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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Series (FP1.6)

International A Level Mathematics · FP1: Further Pure Mathematics 1 (WFM01)

9 questions · 65 marks · Mark schemes included

Past papers October 2020 - January 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.
Click any row below to jump straight to that sub-topic or question.
On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.
A gold bar in the left margin marks exactly where the question sits on the original page.
Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.
Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
FP1.6a SUMS OF SERIES (·R, ·R2, ·R3)		9 Qs · 65 marks	2
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QUESTIONS

9 questions · 65 marks · 1 sub-topic

SUB-TOPIC FP1.6a

SUMS OF SERIES ($\cdot R$, $\cdot R^2$, $\cdot R^3$)

Sub-topic FP1.6a · 9 questions · 65 marks · lowest tariff first



1. Use the standard results for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$ to show that, for all positive integers n

$$\sum_{r=1}^n r^2 (r + 2) = \frac{1}{12} n(n + 1)(an^2 + bn + c)$$

where a , b and c are integers to be determined.

(4)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 4 marks)



(Total for Question 2 is 6 marks)



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Q5

(Total for Question 3 is 7 marks)



Question 5 continued

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Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 8 marks)

Q5



8. (a) Use the standard results for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r$ to show that for all positive integers n

$$\sum_{r=0}^n (r+1)(r+2) = \frac{1}{3}(n+1)(n+2)(n+3) \quad (5)$$

- (b) Hence determine the value of

$$10 \times 11 + 11 \times 12 + 12 \times 13 + \dots + 100 \times 101 \quad (3)$$

Question 8 continued

Lined area for writing the answer to Question 8 continued.





Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)







(Total for Question 6 is 6 marks)



In this question use the standard results for summations.

(a) Show that for all positive integers n

$$\sum_{r=1}^n (12r^2 + 2r - 3) = An^3 + Bn^2$$

where A and B are integers to be determined.

(4)

(b) Hence determine the value of n for which

$$\sum_{r=1}^{2n} r^3 - \sum_{r=1}^n (12r^2 + 2r - 3) = 270$$

(4)

(Total for Question 7 is 6 marks)



Question 4 continued

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Q4



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Notes	Marks
1	$\sum_{r=1}^n r^2 (r+2) = \sum_{r=1}^n r^3 + 2 \sum_{r=1}^n r^2$ or $\sum_{r=1}^n r^3 + \sum_{r=1}^n 2r^2$	Correct split with 2 summations. Could be implied by correct work. Condone missing or incorrect summation limits.	B1
	$= \frac{1}{4} n^2 (n+1)^2 + 2 \times \frac{1}{6} n(n+1)(2n+1)$	Attempts to use both standard results and obtains an expression of the form $pn^2(n+1)^2 + qn(n+1)(2n+1)$ $p, q \neq 0$ Could be implied by immediate expansion	M1
	$= \frac{1}{12} n(n+1)[3n(n+1) + 4(2n+1)]$ $= \frac{1}{12} n(n+1)(3n^2 + 11n + 4)$	dM1: Attempts factorisation to obtain $\frac{1}{12} n(n+1)(an^2 + bn + c)$ $a, b, c \neq 0$. Condone poor algebra. Could follow cubic or quartic. Allow a consistent $a = \dots, b = \dots, c = \dots$ if quadratic never seen simplified Requires previous M mark. A1: Correct expression or $a = 3, b = 11, c = 4$ Allow e.g., $\frac{1}{12} n(n+1)$ written as $\frac{n}{12}(n+1)$	dM1 A1
	Note: $n(n+1)(3n^2 + 11n + 4) = 3n^4 + 14n^3 + 15n^2 + 4n$		Total 4



Question	Scheme	Marks
1.(a)	$\begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix} = \begin{pmatrix} 2+0+6k & 2k+3+6 \\ -2+0+0 & -2k-9+0 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2+6k & 2k+9 \\ -2 & -2k-9 \end{pmatrix}$	A1cao
		(2)
(b)	$\det \mathbf{AB} = (2+6k)(-2k-9) - (-2)(2k+9)$	M1
	$\det \mathbf{AB} = 0 \Rightarrow -12k^2 - 54k = 0 \Rightarrow k = \dots$	dM1
	$k = -\frac{9}{2}$	A1
		(3)
(5 marks)		
Notes:		
(a) M1: Obtains a 2×2 matrix with at least two entries correct, unsimplified. A1cao: Correct matrix with terms simplified. (b) M1: Attempts the determinant, be tolerant of minor slips, such as sign slips with the negatives, if the correct “$ad - bc$” form is apparent. They may give the $-(-2)(\dots)$ as just $+2(\dots)$. Accept if seen as part of the attempt at the inverse matrix. dM1: Expands their determinant to a quadratic, sets equal to zero (may be implied) and achieves a value for k via correct method (allow if a factor k is cancelled, use of formula or calculator (a correct value for their quadratic)). A1: cao for $-\frac{9}{2}$. Accept as decimal or equivalent fractions, such as $-\frac{54}{12}$. Ignore any reference to the 0 solution.		

Question	Scheme	Marks
2.	$(7r-5)^2 = 49r^2 - 70r + 25$	B1
	$\sum_{r=1}^n (7r-5)^2 = 49 \sum_{r=1}^n r^2 - 70 \sum_{r=1}^n r + \sum_{r=1}^n 25$ $= 49 \times \frac{n}{6} (n+1)(2n+1) - 70 \times \frac{n}{2} (n+1) + 25 \times n$	M1 A1ft
	$= \frac{n}{6} (49(2n^2 + 3n + 1) - 210(n+1) + 150)$	M1
	$= \frac{n}{6} (98n^2 - 63n - 11)$	A1
	$= \frac{n(7n+1)(14n-11)}{6}$	A1
		(6)

(6 marks)

Notes:
B1: Correct expansion.

M1: Attempts the summations with at least two of the underlined formulae correct.

A1ft: Fully correct application of all three summations. Follow through on their expansion as long as there are 3 terms.

M1: Attempts to factor out at least the factor of n from their **three term** expansion – must have a common factor of n throughout to be able to score this mark which must be extracted from each term. (If the last term is +25, it is M0.) Allow if there are minor slips but the process must be correct.

 Alternatively allow this mark for an attempt to expand $\frac{n}{6}(7n+1)(An+B)$ and compare coefficients with their expanded equation.

A1: Gathers terms appropriately and achieves the correct quadratic. In the alternative approach allow for $A=14$ and $B=-11$ stated from their comparison.

A1cso: Correct answer from correct work. Any values found from the comparison approach must be substituted back in to achieve the result. Note from a correct unsimplified quadratic to correct answer, A0A1 can be awarded.

Question Number	Scheme	Marks
2		
(a)	$\frac{3}{8} - \frac{\sqrt{71}}{8}i$	B1 (1)
(b)	$\left(x - \frac{3}{8} - \frac{\sqrt{71}}{8}i\right)\left(x - \frac{3}{8} + \frac{\sqrt{71}}{8}i\right) ((x-4)=0)$ $\left(x^2 - \frac{3}{4}x + \frac{5}{4}\right) ((x-4)=0)$ $x^3 - \frac{19}{4}x^2 + \frac{17}{4}x - 5 (=0)$ $4x^3 - 19x^2 + 17x - 20 (=0) \quad p=17, q=-20$	M1A1 dM1 A1 (4)
(a) B1 (b) M1 A1 dM1 A1	Correct answer only Attempt the multiplication of the 2 brackets with the complex terms. Allow $(x \pm \text{root})$ for the brackets. Allow “invisible” brackets. Correct quadratic obtained - may have multiplied by the 4 (or other constant factor) and this is fine. (Need not be fully simplified but must have real terms) Attempt to multiply their quadratic by $(x-4)$ or may divide their quadratic into the cubic or other full method leading to at least one of p or q. Correct values. Values of p and q need not be shown explicitly but may be seen in a cubic, provided the cubic starts $4x^3 - 19x^2$ (isw after a correct cubic)	[5]
	Note if a candidate uses a hybrid method, mark under main scheme unless an Alt scores more marks.	
Alt 1 (b)	$-\frac{q}{4} = 4 \times \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) \times \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) = \dots \rightarrow q = \dots \quad \text{or}$ $\frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $\Rightarrow q = -16 \times \left(\frac{9}{64} + \frac{71}{64}\right) = -20 \quad \text{or} \quad p = 17$ E.g. $f(4) = 0 \Rightarrow 4(4)^3 - 19(4)^2 + 4p - 20 = 0 \Rightarrow p = \dots$ $\text{or } \frac{p}{4} = 4 \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right) + 4 \left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) + \left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) \rightarrow p = \dots$ $p = 17, q = -20$	M1 A1 dM1 A1 (4)



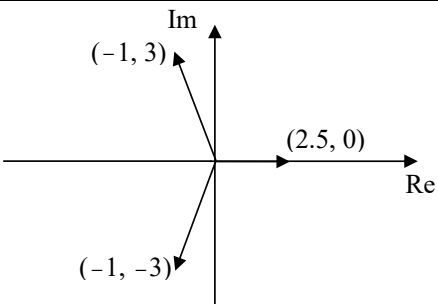
Question Number	Scheme	Notes	Marks
1(b) cont.	4. Differentiation: $\frac{dy}{dp} = 0 \Rightarrow 2p + 2 = 0 \Rightarrow p = -1 \Rightarrow p^2 + 2p + 3 = 2$ Minimum or "U-shape" or $a > 0$ or appropriate sketch, so non-singular M1: For -1 and 2 A1: Full explanation and conclusion. 5. Vertex just stated: Vertex is at $(-1, 2)$ "U-shape" or $a > 0$ or appropriate sketch, so non-singular M1: Correct vertex. If preceded by $p^2 + 2p + 3 = (p+1)^2 + 2$ award at that point for CTS A1: Full explanation and conclusion.		
			(2)
(c)	$\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix} \Rightarrow$ $\{\mathbf{P}^{-1}\} = \frac{1}{p^2 + 2p + 3} \begin{pmatrix} p & -p-1 \\ 3 & p-1 \end{pmatrix}$ or $\begin{pmatrix} \frac{p}{p^2 + 2p + 3} & \frac{-p-1}{p^2 + 2p + 3} \\ \frac{3}{p^2 + 2p + 3} & \frac{p-1}{p^2 + 2p + 3} \end{pmatrix}$	M1: $\frac{1}{p^2 + 2p + 3} \times (\text{a changed } 2 \times 2 \mathbf{P})$ (their changed $\mathbf{P}$ must not be or become constant) OR sight of $\text{Adj}(\mathbf{P})$ i.e. $\begin{pmatrix} p & -p-1 \\ 3 & p-1 \end{pmatrix}$ oe which may be labelled as $\mathbf{P}^{-1}$ A1ft: Correct inverse ft their det $\mathbf{P}$ [from part (a) or reattempted] provided it is (and remains) a function of p and accept det $\mathbf{P}$ unsimplified. Condone if det $\mathbf{P}$ clearly miscopied or rewritten incorrectly e.g., $(p+1)^2 - 2$ Allow $-1(p+1)$ for $-p-1$ but A0 if $-(-3)$ for 3 Is w when a correct or correct ft answer is seen unless a value for p is substituted	M1A1ft
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2(a)	$f(0.3) = \dots \{3.4563\dots\}$ $f(0.4) = \dots \{4.0615\dots\}$	Attempts both $f(0.3)$ and $f(0.4)$ and achieves a positive value for $f(0.3)$ and a negative value for $f(0.4)$	M1
	Sign change oe and $\{f(x) \text{ is}\}$ continuous $\Rightarrow$ root $\{\text{between } x = 0.3 \text{ and } x = 0.4\}$	Both $f(0.3) = \text{awrt } 3.5$ or 3.4 (truncated) & $f(0.4) = \text{awrt } -4.1$ or -4.0 or -4 (truncated), sign change oe, continuity and a minimal conclusion e.g., “root” or “shown”. A graph alone is insufficient. Allow “positive, negative” or $f(0.3) > 0$, $f(0.4) < 0$ or $f(0.3)f(0.4) < 0$ for “sign change”.	A1
			(2)
(b)	$f(x) = x^2 - \frac{7x - 4\sqrt{x}}{x^3} = x^2 - 7x^{-2} + 4x^{-2.5}$ $f'(x) = 2x + 14x^{-3} - 10x^{-3.5}$	Indices must be processed for any marks M1: For $\dots x^n \rightarrow \dots x^{n-1}$ at least once A1: 2 correct terms simplified or unsimplified A1: All correct simplified or unsimplified	M1A1A1
	If quotient/product rule used award M1 for any evidence of $\dots x^n \rightarrow \dots x^{n-1}$ (e.g., $x^2 \rightarrow 2x$). Their final expression in these cases must imply 2 correct terms for the first A mark. Quotient rule leads to $2x - \frac{x^3(7 - 2x^{-\frac{1}{2}}) - 3x^2(7x - 4x^{\frac{1}{2}})}{(x^3)^2} = 2x - \frac{7x^3 - 2x^{\frac{5}{2}} - 21x^3 + 12x^{\frac{5}{2}}}{x^6} = 2x - \frac{-14x^3 + 10x^{\frac{5}{2}}}{x^6}$ Product rule on $x^{-3}(7x - 4\sqrt{x})$ leads to $2x - [x^{-3}(7 - 2x^{-\frac{1}{2}}) + (-3x^{-4})(7x - 4x^{\frac{1}{2}})] = 2x - [7x^{-3} - 2x^{-\frac{7}{2}} - 21x^{-3} + 12x^{-\frac{7}{2}}]$		
			(3)
(c)	$x_1 = 0.3 - \frac{f(0.3)}{f'(0.3)} = \dots$ $\left\{ = 0.3 - \frac{0.3^2 - \frac{7(0.3) - 4\sqrt{0.3}}{0.3^3}}{2(0.3) + 14(0.3)^{-3} - 10(0.3)^{-3.5}} \right\}$ $\left\{ = 0.3 - \frac{3.456304816\dots}{-157.0821698\dots} = 0.3 + 0.02200316\dots = \dots \right\}$	Obtains a value from an attempt to apply the correct Newton-Raphson formula. Allow slips with substitution/miscopying and may be using an incorrectly simplified $f(x)$. Implied by awrt 0.32 (0.322003...) <u>even if $f'(x)$ is incorrect.</u> Not implied by real root which is 0.33 or 0.328 (0.3276079...) If not implied and no substitution is seen accept as minimum $0.3 - \frac{f(0.3)}{f'(0.3)} = \dots$ but $x_0 - \frac{f(x_0)}{f'(x_0)} = \dots$ is only acceptable if $x_0 = 0.3$ is seen. If only a value is seen it must round to 0.32	M1
	0.322	For awrt 0.322. Must be decimal Ignore further iterations $\alpha =$ is not required - just look for awrt 0.322 regardless of how it is labelled	A1
			(2)

Question Number	Scheme	Notes	Marks
2(d)	$f(1.3) = -0.37613... \quad f(1.5) = 0.590438...$ Examples: $\frac{0.590438...}{1.5 - \beta} = \frac{-(-0.37613...)}{\beta - 1.3}$ $\frac{1.5 - \beta}{\beta - 1.3} = \frac{0.590438...}{-(-0.37613...)}$ $\frac{1.5 - \beta}{1.5 - 1.3} = \frac{0.590438... - (-0.37613...)}{0.590438... - (-0.37613...)}$ $\beta = \frac{1.3(0.590438...) - 1.5(-0.37613...)}{0.590438... - (-0.37613...)}$ $\Rightarrow \beta = ...$	Obtains a negative value for $f(1.3)$ and a positive value for $f(1.5)$, forms a correct equation for these values and solves to obtain a value. Apply BOD if only $ f(1.3) $ and $ f(1.5) $ seen. Note $f(1.5)$ may be seen as $\frac{64\sqrt{6} - 93}{108}$. Accept "f(1.3)" & "f(1.5)" in equation if values for these seen. May use $\frac{af(b) - bf(a)}{f(b) - f(a)}$ oe. Allow e.g., x for β . If their variable denotes e.g., the distance between (1.3, 0) and $(\beta, 0)$ then 1.3 must be added later. Implied by awrt 1.378 (1.3778285...). Not by real root of 1.377 (1.376561...). Must be using the correct interval.	M1
	$= 1.378$	For awrt 1.378. Must be decimal. Ignore further iterations " $\beta =$ " is not required - just look for awrt 1.378 regardless of how it is labelled	A1
	Alternative via line equation: $y - (-0.37613...) = \frac{0.590438... - (-0.37613...)}{1.5 - 1.3}(x - 1.3)$ then $y = 0 \Rightarrow x = ...$ M1 for a correct equation with their $f(1.3)$ and $f(1.5)$, setting $y = 0$ and solving for x . May use (1.5, "0.590438...") as the point. Could also see equivalent attempts using $y = mx + c$ (finds c from a correct equation, puts $y = 0$ and solves)		(2)
			Total 9

Question Number	Scheme	Notes	Marks
3	Score B0 in (a) if the roots $\frac{1 \pm \sqrt{14}i}{3}$ are seen and then answers are just written down. The three subsequent method marks require use of the relevant identities. If not, a maximum of 0000010 is likely.		
(a)	$3x^2 - 2x + 5 = 0 \Rightarrow$ $\alpha + \beta = \frac{2}{3}, \quad \alpha\beta = \frac{5}{3}$	Both values correct. Consider in order presented if not labelled	B1
			(1)
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{2}{3}\right)^2 - 2\left(\frac{5}{3}\right) = \dots$	Uses correct identity with their sum and product to obtain a value for $\alpha^2 + \beta^2$	M1
	$= -\frac{26}{9}$	Correct value from correct sum and product	A1
			(2)
(c)	The work for the first two marks may be embedded within a quadratic expression/equation		
	$\alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{3} + \frac{\frac{2}{3}}{\frac{5}{3}} = \dots \left(\frac{16}{15}\right)$	Obtains a value for the new sum from a <u>correct numerical expression (which could be implied)</u> with their sum and product. Allow use of equivalent numerical expressions following use of e.g. $\frac{\alpha\beta(\alpha + \beta) + \alpha + \beta}{\alpha\beta}$	1st M1 (Sum)
	$\left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right) = \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta} = \alpha\beta + \frac{\alpha^2 + \beta^2}{\alpha\beta} + \frac{1}{\alpha\beta}$ $= \frac{5}{3} + \frac{-\frac{26}{9}}{\frac{5}{3}} + \frac{1}{\frac{5}{3}} = \dots \left(\frac{8}{15}\right)$	Obtains a value for the new product via a <u>correct numerical expression (which could be implied)</u> with their sum, product and answer to (b) which may have been reattempted. Allow use of equivalent numerical expressions following use of e.g. $\frac{(\alpha\beta)^2 + \alpha^2 + \beta^2 + 1}{\alpha\beta}$	2nd M1 (Product)
	$x^2 - \frac{16}{15}x + \frac{8}{15} \{= 0\}$	Correctly applies $x^2 - (\text{their new sum})x + \text{their new product}$ with values to obtain a 3TQ. Not dependent. Accept appropriate values for p, q and r for this mark.	M1
	$15x^2 - 16x + 8 = 0$	Correct equation – not just values for p, q and r . Terms in any order but must have " $=0$ ". Allow any integer multiple. Must come from $\alpha + \beta = \frac{2}{3}, \quad \alpha\beta = \frac{5}{3}$ Condone a different variable (e.g., z)	A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
3	$2x^2 - 5x + 7 = 0$		
(a)	$\alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{7}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{3}{4}$	cso – must have scored the B1	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(\frac{5}{2}\right) = -\frac{85}{8}$	cso – must have scored the B1	A1
			(4)
(c)	$\text{Sum} = \frac{1}{\alpha^2 + \beta} + \frac{1}{\beta^2 + \alpha} = \frac{\alpha^2 + \beta + \beta^2 + \alpha}{(\alpha^2 + \beta)(\beta^2 + \alpha)}$ $= \frac{\alpha^2 + \beta^2 + \alpha + \beta}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{-\frac{3}{4} + \frac{5}{2}}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{14}{41} \right)$ Attempts sum – substitutes their into a correct numerator must but allow slips in the denominator as long as 4 terms are produced from the expansion.		M1
	$\text{Product} = \frac{1}{\alpha^2 + \beta} \times \frac{1}{\beta^2 + \alpha} = \frac{1}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{1}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{8}{41} \right)$ Attempts product – must be correct expansion of denominator with their values.		M1
	$x^2 - \frac{14}{41}x + \frac{8}{41} (= 0)$	Applies $x^2 - (\text{their sum})x + \text{their product} (= 0)$ Depends on at least one previous M awarded.	dM1
	$41x^2 - 14x + 8 = 0$	Allow any integer multiple. Must include “=0”	A1
			(4)
			Total 9

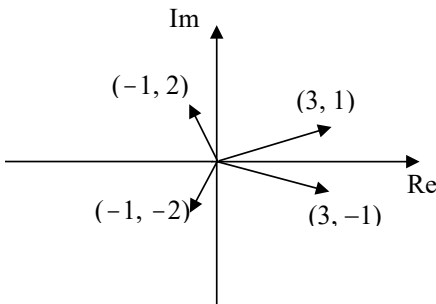
Question Number	Scheme	Notes	Marks
4	$f(z) = 2z^3 - z^2 + az + b$		
(a)	$(z =) -1 + 3i$	Correct complex number	B1
			(1)
(b)	$z = -1 \pm 3i \Rightarrow (z - (-1 + 3i))(z - (-1 - 3i)) \rightarrow z^2 + \dots z + \dots$ Or e.g. Sum = -2, Product = $(-1)^2 - (3i)^2 = 10 \rightarrow z^2 + \dots z + \dots$ Correct strategy to find the quadratic factor		M1
	$z^2 + 2z + 10$	Correct expression	A1
	$f(z) = (z^2 + 2z + 10)(2z - 5)$	Uses an appropriate method to find the linear factor	M1
	$\Rightarrow f(z) = 2z^3 - z^2 + 10z - 50$ or $a = 10, b = -50$	Correct cubic or correct constants	A1
			(4)
(c)		$-1 \pm 3i$ correctly plotted with vectors or dots or crosses etc. May be labelled by coordinates or on axes. Do not be concerned about scale but should look like reflections in the real line.	B1
		$(2.5, 0)$ plotted correctly or follow through their non-zero real root correctly plotted. May be labelled by coordinates or on axes. Do not be too concerned about scale but e.g. $(2.5, 0)$ should be further from O than $(-1, 0)$ is.	B1ft
			(2)
			Total 7
Alt (b)	$f(-1 + 3i) = 0 \Rightarrow 2(-1 + 3i)^3 - (-1 + 3i)^2 + a(-1 + 3i) + b = 0$ $\text{Im}(f(-1 + 3i)) = 0 \Rightarrow 2(9 - 27) - (-6) + 3a = 0 \Rightarrow a = \dots$ Or e.g. $f(-1 + 3i) - f(-1 - 3i) = 0 \Rightarrow 2(2(9i - 27i) - (-6i) + 3ai) = 0 \Rightarrow a = \dots$ Correct full strategy to find one constant.		M1
	$a = 10$ or $b = -50$	One correct value.	A1
	E.g. $\text{Re}(f(-1 + 3i)) = 0 \Rightarrow 2(-1 + 27) - (1 - 9) - a + b = 0 \Rightarrow b = \dots$ Correct method to find the second constant.		M1
	$a = 10$ and $b = -50$ or $f(z) = 2z^3 - z^2 + 10z - 50$	Correct constants or correct cubic	A1
			(4)

Question Number	Scheme	Notes	Marks
5(a)	$r(r-1)(r-3) = r^3 - 4r^2 + 3r$	Correct expansion	B1
	$\sum_{r=1}^n (r^3 - 4r^2 + 3r) = \frac{1}{4}n^2(n+1)^2 - 4\frac{1}{6}n(n+1)(2n+1) + 3 \times \frac{1}{2}n(n+1)$ M1: Attempt to use at least two of the standard formulae correctly A1: Correct expression		M1A1
	$= \frac{1}{12}n(n+1)[3n(n+1) - 8(2n+1) + 18]$	Attempt to factorise $\frac{1}{12}n(n+1)$ from an expression with these factors. Depends on previous M.	dM1
	Note: for attempts that first expand to a quartic this mark may be awarded at the point the relevant factors are taken out provided a suitable quadratic factor is seen before the final answer.		
	$= \frac{1}{12}n(n+1)[3n^2 - 13n + 10]$ $= \frac{1}{12}n(n+1)(n-1)(3n-10)^*$	Cso with $3n^2 - 13n + 10$ (or another appropriate correct quadratic) seen before the final printed answer.	A1*
			(5)
(b)	$\sum_{r=n+1}^{2n+1} r(r-1)(r-3) = \frac{1}{12}(2n+1)(2n+2)2n(6n-7) - \frac{1}{12}n(n+1)(n-1)(3n-10)$ Attempts $f(2n+1) - f(n)$		M1
	$= \frac{1}{12}n(n+1)[4(2n+1)(6n-7) - (n-1)(3n-10)] = \frac{1}{12}n(n+1)(\dots n^2 + \dots n + \dots)$ Attempt to factor out $\frac{1}{12}n(n+1)$ and simplify the rest to 3 term quadratic expression. For attempts expanding to a quartic first, score for reaching an expression of the correct form.		dM1
	$= \frac{1}{12}n(n+1)(45n^2 - 19n - 38)$	Cao	A1
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
8	$\sum_{r=0}^n (r+1)(r+2)$		
(a)	$\sum_{r=0}^n r^2 + 3r + 2 = 2 + \frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + 2n$ M1: Attempt to use at least one of the standard formulae correctly A1: For $\frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + (2n \text{ or } 2n+2)$ A1: Fully correct expression		M1A1A1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + 2n + 2 = (n+1) \left[\frac{1}{6}n(2n+1) + \frac{3}{2}n + 2 \right]$ Attempt to factorise $(n+1)$ It is a “show” question so this must be seen (in any equivalent form). If their expression does not allow for factorising $(n+1)$ score M0		M1
	$\frac{1}{3}(n+1)[n^2 + 5n + 6]$	May obtain a cubic and extract a different factor ie $n+2$ or $n+3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)^*$	Cso At least one intermediate step in the working must be seen.	A1*
			(5)
(a) Way 2	$\sum_{r=0}^n (r+1)(r+2) = \sum_{r=1}^{n+1} r(r+1)$ $= \sum_{r=1}^{n+1} r^2 + r = \frac{1}{6}(n+1)(n+2)(2(n+1)+1) + \frac{1}{2}(n+1)(n+2)$ M1: Attempt to use at least one of the standard formulae correctly with $n = n+1$ A1: For $\frac{1}{6}(n+1)(n+2)(2(n+1)+1)$ or $\frac{1}{2}(n+1)(n+2)$ A1: Fully correct expression		M1A1A1
	$\frac{1}{6}(n+1)(n+2)(2(n+1)+1) + \frac{1}{2}(n+1)(n+2) = (n+1) \left[\frac{1}{6}(n+1)(2n+3) + \frac{1}{2}(n+2) \right]$ Attempt to factorise $(n+1)$ (see additional comments above)		M1
	$\frac{1}{3}(n+1)[n^2 + 5n + 6]$	May obtain a cubic and extract a different factor ie $n+2$ or $n+3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)^*$	Cso At least one intermediate step in the working must be seen.	A1*
(b)	Upper limit = 99	Correct upper limit	B1
	$10 \times 11 + 11 \times 12 + 12 \times 13 + \dots + 100 \times 101 = \sum_{r=0}^{99} (r+1)(r+2) - \sum_{r=0}^8 (r+1)(r+2)$ Fully correct strategy for the sum using their upper limit for the first sum and upper limit 8 for the second in the result from (a). Lower limits 0 or 1		M1
	$= \frac{1}{3}(100)(101)(102) - \frac{1}{3}(9)(10)(11)$ $= 343\,070$	Correct value	A1

A1: Both coordinates correct and no incorrect ones (but ignore (27,6))		
Question	Scheme	Marks
8.(a)	$\sum_{r=1}^n r(2r^2 - 3r - 1) = \sum_{r=1}^n (2r^3 - 3r^2 - r)$ $= 2 \times \frac{1}{4} n^2 (n+1)^2 - 3 \times \frac{1}{6} n(n+1)(2n+1) - \frac{1}{2} n(n+1)$	M1 A1
	$\frac{1}{2} n^2 (n+1)^2 - \frac{1}{2} n(n+1)(2n+1) - \frac{1}{2} n(n+1)$ $= \frac{1}{2} n(n+1)[n(n+1) - (2n+1) - 1]$	M1
	$= \frac{1}{2} n(n+1)[n^2 - n - 2] = \frac{1}{2} n(n+1)(n+1)(n-2)$ $= \frac{1}{2} n(n+1)^2 (n-2)^*$	A1*
		(4)
(b)	$\sum_{r=n}^{2n} r(2r^2 - 3r - 1) = \sum_{r=1}^{2n} r(2r^2 - 3r - 1) - \sum_{r=1}^{n-1} r(2r^2 - 3r - 1)$ $= \frac{1}{2} (2n)(2n+1)^2 (2n-2) - \frac{1}{2} (n-1)(n)^2 (n-3)$	M1
	$= \frac{1}{2} n(n-1)[4(2n+1)^2 - n(n-3)]$	M1
	$= \frac{1}{2} n(n-1)[15n^2 + 19n + 4]$	A1
	$= \frac{1}{2} n(n-1)(15n+4)(n+1)$	A1
		(4)
(Total 8 marks)		
Notes		
(a) Note – attempts at induction score no marks. M1: Expands the bracket and attempt to use at least one of the standard formulae correctly. A1: Fully correct expression M1: Attempts to factorise out at least $n(n+1)$ - both terms must have been common factors in the terms of their expression. If expanded to a quartic, there must be a clear attempt at factorisation in stages, directly to the given answer will be M0A0. (Note if they try to find roots there needs to be evidence that -1 is a repeated root before going direct to the given answer from these.) A1*: cso Must have achieved a suitable correct intermediate stage with a quadratic in their working. (b) M1: Applies $f(2n) - f(k)$ where k is $n-1$ or n with the formula from (a) or allow from restarts using the standard formulae. dM1: Attempts to factorise out $n(n-1)$ - which must be factors of their expression, so use of $f(2n) - f(n)$ will score dM0. Accept for this mark if they expand from a correct expression and achieve the correct answer.		

Question Number	Scheme	Notes	Marks
7(a)	$\sum_{r=1}^n (12r^2 + 2r - 3) = 12 \sum_{r=1}^n r^2 + 2 \sum_{r=1}^n r - 3 \sum_{r=1}^n 1$ $= 12 \times \frac{n}{6}(n+1)(2n+1) + 2 \times \frac{n}{2}(n+1) - 3n$ $[= 2n(n+1)(2n+1) + n(n+1) - 3n]$	M1: Expands summation to at least 2 separate sums with one correct (could be implied), uses $\sum_{r=1}^n r^2 = \frac{n}{6}(n+1)(2n+1)$ (allowing one of the following slips within the formula above: One of the 2 + signs seen as – or a missing first n) and replaces $\sum_{r=1}^n r$ with $\frac{n}{2}(n+1)$ or $\sum_{r=1}^n 1$ with n Condone r used for n for the first three marks only. Allow $\sum$ for $\sum_{r=1}^n$ A1: Fully correct unsimplified expression	M1 A1
	$\sum_{r=1}^n (12r^2 + 2r - 3) = 4n^3 + 6n^2 + 2n + n^2 + n - 3n = \dots$	Expands to a cubic and collects terms. Allow slips. Requires previous M mark.	dM1
	$4n^3 + 7n^2$	Correct expression from correct work Allow $A = 4$, $B = 7$ following "$= An^3 + Bn^2$"	A1
	(4)		
(b)	Full marks in (b) does not require full marks in (a)		
	$\sum_{r=1}^{2n} r^3 = \frac{(2n)^2}{4}(2n+1)^2$	Attempts to use the sum of cubes formula with 2n Allow one of the following two slips: $2n^2$ for $(2n)^2$ Only one of the n's in the formula replaced by 2n	M1
	$\sum_{r=1}^{2n} r^3 - \sum_{r=1}^n (12r^2 + 2r - 3) =$ $4n^4 + 4n^3 + n^2 - "4"n^3 - "7"n^2 [= 270]$ $[\Rightarrow 4n^4 - 6n^2 = 270]$	Correct expanded quartic expression for $\sum_{r=1}^{2n} r^3 - \sum_{r=1}^n (12r^2 + 2r - 3)$ (ft their $An^3 + Bn^2$) No requirement to collect terms but must be correct for their A and B if expression only seen with terms collected. If this is only seen as an equation it must be correct.	A1ft
	$4n^4 - 6n^2 - 270 = 0 \Rightarrow$ $2n^4 - 3n^2 - 135 = (2n^2 + 15)(n^2 - 9) = 0$ $\Rightarrow n^2 = \dots$	Solves their 3TQ in n^2 (usual rules and allow for one correct root if no working). May change variable e.g., $n^2 \rightarrow x$ Ignore the labelling of roots (e.g. "$n = \dots$") Allow for solving as a quartic if one root correct but requires $pn^4 + qn^2 + r = 0$ oe, $p, q, r \neq 0$ Requires previous M mark.	dM1
	$n^2 = 9 \Rightarrow n = 3$	$n = 3$ and no other unrejected solutions. $n = \pm 3$ is A0 Must follow a correct equation.	A1
(4)			
			Total 8

Question Number	Scheme	Notes	Marks
3	$f(z) = z^4 + az^3 + bz^2 + cz + d$		
(a)	$(z =) 3 - i$ or $(z =) -1 + 2i$		B1
	$(z =) 3 - i$ and $(z =) -1 + 2i$		B1
			(2)
(b)		$3 \pm i$ correctly plotted with vectors or dots or crosses etc. or $-1 \pm 2i$ correctly plotted with vectors or dots or crosses etc.	B1
		All 4 correct roots correctly plotted with scaling approximately correct (e.g. $(-1, 2)$ higher than $(3, 1)$, etc.) There should be approximate symmetry about the real axis, but be generous	B1
			(2)
(c)	$z = 3 \pm i \Rightarrow (z - (3 + i))(z - (3 - i)) = \dots$ or $z = -1 \pm 2i \Rightarrow (z - (-1 + 2i))(z - (-1 - 2i)) = \dots$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2 - 6z + 10$ or $z^2 + 2z + 5$	One correct quadratic	A1
	$z^2 - 6z + 10$ and $z^2 + 2z + 5$	Both correct	A1
	$(z^2 - 6z + 10)(z^2 + 2z + 5) = \dots$	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a = -4, b = 3, c = -10, d = 50$ or $f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1
			(5)
			Total 9

(c) Way 2	$(z - (3 + i))(z - (-1 \pm 2i)) = \dots$ or $(z - (3 - i))(z - (-1 \pm 2i)) = \dots$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2 + z(-2 + i) + (-1 - 7i)$ (i) or $z^2 + z(-2 - i) + (-1 + 7i)$ (ii) or $z^2 + z(-2 + 3i) + (-5 - 5i)$ (iii) or $z^2 + z(-2 - 3i) + (-5 + 5i)$ (iv)	One correct quadratic	A1
	(i) and (ii) correct or (iii) and (iv) correct	A correct pair	A1
	e.g $[z^2 + z(-2 + i) + (-1 - 7i)] \times [z^2 + z(-2 - i) + (-1 + 7i)] = \dots$	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a = -4, b = 3, c = -10, d = 50$ or $f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1

(c) Way 3	$\sum \alpha = (3 + i) + (3 - i) + (-1 - 2i) + (-1 + 2i) = \dots$ or $\alpha\beta\gamma\delta = (3 + i)(3 - i)(-1 - 2i)(-1 + 2i) = \dots$	Attempts one of these	M1
	$a = -4$ or $d = 50$		A1
	Both $a = -4$ and $d = 50$		A1
	$\sum \alpha\beta = \dots$ and $\sum \alpha\beta\gamma = \dots$	Attempts $\sum \alpha\beta$ (all 6 terms) and $\sum \alpha\beta\gamma$ (all 4 terms)	M1
	$a = -4, b = 3, c = -10, d = 50$ or $f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1

(c) Way 4	$(3 + i)^4 + a(3 + i)^3 + b(3 + i)^2 + c(3 + i) + d = 0$ $(\dots) + (\dots)i = 0$	Substitutes one of the roots into the given quartic and fully multiplies out	M1
	$(28 + 18a + 8b + 3c + d) + i(96 + 26a + 6b + c)$ or $(-7 + 11a - 3b - c + d) + i(-24 + 2a + 4b - 2c)$	One correct expansion	A1
	$(28 + 18a + 8b + 3c + d) + i(96 + 26a + 6b + c)$ and $(-7 + 11a - 3b - c + d) + i(-24 + 2a + 4b - 2c)$	Obtains a second correct expansion using another root.	A1
	$(28 + 18a + 8b + 3c + d) = 0$, etc leading to $a = \dots, b = \dots, c = \dots, d = \dots$	Solves 4 simultaneous equation to find values of a, b, c and d	M1
	$a = -4, b = 3, c = -10, d = 50$ or $f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$	All correct values or correct quartic	A1

Question Number	Scheme	Notes	Marks
4(a)	$(2r-1)^2 = 4r^2 - 4r + 1$	Correct expansion	B1
	$\sum_{r=1}^n (4r^2 - 4r + 1) = 4 \times \frac{1}{6} n(n+1)(2n+1) - 4 \times \frac{1}{2} n(n+1) + n$ M1: Attempt to use at least one of the standard results correctly A1: Correct expression		M1A1
	$= \frac{1}{3} n [2(n+1)(2n+1) - 6(n+1) + 3]$	Attempt to factorise $\frac{1}{3} n(\dots)$ Condone one slip but there must have been $+n$, not $+1$ in their expression for the sum	M1
	$= \frac{1}{3} n [4n^2 - 1]^*$	Correct proof with no errors. There should be an intermediate step showing the expansion of $(n+1)(2n+1)$, or equivalent	A1*
	Condone poor or incorrect use of notation, e.g. Σ used at every step of the proof		
			(5)
(b)	$2r-1 = 499 \Rightarrow r = 250$	Identifies the correct upper limit (may be implied)	B1
	$2r-1 = 201 \Rightarrow r = 101$	Identifies the correct lower limit (may be implied)	B1
	$\sum_{r=101}^{250} (2r-1)^2 = \frac{1}{3} \times 250(4 \times 250^2 - 1) - \frac{1}{3} \times 100(4 \times 100^2 - 1)$ Uses the result from part (a) together with their upper limit and their lower limit -1 . A common mistake is to assume 500 and 200 are the limits, and in this case the mark is scored if 199 is used		M1
	$= 19\,499\,950$	Cao	A1
			(4)
			Total 9