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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Inequalities (FP2.1)

International A Level Mathematics · FP2: Further Pure Mathematics 2 (WFM02)

13 questions · 97 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

13 questions · 97 marks · 1 sub-topic

SUB-TOPIC FP2.1a

SOLVING INEQUALITIES (ALGEBRAIC & MODULUS)

Sub-topic FP2.1a · 13 questions · 97 marks · lowest tariff first



1. Using algebra, solve the inequality

$$\frac{1}{x+2} > 2x+3$$

(5)



(Total for Question 1 is 5 marks)



5.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**Use algebra to determine the set of values of x for which

$$\frac{x^2 - 9}{|x + 8|} > 6 - 2x$$

(6)





Question 5 continued

(Total for Question 5 is 6 marks)



5.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**Use algebra to determine the values of x for which

$$\frac{x+1}{(x-3)(x+2)} \leq 1 - \frac{2}{x-3}$$

(6)

Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5.



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)



1.

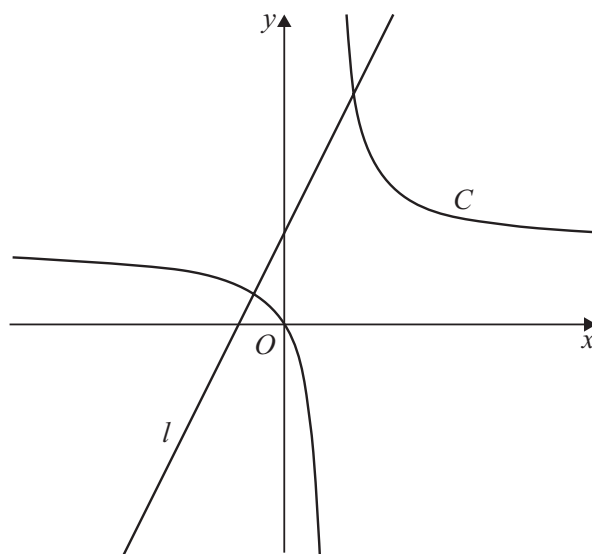


Figure 1

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{10x}{x-6} \quad x \neq 6$$

and the line l with equation $y = 2x + 12$

(a) Use algebra to determine the x coordinates of the points of intersection of C and l (2)

(b) Determine the range of values of x for which

(i) $2x + 12 > \frac{10x}{x-6}$

(ii) $|2x + 12| > \left| \frac{10x}{x-6} \right|$ (4)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)



3. Use algebra to obtain the set of values of x for which

$$|x^2 + x - 2| < \frac{1}{2}(x + 5)$$

(7)

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Question 3 continued

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Question 3 continued

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Q3

(Total 7 marks)



5. Use algebra to find the set of values of x for which

$$|2x^2 + x - 3| > 3(1 - x)$$

(7)

[Solutions based entirely on graphical or numerical methods are not acceptable.]

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Question 5 continued

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Question 5 continued

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Question 5 continued

Leave
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(Total 7 marks)

Q5



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 7 marks)







(Total for Question 2 is 7 marks)



2. Use algebra to determine the set of values of x for which

$$\frac{x}{2-x} \leq \frac{x+3}{x}$$

(Solutions relying entirely on graphical methods are not acceptable.)

(8)

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Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total 8 marks)

Q2



2. (a) Use algebra to determine the set of values of x for which

$$x - 5 < \frac{9}{x + 3}$$

(6)

(b) Hence, or otherwise, determine the set of values of x for which

$$x - 5 < \frac{9}{|x + 3|}$$

(2)

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 8 marks)



3. Use algebra to obtain the set of values of x for which

$$\left| \frac{x^2 + 3x + 10}{x + 2} \right| < 7 - x$$

(9)

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Question 3 continued

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(Total 9 marks)

Q3



8.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

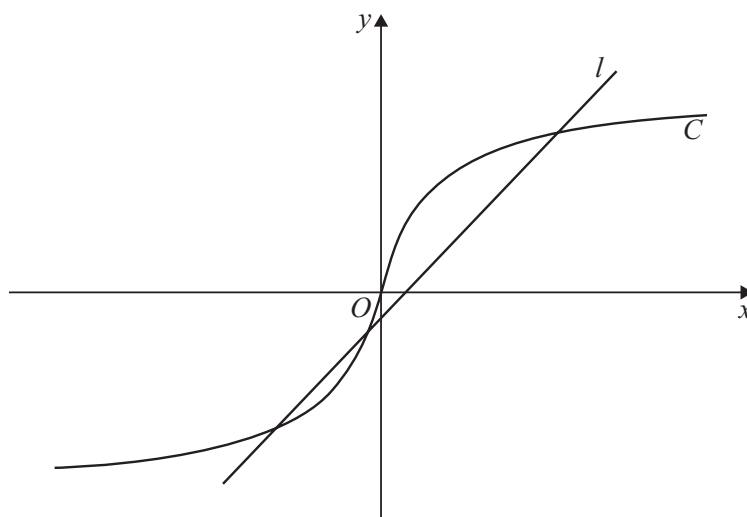


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \frac{15x}{|x| + 4}$ and the line l with equation $y = x - 2$

(a) Use algebra to determine the exact values of x for which

$$x - 2 > \frac{15x}{|x| + 4} \quad (6)$$

(b) Hence use algebra to determine the exact values of x for which

$$|x - 2| > \left| \frac{15x}{|x| + 4} \right| \quad (4)$$

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

(Total for Question 8 is 10 marks)



Question 3 continued

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Question 3 continued

Leave
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Question 3 continued

Leave
blank

Q3

(Total 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, score for their best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$, otherwise must divide by a first – allow equivalent work if a is a square number)

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$



Question Number	Scheme	Notes	Marks
5	$(x =) -8$	This cv stated or used	B1
	For cv's $3, -\frac{19}{3}$ OR For cv's $3, -13$		
	Examples: $x^2 - 9 = (x+8)(6-2x) \Rightarrow x = \dots$ or $(x^2 - 9)(x+8) = (x+8)^2(6-2x) \Rightarrow x = \dots$ or $\frac{x^2 - 9}{(x+8)} - (6-2x) = 0 \Rightarrow x = \dots >$ NB leads to $3x^2 + 10x - 57 = 0$	Examples: $x^2 - 9 = -(x+8)(6-2x) \Rightarrow x = \dots$ or $-(x^2 - 9)(x+8) = (x+8)^2(6-2x) \Rightarrow x = \dots$ or $\frac{x^2 - 9}{-(x+8)} - (6-2x) = 0 \Rightarrow x = \dots >$ NB leads to $x^2 + 10x - 39 = 0$	M1
	For cv's $3, -\frac{19}{3}$ AND For cv's $3, -13$		
	Examples: $x^2 - 9 = (x+8)(6-2x) \Rightarrow x = \dots$ or $(x^2 - 9)(x+8) = (x+8)^2(6-2x) \Rightarrow x = \dots$ or $\frac{x^2 - 9}{(x+8)} - (6-2x) = 0 \Rightarrow x = \dots >$ NB leads to $3x^2 + 10x - 57 = 0$	Examples: $x^2 - 9 = -(x+8)(6-2x) \Rightarrow x = \dots$ or $-(x^2 - 9)(x+8) = (x+8)^2(6-2x) \Rightarrow x = \dots$ or $\frac{x^2 - 9}{-(x+8)} - (6-2x) = 0 \Rightarrow x = \dots >$ NB leads to $x^2 + 10x - 39 = 0$	M1
	Any two of: $x = -13, -\frac{19}{3}, 3$	For any two of these cv's. May be seen embedded in their inequalities. Depends on at least one previous M mark.	A1
	$-13 < x < -8, -8 < x < -\frac{19}{3}, x > 3$ A1: Any 2 of these inequalities. Note that $-13 < x < -\frac{19}{3}, x \neq -8$ would count as 2 correct inequalities. Also condone $-13 < x < -\frac{19}{3}, x > 3$ as 2 correct inequalities. Depends on at least one previous M mark. A1: All correct and no other regions. Depends on all previous marks.		A1 A1
	Allow equivalent notation for the inequalities e.g. for $-13 < x < -8$ allow $x > -13$ and $x < -8, x > -13, x < -8, -8 > x > -13, (-13, -8), \{x : x > -13 \cap x < -8\}$ But not $x > -13$ or $x < -8$ Note that $-13 < x < -\frac{19}{3}, x \neq -8, x > 3$ is fully correct.		
			(6)
			Total 6

Note that it is possible to find all the cv's by squaring both sides of the equation:

	$(x =) -8$	This cv stated or used	B1
	$\frac{(x^2 - 9)^2}{(x + 8)^2} = (6 - 2x)^2 \Rightarrow x^4 - 18x^2 + 81 = (36 - 24x + 4x^2)(x^2 + 16x + 64)$ $\Rightarrow 3x^4 + 40x^3 - 74x^2 - 960x + 2223 = 0 \Rightarrow x = \dots$ M2 Requires a complete attempt to square both sides, multiply up to obtain a quartic equation and an attempt to solve to find at least 1 critical value other than $x = -8$		M1M1
	Any two of: $x = -13, -\frac{19}{3}, 3$	For any two of these cv's. May be seen embedded in their inequalities. Depends on both previous M marks.	A1
	$-13 < x < -8, -8 < x < -\frac{19}{3}, x > 3$ A1: Any 2 of these inequalities. Note that $-13 < x < -\frac{19}{3}, x \neq -8$ would count as 2 correct inequalities. Also condone $-13 < x < -\frac{19}{3}, x > 3$ as 2 correct inequalities. Depends on at least one previous M mark. A1: All correct and no other regions. Depends on all previous marks.		A1 A1
	Allow equivalent notation for the inequalities e.g. for $-13 < x < -8$ allow $x > -13$ and $x < -8, x > -13, x < -8, -8 > x > -13, (-13, -8), \{x : x > -13 \cap x < -8\}$ But not $x > -13$ or $x < -8$ Note that $-13 < x < -\frac{19}{3}, x \neq -8, x > 3$ is fully correct.		

Question Number	Scheme	Marks
2(a)	$x \frac{dy}{dx} - y^3 = 4 \Rightarrow \frac{dy}{dx} + x \frac{d^2y}{dx^2} - 3y^2 \frac{dy}{dx} = 0$	M1A1
	Alt: $\frac{dy}{dx} = \frac{y^3}{x} + \frac{4}{x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{y^3}{x^2} + \frac{3y^2}{x} \frac{dy}{dx} - \frac{4}{x^2}$	
	$\Rightarrow \frac{d^2y}{dx^2} + x \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 6y \left(\frac{dy}{dx} \right)^2 - 3y^2 \frac{d^2y}{dx^2} = 0$	dM1
	Alt: $\Rightarrow \frac{d^3y}{dx^3} = \frac{2y^3}{x^3} - \frac{3y^2}{x^2} \frac{dy}{dx} - \frac{3y^2}{x^2} \frac{dy}{dx} + \frac{1}{x} \left(6y \left(\frac{dy}{dx} \right)^2 + 3y^2 \frac{d^2y}{dx^2} \right) + \frac{8}{x^3}$	
	$\Rightarrow x \frac{d^3y}{dx^3} = 6y \left(\frac{dy}{dx} \right)^2 + (3y^2 - 2) \frac{d^2y}{dx^2}$	A1
		(4)
(b)	$x = 2, y = 1 \Rightarrow \left(\frac{dy}{dx} \right)_{x=2} = \frac{4+1}{2} = \frac{5}{2}, \left(\frac{d^2y}{dx^2} \right)_{x=2} = \frac{3(1) \left(\frac{5}{2} \right) - \frac{5}{2}}{2} = \frac{5}{2}$	M1
	$\left(\frac{d^3y}{dx^3} \right)_{x=2} = \frac{6(1) \left(\frac{25}{4} \right) + \frac{5}{2}}{2} = 20$	
	$(y =) f(2) + (x-2)f'(2) + \frac{(x-2)^2}{2} f''(2) + \frac{(x-2)^3}{6} f'''(2) + \dots$ $= 1 + \frac{5}{2}(x-2) + \frac{5}{2} \frac{(x-2)^2}{2} + 20 \frac{(x-2)^3}{6} + \dots$	M1
	$(y =) 1 + \frac{5}{2}(x-2) + \frac{5}{4}(x-2)^2 + \frac{10}{3}(x-2)^3 + \dots$	A1
		(3)
		Total 7

Notes: Accept other derivative notations, e.g. y' , throughout.

(a)

M1: $x \frac{dy}{dx} \rightarrow \frac{dy}{dx} + x \frac{d^2y}{dx^2}$ or $y^3 \rightarrow ky^2 \frac{dy}{dx}$ Alternatively for $\frac{y^3}{x} \rightarrow \pm \alpha \frac{y^3}{x^2} + \frac{ky^2}{x} \frac{dy}{dx}$ (oe via quotient rule), or any correct attempt at implicit differentiation on the equation (coefficient slips condoned).

A1: Any fully correct second derivative expression. The two most common are shown in the scheme.

dM1: Differentiates again using the product rule correctly at least once. Depends on the first method mark. Again, look for correct forms, but allow if coefficients are incorrect.

There may be alternatives where they substitute for $\frac{dy}{dx}$ before differentiating again.

The scheme follows the same principle. If unsure, use the Review system.

A1: Correct answer achieved from correct work. In the Alt they need to substitute the second derivative expression before reaching the final answer.

(b)

M1: Attempts to find values for the first 3 derivatives at $x = 2$. Working need not be seen, in which case accept if all are attempted with at least y' correct providing expressions involving the other derivatives were actually found in part (a).

M1: Correct application of Taylor's theorem with their values up to at least the $(x-2)^2$ term.

A1: Correct simplified expression. May be called y or $f(x)$ or unlabelled for this part.

Question Number	Scheme	Notes	Marks
1(a)	$\frac{10x}{x-6} = 2x+12 \Rightarrow 10x = 2x^2 - 72$ $x^2 - 5x - 36 = 0 \Rightarrow (x+4)(x-9) = 0$ $\Rightarrow x = -4, 9$	M1: Cross multiplies, forms 3TQ and solves (usual rules) A1: Both correct values	M1 A1
(2)			
(b)(i)	$-4 < x < 6, \quad x > 9$	$a < x < 6, \quad x > b$ with their $a < 0$ and $b > 6$	B1ft
(b)(ii)	$\frac{10x}{x-6} = -(2x+12) \Rightarrow$ $x^2 + 5x - 36 = 0 \Rightarrow x = -9, 4$	From a correct equation, cross multiplies, forms 3TQ and solves (usual rules)	M1
	$x < -9, \quad -4 < x < 4, \quad x > 9$	M1: Correct regions $x < \alpha, \beta < x < \gamma, \quad x > \delta$ for their values with $\alpha < \beta < 0 < \gamma < \delta$ condoning any non-strict inequality signs and $x \neq 6$ A1: Fully correct range of values in any suitable notation	M1 A1
(4)			
Total 6			

Additional Notes

(a)+(b) Answers only without an algebraic method will score M0, but the roots (product = -36, sum = ±5) are deducible from the simplified (leading coefficient 1) quadratic in this case, so accept correct answers (or allow sign error) following the correct simplified quadratic to imply the M. If an error means a quadratic does not have simple roots, a method must be shown to solve. However, allow answers following an application of the formula even if the simplification would seem to warrant a calculator.

(a)

M1: Alternative to cross multiply is putting over a common denominator of $x-6$ and forming a quadratic in the numerator, or multiplying through by $(x-6)^2$, expanding, gathering and then factoring out the $x-6$ to leave a quadratic. Method must be clear, solutions by calculator are not acceptable.

A1: Both correct values and no spurious extras.

(b) **Note on the grids (b)(ii) will appear as MAA but is being marked as MMA**

B1ft: Correct region following their values as long as they satisfy that one is less than 0 and the other greater than 6 to match the given sketch. Accept set notation or interval notation. Accept with “and” or “or” between but set notation must use union, not intersection.

M1: Must be attempting to solve the correct initial equation. Same notes as the M1 in part (a) apply.

M1: This mark is essentially for demonstrating they understand the shape of the region that is needed. They may draw on the sketch to help. The solutions to their quadratics will have to fit the indicated pattern but if the made sign errors such that they had -9 and 4 in (a) and -4 and 9 in (b) this mark is recoverable (they will have lost the A in (a) and B1ft in (b) already).

A1: Correct answer, allowing the similar recovery as noted above. Allow if scored from calculator solutions to the quadratic equation. (Same criteria.)

Note: Candidates can recover all the marks in (b)(ii) if they start again by squaring both sides, but algebra would have to be complete and non-calculator method.

$$(2x+12)^2 > \left(\frac{10x}{x-6}\right)^2 \Rightarrow 4((x-6)(x+6))^2 > 100x^2 \Rightarrow (x^2-36)^2 - 25x^2 > 0 \Rightarrow x^4 - 72x^2 + 36^2 - 25x^2 > 0$$

$$\Rightarrow x^4 - 97x^2 + 4^2 \times 9^2 > 0 \Rightarrow (x^2-16)(x^2-81) > 0 \Rightarrow \text{CVs } x = \pm 4, \pm 9 \Rightarrow -9 < x, -4 < x < 4, x > 9$$

Answers are not likely to be this concise and will probably not succeed in reaching the answers via non-calculator means.



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
1.	$i(1+\sqrt{3}) = \frac{i(1+\sqrt{3}) + pi}{i^2(1+\sqrt{3}) + 3}$ $-i(1+\sqrt{3})^2 + 3i(1+\sqrt{3}) = i(1+\sqrt{3}) + pi$ $-1 - 2\sqrt{3} - 3 + 3 + 3\sqrt{3} = 1 + \sqrt{3} + p$ $p = -2$	M1 dM1 A1 [3]
M1 dM1 A1	Substitute $i(1+\sqrt{3})$ for w and z Solve to $p = \dots$ Correct value for p Some solve for p first: Obtain an expression for p in terms of w and/or z M1 Substitute $i(1+\sqrt{3})$ for w and z dM1 Correct value for p A1	

Question Number	Scheme	Marks
2		
(a)	$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{(r+2)^2 - r(r+3)}{r(r+1)(r+2)}$ $= \frac{r^2 + 4r + 4 - r^2 - 3r}{r(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)} \quad *$	M1 A1* (2)
(b)	$r=1 \quad \frac{3}{1 \times 2} - \frac{4}{2 \times 3} \qquad r=n-1 \quad \frac{n+1}{(n-1)n} - \frac{n+2}{n(n+1)}$ $r=2 \quad \frac{4}{2 \times 3} - \frac{5}{3 \times 4} \qquad r=n \quad \frac{n+2}{n(n+1)} - \frac{n+3}{(n+1)(n+2)}$ $r=3 \quad \frac{5}{3 \times 4} - \frac{6}{4 \times 5}$ $\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)}$ $\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{3(n+1)(n+2) - 2n - 6}{2(n+1)(n+2)} = \frac{n(3n+7)}{2(n+1)(n+2)}$	M1 A1 dM1 A1cao (4)
(a) M1 A1*	Attempt a single fraction with the correct denominator (or 2 separate fractions with the correct common denominator) Correct result obtained with no errors in the working. Must include LHS as shown in question or LHS = ...	
(b) M1 A1 dM1 A1cao	Show sufficient terms to demonstrate the cancelling, min 3 at start and 1 at end or 2 at start and 2 at end. Award by implication if the correct 2 remaining terms are seen Extract the correct 2 remaining terms Attempt common denominator of the form $k(n+1)(n+2)$ Correct result obtained. No need to show a , b and c explicitly.	

[6]

Question Number	Scheme	Marks
1(a)	Special Case: $\frac{2}{r(r^2-1)} = \frac{2r}{r^2-1} - \frac{2}{r}$ seen, award M1A1A0 Award M1A0A0 provided of the form $\frac{2}{r(r^2-1)} = \frac{Ar}{r^2-1} - \frac{B}{r}$	
1(b)	Terms listed as described above – award M1M1. Further progress unlikely as too many terms needed to establish the cancellation.	



Question Number	Scheme	Marks
2	$w = \frac{z+2}{z-i} \quad z \neq i$ $z = \frac{2+iw}{w-1}$ $ z = 2 \Rightarrow \left \frac{2+iw}{w-1} \right = 2 \Rightarrow 2+iw = 2 w-1 $ $ 2+iu-v = 2 u+iv-1 $ $(2-v)^2 + u^2 = 4((u-1)^2 + v^2)$ $3u^2 + 3v^2 - 8u + 4v = 0 \quad \text{oe}$ $\left(u - \frac{4}{3}\right)^2 + \left(v + \frac{2}{3}\right)^2 = \frac{20}{9} \quad \text{or} \quad u^2 + v^2 - \frac{8}{3}u + \frac{4}{3}v = 0$ (i) centre is $\left(\frac{4}{3}, -\frac{2}{3}\right)$ (ii) radius is $\frac{2\sqrt{5}}{3}$ oe	M1 M1 A1 M1 A1 dM1 A1 A1 [8]
M1 M1 A1 M1 A1 dM1 (i)A1 (ii)A1 ALT 1 M1 M1 A1	Rearrange equation to $z = \dots$ Change w to $u + iv$ and use $z = 2$ Allow if a different pair of letters used. Correct equation Correct use of Pythagoras on either side. Allow with 2 or 4 (RHS) Correct unsimplified equation Attempt the circle form. Coefficients for u^2 and v^2 must be 1. Depends on all 3 previous M marks Correct centre given (no decimals) (Use of rounded decimals changes the values) Correct radius given, any equivalent form (but no decimals) NB: These 2 A marks can only be awarded if the results have been deduced from a correct circle equation. Change w to $u + iv$ Allow a different pair of letters. Rearrange equation to $z = \dots$ and use $z = 2$ Correct equation Then as above.	
ALT 2	Very rare but may be seen: i maps to $\infty \Rightarrow \pm 2i$ map to a diameter of C So $\frac{2i+2}{i}$ and $\frac{-2i+2}{-3i}$ are ends of a diameter Calculate centre and radius	M1A1 M2A1 M1A1A1



Question Number	Scheme	Marks
3(a)	$y = r \sin \theta = \sin \theta + \sin \theta \cos \theta$ OR $r \sin \theta = \sin \theta + \frac{1}{2} \sin 2\theta$ $\frac{dy}{d\theta} = \cos \theta - \sin^2 \theta + \cos^2 \theta$ OR $\frac{dy}{d\theta} = \cos \theta + \cos 2\theta$ $0 = \cos \theta + 2 \cos^2 \theta - 1 = (2 \cos \theta - 1)(\cos \theta + 1)$ $\cos \theta = \frac{1}{2}$ ($\cos \theta = -1$ outside range for θ) $\theta = \frac{\pi}{3}$ A is $\left(1\frac{1}{2}, \frac{\pi}{3}\right)$	B1 M1 M1 A1 (4)
3(b)	$\text{Area} = \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 + \cos \theta)^2 d\theta$ $= \frac{1}{2} \int \left(1 + 2 \cos \theta + \frac{1}{2}(\cos 2\theta + 1)\right) d\theta$ $= \frac{1}{2} \left[\frac{3}{2}\theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta \right]_0^{\frac{\pi}{3}}$ $= \frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16} = \frac{\pi}{4} + \frac{9\sqrt{3}}{16}$	B1 M1A1 dM1A1 A1 (6) [10]
(a) B1 M1 M1 A1	Use of $r \sin \theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen. Differentiate $r \sin \theta$ or $r \cos \theta$ Set $\frac{d(r \sin \theta)}{d\theta} = 0$ and solve the resulting equation. Only the solution used need be shown. Correct coordinates of A	
(b)B1 M1 A1 dM1 A1 A1	Use of $\text{Area} = \frac{1}{2} \int r^2 d\theta$ with $r = 1 + \cos \theta$, limits not needed. Attempt $(1 + \cos \theta)^2$ (minimum accepted is $(1 + k \cos \theta + \cos^2 \theta)$) and change $\cos^2 \theta$ to an expression in $\cos 2\theta$ using $\cos^2 \theta = \frac{1}{2}(\pm \cos 2\theta \pm 1)$ Correct integrand; limits not needed. $\frac{1}{2}$ may be missing. Attempt to integrate all terms. $\cos 2\theta \rightarrow \pm \frac{1}{k} \sin 2\theta$ $k = \pm 1$ or ± 2 Limits not needed. Depends on the previous M mark Correct integration and correct limits seen Substitute correct limits and obtain the correct answer in the required form.	



Question Number	Scheme	Marks
	<i>Alternative for (b) using integration by parts (Very rare but may be seen)</i> $\text{Area} = \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 + \cos \theta)^2 d\theta$ $= \frac{1}{2} \left[\int (1 + 2\cos \theta) d\theta + \int \cos^2 \theta d\theta \right]$ $= \frac{1}{2} \left[\int (1 + 2\cos \theta) d\theta + \cos \theta \sin \theta + \int \sin^2 \theta d\theta \right]$ $= \frac{1}{2} \left[\theta + 2\sin \theta + \sin \theta \cos \theta + \int (1 - \cos^2 \theta) d\theta \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{2} \left[\theta + 2\sin \theta + \frac{1}{2} (\sin \theta \cos \theta + \theta) \right]_0^{\frac{\pi}{3}}$ $= \frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16} = \frac{\pi}{4} + \frac{9\sqrt{3}}{16}$	B1 M1A1 dM1A1 A1
B1 M1 A1 dM1 A1A1	Use of $\text{Area} = \frac{1}{2} \int r^2 d\theta$ with $r = 1 + \cos \theta$, limits not needed. Attempt $(1 + \cos \theta)^2$ (minimum accepted is $(1 + k \cos \theta + \cos^2 \theta)$) and attempt first stage of $\int \cos^2 \theta d\theta$ by parts. Reach $\int \cos^2 \theta d\theta = \cos \theta \sin \theta \pm \int \sin^2 \theta d\theta$ Limits not needed Correct so far. Limits not needed. Attempt to integrate all terms. $\int (1 + 2\cos \theta) d\theta$ and attempt to complete $\int \cos^2 \theta d\theta$ using Pythagoras identity. Limits not needed. Depends on the previous M mark As main scheme	



Question Number	Scheme	Marks
4 (a)	$\frac{d^2 y}{dx^2} = \frac{4}{y} \left(\frac{dy}{dx} \right)^2 - 3$ $\frac{d^3 y}{dx^3} = -\frac{4}{y^2} \left(\frac{dy}{dx} \right)^3 + \frac{8}{y} \times \frac{d^2 y}{dx^2} \times \frac{dy}{dx}$ $\frac{d^3 y}{dx^3} = -\frac{4}{y^2} \left(\frac{dy}{dx} \right)^3 + \frac{8}{y} \left(\frac{4}{y} \left(\frac{dy}{dx} \right)^2 - 3 \right) \left(\frac{dy}{dx} \right)$ $\frac{d^3 y}{dx^3} = \frac{28}{y^2} \left(\frac{dy}{dx} \right)^3 - \frac{24}{y} \left(\frac{dy}{dx} \right) \quad *$	M1 M1A1A1 A1* (5)
ALT	$\frac{dy}{dx} \frac{d^2 y}{dx^2} + y \frac{d^3 y}{dx^3} - 8 \frac{dy}{dx} \times \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} = 0$ $\frac{d^3 y}{dx^3} = \frac{1}{y} \left(7 \frac{dy}{dx} \right) \left(\frac{4}{y} \left(\frac{dy}{dx} \right)^2 - 3 \right) - \frac{3}{y} \frac{dy}{dx}$ $\frac{d^3 y}{dx^3} = \frac{28}{y^2} \left(\frac{dy}{dx} \right)^3 - \frac{24}{y} \left(\frac{dy}{dx} \right) \quad *$	M1A1A1 M1 A1* (5)
4(b)	At $x = 0$ $\frac{d^2 y}{dx^2} = \frac{4}{8}(1)^2 - 3 = -\frac{5}{2}$ oe $\frac{d^3 y}{dx^3} = \frac{28}{64} \times 1^3 - \frac{24}{8} \times 1 = -\frac{41}{16}$ $y = 8 + x - \frac{5}{2} \times \frac{x^2}{2!} - \frac{41}{16} \times \frac{x^3}{3!} + \dots$ $y = 8 + x - \frac{5}{4} x^2 - \frac{41}{96} x^3 + \dots$	B1 M1 M1 A1 (4) [9]



Question Number	Scheme	Marks
5(a)		
M1	Divide through by y No need to re-arrange the equation until later	
M1	Attempt the differentiation using product rule and chain rule and obtain $\frac{d^3y}{dx^3} = \dots$	
A1A1	A1 Either RHS term correct A1 Second RHS term correct and no extras	
A1*	Eliminate $\frac{d^2y}{dx^2}$ and obtain the given result	
ALT		
M1	Re-arrange the equation (Will probably be seen later in work)	
M1	Attempt the differentiation using product rule and chain rule	
A1A1	A1 Two terms correct A1 All correct and no extras	
A1*	Eliminate $\frac{d^2y}{dx^2}$ and obtain the correct result	
5(b)B1	Correct value for $\frac{d^2y}{dx^2}$	
M1	Use the <i>given</i> expression from (a) to obtain a value for $\frac{d^3y}{dx^3}$ Award if correct value seen.	
M1	Taylor's series formed using their values for the derivatives (2! or 2, 3! or 6)	
A1	Correct series, must start (or end) $y = \dots$ Correct terms must be seen, order may be different. Can have $f(x) = \dots$ provided $f(x) = y$ is defined somewhere.	



Question Number	Scheme	Marks
5 NB	Question states "Use algebra..." so purely graphical solutions (using calculator?) score 0/7. A sketch and some algebra to find intersection points can score. $2x^2 + x - 3 \geq 0$ $2x^2 + x - 3 = 3(1-x) \Rightarrow 2x^2 + 4x - 6 = 0$ $2x^2 + 4x - 6 \Rightarrow x^2 + 2x - 3 = (x+3)(x-1) = 0$ $x = -3, 1$ $2x^2 + x - 3 \leq 0$ $-2x^2 - x + 3 = 3(1-x) \Rightarrow 2x^2 - 2x = 0$ $2x(x-1) = 0, x = 0, 1$ $x < -3 \quad 0 < x < 1 \quad x > 1$	M1 A1 M1 A1 dM1A1A1 [7]
M1 A1 M1 A1 dM1 A1 A1	The first 4 marks can be awarded with any inequality sign or = Assume $2x^2 + x - 3 \geq 0$ and obtain a 3TQ Correct CVs obtained from a correct equation. Assume $2x^2 + x - 3 \leq 0$ and obtain a 2 or 3TQ Correct CVs obtained from a correct equation. Form 3 distinct inequalities with their 3 CVs. Can have $<$ or $\leq$, $>$ or $\geq$. Must have scored both previous M marks. Accept $x < -3 \quad 0 < x \quad x \neq 1$ All 3 correct CVs used correctly Inequalities fully correct. "and" between the inequalities is acceptable. If $\cap$ is used, award A0 here. Fully correct set language accepted.	
ALT	<i>Squaring both sides</i> $(2x^2 + x - 3)^2 > 9(1-x)^2$ $4x^4 + 4x^3 - 20x^2 + 12x > 0$ $x(x+3)(x-1)(x-1) > 0$ CVs: $x = 0, -3, 1$ Then as main scheme	M1A1 M1 A1
M1 A1 M1 A1	These 4 marks can be awarded with any inequality sign or = Square both sides and collect terms to obtain a quartic with 4 or 5 terms Correct quartic Factorise their quartic 3 correct CVs	



Question Number	Scheme	Marks
1(a)	$\frac{2}{\sqrt{r} + \sqrt{r-2}} \times \frac{\sqrt{r} - \sqrt{r-2}}{\sqrt{r} - \sqrt{r-2}}$	M1
	$\frac{2(\sqrt{r} - \sqrt{r-2})}{r - (r-2)} = \sqrt{r} - \sqrt{r-2} *$	A1*
		(2)
(b)	$r = 2: \sqrt{2} - \sqrt{2-2} (= \sqrt{2} - 0) \quad \dots r = n-2: \sqrt{n-2} - \sqrt{n-4}$ $r = 3: \sqrt{3} - \sqrt{3-2} (= \sqrt{3} - 1) \quad r = n-1: \sqrt{n-1} - \sqrt{n-3}$ $r = 4: \sqrt{4} - \sqrt{4-2} (= 2 - \sqrt{2}) \quad r = n: \sqrt{n} - \sqrt{n-2}$	M1
	$\left[\sum_{r=2}^n \frac{2}{\sqrt{r} + \sqrt{r-2}} \right] = \sqrt{n} + \sqrt{n-1} - 1$	A1 A1
		(3)
(c)	$\left[\sum_{r=4}^{50} \frac{2}{\sqrt{r} + \sqrt{r-2}} \right] f(50) - f(3) = \sqrt{50} + \sqrt{49} - 1 - (\sqrt{3} + \sqrt{2} - 1)$	M1
	$= 5\sqrt{2} + 7 - 1 - \sqrt{3} - \sqrt{2} + 1 = 7 + 4\sqrt{2} - \sqrt{3}$	A1
		(2)
		Total 7
Notes		
(a) M1: Indicates intention to multiply either side by a correct fraction, may use $\frac{\sqrt{r-2} - \sqrt{r}}{\sqrt{r-2} - \sqrt{r}}$. The “2” may be missing. May work in reverse from right hand side to left, and this is fine. Alternatively, may multiply the initial expression through by $\sqrt{r} + \sqrt{r-2}$ and use a sequence of equivalences (though accept with $\Rightarrow$ or nothing between lines). A1: Fully correct proof. A result from rationalisation that is not the given answer must be seen. If using a sequences of equivalences there must be a minimal conclusion. (b) M1: Correct process of differences evidenced in their work, e.g. attempts any three of the 6 expression shown. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at any of $r = 0$ or 1 if they start earlier than $r = 2$. A1: Correct algebraic terms or correct constant term(s) extracted. Accept unsimplified expressions such as “$-\sqrt{1} - \sqrt{0}$” A1: Fully correct simplified expression. Must have simplified the $\sqrt{1}$ to 1 (c) M1: Attempts $f(50) - f(3)$ using their answer to (b). Must be indication of subtraction. The ‘-1’s may be omitted, and allow if a slip is made. They may subtract the sum of their results from $r = 2$ and $r = 3$ from (b) or by using (a) again to obtain $f(3)$ but their answer to (b) must be used for $f(50)$. Allow from attempts starting at $r = 1$ in their summations. $f(50) - f(4)$ is M0. A1: Correct expression or $A = 7, B = 4, C = -1$ following a correct answer to (b). The decimal answer 10.92480344... without evidence of the M mark is 0/2		

Question	Scheme	Marks
1(a)	$\frac{dy}{dx} - 3y \tan x = \sec^2 x$ with $y = 4$ when $x = \frac{\pi}{4}$	
	Mark parts (a) and (b) together	
	$IF = \exp\left(\int -3 \tan x \, dx\right) = \dots$	M1
	$= \exp(3 \ln \cos x) = \cos^3 x$ *	A1*
		(2)
Alt (a)	$\frac{d}{dx}(y \cos^3 x) = \frac{dy}{dx} \cos^3 x + y \times 3 \cos^2 x \times -\sin x$	M1
	$= \cos^3 x \left(\frac{dy}{dx} - 3y \tan x\right)$ hence integrating factor	A1*
		(2)
(b)	$\Rightarrow y \cos^3 x = \int \sec^2 x \times \cos^3 x \, dx$	M1
	$\Rightarrow y \cos^3 x = \sin x (+c)$ o.e. OR $y = \sin x \sec^3 x + (c \sec^3 x)$ OR $y = \tan x \sec^2 x + (c \sec^3 x)$ o.e.	A1
	$\Rightarrow 4 \cos^3 \frac{\pi}{4} = \sin \frac{\pi}{4} + c \Rightarrow c = \frac{\sqrt{2}}{2} \Rightarrow y = \tan x \sec^2 x + \frac{\sqrt{2}}{2} \sec^3 x$ o.e.	M1 A1
		(4)
		(6 marks)
Notes:		
(a)		
M1: Attempts the integrating factor directly. Allow for an attempt at $\exp\left(\pm \int k \tan x\right)$ but no need to evaluate the integral for this mark. Must be shown, but could be implied by an integrated expression of the correct form e.g. $\exp(-3 \ln \sec x)$ or $\exp(3 \ln \cos x)$. If $(\sec x)^{-3}$ is all that is seen then accept for M1 only A1*: Correct simplified integrating factor. Correct processing seen, so exp and ln inverse applied correctly, and correct integration of $-3 \tan x \rightarrow -3 \ln \sec x$ or $3 \ln \cos x$ seen. No sight of exp and ln being correctly processed, or no evidence of correct integration is max M1A0		

Alt by verification

M1: Attempts to differentiate $y \cos^3 x \rightarrow \cos^3 x \frac{dy}{dx} \pm ky \sin x \cos^2 x$

A1*: Shows correctly that the derivative is $\cos^3 x$ times equation (I) **and makes minimal conclusion.**

(b)

M1: Applies the integrating factor. Look for $y \times \text{their IF} = \int \sec^2 x \times \text{their IF} \, dx$ or equivalent

A1: Correct general form. The constant may be missing for this mark.

M1: Applies the initial condition to find a value or c and treats it correctly (i.e. having the correct position, and not just appearing in isolation). Incorrect treatment of c is M0 (e.g. $y = \tan x \sec^2 x + c \rightarrow c = \dots$ is M0)

A1: Correct answer. Accept alternatives in terms of $\cos x$ etc that are in the form $y = \dots$ so e.g.

$$y = \frac{\sin x}{\cos^3 x} + \frac{\sqrt{2}}{2\cos^3 x}$$

etc

Note: c could be given as $\frac{1}{\sqrt{2}}$. ISW once a correct answer is seen.



Question	Scheme	Marks
2(a)	$\frac{2x}{x^2+1} = \frac{1}{x+4} \Rightarrow 2x(x+4) = x^2+1$ OR $\frac{2x}{x^2+1} - \frac{1}{x+4} = 0 \Rightarrow \frac{2x(x+4) - (x^2+1)}{(x^2+1)(x+4)} = 0$	M1
	$(\Rightarrow x^2 + 8x - 1 = 0) \Rightarrow x = \frac{-8 \pm \sqrt{64 - 4 \times 1 \times -1}}{2} = \dots$	dM1
	$x = -4 \pm \sqrt{17}$	A1
		(3)
(b)	$x < -4 - \sqrt{17} \text{ or } -4 < x < -4 + \sqrt{17}$	M1
	Both $x < -4 - \sqrt{17}, -4 < x < -4 + \sqrt{17}$	A1
		(2)
(c)	$x < -4 + \sqrt{17}, x \neq -4$ OR $x < -4, -4 < x < -4 + \sqrt{17}$ OR $x \in (-\infty, -4) \cup (-4, -4 + \sqrt{17})$	M1 A1
		(2)
		(7 marks)

Notes:**(a)**

M1: Sets equal and cross multiplies to reach an unsimplified quadratic equation, OR takes terms to one side, combines together with common denominator to reach an unsimplified quadratic on their numerator.

dM1: Solves the quadratic. Usual rules. May use a calculator. **Dependent on the previous M Mark.**

Note: If the answer given is incorrect, and no QF is quoted then M0. But if the correct formula is quoted and there are errors in the substitution of values or incorrect answer given after formula quoted then the M mark can still be scored.

A1: Correct roots. Must be exact.

(b)

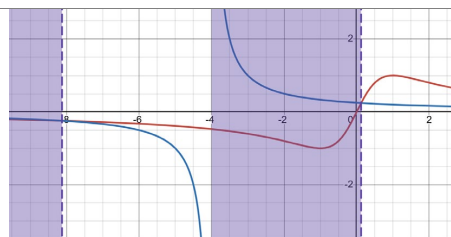
NOTE: decimal answers can score M marks only in parts (b) and (c)

M1: For $x < \alpha$ OR $-4 < x < \beta$ for their roots α, β , where $\alpha < \beta$ and their $\alpha < -4$, their $\beta > -4$ (their roots need to be on opposite sides of the asymptote)

Selects a solution set from one of those above with their critical values (provided their CVs are within the parameters allowed). Allow if any non-strict inequalities are used for this mark.

A1: Correct answer. **Must be exact.**





Useful graph of solution

(c)

M1: Deduces partially correct range by including all the values less than -4 in their answer (with possible exception of their negative root α). May also include the -4 and boundary value(s) for this mark. **i.e. allow boundary values to be included/excluded as long as the reflected part of the reciprocal graph is included in the solution.**

e.g. $x \leq -4$ **M1A0** (values less than -4 included, boundary allowed, but incomplete solution)

$x < -4$ **M1A0** (values less than -4 included, but incomplete solution)

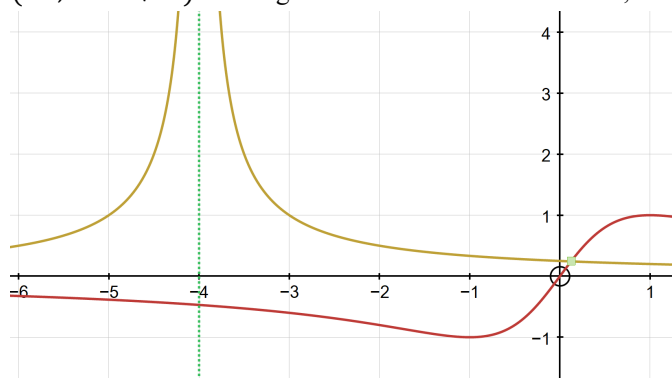
$-4 - \sqrt{17} < x < -4$ and $x < -4 - \sqrt{17}$ **M1A0** (values less than -4 included, excluding the negative root)

$-4 - \sqrt{17} < x < -4$ **M0A0** (values less than -4 not included in the solution set)

$x \leq -4 + \sqrt{17}$, $x \neq -4$ **M1A0** (values less than -4 included, but includes end point)

$x \leq -4 + \sqrt{17}$ **M1A0** (values less than -4 included, but incomplete solution)

A1: Deduces fully correct range. Allow equivalent notations such as $x \in (-\infty, -4) \cup (-4, -4 + \sqrt{17})$ with e.g. 'or/and' between the brackets, but not $\cap$. **Values must be exact.**



Useful graph of solution

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$\frac{d^3y}{dx^3} + 3\frac{dy}{dx} + 3x\frac{d^2y}{dx^2} = -2\sin x$ $\frac{d^3y}{dx^3} = -2\sin x - 3\frac{dy}{dx} - 3x\frac{d^2y}{dx^2}$	M1M1
(b)	$\frac{d^3y}{dx^3} = -3 \times 5 = -15$	A1 (3)
(c)	$\frac{d^2y}{dx^2} = -3 \times 0 \times 5 + 2 = 2$ $y = 2 + 5x + x^2 - \frac{5}{2}x^3$	B1
		M1A1 (3)
		[7]
(a)	Accept the dashed notation throughout this question.	
M1	Differentiate $3x\frac{dy}{dx}$ with respect to x . The product rule must be used for $x\frac{dy}{dx}$ with at least one term correct	
M1	Differentiate $\frac{d^2y}{dx^2}$ and $2\cos x$. $\frac{d^2y}{dx^2} \rightarrow \frac{d^3y}{dx^3}$ $2\cos x \rightarrow \pm 2\sin x$	
A1	$\frac{d^3y}{dx^3} = -3\left(x\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) - 2\sin x$. Give A0 if not rearranged to have $\frac{d^3y}{dx^3} = \dots$	
(b)		
B1	$\frac{d^3y}{dx^3} = -15$ provided 3 terms in result in (a)	
(c)		
B1	$\frac{d^2y}{dx^2} = 2$ can be implied by a correct x^2 term in the expansion	
M1	Use of a correct Taylor expansion with their values for $\frac{d^3y}{dx^3}$ and $\frac{d^2y}{dx^2}$ $2!$ or 2 , $3!$ or 6 .	
A1	$y = 2 + 5x + x^2 - \frac{5}{2}x^3$ Must include $y = \dots$ or $f(x) = \dots$ provided $f(x)$ has been defined to be y somewhere in the work.	

Question Number	Scheme	Notes	Marks
7	$r = \sqrt{3} \sin \theta \quad (0 \leq \theta \leq \pi) \quad r = 3 \cos \theta \quad \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$		
(a)	$\sqrt{3} \sin \theta = 3 \cos \theta \Rightarrow \tan \theta = \sqrt{3}$ $\Rightarrow \theta = \frac{\pi}{3}, \quad r = \frac{3}{2} \quad \text{or} \quad \left(\frac{3}{2}, \frac{\pi}{3}\right)$	B1: Either correct coordinate B1: Both correct coordinates and no others	B1 B1
(2)			
(b)	$\int (\sqrt{3} \sin \theta)^2 d\theta = K \int \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta d\theta = \dots$ or $\int (3 \cos \theta)^2 d\theta = M \int \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta d\theta = \dots$	Uses one of $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ in an attempt to integrate r^2 for one of the equations.	M1
	$\int (\sqrt{3} \sin \theta)^2 d\theta = K \int \frac{1}{2} - \frac{1}{2} \cos 2\theta d\theta = \dots$ and $\int (3 \cos \theta)^2 d\theta = M \int \frac{1}{2} + \frac{1}{2} \cos 2\theta d\theta = \dots$	Uses both $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ and $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ in an attempt to integrate r^2 for both of the equations to correct form.	M1
	$\int (\sqrt{3} \sin \theta)^2 d\theta = \frac{3}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right)$ and $\int (3 \cos \theta)^2 d\theta = \frac{9}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right)$	Fully correct expression after integration ignoring limits (accept equivalent expressions)	A1
	$\frac{1}{2} \int_0^{\frac{\pi}{3}} (\sqrt{3} \sin \theta)^2 d\theta = \frac{3}{4} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{3}} = \dots$ or $\frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3 \cos \theta)^2 d\theta = \frac{9}{4} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \dots$	Full process for at least one of the two areas required, including correct area formula, with their attempt to apply the double angle formula, and evidence of correct limits applied.	M1
	Area of $R = \frac{1}{2} \int_0^{\frac{\pi}{3}} 3 \sin^2 \theta d\theta + \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9 \cos^2 \theta d\theta$ $= \frac{3}{4} \left[\frac{\pi}{3} - \frac{1}{2} \sin \frac{2\pi}{3} - (0 - 0) \right] + \frac{9}{4} \left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{3} + \frac{1}{2} \sin \frac{2\pi}{3} \right) \right]$	Full and correct process for the area of R . Formulae and limits must all be correct, and evidence of limits applied, with areas added.	DM1
	$\frac{\pi}{4} - \frac{3\sqrt{3}}{16} + \frac{3\pi}{8} - \frac{9\sqrt{3}}{16} = \frac{5\pi}{8} - \frac{3\sqrt{3}}{4}$	Correct answer. Allow $a = \frac{5}{8}, \quad b = -\frac{3}{4}$	A1
(6)			
Total 8			

Additional Notes

(a)

B1: One correct coordinate, do not be concerned about the method for this mark. Allow for $\theta = \frac{\pi}{3}$ seenand isw if they go on to use $\theta = \frac{\pi}{3}$ for this mark.

B1: Both coordinates correct, and no extras. Again, do not be concerned about the method. Condone if given in the wrong order in a coordinate pair.

(b)



M1: Starts correct process to find one of the two areas. Must be using r^2 for one of the two curves, attempt a double angle formula $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ and any attempt to integrate the result (no matter how poor, ie a changed function). Condone errors with the coefficient not being squared.

M1: A full process to find the integral of r^2 for **both** curves with **correct** double angle formulae used, so $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ and $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ and reaches correct for each, but again allow if slips occur in the coefficient.

A1: Fully correct integration for r^2 for **both** curves. The coefficients must have been correctly dealt with.

Note the $\frac{1}{2}$ may be included throughout.

M1: For a full correct process to find at least one of the areas. Must be using $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$ including the $\frac{1}{2}$

and with **correct** limits for the area applied (condone not showing subtracted 0's), but condone slips in the coefficients when squaring and condone if incorrect double angle formula attempts were made.

Note: Some candidates may attempt to find only one area and double the result. If the area formula is clearly seen or applied before doubling this is acceptable as an attempt for the full process for one area.

DM1: Depends on all previous method marks. Fully correct process for **both** areas and adds the results. This may be done separately or in one expression throughout the working.

A1: Correct answer.

Question Number	Scheme	Notes	Marks
8(a)	For intersection when $x > 0$: $\frac{15x}{x+4} = x-2$ $15x = x^2 + 2x - 8$ $x^2 - 13x - 8 = 0$ $\Rightarrow x = \frac{13 + \sqrt{201}}{2}$	M1: Solves 3TQ obtained from a correct equation (usual rules) for $x > 0$ A1: Correct exact positive value (condone negative root also included)	M1 A1
	For intersections when $x < 0$: $\frac{15x}{-x+4} = x-2$ $15x = -x^2 + 6x - 8$ $x^2 + 9x + 8 = 0$ $(x+8)(x+1) = 0$ $\Rightarrow x = -8, -1$	M1: Solves 3TQ obtained from a correct equation (usual rules) for $x < 0$ A1: Correct values	M1 A1
	$-8 < x < -1, \quad x > \frac{13 + \sqrt{201}}{2}$	M1: For $a < x < b, \quad x > c$ oe with cvs $a < b < 0 < c$ from the correct equations. A1: Fully correct set of values in any form	M1 A1
(6)			
(b)	$\frac{15x}{x+4} = -x+2$ $15x = -x^2 - 2x + 8$ $x^2 + 17x - 8 = 0$ $\Rightarrow x = \frac{-17 + \sqrt{321}}{2}$	M1: Solves 3TQ obtained from a correct equation (usual rules) A1: Correct exact positive value (condone negative root also included)	M1 A1
	$x < "-8", \quad x > "\frac{13 + \sqrt{201}}{2}"$	Correct regions ft their $a < 0$ and $c > 0$	B1ft
	$x < "-8", \quad "-1" < x < \frac{-17 + \sqrt{321}}{2}, \quad x > "\frac{13 + \sqrt{201}}{2}"$	Correct set of values in any form. Allow ft on their cvs a, b and c provided $a < b < 0$ and $c > \frac{-17 + \sqrt{321}}{2} (= 0.458...)$	A1ft
(4)			
Total 10			

Additional notes.

“Usual Rules” : as well as the general information, we will accept here solutions by calculator having reached a quadratic equation but they must be correct solutions for the stated quadratic if no method is shown (you may need to check).

(a)

Accept with any inequalities used instead of equalities for the solving of the intersection points. For methods via putting over common denominators score the Ms for the process of putting over a common denominator and solving for the numerator = 0.

May see attempts via squaring

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
1(a)	$r = \sqrt{(-4)^2 + (-4\sqrt{3})^2} = \dots$	M1
	$\tan \theta = \frac{-4\sqrt{3}}{-4} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) \pm \pi$	M1
	$8 \left(\cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right) \right)$	A1
		(3)
(b)	$z = re^{i\theta} \Rightarrow (re^{i\theta})^3 = -4 - 4\sqrt{3} \Rightarrow r^3 (e^{3i\theta}) = 8e^{-i\frac{2\pi}{3}}$	
	$\Rightarrow r = \sqrt[3]{8} = 2$	M1
	$3\theta = -\frac{2\pi}{3} (+2k\pi) \Rightarrow \theta = -\frac{2\pi}{9} + \left(\frac{2k\pi}{3}\right)$	M1
	So $z = 2e^{-\frac{8\pi}{9}i}, 2e^{-\frac{2\pi}{9}i}, 2e^{\frac{4\pi}{9}i}$	A1ft A1
		(4)
(7 marks)		

Notes:**(a)**

M1: For a correct attempt at the modulus, implied by a correct modulus if no method seen and allow recovery if correct answer follows a minor slip in notation.

M1: For an attempt to find a value of θ in the correct quadrant. Accept $\tan^{-1}(\sqrt{3}) \pm \pi$ or $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \pm \pi$

May be implied by sight of an of $-\frac{2}{3}\pi, \frac{4}{3}\pi, -\frac{5}{6}\pi, \frac{7}{6}\pi$.

A1: cao as in scheme, no other solution.

(b)

M1: Applies De Moivre's Theorem and proceeds to find a value for r ie (their 8) ^{$\frac{1}{3}$}

M1: Proceeds to find at least one value for θ – ie their argument/3.

A1ft: At least two roots correct for their r and θ . (Must come from correct method, watch for correct roots coming from an incorrect angle due to errors.)

A1: All three correct roots and no others. Accept e.g $2e^{i\frac{8\pi}{9}}$ as a slip in notation, so allow marks.

