2p^3 + 4p$ $y + qx = 2q^3 + 4q$ $px - qx = 2p^3 + 4p - 2q^3 - 4q$	Attempt to solve simultaneous equations. A correct equation in only one variable	M1 A1
	$(p - q)x = 2((p - q)(p^2 + pq + q^2) + 2(p - q))$	Attempts to simplify the expression to required form. E.g. factorise difference of two cubes, $p^3 - q^3 = (p - q)(p^2 + pq + q^2)$ or equivalent work to enable $p - q$ to cancel.	M1
	$x = 2(p^2 + pq + q^2 + 2)^*$	cso for reaching the correct x coordinate.	A1
	$y + 2p^3 + 2p^2q + 2pq^2 + 4p = 2p^3 + 4p$ $y = -2pq(p + q)^*$	cso for reaching both coordinates correctly.	A1
			(5)
ALT	$qy + pqx = 2p^3q + 4pq$ $py + pqx = 2pq^3 + 4pq$ $py - qy = 2pq^3 - 2p^3q$	Attempt to solve simultaneous equations. A correct equation in only one variable	M1 A1

	$(p-q)y = -2pq(p^2 - q^2)$ $(p-q)y = -2pq(p-q)(p+q)$	Attempts to simplify the expression to required form. E.g. Attempt to factorise difference of two squares. $p^2 - q^2 = (p-q)(p+q)$ or equivalent work to enable $p-q$ to cancel.	M1
	$y = -2pq(p+q)^*$	cso for reaching the correct y coordinate.	A1
	$-2p^2q^2 - 2pq^3 + pqx = 2p^3q + 4pq$ $x = 2(p^2 + pq + q^2 + 2)^*$	cso for reaching both coordinates correctly.	A1
			(5)
ALT 2	$-2pq(p+q) + 2p(p^2 + pq + q^2 + 2) =$ $-2p^2q - 2pq^2 + 2p^3 + 2p^2q + 2pq^2 + 4p$	Substitutes both coordinates into the equation for normal at P and expands brackets.	M1
	$= 2p^3 + 4p$ so N is on normal at P	Simplifies correctly to $2p^3 + 4p$ and deduces N is on the normal at P	A1
	$-2pq(p+q) + 2q(p^2 + pq + q^2 + 2) =$ $-2p^2q - 2pq^2 + 2p^2q + 2pq^2 + 2q^3 + 4q$	Substitutes both coordinates into the equation for normal at Q and expands brackets.	M1
	$= 2q^3 + 4q$ so N is on normal at Q	Simplifies correctly to $2q^3 + 4q$ and deduces N is on the normal at Q but penalise only the first instance for missing the deduction.	A1
	As N is on both normals it is therefore the intersection point of them.	Makes a suitable conclusion	A1
			(5)
6(d)	$\text{Grad } PQ = \frac{4q - 4p}{2q^2 - 2p^2} \left(= \frac{2}{p+q} \right)$	Use of $\frac{y_2 - y_1}{x_2 - x_1}$ and substituting.	M1A1
	$\text{Grad } ON = \frac{-2pq(p+q)}{2(p^2 + pq + q^2 + 2)}$	Can be unsimplified.	B1
	$\text{Grad } PQ \times \text{Grad } ON$ $= \frac{2}{p+q} \times \frac{-2pq(p+q)}{2(p^2 + pq + q^2 + 2)} = -1$	Using product of their gradients $= -1$	M1
	$(p+q)^2 - 3pq = -2$	-2	A1
			(5)
			Total 16