



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Matrix Algebra (FP1.5)

International A Level Mathematics · FP1: Further Pure Mathematics 1 (WFM01)

21 questions · 155 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

 guidemygrades.com

 @Guidemygrades

 @Guidemygrades

 @Guidemygrades

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
FP1.5a	MATRICES, DETERMINANTS, INVERSES & TRANSFORMATIONS	21 Qs · 155 marks	2
1. Q3	IAL FP1 (WFM01) January 2021	4	3
2. Q3	IAL FP1 (WFM01) June 2022	4	5
3. Q1	IAL FP1 (WFM01) October 2021	5	7
4. Q1	IAL FP1 (WFM01) January 2022	5	9
5. Q1	IAL FP1 (WFM01) January 2023	5	11
6. Q1	IAL FP1 (WFM01) January 2024	5	13
7. Q1	IAL FP1 (WFM01) June 2024	6	15
8. Q1	IAL FP1 (WFM01) January 2025	6	17
9. Q1	IAL FP1 (WFM01) June 2025	6	19
10. Q4	IAL FP1 (WFM01) January 2024	7	23
11. Q5	IAL FP1 (WFM01) January 2022	8	25
12. Q4	IAL FP1 (WFM01) June 2023	8	29
13. Q2	IAL FP1 (WFM01) June 2025	8	33
14. Q7	IAL FP1 (WFM01) January 2021	9	37
15. Q7	IAL FP1 (WFM01) October 2021	9	41
16. Q7	IAL FP1 (WFM01) June 2022	9	45
17. Q6	IAL FP1 (WFM01) June 2024	9	49
18. Q6	IAL FP1 (WFM01) October 2020	10	53
19. Q3	IAL FP1 (WFM01) June 2021	10	57
20. Q7	IAL FP1 (WFM01) January 2023	11	61
21. Q7	IAL FP1 (WFM01) January 2025	11	65



QUESTIONS

21 questions · 155 marks · 1 sub-topic

SUB-TOPIC FP1.5a

MATRICES, DETERMINANTS, INVERSES & TRANSFORMATIONS

Sub-topic FP1.5a · 21 questions · 155 marks · lowest tariff first



3. The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of k for which $\mathbf{M}$ is singular.

(2)

Given that $\mathbf{M}$ is non-singular,

(b) find $\mathbf{M}^{-1}$ in terms of k .

(2)

Leave
blank



Question 3 continued

Leave
blank

(Total 4 marks)

Q3



3.

$$\mathbf{M} = \begin{pmatrix} k & k \\ 3 & 5 \end{pmatrix} \quad \text{where } k \text{ is a non-zero constant}$$

where k is a non-zero constant

(a) Determine $\mathbf{M}^{-1}$, giving your answer in simplest form in terms of k .

(2)

Hence, given that $\mathbf{N}^{-1} = \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix}$

(b) determine $(\mathbf{MN})^{-1}$, giving your answer in simplest form in terms of k .

(2)

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 4 marks)



Leave
blank

1. $\mathbf{A} = \begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix}$

where a is a non-zero constant and $a \neq 3$

(a) Determine $\mathbf{A}^{-1}$ giving your answer in terms of a .

(2)

Given that $\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a .

(3)

Leave
blank

(Total 5 marks)

Q1



1.

$$\mathbf{M} = \begin{pmatrix} 3x & 7 \\ 4x + 1 & 2 - x \end{pmatrix}$$

Find the range of values of x for which the determinant of the matrix $\mathbf{M}$ is positive.

(5)

Leave
blank

Leave
blank

Q1

1. Given that

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix}$$

where k is a non-zero constant,

(a) determine the matrix $\mathbf{AB}$

(2)

(b) determine the value of k for which $\det(\mathbf{AB}) = 0$

(3)

Question 1 continued

(Total for Question 1 is 5 marks)



1.

$$\mathbf{M} = \begin{pmatrix} 2k+1 & k \\ k+7 & k+4 \end{pmatrix} \text{ where } k \text{ is a constant}$$

- (a) Show that $\mathbf{M}$ is non-singular for all real values of k .

(3)

- (b) Determine $\mathbf{M}^{-1}$ in terms of k .

(2)

(Total for Question 1 is 5 marks)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)



1. $\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix}$ where p is a constant

- Determine $\det \mathbf{P}$ in simplest form in terms of p .
(2)
- Hence show that $\mathbf{P}$ is non-singular for all real values of p .
(2)
- Determine $\mathbf{P}^{-1}$ in terms of p .
(2)

Question 1 continued

(Total for Question 1 is 6 marks)



1.

$$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix}$$

where a is a constant, $a \neq -\frac{5}{3}$

(a) Determine $\mathbf{M}^{-1}$ in terms of a .

(3)

Given that $\mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a .

(3)



Question 1 continued

Lined area for writing the answer to Question 1 continued.



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

- (a) Describe the single geometrical transformation represented by the matrix $\mathbf{A}$.

(2)

The matrix **B** represents a rotation of 210° anticlockwise about centre $(0, 0)$.

- (b) Write down the matrix $\mathbf{B}$, giving each element in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

- (c) Find $\mathbf{C}$.

(2)

The hexagon H is transformed onto the hexagon H' by the matrix C .

- (d) Given that the area of hexagon H is 5 square units, determine the area of hexagon H'

(2)

(Total for Question 4 is 7 marks)

Leave
blank

Leave
blank



Question 5 continued

Leave
blank

(Total 8 marks)

Q5



4. (i) $\mathbf{A} = \begin{pmatrix} -3 & 8 \\ -3 & k \end{pmatrix}$ where k is a constant

The transformation represented by $\mathbf{A}$ transforms triangle T to triangle T'

The area of triangle T' is three times the area of triangle T

Determine the possible values of k

(4)

(ii) $\mathbf{B} = \begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{BC} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix}$ where a is a constant

Determine, in terms of a , the matrix $\mathbf{C}$

(4)



Question 4 continued

Lined area for writing the answer to Question 4 continued.







Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 8 marks)



Question 7 continued

Leave
blank



Leave
blank

Question 7 continued

Leave
blank

(Total 9 marks)

Q7



Question 7 continued

Leave
blank



Leave
blank



Question 7 continued

Leave
blank

(Total 9 marks)

Q7



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 9 marks)







(Total for Question 6 is 9 marks)

Question 6 continued

Leave
blank



Leave
blank



Question 6 continued

Leave
blank

Q6

(Total 10 marks)



Leave
blank

- 3.** The triangle T has vertices $A(2, 1)$, $B(2, 3)$ and $C(0, 1)$.

The triangle T' is the image of T under the transformation represented by the matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- (a) Find the coordinates of the vertices of T' (2)

- (b) Describe fully the transformation represented by $\mathbf{P}$ (2)

The 2×2 matrix **Q** represents a reflection in the x -axis and the 2×2 matrix **R** represents a rotation through 90° anticlockwise about the origin.

- (c) Write down the matrix $\mathbf{Q}$ and the matrix $\mathbf{R}$ (2)

- (d) Find the matrix $\mathbf{RQ}$ (2)

- (e) Give a full geometrical description of the single transformation represented by the answer to part (d). (2)



Leave
blank



Question 3 continued

Leave
blank

(Total 10 marks)

Q3





Question 7 continued



Question 7 continued

(Total for Question 7 is 11 marks)







(Total for Question 7 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
1.(a)	$f(0.2) = \dots$ and $f(0.6) = \dots$ $f(0.2) = -0.5973\dots$ and $f(0.6) = 0.2707\dots$ Continuous function with change of sign so root (in given interval)	M1 A1 (2)
(b)	$f(0.4) = -0.1788$ $f(0.5) = \dots$ $f(0.5) = 0.04114\dots \Rightarrow 0.4 \leq \alpha \leq 0.5$	B1 M1 A1 (3) [5]
Notes		
(a) M1 A1	Must see correct values for the accuracy marks. But allow signs only for attempts at values for the method marks. Attempts both values, accept $f(0.2) = \dots$ and $f(0.6) = \dots$ with any values. (NB $f(0.2) = -0.2069\dots$, $f(0.6) = 1.379$ are the values with calculator in degrees mode.) Both values correct (rounded or truncated to 1d.p.) and a correct conclusion (continuous and sign change). Allusion to continuity must be mentioned somewhere in the solution. Allow other ways to show sign change e.g. $<0, >0$ etc.	
(b) B1 M1 A1	Correct value of $f(0.4)$; may be rounded or truncated to 1 dp Attempt value of $f(0.5)$ or attempt value of $f(0.3)$ if relevant for their sign of $f(0.4)$. Correct value of $f(0.5)$ which may be rounded to 2 dp and correct interval. Allow as open or closed interval. Accept any valid notation for the interval. Accept e.g. $0.4 < x < 0.5$	

Question Number	Scheme	Marks
2		
(a)	$\frac{3}{8} - \frac{\sqrt{71}}{8}i$	B1 (1)
(b)	$\left(x - \frac{3}{8} - \frac{\sqrt{71}}{8}i\right)\left(x - \frac{3}{8} + \frac{\sqrt{71}}{8}i\right) ((x-4)=0)$ $\left(x^2 - \frac{3}{4}x + \frac{5}{4}\right) ((x-4)=0)$ $x^3 - \frac{19}{4}x^2 + \frac{17}{4}x - 5 (=0)$ $4x^3 - 19x^2 + 17x - 20 (=0) \quad p=17, q=-20$	M1A1 dM1 A1 (4)
(a) B1 (b) M1 A1 dM1 A1	Correct answer only Attempt the multiplication of the 2 brackets with the complex terms. Allow $(x \pm \text{root})$ for the brackets. Allow “invisible” brackets. Correct quadratic obtained - may have multiplied by the 4 (or other constant factor) and this is fine. (Need not be fully simplified but must have real terms) Attempt to multiply their quadratic by $(x-4)$ or may divide their quadratic into the cubic or other full method leading to at least one of p or q . Correct values. Values of p and q need not be shown explicitly but may be seen in a cubic, provided the cubic starts $4x^3 - 19x^2$ (isw after a correct cubic)	[5]
	Note if a candidate uses a hybrid method, mark under main scheme unless an Alt scores more marks.	
Alt 1 (b)	$-\frac{q}{4} = 4 \times \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) \times \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) = \dots \rightarrow q = \dots$ or $\frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $\Rightarrow q = -16 \times \left(\frac{9}{64} + \frac{71}{64}\right) = -20$ or $p = 17$ E.g. $f(4) = 0 \Rightarrow 4(4)^3 - 19(4)^2 + 4p - 20 = 0 \Rightarrow p = \dots$ or $\frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $p = 17, q = -20$	M1 A1 dM1 A1 (4)

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

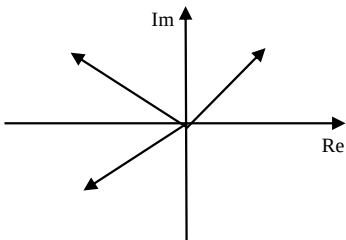
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1(a)	$z_1 = 3 + 3i$ $z_2 = p + qi$ $p, q \in \mathbb{R}$		
	$ z_1 = \sqrt{3^2 + 3^2}$ $ z_1 z_2 = z_1 z_2 \Rightarrow z_2 \sqrt{18} = 15\sqrt{2} \Rightarrow z_2 = \dots$	Attempts $ z_1 $ using Pythagoras and uses $ z_1 z_2 = z_1 z_2 $ to find $ z_2 $	M1
	$ z_2 = 5$	Cao	A1
			(2)
ALT	$ z_1 z_2 = 15\sqrt{2}$ $ (3p - 3q) + i(3p + 3q) = 15\sqrt{2}$ $\sqrt{18p^2 + 18q^2} = 15\sqrt{2}$ $p^2 + q^2 = 25$ $ z_2 = \sqrt{p^2 + q^2} = 5$	Uses $ z_1 z_2 = z_1 z_2 $ to reach $p^2 + q^2 = \dots$	M1
			A1
			(2)
(b)	$ z_2 = 5 \Rightarrow p^2 + q^2 = 25$ $\Rightarrow (-4)^2 + q^2 = 25 \Rightarrow q = \dots$	Uses $p^2 + q^2 = 25$ with $p = \pm 4$ leading to a value for q .	M1
	$q = \pm 3$	Both values. Must be clear $p = 4$ has not been used	A1
			(2)
(c)	 Points to be in the correct quadrants and either with correct numbers on the axes or labelled correctly	3 + 3i plotted correctly and labelled	B1
		Vectors/ lines not needed; point(s) alone are sufficient	
		A conjugate pair plotted correctly following through their q .	B1ft
			(2)
			Total 6

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Question	Scheme	Marks
1.(a)	$\begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix} = \begin{pmatrix} 2+0+6k & 2k+3+6 \\ -2+0+0 & -2k-9+0 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2+6k & 2k+9 \\ -2 & -2k-9 \end{pmatrix}$	A1cao
		(2)
(b)	$\det \mathbf{AB} = (2+6k)(-2k-9) - (-2)(2k+9)$	M1
	$\det \mathbf{AB} = 0 \Rightarrow -12k^2 - 54k = 0 \Rightarrow k = \dots$	dM1
	$k = -\frac{9}{2}$	A1
		(3)
(5 marks)		
Notes:		
(a) M1: Obtains a 2×2 matrix with at least two entries correct, unsimplified. A1cao: Correct matrix with terms simplified. (b) M1: Attempts the determinant, be tolerant of minor slips, such as sign slips with the negatives, if the correct “$ad - bc$” form is apparent. They may give the $-(-2)(\dots)$ as just $+2(\dots)$. Accept if seen as part of the attempt at the inverse matrix. dM1: Expands their determinant to a quadratic, sets equal to zero (may be implied) and achieves a value for k via correct method (allow if a factor k is cancelled, use of formula or calculator (a correct value for their quadratic)). A1: cso for $-\frac{9}{2}$. Accept as decimal or equivalent fractions, such as $-\frac{54}{12}$. Ignore any reference to the 0 solution.		

Question	Scheme	Marks
1.(a)	$\det \mathbf{M} = (2k+1)(k+4) - k(k+7)$	M1
	$\det \mathbf{M} = k^2 + 2k + 4 \Rightarrow b^2 - 4ac = 4 - 16$ or $\det \mathbf{M} = k^2 + 2k + 4 = (k+1)^2 + 3$ or $\frac{d(\det \mathbf{M})}{dk} = 2k + 2 = 0 \Rightarrow k = -1$	M1
	$b^2 - 4ac < 0 \Rightarrow k^2 + 2k + 4 > 0$ or $\det \mathbf{M} = (k+1)^2 + 3 \dots 3$ or $k = -1$ at minimum so $\det \mathbf{M} \dots 3$ Hence $\mathbf{M}$ is non-singular for all real values of k	A1
		(3)
(b)	$\mathbf{M}^{-1} = \frac{1}{k^2 + 2k + 4} \begin{pmatrix} k+4 & -k \\ -k-7 & 2k+1 \end{pmatrix}$	M1
		A1
		(2)
(Total 5 marks)		
Notes		
(a) M1: Attempts the determinant of $\mathbf{M}$. Must see evidence of the attempt at subtracting but allow e.g. minor sign slips inside the brackets. Must be seen in (a). M1: Begins a correct strategy for attempting to establish that the determinant is non-zero, must follow a valid attempt at the determinant involving all four terms.. May find the discriminant, complete the square (usual rules) on the determinant, attempt to solve the quadratic via formula, or attempt a minimisation process. There must be an attempt at a calculation. A1: Full and correct reasoning and conclusion. Must see consideration of the sign or non-zero oe, but accept e.g. $\dots 3$ (condone >3) as being sufficient to show $\det \mathbf{M}$ cannot be zero for the reason. Showing the roots $(-1 \pm i\sqrt{3})$ are not real is acceptable The final deduction must refer to non-singular, but no need to mention $\mathbf{M}$ and condone $\det \mathbf{M}$ is non-singular as conclusion. (b) M1: For applying $\mathbf{M}^{-1} = \frac{1}{\det \mathbf{M}} \times \text{adj}(\mathbf{M})$ with their determinant. At least three entries in $\text{adj}(\mathbf{M})$ must be correct initially. A1: Correct matrix, and isw after a correct answer.		

Question Number	Scheme	Notes	Marks
1(i)	$\mathbf{A} = \begin{pmatrix} 3k & 4k-1 \\ 2 & 6 \end{pmatrix}$		
(a)	$3k \times 6 - 2(4k-1) = 0 \Rightarrow k = \dots$ Forms $\det \mathbf{A} = 0$ and solves for k . The “= 0” can be implied by a solution for k . Award for $3k \times 6 - 2(4k-1) = 0 \Rightarrow k = \dots$ If LHS is only seen expanded 2 terms of $18k - 8k + 2$ must be correct (implied by $10k$) May use $ad = bc$ and condone $\det \mathbf{A} = bc - ad = 0$ but clear use of $ad + bc$ is M0		M1
	$(10k + 2 = 0 \Rightarrow k = -\frac{1}{5} \text{ or } -0.2$	A1: Correct value. Accept $-\frac{2}{10}$	A1
(2)			
(b)	$(\mathbf{A}^{-1} =) \frac{1}{10k+2} \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix} \text{ or } \begin{pmatrix} \frac{6}{10k+2} & \frac{1-4k}{10k+2} \\ \frac{-2}{10k+2} & \frac{3k}{10k+2} \end{pmatrix} \text{ or e.g., } \begin{pmatrix} \frac{3}{5k+1} & \frac{1-4k}{10k+2} \\ \frac{-1}{5k+1} & \frac{3k}{10k+2} \end{pmatrix}$ M1: for $\dots \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix}$ Ignore any multiplier and accept without one and condone if this matrix is labelled as $\mathbf{A}^{-1}$. Allow unsimplified e.g., $\dots \begin{pmatrix} 6 & -(4k-1) \\ -2 & 3k \end{pmatrix}$ Allow if determinant incorporated provided it is clear that the elements of $\text{Adj}(\mathbf{A})$ are correct A1ft: $\frac{1}{"10k+2"} \begin{pmatrix} 6 & 1-4k \\ -2 & 3k \end{pmatrix}$ Fully correct inverse ft their determinant in form $ak + b$ $a, b \neq 0$ and simplified but if determinant incorporated there is no requirement to write e.g., $\frac{6}{10k+2}$ as $\frac{3}{5k+1}$. Allow different brackets e.g., $[\dots]$, $\{\dots\}$ but $ \dots $ is M0 if followed by an attempt at $\det(\text{Adj}(\mathbf{A}))$. Allow if “ $\times$ ” is between fraction and matrix and allow fraction to appear on the right of the matrix. Isw when a correct answer is seen but this mark is not available if they substitute a value of k into the determinant and/or matrix.		M1 A1ft
(2)			
(ii)(a)	$p = q = -2 \text{ or } (\mathbf{B} =) \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	Both values identified or correct matrix (any or no bracket). Allow “Both are -2” or “-2, -2”	B1
(b)	$p = -1 \quad q = 1 \text{ or } (\mathbf{B} =) \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Both values identified or correct matrix (any or no bracket). Allow “-1, +1” (Mark in order presented). No trig expressions.	B1
(2)			
Total 6			

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the mark scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to $x = \dots$ (may be unsimplified).

3. Completing the square (where a = 1; if a ≠ 1 must divide by a first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$



4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the mark scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation but refer to scheme.

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ or}$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$

both leading to at least one solution $x = \dots$

2. Formula

Correctly use the correct formula with values for a , b and c to obtain at least one solution $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$; if $a \neq 1$ must divide by a first but allow equivalent work e.g., if a is a perfect square)

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$$

leading to at least one solution $x = \dots$



Question	Scheme	Marks
4.(a)	Stretch – SF 3 parallel to the y-axis	M1 A1
		(2)
(b)	$\mathbf{B} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	M1
	$\mathbf{C} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{3}{2} \\ -\frac{1}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix}$	A1
		(2)
(d)	$\det \mathbf{C} = -\frac{\sqrt{3}}{2} \times -\frac{3\sqrt{3}}{2} - \frac{3}{2} \left(-\frac{1}{2} \right) = 3$ So area of $H\phi$ is $5 \times \det \mathbf{C} = \dots$	M1
	$= 15$	A1
		(2)
(Total 7 marks)		

Notes

(a)

M1: Identifies one correct aspect, EITHER **stretch** (or allow scaling)OR scale factor 3 **and** correct direction indicated (need not be precise).

A1: Fully correct description with both aspects correct. Must mention stretch or scaling, but be tolerant with the description of direction and scale factor as long as both are clear. Accept e.g. parallel to the y-axis, y direction, in y axis, or vertically for direction, but not “about y” (reflection implied). Accept e.g. scale factor 3, by 3, $\times 3$ or three times for the scale factor. Ignore references to “about origin” or additional references to stretch of factor 1 parallel to the x-axis.

Some examples:

- “stretched by 3 in direction of y axis”.
- “stretch the y for scale factor of 3.”

Both the above score M1A1. Stretch stated, direction and scale factor 3 both indicated.

- “enlargement scale factor 3 for y-axis.”
- “A enlarge by three times paralleled to y-axis.”

Both score M1A0 Indicates direction and scale factor but does not mention stretch or scaling.

- “the y-axis will be enlarged by three times, whereas the x-axis stay the same.”

M1A0 Indicates direction and scale factor but does not mention stretch or scaling, the reference to the x-axis is ignored as not incorrect.



(b)

B1: Correct matrix. Must be exact (trig terms evaluated) and seen in (b).

(c)

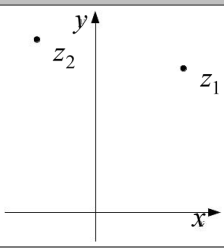
M1: Attempts to multiply matrices the right way around. Implied by 3 correct entries if no product shown.

A1: Correct matrix. Must be exact.

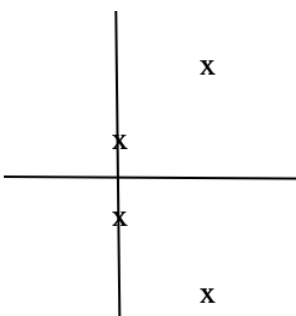
(d)

M1: Attempts determinant of **C** (or deduces area scale factor is 3) and multiplies by 5. Implied by a correct answer if no incorrect working is seen.

A1: Cao. Allowed if scored from a **C** arising from multiplication the wrong way round in (c) or an incorrect **B** that has determinant ± 1

Question	Scheme	Marks
2(a)		M1 A1
		(2)
(b) (i)	$ z_1 = \sqrt{3^2 + 5^2} = \sqrt{34}$	B1
		(1)
(ii)	$\frac{z_1}{z_2} = \frac{3+5i}{-2+6i} \times \frac{-2-6i}{-2-6i} = \dots$	M1
	$= \frac{-6-18i-10i+30}{40}$	A1
	$= \frac{3}{5} - \frac{7}{10}i$	A1
		(3)
(c)	$\arg \frac{z_1}{z_2} = \arctan \frac{-7/10}{3/5} = \arctan \frac{-7}{6} = \dots$ but allow $\arctan \frac{7}{6}$ for M1	M1
	$= -0.86$ or 5.42 (awrt)	A1
		(2)
(8 marks)		
Notes:		
(a) M1: Points in correct quadrants - z_1 in quadrant 1 and z_2 in quadrant 2. Must be clearly labelled either eg z_1 or $3 + 5i$ or correct numbers on the axes. (Accept with vector arrows.) A1: Correct diagram, z_1 in first quadrant further away from real axis than imaginary and z_2 in second quadrant, closer to imaginary axis but above z_1 OR correct nos on their axes (imag axis may include i), but not dashes w/o any indication of scale. Allow M1A0 for points unlabelled but diagram otherwise correct. (b)(i) B1: Correct modulus. Must be evaluated to $\sqrt{34}$ Question says “without using your calculator” so decimal answers can be ignored (isw) but exact answer must be seen somewhere. (ii) M1: Multiplies numerator and denominator by the conjugate of their denominator. A1: Correct unsimplified (or simplified) numerator, with the i^2 correctly dealt with, and correct denominator. A1: Correct answer. Allow as shown, $\frac{6}{10} - \frac{7}{10}i$, or $0.6 - 0.7i$.		

Question Number	Scheme	Notes	Marks
1	$\sum_{r=1}^n r^2 (r+2) = \sum_{r=1}^n r^3 + 2 \sum_{r=1}^n r^2$ or $\sum_{r=1}^n r^3 + \sum_{r=1}^n 2r^2$	Correct split with 2 summations. Could be implied by correct work. Condone missing or incorrect summation limits.	B1
	$= \frac{1}{4} n^2 (n+1)^2 + 2 \times \frac{1}{6} n(n+1)(2n+1)$	Attempts to use both standard results and obtains an expression of the form $pn^2(n+1)^2 + qn(n+1)(2n+1)$ $p, q \neq 0$ Could be implied by immediate expansion	M1
	$= \frac{1}{12} n(n+1)[3n(n+1) + 4(2n+1)]$ $= \frac{1}{12} n(n+1)(3n^2 + 11n + 4)$	dM1: Attempts factorisation to obtain $\frac{1}{12} n(n+1)(an^2 + bn + c)$ $a, b, c \neq 0$. Condone poor algebra. Could follow cubic or quartic. Allow a consistent $a = \dots, b = \dots, c = \dots$ if quadratic never seen simplified Requires previous M mark. A1: Correct expression or $a = 3, b = 11, c = 4$ Allow e.g., $\frac{1}{12} n(n+1)$ written as $\frac{n}{12}(n+1)$	dM1 A1
	Note: $n(n+1)(3n^2 + 11n + 4) = 3n^4 + 14n^3 + 15n^2 + 4n$		Total 4

Question Number	Scheme	Notes	Marks
2	$2x^4 - 8x^3 + 29x^2 - 12x + 39 = 0, \quad x = 2 + 3i$ Condone work in e.g., z throughout		
(a)	$2-3i$	Correct conjugate	B1
			(1)
(b)	$(x - (2 - 3i))(x - (2 + 3i)) = \dots \{x^2 - 4x + 13\}$ or $(x - 2 + 3i)(x - 2 - 3i)$ or sum = 4, product = 13 $\Rightarrow x^2 \pm 4x \pm 13$ or $x^2 \pm 13x \pm 4$ or $x^2 - (2 + 3i + 2 - 3i)x + (2 + 3i)(2 - 3i)$ $\Rightarrow \dots \{x^2 - 4x + 13\}$	Attempts to multiply the two correct factors to obtain a 3 term quadratic with real coefficients. Could use $(x - 2)^2 = (\pm 3i)^2$ or $x^2 - 2ax + a^2 + b^2$ with $a = 2, b = \pm 3$ Or uses the correct sum and product of the roots to obtain an expression of the form shown (must be some minimal working – but if just a quadratic is given the next 2 marks are available) or $x^2 - (\alpha + \beta)x + \alpha\beta$ to obtain a 3 term quadratic with real coefficients.	M1
	$2x^4 - 8x^3 + 29x^2 - 12x + 39 \Rightarrow (x^2 - 4x + 13)(2x^2 + 3)$	Uses their 2 or 3 term quadratic factor with real coefficients to obtain a second 2 or 3 term quadratic of the form $2x^2 + \dots$ by long division, equating coefficients or inspection. Ignore any remainder from long division. Can follow M0	M1
	$2x^2 + 3 = 0 \Rightarrow$ $x = \pm \frac{\sqrt{6}}{2}i$ or $\pm i\sqrt{\frac{3}{2}}$ or $\pm \frac{\sqrt{3}}{\sqrt{2}}i$ or $\sqrt{1.5}i$ $\sqrt{1.5}i$ is M0 1.2247...i is M1 A0	dM1: Solves their second quadratic factor = 0. If 2 term must get one correct non-zero root. (Usual rules if 3TQ and one correct root if no working) Could be inexact. Requires previous method mark. A1: Both correct exact roots with “i” Requires all previous marks.	dM1 A1
	Solving by calculator, sometimes followed by attempts to reconstruct factors. e.g., $f(x) = (x^2 - 4x + 13)\left(x^2 + \frac{3}{2}\right)$ is first M1 only and working for the 3TQ must be seen		(4)
(c)		Allow ft on their answers to (b) if they are of the form $\pm ki$ or $\pm k\sqrt{-1}$, $k \neq 0$ regardless of how they were obtained 1st B1: One of the two pairs of roots in correct positions 2nd B1: Both pairs of roots in correct positions and correct relative to each other for their k Allow any suitable indication of the roots such as vectors. Ignore all labelling and scaling but each pair should be reasonably symmetric in x -axis for any marks (for each pair -distance of one to x -axis not less than $\frac{1}{2}$ of the other)	B1 B1 (ft on (b))
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$y = 9x^{-1} \Rightarrow \frac{dy}{dx} = -9x^{-2} \left\{ = -\frac{9}{(3t)^2} \right\}$ or $xy = 9 \Rightarrow x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} \left\{ = -\frac{\frac{3}{t}}{3t} \right\}$ or $x = 3t, y = 3t^{-1} \Rightarrow \frac{dx}{dt} = 3, \frac{dy}{dt} = -3t^{-2} \Rightarrow \frac{dy}{dx} = \frac{-3t^{-2}}{3}$	Any correct expression for $\frac{dy}{dx}$ but allow e.g., $\frac{dx}{dy} = -9y^{-2}$ Calculus must be seen so there is no credit for just a statement e.g., $m_t = -\frac{1}{t^2}$	B1
	e.g., $m_N = \frac{(3t)^2}{9}$ or $\frac{3t}{\frac{3}{t}}$ or $\frac{3}{3t^{-2}} \left\{ = t^2 \right\}$	Uses the perpendicular gradient rule to obtain the gradient of the normal in terms of t correct for their m_t Implied by correct use of $-\frac{dx}{dy}$	M1
	$y - \frac{3}{t} = t^2(x - 3t) \text{ or } \frac{3}{t} = t^2(3t) + c \Rightarrow c = \dots$ $\left\{ c = \frac{3}{t} - 3t^3 \right\}$	Applies straight line method correctly with their normal (changed) gradient in terms of t . If using $y = mx + c$ coordinates must be correctly placed and $c = \dots$ reached	M1
	$ty - t^3x = 3 - 3t^4$ Intermediate step not required. Allow recovery from a slip.	Correct equation or $f(t)$. Must be seen in (a). Accept equivalents for $f(t)$ e.g., $3(1 - t^4)$, $-3(t^4 - 1)$	A1
	Allow work with $xy = c^2$ but the final mark requires use of $c^2 = 9$ No calculus scores a maximum of 0111 if m_t is stated and 0011 if m_N is stated		(4)
(b)	$xy = 9, 2y - 8x = 3 - 3 \times 16$ e.g., $\Rightarrow y = 4x - \frac{45}{2}$ or $x = \frac{45}{8} + \frac{y}{4}$ $\Rightarrow x \left(4x - \frac{45}{2} \right) = 9 \text{ or } y \left(\frac{45}{8} + \frac{y}{4} \right) = 9$	Uses $t = 2$ in their $ty - t^3x = f(t) \neq 0$ and the equation of H to obtain an unsimplified three term quadratic equation in x or y (no variables in denominators). Only allow $f(t) = \frac{9}{t}$ if stated first	M1
	$8x^2 - 45x - 18 = 0 \text{ or } 2y^2 + 45y - 72 = 0$ $\{ \Rightarrow (8x + 3)(x - 6) = 0 \text{ or } (2y - 3)(y + 24) = 0 \}$ $\Rightarrow x = \dots \left\{ -\frac{3}{8}, 6 \right\} \text{ or } y = \dots \left\{ \frac{3}{2}, -24 \right\}$	Solves their 3TQ to find a value for x or y – apply usual rules. One root correct if no working. Can award for P provided it has come from quadratic. Requires previous method mark.	dM1
	$\left(-\frac{3}{8}, -24 \right) \text{ or } (-0.375, -24)$	Correct exact coordinates in simplest form from correct work. Allow $x = \dots, y = \dots$ Ignore $(6, \frac{3}{2})$ but A0 for any other point shown or incorrect x or y value.	A1
	Solving in terms of t: M1: $\Rightarrow$ Unsimplified 3TQ e.g., $t^2x^2 + \left(\frac{3}{t} - 3t^3 \right)x - 9 = 0$ M1 M1: Solves e.g., $x = \frac{-\frac{3}{t} + 3t^3 \pm \sqrt{\left(\frac{3}{t} - 3t^3 \right)^2 + 36t^2}}{2t^2} \left\{ \Rightarrow \left(-\frac{3}{t^3}, -3t^3 \right) \right\}$ A1: $t = 2 \Rightarrow \left(-\frac{3}{8}, -24 \right)$		(3)
	Correct final answer with no incorrect work is 111 provided $f(t)$ was correct		Total 7

Question Number	Scheme	Notes	Marks
4	$\mathbf{A} = \begin{pmatrix} -3 & 8 \\ -3 & k \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix} \quad \mathbf{BC} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix}$		
(i)	$\det \mathbf{A} = -3k - 8(-3) \{ = -3k + 24 \}$ Could be implied	Attempts $\det \mathbf{A}$ and obtains $\pm 3k \pm 8(\pm 3)$ or $\pm 3k \pm 24$	M1
	$-3k + 24 = 3$ or $-3k + 24 = -3$ $\Rightarrow k = \dots$ May see $(-3k + 24)^2 = +9 \Rightarrow 9k^2 - 144k + 567 = 0 \Rightarrow \dots$	Equates their $\det \mathbf{A}$ of form $ak + b$, $a, b \neq 0$ to 3 or -3 or equivalent work and solves for k (usual rules if quadratic and must use +9)	M1
	$k = 7, k = 9$ 1st A1: Either correct value of k from correct work. Allow e.g., $\frac{-21}{-3}$ or $\frac{-27}{3}$ 2nd A1: Both correct values of k from correct work. 7 and 9 only. No extra		A1 A1
			(4)
(ii)	$\det \mathbf{B} = 1 \times 3a - (-4) \times 2 \{ = 3a + 8 \}$	Correct unsimplified expression for $\det \mathbf{B}$. Could be implied	B1
	$\mathbf{B}^{-1} = \frac{1}{3a+8} \begin{pmatrix} 3 & 4 \\ -2 & a \end{pmatrix}$	Correct $\mathbf{B}^{-1}$ with their $\det \mathbf{B}$. $\text{Adj}(\mathbf{B})$ to be correct but allow elements to have their $\det \mathbf{B}$ as denominators if incorporated.	M1
	$\mathbf{C} = \mathbf{B}^{-1}\mathbf{BC} = \frac{1}{3a+8} \begin{pmatrix} 3 & 4 \\ -2 & a \end{pmatrix} \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix} = \dots$ Access to this mark is allowed if there is no determinant or if $\mathbf{B}^{-1} = \det \mathbf{B} \times \text{Adj}(\mathbf{B})$ used	Multiplies $\mathbf{BC}$ by their $\mathbf{B}^{-1}$ (changed – and not just by incorporation of their determinant) the correct way round . Expect four correct elements for their matrices if the method is unclear. The incorrect order scores M0 even if the correct result is obtained.	M1
	$\mathbf{C} = \frac{1}{3a+8} \begin{pmatrix} 10 & 31 & 11 \\ a-4 & 4a-10 & 2a-2 \end{pmatrix}$ Ignore any reference to inapplicable values of a ($a \neq -\frac{8}{3}$)	Correct $\mathbf{C}$ or equivalent with like terms collected and single fractions if necessary. e.g., $\begin{pmatrix} \frac{10}{3a+8} & \frac{31}{3a+8} & \frac{11}{3a+8} \\ \frac{a-4}{3a+8} & \frac{2(2a-5)}{3a+8} & \frac{2(a-1)}{3a+8} \end{pmatrix}$	A1
			(4)
Alt Sim. equations	$\begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} p & q & r \\ s & t & u \end{pmatrix} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix} \Rightarrow \begin{matrix} ap - 4s = 2 & aq - 4t = 5 & ar - 4u = 1 \\ 2p + 3s = 1 & 2q + 3t = 4 & 2r + 3u = 2 \end{matrix}$ Multiplies in the correct order to obtain at least three correct equations		B1
	$(3a+8)p = 10 \quad (3a+8)q = 31 \quad (3a+8)r = 11$ $p = \frac{10}{3a+8} \quad q = \frac{31}{3a+8} \quad r = \frac{11}{3a+8}$ $s = \frac{1}{3} \left(1 - \frac{20}{3a+8} \right) \quad t = \frac{1}{3} \left(4 - \frac{62}{3a+8} \right) \quad u = \frac{1}{3} \left(2 - \frac{22}{3a+8} \right) \Rightarrow \begin{pmatrix} \frac{10}{3a+8} & \frac{31}{3a+8} & \frac{11}{3a+8} \\ \frac{a-4}{3a+8} & \frac{4a-10}{3a+8} & \frac{2a-2}{3a+8} \end{pmatrix}$ $s = \frac{a-4}{3a+8} \quad t = \frac{4a-10}{3a+8} \quad u = \frac{2a-2}{3a+8}$ M1: Solves their equations to find expressions in terms of a for three elements M1: Finds expressions in terms of a for all six elements A1: Correct matrix – like terms collected and single fractions		M1 M1 A1
			Total 8

Question Number	Scheme	Marks
4		
(a)	$\alpha + \beta = -\frac{5}{2} \quad \alpha\beta = \frac{7}{2}$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \left(-\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(-\frac{5}{2}\right)$ $= \frac{85}{8}$	B1 M1 A1 (3)
(b)	$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \dots$ $= \left(\frac{85}{8}\right) \times \left(\frac{2}{7}\right) = \frac{85}{28}$ $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = \frac{7}{2}$ $x^2 - \frac{85}{28}x + \frac{7}{2} (= 0)$ $28x^2 - 85x + 98 = 0$	M1 A1 B1ft M1 A1 (5) [8]
(a)	Note: if a candidate solves the equation and uses the roots to answer the question, then send to review.	
B1	Both correct. (Seen anywhere in the working)	
M1	Uses their sum and product of roots in a correct expression for $\alpha^3 + \beta^3$.	
A1	Correct value. Must be exact. Accept 10.625	
(b)		
M1	$\alpha^3 + \beta^3 \quad \alpha\beta \quad \frac{\alpha^3 + \beta^3}{\alpha\beta} = \dots$ substitutes their values for $\alpha^3 + \beta^3$ and $\alpha\beta$ into $\frac{\alpha^3 + \beta^3}{\alpha\beta}$ (allow slips in substitution).	
A1	Correct sum as a single fraction (may be seen or implied in their equation)	
B1ft	Correct product or follow through their product	
M1	Use $x^2 - \text{sum of roots} \times x + \text{product of roots}$ with their values for sum and product. “= 0” may be missing.	
A1	A correct final equation as shown or any integer multiple of this. “= 0” must be included.	



Question Number	Scheme	Notes	Marks
7(i)(a)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	B1
			(1)
(b)	$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$	Correct matrix	B1
			(1)
(c)	$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Attempt to multiply the right way round. Implied by a correct answer (for their (a) and (b)) if no working is shown, but M0 if incorrect with no working.	M1
	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{5}{2} & -\frac{5\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)(a)	$\begin{vmatrix} k & k+3 \\ -5 & 1-k \end{vmatrix} = k(1-k) - (-5)(k+3)$	Correct method for the determinant. (Allow miscopy slips only. So $k(1-k) - 5(k+3)$ is M0 without further evidence.)	M1
	$= -k^2 + 6k + 15$	Correct simplified expression	A1
			(2)
(b)	$-k^2 + 6k + 15 = \frac{16k}{2} \Rightarrow k = \dots$ or $-k^2 + 6k + 15 = -\frac{16k}{2} \Rightarrow k = \dots$	Correct strategy for establishing at least one value for k	M1
	One of $k = -5, 3, -1, 15$	Any one correct value. Note that the negative values may be rejected here.	A1
	$k = 3$ and $k = 15$ or $k = -5, 3$ and $k = -1, 15$	Both correct positive values and no others. Condone the inclusion of the negative values if given.	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	$\mathbf{A}^2 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct matrix	B1
			(1)
(b)	Rotation -60° (anticlockwise) about the origin	Rotation	M1
		-60° (anticlockwise) (Or 60° clockwise or 300° (anticlockwise)) about (0, 0)	A1
			(2)
(c)	$n = 12$	Cao but can be embedded ie $A^{12} = I$	B1
			(1)
(d)	$\mathbf{B} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix	B1
			(1)
(e)	$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Multiplies the right way round.	M1
	$\mathbf{C} = \begin{pmatrix} -2\sqrt{3} & -2 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix Accept unsimplified	A1
			(2)
(f)	$\det \mathbf{C} = -2\sqrt{3} \times -\frac{\sqrt{3}}{2} - \frac{1}{2}(-2) = 4$ So area of P is $\frac{20}{\det \mathbf{C}} = \dots$	Attempts determinant of $\mathbf{C}$ (or deduces area scale factor is 4) and divides into 20	M1
	$= 5$	Cao Must follow a correct matrix in (e)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
6(a)	$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & (2^n - 1)r \\ 0 & 2^n \end{pmatrix}$		
	Evaluates LHS & RHS for $n = 1$. LHS & RHS indicated (or “true” seen) if not equated $(\text{LHS}) = \begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^1 \text{ or } \begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & (2^1 - 1)r \\ 0 & 2^1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & (2 - 1)r \\ 0 & 2 \end{pmatrix} (= \text{RHS})$		B1
	Assume true for $n = k$, i.e., $\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^k = \begin{pmatrix} 1 & (2^k - 1)r \\ 0 & 2^k \end{pmatrix}$		
	$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & (2^k - 1)r \\ 0 & 2^k \end{pmatrix}$	Uses $n = k$ result to form expression for $\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^{k+1}$ Implied by 3 correct elements if they immediately multiply provided the result is not just the “given” answer and allow this to be the intermediate step	M1
	$= \begin{pmatrix} 1 & (2^k - 1)r + 2^k r \\ 0 & 2(2^k) \end{pmatrix} = \begin{pmatrix} 1 & (2^{k+1} - 1)r \\ 0 & 2^{k+1} \end{pmatrix}$	Correct result with intermediate step that involves the top right element and no errors seen in the algebra. Allow “meet in the middle” proofs. Only allow $(2^{k+1} - 1)r$ written as $r(2^{k+1} - 1)$ or $(-1 + 2^{k+1})r$ or $r(-1 + 2^{k+1})$. No $2(2^k)$ s for 2^{k+1}	A1
	Alternatively: $\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & (2^k - 1)r \\ 0 & 2^k \end{pmatrix} \begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & r + 2(2^k - 1)r \\ 0 & 2(2^k) \end{pmatrix} = \begin{pmatrix} 1 & (2^{k+1} - 1)r \\ 0 & 2^{k+1} \end{pmatrix}$		
True for $n = 1$, if true for $n = k$ then true for $n = k + 1$, true for all (positive integers) n Correct conclusion or narrative. Minimum in bold . “Assume true for $n = k$... true for $n = k + 1$ ” is sufficient for the “ then ” The two previous marks are required and this mark can only follow B0 if the B mark was only withheld for insufficient working provided there was an attempt with $n = 1$. Ignore further verifications for $n = 2$ etc. Condone “for all $n \in \mathbb{Z}$ ” but not $n \in \mathbb{R}$ Condone work with n used for k .			A1
(4)			
(b)(i)	$\begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^4 = \begin{pmatrix} 1 & (2^4 - 1)(-2) \\ 0 & 2^4 \end{pmatrix} = \begin{pmatrix} 1 & -30 \\ 0 & 16 \end{pmatrix}$	Correct matrix N . Could come from manual multiplication or calculator	B1
(ii)	$\mathbf{B} = \mathbf{NM} = \begin{pmatrix} 1 & -30 \\ 0 & 16 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} = \dots$	Attempts NM with their N . Must not be MN . The N must have exactly three non-zero elements with 0 as the first element in the second row and their NM must have three elements correct for their matrices	M1
	$\begin{pmatrix} 4 & -150 \\ 0 & 80 \end{pmatrix}$	Correct matrix B	A1
(3)			
(c)	$\det \mathbf{B} = 4 \times 80 - (0 \times (-150)) = 320$ $\text{area } S = \frac{720}{320}$	A correct non-zero value for the determinant of their B (no more than two zero elements) and divides this result into 720 to obtain a value for the area	M1
	$\frac{9}{4} \text{ or } 2\frac{1}{4} \text{ or } 2.25$	Correct area. Any exact equivalent. <u>Must follow a correct B</u> . Answer only is M1A1 if B correct.	A1
(2)			
Total 9			

Question Number	Scheme	Notes	Marks
6(i)(a)	Stretch scale factor 3 parallel to the y-axis	Stretch (<u>not</u> enlargement)	B1
		Scale factor 3 parallel to the y-axis. Allow, e.g. '3 times y values', 'y increased by 3 factor', or similar. Allow, e.g. 'direction of y', 'along y', 'vertical', or similar. Ignore any mention of the origin. If additional transformations are included, send to Review	B1
			(2)
(b)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix. $\frac{1}{\sqrt{2}}$ may be seen rather than $\frac{\sqrt{2}}{2}$	B1
			(1)
(c)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	Attempt to multiply the right way round, i.e. BA , not AB At least two correct terms (for their matrix B) are needed to indicate a correct multiplication attempt	M1
	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \end{pmatrix}$ or equiv. e.g. $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	Trapezium area = $\frac{1}{2}(5+2)(k+8)$	Correct method for the area of the trapezium	M1
	$\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix} = 5 \times 3 - (-2) \times 1 = 17$	Correct method for the determinant	M1
		17 (Allow ± 17)	A1
	$\frac{1}{2}(5+2)(k+8) \times 17 = 510 \Rightarrow k = \dots$	Multiplies their trapezium area by their determinant, sets equal to 510 and solves for k . Or equivalently: Equates their trapezium area to $(510 \div \text{determinant})$ and solves for k	M1
	$k = \frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			(5)

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Summary of changes from Provisional Mark Scheme

A few minor changes were made to the Mark Scheme before marking on the marking service began.

Question Number	Summary of changes
Q01	Corrected typing error in second equation in (ii) second B1: $0 - 1.4$ in the denominator corrected to $0 - g(1.4)$
Q05b	added “or f.t. their roots” to both B marks, and amended note to “Correct roots, or f.t. their answer” for first B, “Correct roots for the given quadratic, or f.t. their 3TQ”, to allow for students restating from the original equation with $d = 4$.
Q06	Removed comment about ePEN marking traits change.
Q07	Added a special case to the first A mark in (a): “SC allow dM1A0 for fully factorising to a cubic expression and going direct to the fully factorised expression with no intermediate quadratic seen.”

Question Number	Scheme	Notes	Marks
1	$f(x) = x^3 + 4x - 6$		
1(i)(a)	$f(1) = -1$ $f(1.5) = 3.375 \left(= \frac{27}{8} \right)$	Attempts to evaluate at both end points. If substitution not seen accept $f(1) = -1$ or $f(1.5) = 3.375$ as evidence, with any value for the other end.	M1
	Sign change and $f(x)$ continuous therefore α is between $x = 1$ and $x = 1.5$	$f(1) = -1$ and $f(1.5) = 3.375$ <u>both correct</u> and mentions/indicates <u>sign change</u> , <u>continuous</u> and <u>conclusion</u> .	A1
			(2)
1(i)(b)	$f'(x) = 3x^2 + 4$	Correct derivative. Can be implied by a correct expression seen later such as $3(1.5)^2 + 4$	B1
	$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{3.375}{10.75}$	Attempt Newton-Raphson using the correct formula.	M1
	$x_2 = 1.186...$	Accurate first application either awrt 1.186, $\frac{51}{43}$ or a correct numerical expression.	A1
	$x_3 = \left(1.186... - \frac{0.412...}{8.220...} \right) = 1.1358...$		
	$\alpha \approx 1.136$	cao	A1
			(4)
1(ii)	$g(1.4) = 3.442116... \text{ or } g(1.5) = -3.601419...$	Evidence of at least one value correct to 3 d.p. or better. May be implied by correct answer if never seen.	B1
	$\frac{1.5 - \beta}{\beta - 1.4} = \frac{0 - g(1.5)}{g(1.4) - 0}$ or $\frac{\beta - 1.4}{0 - g(1.4)} = \frac{1.5 - 1.4}{g(1.5) - g(1.4)}$ oe	A correct linear interpolation statement as shown oe (with correct signs). May omit the zeroes or use evaluated values (e.g 0.1 instead of $1.5 - 1.4$). Other forms are possible.	B1
	$\Rightarrow \beta \approx 1.448869...$	Evaluates β from an attempt at a linear interpolation statement, allow if signs are incorrect. $\beta \approx$ awrt 1.449	M1 A1
			(4)
			Total 10

Question	Scheme	Marks
7(i)(a)	Reflection or in the line $y = -x$	M1
	Reflection in the line $y = -x$	A1
		(2)
(b)	$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ or $6 \times \begin{pmatrix} \pm \cos 240^\circ & \pm \sin 240^\circ \\ \pm \sin 240^\circ & \pm \cos 240^\circ \end{pmatrix}$	M1
	$\begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}$	A1
		(2)
(c)	$\mathbf{R} = \mathbf{QP} = \begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \dots$	M1
	$= \begin{pmatrix} -3\sqrt{3} & 3 \\ 3 & 3\sqrt{3} \end{pmatrix}$ QP correctly found	A1
		(2)
(ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix} \begin{pmatrix} \lambda \\ 1 \end{pmatrix} = \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} -2\lambda + 2\sqrt{3} \\ 2\lambda\sqrt{3} + 2 \end{pmatrix} = \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$	M1
	$-2\lambda + 2\sqrt{3} = 4\lambda$ or $2\lambda\sqrt{3} + 2 = 4$	A1
	$\Rightarrow 6\lambda = 2\sqrt{3} \Rightarrow \lambda = \dots$ or $2\sqrt{3}\lambda = 2 \Rightarrow \lambda = \dots$	dM1
	$\lambda = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ oe	A1
	Both $-2\lambda + 2\sqrt{3} = 4\lambda$ and $2\lambda\sqrt{3} + 2 = 4$ solved leading to $\lambda = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	A1
		(5)
(11 marks)		
Notes:		
(a)		
M1: Identifies the transformation as a reflection or identifies the correct line of reflection.		

A1: Fully correct description, with the equation of the line of reflection or suitable description (e.g. in the line through angle 135° with the positive x -axis). Ignore any references to a centre of reflection.

(b)

M1: Either the correct matrix for the rotation (with trig ratios evaluated) or an attempt at scaling a matrix of form shown by a factor 6 (need not evaluate ratio) – if no trig ratios seen this may be implied by the exact values in the right places. The correct answer implies the M.

A1: Correct matrix.

(c)

M1: Attempts to multiply **Q** and **P** in the correct order.

A1: **QP** correct

(ii)

M1: Attempts the product $\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix} \begin{pmatrix} \lambda \\ 1 \end{pmatrix}$ and sets equal to $\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$. Allow for poor notation as

long as the intention is clear, and it may be implied by one correct equation or follow through equation.

A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.

dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.

A1: Correct value for λ from at least one equation and isw if incorrectly simplified (allow if their second equation does not concur).

A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Alt (ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix} \begin{pmatrix} \lambda \\ 1 \end{pmatrix} = \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda \\ 1 \end{pmatrix} = \frac{1}{-4-12} \begin{pmatrix} 2 & -2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$	M1
	$\Rightarrow \begin{pmatrix} \lambda \\ 1 \end{pmatrix} = -\frac{1}{16} \begin{pmatrix} 8\lambda - 8\sqrt{3} \\ -8\lambda\sqrt{3} - 8 \end{pmatrix}$	
	$2\lambda = \sqrt{3} - \lambda \text{ or } 2 = \lambda\sqrt{3} + 1$	A1
	$\Rightarrow 3\lambda = \sqrt{3} \Rightarrow \lambda = \dots \text{ or } \sqrt{3}\lambda = 1 \Rightarrow \lambda = \dots$	dM1
	$\lambda = \frac{\sqrt{3}}{3} \text{ or } \frac{1}{\sqrt{3}}$	A1
	Both $2\lambda = \sqrt{3} - \lambda$ and $2 = \lambda\sqrt{3} + 1$ solved leading to $\lambda = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	A1
		(5)

Notes:

M1: Correct attempt at inverse, attempts the product $\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}^{-1} \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$ and sets equal to $\begin{pmatrix} \lambda \\ 1 \end{pmatrix}$.

Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.

A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.

dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.

A1: Correct value for λ from at least one equation (allow if their second equation does not concur).

A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.



Question Number	Scheme	Notes	Marks
4	$f(z) = 6z^3 + Az^2 + Bz + C$ Condone work in x throughout		
(a)	$(z =) \frac{2}{3} - \frac{\sqrt{17}}{3}i$	Correct conjugate. Must be seen in (a)	B1
			(1)
(b)	$\left(z - \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right) = \dots \left\{z^2 - \frac{4}{3}z + \frac{7}{3}\right\} \quad \text{or e.g.}$ $\alpha + \beta = \frac{2}{3} + \frac{\sqrt{17}}{3}i + \frac{2}{3} - \frac{\sqrt{17}}{3}i = \dots \left\{\frac{4}{3}\right\}, \quad \alpha\beta = \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right) = \dots \left\{\frac{4}{9} + \frac{17}{9} = \frac{7}{3}\right\} \Rightarrow \dots \left\{z^2 - \frac{4}{3}z + \frac{7}{3}\right\}$ M1: Completes a correct strategy for finding a quadratic factor and obtains a 3TQ with real coefficients. May be slips with the expansion or calculation/forming quadratic but score M0 if the starting point is clearly $\left(z + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z + \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right)$ A1: Any correct three term quadratic factor. Allow any multiple of $z^2 - \frac{4}{3}z + \frac{7}{3}$ e.g., $3z^2 - 4z + 7$ May see $(3z - (2 + \sqrt{17}i))(3z - (2 - \sqrt{17}i))$ or $(3z - 2)^2 = (\pm\sqrt{17}i)^2 \Rightarrow \dots \{9z^2 - 12z + 21\}$		M1A1
	$6\left(z + \frac{3}{2}\right)\left(z^2 - \frac{4}{3}z + \frac{7}{3}\right) = \dots \text{ or e.g., } (2z + 3)(3z^2 - 4z + 7) = \dots$ Multiplies their 3 term quadratic factor with real coefficients (or multiple) by $z + \frac{3}{2}$ (or multiple) to obtain a 4TC with real coefficients so allow $\left(z + \frac{3}{2}\right)\left(z^2 - \frac{4}{3}z + \frac{7}{3}\right) = \dots \left\{z^3 + \frac{z^2}{6} + \frac{z}{3} + \frac{7}{2}\right\}$		M1
	$\{f(z) = \} 6z^3 + z^2 + 2z + 21 \quad \text{or} \quad A = 1, B = 2, C = 21$	A1: Any two correct values for A, B or C (could be embedded) A1: Fully correct expression (ignore an “=0”) or three correct values	A1A1
	Note that if the complex factors are not multiplied first e.g., $\left(z - \frac{3}{2}\right)\left(z - \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right) = \left(z^2 + \left(\frac{5}{6} - \frac{\sqrt{17}}{3}i\right)z - 1 - \frac{\sqrt{17}}{2}i\right)\left(z - \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right)$ $= z^3 + \frac{1}{6}z^2 + \frac{1}{3}z + \frac{7}{2} \Rightarrow 6z^3 + z^2 + 2z + 21$ Score the first 2 M marks together for obtaining a 4TC with real coefficients. M0M1 is possible e.g., with $\left(z + \frac{3}{2}\right)\left(z + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\right)\left(z + \left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right)\right)$ Any correct multiple of the 4TC scores the first A then score as main scheme		
			(5)

Question Number	Scheme/Notes	Marks
4(b)	Alternative 1: Substituting to obtain simultaneous equations $z = -\frac{3}{2} \Rightarrow -\frac{81}{4} + \frac{9}{4}A - \frac{3}{2}B + C = 0$ $z = \frac{2}{3} \pm \frac{\sqrt{17}}{3}i \Rightarrow -\frac{188}{9} \mp \frac{10\sqrt{17}}{9}i + A\left(-\frac{13}{9} \pm \frac{4\sqrt{17}}{9}i\right) + B\left(\frac{2}{3} \pm \frac{\sqrt{17}}{3}i\right) + C = 0$ $\Rightarrow -\frac{188}{9} - \frac{13}{9}A + \frac{2}{3}B + C = 0, \quad \pm\left(-\frac{10\sqrt{17}}{9} + \frac{4\sqrt{17}}{9}A + \frac{\sqrt{17}}{3}B\right) = 0$ M1: Substitutes $-\frac{3}{2}$ to obtain an equation and substitutes one of $\frac{2}{3} \pm \frac{\sqrt{17}}{3}i$ and equates real and imaginary parts to obtain two further equations. All equations must have real coefficients and each variable must appear in at least one equation. A1: All three correct equations $13A - 6B - 9C = -188, \quad 4A + 3B = 10, \quad 9A - 6B + 4C = 81$ $\Rightarrow A = 1, \quad B = 2, \quad C = 21$ M1: Solves to obtain real values for A, B and C A1: Two correct values A1: All three correct values Alternative 2: Sum/product/pairwise product sum of roots of cubic $\text{sum} = \frac{2}{3} + \frac{\sqrt{17}}{3}i + \frac{2}{3} - \frac{\sqrt{17}}{3}i - \frac{3}{2} = \dots \left\{ -\frac{1}{6} \right\} = -\frac{A}{6}$ $\text{pairwise product sum} = -\frac{3}{2}\left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right) - \frac{3}{2}\left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right) + \left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right) = \dots \left\{ \frac{1}{3} \right\} = \frac{B}{6}$ $\text{product} = -\frac{3}{2}\left(\frac{2}{3} + \frac{\sqrt{17}}{3}i\right)\left(\frac{2}{3} - \frac{\sqrt{17}}{3}i\right) = \dots \left\{ -\frac{7}{2} \right\} = -\frac{C}{6}$ M1: Obtains one equation in A, one in B and one in C all with real coefficients A1: All three correct equations $-\frac{1}{6} = -\frac{A}{6} \Rightarrow A = 1, \quad \frac{1}{3} = \frac{B}{6} \Rightarrow B = 2, \quad -\frac{7}{2} = -\frac{C}{6} \Rightarrow C = 21$ M1: Solves to obtain real values for A, B and C Note that A, B and C would be implied by e.g., $-\frac{1}{6}, \frac{1}{3}, -\frac{7}{2} \Rightarrow x^3 + \frac{1}{6}x^2 + \frac{1}{3}x + \frac{7}{2}$ A1: Two correct values A1: All three correct values If real values for any of the sum/pairwise product sum/product are not explicitly seen allow the M marks if real values for A, B and C (which could be embedded) are obtained. The first A mark would then require all values correct (or a correct cubic) to be awarded. It is possible to e.g., find A and C as above and then use e.g., $f\left(-\frac{3}{2}\right) = 0$ to determine B. In such cases the first M mark is scored when three equations have been attempted.	