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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Series: the Method of Differences (FP2.2)

International A Level Mathematics · FP2: Further Pure Mathematics 2 (WFM02)

13 questions · 103 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

13 questions · 103 marks · 1 sub-topic

SUB-TOPIC FP2.2a

METHOD OF DIFFERENCES

Sub-topic FP2.2a · 13 questions · 103 marks · lowest tariff first



Question 1 continued

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Question 1 continued

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Question 1 continued

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(Total for Question 1 is 5 marks)



2. (a) Show that, for $r > 0$

$$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)} \quad (2)$$

(b) Hence show that

$$\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{n(an+b)}{c(n+1)(n+2)}$$

where a, b and c are integers to be determined.

(4)

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Q2



Question 2 continued

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Question 2 continued

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(Total for Question 2 is 6 marks)



Question 1 continued

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Question 1 continued

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Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 7 marks)



Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total for Question 3 is 7 marks)



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Q1

Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total for Question 3 is 8 marks)



Question 5 continued

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Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

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(Total for Question 5 is 8 marks)



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Q2

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$$n^5 - (n-1)^5 \equiv 5n^4 - 10n^3 + 10n^2 - 5n + 1 \quad (2)$$

(b) Hence, using the method of differences, show that for all integer values of n ,

$$\sum_{r=1}^n r^4 = \frac{1}{30} n(n+1)(2n+1)(an^2 + bn + c)$$

where a , b and c are integers to be determined. (7)

Question 9 continued

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Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

Q9

Q9 mark box





Question 4 continued

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Question 4 continued

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(Total for Question 4 is 9 marks)



Question 5 continued

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Question 5 continued

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(Total for Question 3 is 10 marks)



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Question 6 continued

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(Total 11 marks)

Q6

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MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Question Number	Scheme	Notes	Marks
2(a)	$\frac{1}{(2n-1)(2n+1)(2n+3)} \equiv \frac{A}{2n-1} + \frac{B}{2n+1} + \frac{C}{2n+3}$ $\Rightarrow A = \dots, B = \dots, C = \dots$ Correct partial fraction attempt to obtain values for A, B and C		M1
	$\frac{1}{8(2n-1)} - \frac{1}{4(2n+1)} + \frac{1}{8(2n+3)} \text{ or e.g. } \frac{1}{16n-8} - \frac{1}{8n+4} + \frac{1}{16n+24}$ $\text{or e.g. } \frac{\frac{1}{8}}{(2n-1)} - \frac{\frac{1}{4}}{(2n+1)} + \frac{\frac{1}{8}}{(2n+3)}$ Correct partial fractions. (May be seen in (b)) This mark is not for the correct values of A, B and C, it is for the correct fractions.		A1
			(2)
(b)	$\frac{1}{8} \sum_{r=1}^n \left(\frac{1}{2r-1} - \frac{2}{2r+1} + \frac{1}{2r+3} \right) =$ $\frac{1}{8} \left(\frac{1}{1} - \frac{2}{3} + \frac{1}{5} \right.$ $+ \frac{1}{3} - \frac{2}{5} + \frac{1}{7}$ $+ \frac{1}{5} - \frac{2}{7} + \frac{1}{9}$ $\dots$ $+ \frac{1}{2n-3} - \frac{2}{2n-1} + \frac{1}{2n+1}$ $\left. + \frac{1}{2n-1} - \frac{2}{2n+1} + \frac{1}{2n+3} \right)$ Uses the method of differences to find sufficient terms to establish cancelling. E.g. 3 rows at the start and 2 rows at the end or vice versa This may be implied if they extract the correct non-cancelling terms.		M1
	$= \frac{1}{8} \left(1 - \frac{1}{3} - \frac{1}{2n+1} + \frac{1}{2n+3} \right) \text{ or e.g. } = \frac{1}{8} \left(1 - \frac{2}{3} + \frac{1}{3} + \frac{1}{2n+1} - \frac{2}{2n+1} + \frac{1}{2n+3} \right)$ Identifies the correct non-cancelling terms. May be unsimplified.		A1
	$= \frac{1}{8} \left(\frac{2(2n+1)(2n+3) - 3(2n+3) + 3(2n+1)}{3(2n+1)(2n+3)} \right) = \dots$ Attempts to combine terms into one fraction. There must have been at least one constant term and at least 2 different algebraic terms with at least 3 terms in the numerator when combining the fractions.		dM1

	$= \frac{n(n+2)}{3(2n+1)(2n+3)}$	Cao	A1
			(4)
			Total 6



Question Number	Scheme	Marks
1(a)	$\frac{2}{\sqrt{r} + \sqrt{r-2}} \times \frac{\sqrt{r} - \sqrt{r-2}}{\sqrt{r} - \sqrt{r-2}}$	M1
	$\frac{2(\sqrt{r} - \sqrt{r-2})}{r - (r-2)} = \sqrt{r} - \sqrt{r-2} *$	A1*
		(2)
(b)	$r = 2: \sqrt{2} - \sqrt{2-2} (= \sqrt{2} - 0) \quad \dots r = n-2: \sqrt{n-2} - \sqrt{n-4}$ $r = 3: \sqrt{3} - \sqrt{3-2} (= \sqrt{3} - 1) \quad r = n-1: \sqrt{n-1} - \sqrt{n-3}$ $r = 4: \sqrt{4} - \sqrt{4-2} (= 2 - \sqrt{2}) \quad r = n: \sqrt{n} - \sqrt{n-2}$	M1
	$\left[\sum_{r=2}^n \frac{2}{\sqrt{r} + \sqrt{r-2}} \right] = \sqrt{n} + \sqrt{n-1} - 1$	A1 A1
		(3)
(c)	$\left[\sum_{r=4}^{50} \frac{2}{\sqrt{r} + \sqrt{r-2}} \right] f(50) - f(3) = \sqrt{50} + \sqrt{49} - 1 - (\sqrt{3} + \sqrt{2} - 1)$	M1
	$= 5\sqrt{2} + 7 - 1 - \sqrt{3} - \sqrt{2} + 1 = 7 + 4\sqrt{2} - \sqrt{3}$	A1
		(2)
		Total 7
Notes		
(a) M1: Indicates intention to multiply either side by a correct fraction, may use $\frac{\sqrt{r-2} - \sqrt{r}}{\sqrt{r-2} - \sqrt{r}}$. The “2” may be missing. May work in reverse from right hand side to left, and this is fine. Alternatively, may multiply the initial expression through by $\sqrt{r} + \sqrt{r-2}$ and use a sequence of equivalences (though accept with $\Rightarrow$ or nothing between lines). A1: Fully correct proof. A result from rationalisation that is not the given answer must be seen. If using a sequences of equivalences there must be a minimal conclusion. (b) M1: Correct process of differences evidenced in their work, e.g. attempts any three of the 6 expression shown. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at any of $r = 0$ or 1 if they start earlier than $r = 2$. A1: Correct algebraic terms or correct constant term(s) extracted. Accept unsimplified expressions such as “$-\sqrt{1} - \sqrt{0}$” A1: Fully correct simplified expression. Must have simplified the $\sqrt{1}$ to 1 (c) M1: Attempts $f(50) - f(3)$ using their answer to (b). Must be indication of subtraction. The ‘-1’s may be omitted, and allow if a slip is made. They may subtract the sum of their results from $r = 2$ and $r = 3$ from (b) or by using (a) again to obtain $f(3)$ but their answer to (b) must be used for $f(50)$. Allow from attempts starting at $r = 1$ in their summations. $f(50) - f(4)$ is M0. A1: Correct expression or $A = 7, B = 4, C = -1$ following a correct answer to (b). The decimal answer 10.92480344... without evidence of the M mark is 0/2		

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, score for their best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$, otherwise must divide by a first – allow equivalent work if a is a square number)

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$



January 2024
WFM02 Further Pure Mathematics F2 Mark Scheme

Question Number	Scheme	Notes	Marks
1	$\frac{1}{x+2} > 2x+3$		
	Examples: $\frac{1-(x+2)(2x+3)}{x+2} > 0 \Rightarrow 2x^2+7x+5=0$ $x+2 > (2x+3)(x+2)^2$ $\Rightarrow (x+2)(2x^2+7x+5)=0$ or $2x^3+11x^2+19x+10=0$ $\frac{1}{x+2}=2x+3 \Rightarrow (2x+3)(x+2)-1=2x^2+7x+5=0$ Uses algebra to obtain a 3TQ, $(x+2)$ multiplied by a 3TQ or a 4TC. Allow slips and condone incorrect inequality signs but the first algebraic step should be otherwise appropriate so do not accept work with e.g., $(2x+3)(x+2)=0$. The “= 0” can be implied by solutions. Graphical attempts require intersections to be found algebraically. Squaring first is acceptable so allow M1 for obtaining a 5TQ $(4x^4+28x^3+73x^2+84x+35=0)$		M1
	e.g., $(2x+5)(x+1)=0 \Rightarrow$ $x=-\frac{5}{2}, -1$	Both -1 and $-\frac{5}{2}$ from appropriate work and no extra incorrect cvs. May only be seen in the solution set. Allow solving a 3TQ etc. by calculator.	A1
	$x=-2$	Identifies -2 as a critical value. May only be seen in solution set. This is the only mark available if there is no algebraic manipulation seen. Allow from any or no working e.g., from $(2x+3)(x+2)=0$	B1
	$\Rightarrow x < -\frac{5}{2}, -2 < x < -1$ or e.g., $(-\infty, -2.5), (-2, -1)$ M1: For the regions $x < a, -2 < x < b$ with real cvs $a < -2$ and $b > -2$ but condone $b < x < -2$ as a notational slip for this mark. Condone any non-strict inequality signs and poor notation for this mark. Not dependent but must follow an attempt at algebraic manipulation. A1: Correct solution set in any form. Do not isw if the correct inequalities are subsequently incorrectly amended. Allow all marks even if an incorrect inequality sign was seen earlier in the working.		M1 A1
	Examples: $-\frac{5}{2} > x$ or $-2 < x < -1$ M1 A1 $x < -\frac{5}{2}$ and $-2 < x < -1$ M1 A1 (Accept any word between the two correct regions) $x < -\frac{5}{2}, -1 < x < -2$ M1 A0 (notational slip) $(-\infty, -\frac{5}{2}) \cap (-2, -1)$ M1A0 (incorrect symbol – allow “and”) $[-\infty, -\frac{5}{2}] \cup [-2, -1]$ M1A0 $x < -\frac{5}{2} \quad -2 < x \quad x < -1$ M0 A0 (insufficient)		
			(5)
			Total 5

Question Number	Scheme	Notes	Marks
2(a)	(i) $z = 6 - 6\sqrt{3}i \Rightarrow z = \sqrt{6^2 + (6\sqrt{3})^2} = 12$	+12 only. Accept if just stated	B1
	(ii) e.g., $\arg z = -\arctan \frac{6\sqrt{3}}{6}$ Attempts an expression for a relevant angle. Look for $\pm \arctan \left(\pm \frac{6\sqrt{3}}{6} \right)$ or e.g., $\pm \tan^{-1} \left(\pm \frac{1}{\sqrt{3}} \right)$ If arctan is not seen allow e.g., $\tan \alpha = \frac{6\sqrt{3}}{6} \Rightarrow \alpha = \frac{\pi}{3}$ with α correct for their $\tan \alpha$ If using sin or cos the hypotenuse must be their 12		M1
	$\arg z$ or $\arg$ or $\text{argument (of } z) = -\frac{\pi}{3}$ * A correct proof with no incorrect work/statements. LHS required. Allow " $\theta =$ " if consistent , e.g., $\theta = -\frac{\pi}{3}$ cannot follow " $\tan \theta = +\sqrt{3}$ "		A1*
(ii) Way 2	$z = 12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 12\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ or $12e^{-\frac{\pi}{3}i}$ or $\cos \theta = \frac{1}{2}$ or $\sin \theta = -\frac{\sqrt{3}}{2}$ [M1] $\Rightarrow \arg z = -\frac{\pi}{3}$ [A1*] M1: Factorises out 12 and writes in trig or exp form or identifies $\cos \theta = \frac{1}{2}$ and $\sin \theta = -\frac{\sqrt{3}}{2}$ A1: Acceptable statement with all work correct		
(ii) Way 3	$z = 12\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ or $12e^{-\frac{\pi}{3}i}$ or $12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 6 - 6\sqrt{3}i$ [M1] $\Rightarrow \arg z = -\frac{\pi}{3}$ [A1*] M1: Assumes result, writes correctly for their 12 and attempts $a + ib$ form A1: Obtains $6 - 6\sqrt{3}i$ and makes acceptable statement with all work correct		
			(3)
(b)	$z = "12" \left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right) \right)$ or $"12" e^{-\frac{\pi}{3}i}$ [no missing "i" unless recovered] Correct trig or exp. form with their 12. Could be implied by their z^4 in trig or exp. form e.g., $("12" e^{-\frac{\pi}{3}i})^4$ Allow equivalent values of θ e.g. $\frac{5\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$. Condone poor bracketing. Allow this mark if $+2k\pi, -2k\pi, \pm 2k\pi$ appears with argument		M1
	$z^4 = 20736 \left(\cos\left(-\frac{4\pi}{3}\right) + i\sin\left(-\frac{4\pi}{3}\right) \right)$ or $20736 \left(\cos -\frac{4\pi}{3} + i\sin -\frac{4\pi}{3} \right)$ or $20736 e^{-\frac{4\pi}{3}i}$ Correct z^4 in any form. 12^4 evaluated and arg. of $-\frac{4\pi}{3}$ (not just $4 \times -\frac{\pi}{3}$) or $\frac{2\pi}{3}$ only although may use e.g., $\sin\left(-\frac{4\pi}{3}\right) = -\sin\left(\frac{4\pi}{3}\right)$. No "k"s. Condone an "unclosed" bracket. Only accept $-10368 + 10368\sqrt{3}i$ or $20736\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ provided evidence of de Moivre.		A1
			(2)



General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

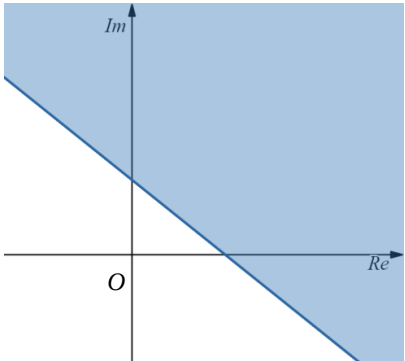
Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Question Number	Scheme	Marks
1(a)	$ z - 3 - 4i = z + 1 + i \Rightarrow x + iy - 3 - 4i = x + iy + 1 + i $ $(x - 3)^2 + (y - 4)^2 = (x + 1)^2 + (y + 1)^2$	M1
	$x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 + 2x + 1 + y^2 + 2y + 1 \Rightarrow ax + by + c = 0$	dM1
	$8x + 10y - 23 = 0$	A1
		(3)
(a) ALT	Mid-point $\left(\frac{3-1}{2}, \frac{4-1}{2}\right)$ and $m = \frac{4+1}{3+1}$	M1
	$y - \frac{3}{2} = -\frac{4}{5}(x - 1) \Rightarrow ax + by + c = 0$	dM1
	$8x + 10y - 23 = 0$	A1
(b)		B1
		(1)
		Total 4

Notes:

(a)

M1: Introduces $z = x + iy$ (or with other variables) and attempts a correct modulus method – the i's should be dealt with correctly but allow for e.g. minor slips in notation with them before applying Pythagoras. Allow sign slips in the i coordinates, e.g.

$(x - 3)^2 + (y \pm 4)^2 = (x + 1)^2 + (y \pm 1)^2$ Must be + between the brackets.

dM1: Expands and attempts the required form.

A1: This equation or any integer multiple.

Alternative: via perpendicular bisector

M1: Uses the points $(3, \pm 4)$ and $(-1, \pm 1)$ to find the midpoint and gradient between them.

dM1: Correct attempt at the equation of the perpendicular bisector and rearranges to the required form. Must be using negative reciprocal of the gradient between points (but watch for those who go directly to this).

A1: This equation or any integer multiple.

(b) May be scored independently of (a) – using the perpendicular bisector.

B1: Shades the area above an appropriate line, must be in quadrants 1, 2 and 4. Accept with solid, dashed or dotted line. Allow for any line with negative gradient and positive y intercept. Do not be concerned about labelling. Part (a) need not have been correct.



Question Number	Scheme	Notes	Marks
5(a)	$1 + \frac{2}{2r+5} = \frac{2r+5+2}{2r+5} = \frac{2r+7}{2r+5}$	Obtains $\frac{2r+7}{2r+5}$	B1
(1)			
(b)	$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right) = \sum_{r=1}^n \log_3 \left(\frac{2r+7}{2r+5} \right) = \sum_{r=1}^n (\log_3 (2r+7) - \log_3 (2r+5))$ $\begin{array}{l} r=1 \quad \log_3 9 (=2) \quad - \quad \log_3 7 \\ r=2 \quad \log_3 11 \quad - \quad \log_3 9 (=2) \\ \dots \\ r=n \quad \log_3 (2n+7) \quad - \quad \log_3 (2n+5) \end{array}$		M1
	$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right) = \log_3 (2n+7) - \log_3 7 = \log_3 \left(\frac{2n+7}{7} \right)$	Correct expression in correct form	M1 A1
(3)			
Alt (b)	$\sum_{r=1}^n \log_3 \left(\frac{2r+7}{2r+5} \right) = \log_3 \frac{9}{7} + \log_3 \frac{11}{9} + \dots + \log_3 \frac{2n+7}{2n+5}$ $= \log_3 \left(\frac{\cancel{9}}{7} \times \frac{11}{\cancel{9}} \times \dots \times \frac{2n+7}{\cancel{2n+5}} \right)$	Combines sum of log's to show cancelling fractions or implied cancelling.	M1
	$= \log_3 \left(\frac{2n+7}{7} \right)$	Cancels their fractions to achieve correct form, A1 for correct answer.	M1 A1
(3)			
(c)	$\sum_{r=n+2}^{10n} \log_3 \left(1 + \frac{2}{2r+5} \right) = \log_3 \left(\frac{2(10n)+7}{7} \right) - \log_3 \left(\frac{2(n+1)+7}{7} \right)$	Forms $g(10n) - g(n+a)$ where $a = 1, 2$	M1
	$\Rightarrow \frac{20n+7}{2n+9} = 3^2$	Correct removal of logs from their equation, from an attempt at difference of sums	M1
	$\begin{array}{l} 20n+7=18n+81 \\ 2n=74 \\ n=37 \end{array}$	dM1: Solves an equation of the correct form. Requires previous method mark A1: $n = 37$ only	dM1 A1
(4)			
Total 8			

Additional Notes

(a)

B1: Correct answer.

(b)

M1: For a valid attempt at the method of differences evidenced by at least two rows to show one cancelling pair of terms, or by correct extraction for the non-cancelling terms with clear indication of the cancelling (e.g. by extraction of the terms if no crossing out seen).

M1: Extracts their non-cancelling terms for their summation and combines the terms to a single log term.

A1: Correct expression in form required.

Note M0M1A1 is possible if the correct answers is attained by implication with the method of differences not clearly being shown.

Alt Method:



M1: Combines the log terms in the summations to a single log expression with cancelling fractions shown.

M1: Cancels the common terms in their expression to simplify to the required form.

A1: Correct expression in form required.

Note the combination of logs is not always being clearly shown in this method. Answers such as

$$r = 1 \quad \log_3 \left(\frac{\cancel{9}}{7} \right)$$

$$r = 2 \quad \log_3 \left(\frac{\cancel{18}}{\cancel{9}} \right)$$

⋮

$$r = n \quad \log_3 \left(\frac{2n+7}{\cancel{2n+5}} \right) \Rightarrow \log_3 \left(\frac{2n+7}{7} \right)$$

Can score M0M1A1

(c)

M1: Correct attempt at finding the sum from $n + 2$ to $10n$. Accept $g(10n) - g(n + a)$ where $a = 1$ or 2 for a valid attempt at the method, with their answer to (b), where $g(n)$ is the answer to (b).

Allow for answers which “use” part (b) to deduce the new first and last terms/new non-cancelling terms and proceed to form the correct expression.

M1: Correct method to eliminate logs and achieve an equation in n having made some attempt at a difference in summations to evaluate the sum. (Must be 3^2 and not 2^3 .) Note: solutions via calculator without showing removal of logs will score M0dM0A0.

dM1: Depends on previous M. Solves the resulting equation in n

A1: Correct

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Question Number	Scheme	Notes	Marks
8(a)	$x = r \cos \theta = (1 + \sin \theta) \cos \theta$ $\Rightarrow \frac{dx}{d\theta} = \cos^2 \theta - (1 + \sin \theta) \sin \theta$	Differentiates $r \cos \theta$ using product rule or double angle formula	M1
	or $\Rightarrow \frac{dx}{d\theta} = -\sin \theta + \cos 2\theta$	Correct derivative in any form	A1
	$\cos^2 \theta - (1 + \sin \theta) \sin \theta = 0 \Rightarrow 1 - \sin^2 \theta - \sin \theta - \sin^2 \theta = 0 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ or $-\sin \theta + \cos 2\theta = 0 \Rightarrow -\sin \theta + 1 - 2 \sin^2 \theta = 0 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ Sets $\frac{dx}{d\theta} = 0$ and proceeds to a 3TQ in $\sin \theta$ Depends on the first M mark		dM1
	$\Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ $\Rightarrow \sin \theta = \frac{1}{2}, (-1) \Rightarrow \theta = \dots$	Solves for θ . Depends on both M marks above.	ddM1
	$\left(\frac{3}{2}, \frac{\pi}{6} \right)$	Correct coordinates and no others. Need not be in coordinate brackets.	A1
			(5)
(b)	$\int (1 + \sin \theta)^2 d\theta = \int (1 + 2 \sin \theta + \sin^2 \theta) d\theta$ $= \int \left(1 + 2 \sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta \right) d\theta$	Attempts $\left(\frac{1}{2} \right) \int r^2 d\theta$ and applies $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ Ignore any limits shown	M1
	$\int (1 + \sin \theta)^2 d\theta = \frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta (+c)$	Correct integration (Ignore limits)	A1
	$\frac{1}{2} \left[\frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2} \left[\frac{3\pi}{4} - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8} \right) \right] \left(= \frac{\pi}{4} + \frac{9\sqrt{3}}{16} \right)$	Applies the limits of $\frac{\pi}{2}$ and their $\frac{\pi}{6}$ Substitution must be shown but no simplification needed	M1
	Trapezium: $\frac{1}{2} \left(2 + \left(2 - \frac{3}{2} \sin \frac{\pi}{6} \right) \right) \times \frac{3}{2} \cos \frac{\pi}{6}$ $\left(= \frac{39\sqrt{3}}{32} \right)$	Uses a correct strategy for the area of trapezium $OQSP$	M1
	$\text{Area of } R = \frac{39\sqrt{3}}{32} - \frac{\pi}{4} - \frac{9\sqrt{3}}{16}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32} (21\sqrt{3} - 8\pi)$	Cao	A1
			(6)

Question Number	Scheme	Notes	Marks
9(a)	$n^5 - (n-1)^5 = n^5 - (n^5 - 5n^4 + 10n^3 - 10n^2 + 5n - 1) = \dots$ Starts the proof by expanding the bracket		M1
	$5n^4 - 10n^3 + 10n^2 - 5n + 1^*$	Correct proof with no errors. Full expansion of $(n-1)^5$ must be shown.	A1*
			(2)
(b)	$1^5 - 0^5 = 5(1)^4 - 10(1)^3 + 10(1)^2 - 5(1) + 1$ $2^5 - 1^5 = 5(2)^4 - 10(2)^3 + 10(2)^2 - 5(2) + 1$ $\dots\dots\dots$ $(n-1)^5 - (n-2)^5 = 5(n-1)^4 - 10(n-1)^3 + 10(n-1)^2 - 5(n-1) + 1$ $(n)^5 - (n-1)^5 = 5(n)^4 - 10(n)^3 + 10(n)^2 - 5(n) + 1$ $n^5 = 5 \sum_{r=1}^n r^4 - 10 \sum_{r=1}^n r^3 + 10 \sum_{r=1}^n r^2 - 5 \sum_{r=1}^n r + n$ M1: Applies the result from part (a) between 1 and n and sums both sides Min 3 lines shown A1: Correct equation If only the last line is seen, award M1A1 These marks can be implied by a correct following stage.		M1A1
	$n^5 = 5 \sum_{r=1}^n r^4 - 10 \times \frac{1}{4} n^2 (n+1)^2 + 10 \times \frac{1}{6} n(n+1)(2n+1) - 5 \times \frac{1}{2} n(n+1) + n$ M1: Introduces at least 2 correct summation formulae A1: Correct equation		M1A1
	$5 \sum_{r=1}^n r^4 = \frac{5}{2} n^2 (n+1)^2 - \frac{5}{3} n(n+1)(2n+1) + \frac{5}{2} n(n+1) + n^5 - n = \dots$ $5 \sum_{r=1}^n r^4 = n(n+1) \left[\frac{5}{2} n(n+1) - \frac{5}{3} (2n+1) + \frac{5}{2} + n^3 - n^2 + n - 1 \right]$ Makes $5 \sum_{r=1}^n r^4$ or $\sum_{r=1}^n r^4$ the subject and takes out a factor of $n(n+1)$		M1
	$\sum_{r=1}^n r^4 = \frac{1}{30} n(n+1) [15n(n+1) - 10(2n+1) + 15 + 6(n^3 - n^2 + n - 1)]$ $= \frac{1}{30} n(n+1) [6n^3 + 9n^2 + n - 1] = \frac{1}{30} n(n+1)(2n+1)(\dots)$ Takes out a factor of $n(n+1)(2n+1)$ Depends on all previous method marks		dM1
	$= \frac{1}{30} n(n+1)(2n+1)(3n^2 + 3n - 1)$	cao	A1
			(7)
			Total 9

Question	Scheme	Marks
4(a)	$\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$	B1
	$\Rightarrow r+4 \equiv A(r+1)(r+2) + Br(r+2) + Cr(r+1) ; r=0 \Rightarrow A=...$	M1
	Two of $A=2, B=-3, C=1$	A1
	$\frac{r+4}{r(r+1)(r+2)} \equiv \frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}$	A1
		(4)
(b)	$\begin{aligned} \sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} &= \sum_{r=1}^n \left(\frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2} \right) \\ &= \left(\frac{2}{1} - \frac{3}{2} + \frac{1}{3} \right) \\ &+ \left(\frac{2}{2} - \frac{3}{3} + \frac{1}{4} \right) \\ &+ \left(\frac{2}{3} - \frac{3}{4} + \frac{1}{5} \right) \\ &+ \dots \\ &+ \left(\frac{2}{n-1} - \frac{3}{n} + \frac{1}{n+1} \right) \\ &+ \left(\frac{2}{n} - \frac{3}{n+1} + \frac{1}{n+2} \right) \end{aligned}$	M1A1
	$= \left(\frac{2}{1} - \frac{1}{2} \right) + \left(-\frac{2}{n+1} + \frac{1}{n+2} \right) \text{ o.e.}$	A1
	$= \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2} = \frac{3(n+1)(n+2) - 4(n+2) + 2(n+1)}{2(n+1)(n+2)} = ...$	M1
	$= \frac{n(3n+7)}{2(n+1)(n+2)}$	A1
	All marks can be awarded if working in terms of r and then replaced at the end. If no replacement made, then deduct the final A mark	
		(5)
	(9 marks)	
Notes:		
(a)		
B1: Correct form for the partial fractions attempted. Note: some candidates may start with $\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ and proceed to split further into four partial fractions, equivalent to the form stated. If in doubt about this method, please send to review.		
M1: Method to <u>find at least one</u> of the coefficients, may be the cover-up rule		
A1: Two correct coefficients		

A1: Fully correct partial fractions found (not just stating the values), but accept if seen in (b) if only the coefficients are found in (a).

(b)

M1: Applies the method of differences, with at least three of the given brackets evaluated, and showing some evidence of appropriate cancellation of terms on the ‘diagonal’ e.g. $\frac{1}{3} - \frac{3}{3} + \frac{2}{3}$ etc

A1: Shows sufficient terms in the method of differences to establish complete cancellation of terms on the main diagonal (so e.g. at least 3 terms at start and 2 terms at the end or for 2 terms at the start and 3 terms at the end)

Alt: establishes the correct non-cancelling numeric (accept e.g. $3/2$ for the numeric terms) **OR** algebraic terms (which would imply M1A1)

A1: Correct non-cancelling terms extracted.

M1: Puts over the correct common denominator for their remaining terms and attempts to simplify the numerator to achieve a quadratic of the form $an^2 + bn$ but accept $an^2 + bn + c$ with $a \neq 0$

A1: Correct answer or values for P , Q , R and S .

Allow recovery if the terms are given in terms of r instead of n and recovered at the end. If no recovery, then score final A0.

Alt for the first 3 marks in (b)

Re-writing the PFs $\frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}$ as $\frac{2}{r} - \frac{2}{r+1} + \frac{-1}{r+1} + \frac{1}{r+2}$ and then using method of differences on two separate sums

(b) Alt	$\sum_{r=1}^n \left[\frac{2}{r} - \frac{2}{r+1} \right] + \sum_{r=1}^n \left[-\frac{1}{r+1} + \frac{1}{r+2} \right]$ $= \left(\frac{2}{1} - \frac{2}{2} \right) + \left(-\frac{1}{2} + \frac{1}{3} \right)$ $+ \left(\frac{2}{2} - \frac{2}{3} \right) + \left(-\frac{1}{3} + \frac{1}{4} \right) + \dots$ $+ \left(\frac{2}{n-1} - \frac{2}{n} \right) + \left(-\frac{1}{n} + \frac{1}{n+1} \right)$ $+ \left(\frac{2}{n} - \frac{2}{n+1} \right) + \left(-\frac{1}{n+1} + \frac{1}{n+2} \right)$	M1A1
	$= \frac{2}{1} - \frac{2}{n+1} - \frac{1}{2} + \frac{1}{n+2}$	A1
	$= \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2} = \frac{3(n+1)(n+2) - 4(n+2) + 2(n+1)}{2(n+1)(n+2)} = \dots$	M1
	$= \frac{n(3n+7)}{2(n+1)(n+2)}$	A1
		(5)

Notes:**(b)**

M1: Applies the method of differences, for both summations, with at least two of the brackets evaluated for both, and some evidence of cancellation of terms on the ‘diagonal’

A1: Shows sufficient terms in the method of differences for the cancelling of terms on the main diagonal (e.g. first two and last, or first and last two etc)

Alt: correct non-cancelling numeric OR algebraic terms extracted (which would imply M1A1)

A1: Correct non-cancelling terms extracted.

M1: Puts over the correct common denominator for their remaining terms and attempts to simplify, to achieve a quadratic of the form $an^2 + bn$ but accept $an^2 + bn + c$ with $a \neq 0$

A1: Correct answer or values for P , Q , R and S .

Allow recovery if the terms are given in terms of r instead of n and recovered at the end. If no recovery, then score final A0.

Question Number	Scheme	Notes	Marks
2	$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y^2 = e^{x^2-9}$		
	$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 10y\frac{dy}{dx} = 2xe^{x^2-9}$	M1: Differentiates to obtain three correct terms A1: Fully correct differentiated equation	M1 A1
	$y(3) = 2, y'(3) = -1$ $y''(3) - 4(-1) - 5 \times 2^2 = 1 \Rightarrow y''(3) = 17$ $y'''(3) - 4(17) - 10(2)(-1) = 6 \Rightarrow y'''(3) = 54$	Uses the given equation and their equation to obtain values for $y''(3)$ and $y'''(3)$	M1
	$(y =) y(3) + y'(3)(x-3) + \frac{y''(3)}{2}(x-3)^2 + \frac{y'''(3)}{3!}(x-3)^3 \Rightarrow$ $2 - 1(x-3) + \frac{17}{2}(x-3)^2 + \frac{54}{3!}(x-3)^3$	A correct unsimplified Taylor series for their $y''(3)$ and $y'''(3)$	M1
	$(y =) 2 - (x-3) + \frac{17}{2}(x-3)^2 + 9(x-3)^3$	Correct simplified series. "y = ..." is not required	A1
			(5)
			Total 5

Additional Notes

M1: Attempts to differentiate using the chain rule at least once – at least three terms correct, allowing for slips in rearranging before differentiation.

A1: Any correct third derivative expression. They may have rearranged first.

M1: Finds the values of the second and third derivatives using the given equation and their third derivative expression (which must have at least three terms). Condone one slip in substitutions as long as evidence of the correct values used is seen. If substitution is not seen, correct values can imply the method.

M1: Applies the Taylor Series formula correctly with the given values of y and $\frac{dy}{dx}$ and with their values for

the second and third derivatives – the first two terms must be correct. If the values of their second and third derivatives are not explicitly stated but embedded in the series, it must be clear that a correct attempt at the series has been used – the denominators 2 and 3! (or 6) must be seen – but their values need not be correct. Condone a miscopy of one of **their** values if it is clearly a correct attempt to apply their value to the series.

A1: Correct fully simplified series expansion. The $y =$ is not required and condone if $f(x)$ is used.



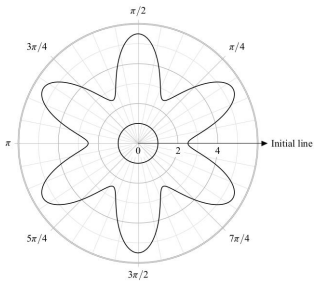
Question	Scheme	Marks
2	$2m^2 - 5m - 3 = 0 \Rightarrow (2m+1)(m-3) = 0 \Rightarrow m = \dots$	M1
	So C.F. is $(y_{CF} =) Ae^{-\frac{1}{2}x} + Be^{3x}$	A1
	P.I. is $y_{PI} = axe^{3x}$	B1
	$\frac{dy_{PI}}{dx} = 3axe^{3x} + ae^{3x}, \frac{d^2y_{PI}}{dx^2} = 9axe^{3x} + 3ae^{3x} + 3ae^{3x}$ $\Rightarrow 2(9ax + 6a)e^{3x} - 5(3ax + a)e^{3x} - 3axe^{3x} = 2e^{3x} \Rightarrow a = \dots$	M1
	$a = \frac{2}{7}$	A1
	General solution is $y = Ae^{-\frac{1}{2}x} + Be^{3x} + \frac{2}{7}xe^{3x}$	B1ft
		(6)
(6 marks)		
Notes:		
M1: Forms and solves the auxiliary equation. A1: Correct complementary function (no need for $y = \dots$) B1: Correct form for the particular integral. Accept any PI that includes axe^{3x}, so e.g. $(ax+b)e^{3x}$ is fine. M1: Attempts to differentiate their PI twice and substitutes into the left hand side of the equation. The derivatives must be changed functions. There is no need to reach a value for the unknown(s) but their PI must contain an unknown constant. A1: Correct value of a (and any other coefficients as zero). Must have had a suitable PI B1ft: For $y =$ their CF + their PI. Must include the $y =$. The PI must be a function of x that matches their initial choice of PI, with their constants substituted.		

Question	Scheme	Marks
3(a)	Meet when $x^2 - 8x = \frac{4x}{4-x} \Rightarrow (x^2 - 8x)(4-x) = 4x \Rightarrow x(4x - 32 - x^2 + 8x - 4) = 0$	M1
	(so $x = 0$ or) $x^2 - 12x + 36 = 0$	A1
	$\Rightarrow x(x-6)^2 = 0 \Rightarrow x = \dots$	M1
	Meet at (6, -12)	A1
	e.g. touch at (6, -12) as repeated root.	B1
		(5)
Alt	$\frac{d}{dx}(x^2 - 8x) = 2x - 8$ and $\frac{d}{dx}\left(\frac{4x}{4-x}\right) = \frac{4(4-x) - 4x(-1)}{(4-x)^2} = \frac{16}{(4-x)^2}$	M1A1
	$2x - 8 = \frac{16}{(4-x)^2} \Rightarrow (x-4)^3 = 8 \Rightarrow x = \dots$	M1
	Meet at (6, -12)	A1
	e.g. $6^2 - 6 \times 9 = -12$ and $\frac{4 \times 6}{4-6} = -12$, so curves meet at tangent at (6, -12)	B1
		(5)
(b)	$x^2 - 8x = \frac{4x}{4+x} \Rightarrow x(x-8)(4+x) - 4x = 0 \Rightarrow x(x^2 - 4x - 36) = 0 \Rightarrow x = \dots$	M1
	$x = (0), 2 \pm 2\sqrt{10} \Rightarrow$ critical value is (0 and) $2 - 2\sqrt{10}$	A1
	Other C.V.'s are 0, ± 4	B1
	E.g. extremes are $x < 2 - 2\sqrt{10}$ and $x > 6$ or any two suitable ranges.	M1
	Solution is $x < 2 - 2\sqrt{10}, -4 < x < 0, 4 < x < 6, x > 6$	A1A1
		(6)
(11 marks)		

Notes:**(a)****M1:** Attempts to find intersection by setting equations equal and cross multiplies and factorises the x out or cancels.**A1:** Correct quadratic reached. May be implied by solutions of 0,6 seen from the cubic (by calculator)**M1:** Solves the quadratic to find roots.**A1:** Obtains the correct point where the curves meet.**B1:** Correct reason given for why the curves touch. Accepted “repeated root” as reason. As a minimum, accept “ $(x-6)^2 = 0$ therefore touches”. Alternatively, accept discriminant = 0 shown with conclusion, or may find gradient at both points and show equal, with conclusion.

Alt:

M1: Attempts derivatives of both curves**A1:** Both derivatives correct.**M1:** Sets derivatives equal and solves to find x value where gradients agree.**A1:** Obtains the correct point where the curves meet.**B1:** Correct value checked in both curves with conclusion that they meet at a tangent or equivalent working as per main scheme.**(b)****M1:** Attempts to find the intersection of the other branch of $\frac{4x}{4-|x|}$ with $x^2 - 8x$. Allow for any attempt atsolving $\frac{4x}{4+x} = x^2 - 8x$ that reaches a value for x **A1:** Correct value of $2 - 2\sqrt{10}$ identified. (No need to see the second root rejected for this mark.)**B1:** Both 0 and ± 4 identified as critical values for the ranges needed at some stage in working.**M1:** Forms at least two suitable ranges from their critical values (allow if e.g. $\leq$ is used instead of $<$). Likely to be the extreme ranges, so look for $x <$ their $2 - 2\sqrt{10}$ and $x >$ their 6. However, allow if this latter is included as part of the range $x > 4$ for this mark.**A1:** At least two correct ranges.**A1:** Fully correct answer as in scheme.

Question	Scheme	Marks
4(a)		M1
	Completes to a closed loop with “petals” containing circle of radius 1 (whether the circle is drawn or not)	A1
	Fully correct – 6 petals in roughly the right places, but allow if curvature is not quite smooth.	B1
	Circle centre O radius 1.	
		(3)
(b)	$\left(\frac{1}{2}\right) \int r^2 d\theta = \left(\frac{1}{2}\right) \int \left(16 - 12 \cos 6\theta + \frac{9}{4} \cos^2 6\theta\right) d\theta$	M1
	$= \frac{1}{2} \int_0^{2\pi} \left(16 - 12 \cos 6\theta + \frac{9}{8} (1 + \cos 12\theta)\right) d\theta$	M1
	$= \frac{1}{2} \left[16\theta - 2 \sin 6\theta + \frac{9}{8} \left(\theta + \frac{1}{12} \sin 12\theta\right)\right]_0^{2\pi}$	M1 A1
	$A_{outer} = \frac{1}{2} \int_0^{2\pi} r^2 d\theta = \frac{1}{2} \int_0^{2\pi} \left(16 - 12 \cos 6\theta + \frac{9}{4} \cos^2 6\theta\right) d\theta$ $= \frac{1}{2} \left(32\pi - 0 + \frac{9}{8} (2\pi + 0) - (0)\right)$	dM1
	So Area required is $\frac{1}{2} \left(32\pi + \frac{9\pi}{4}\right) - \pi(1^2) = \dots$	B1
	$= \frac{129}{8} \pi$	A1
		(7)
(10 marks)		
Notes:		
(a) M1: Allow for any closed loop that oscillates, though may not have the correct number of “petals” but require at least 4. Need not have correct places of maximum radius. A1: Fully correct sketch, 6 “petals” in the right places, with maximum radius between the 5 and 6 radius lines, minimum between the 2 and 3 radius lines. B1: For a circle of radius 1 and centre O drawn. (b) M1: Attempts to square r as part of an integral for the outer curve, achieving a 3 term quadratic in $\cos 6\theta$ M1: Applies the double angle formula to the $\cos^2$ term from their expansion (not dependent on the first M, but must have a $\cos^2$ term). Accept $\cos^2 6\theta \rightarrow \frac{1}{2}(\pm 1 \pm \cos 12\theta)$ M1: Attempts to integrate, achieving the form $\alpha\theta + \beta \sin 6\theta + \gamma \sin 12\theta$ where $\alpha, \beta, \gamma \neq 0$		

A1: Correct integration – limits and the $\frac{1}{2}$ not needed. Look for $16\theta - 2\sin 6\theta + \frac{9}{8}\left(\theta + \frac{1}{12}\sin 12\theta\right)$ oe.

dM1: Depends on at least two of the previous M's being scored. For a correct overall strategy for the area contained in the outer loop, with an attempt at the r^2 (should be 3 term expansion). Correct appropriate limits and the $\frac{1}{2}$ should be present or implied by working, but note variations on the scheme are possible, e.g.

$2 \times \frac{1}{2} \int_0^\pi r^2 d\theta$, in which the $2 \times \frac{1}{2}$ may be implied rather than seen.

B1: Subtracts correct area of π for inner circle

A1: cso. Check carefully the integration was correct as the sin terms disappear with the limits.



Question	Scheme	Marks
5(a)	$\frac{dy}{dx} = \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x}$	M1 A1
	$\frac{d^2y}{dx^2} = \frac{1}{2} \frac{0 - \left(\sqrt{4 + \ln x} + x \times \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x} \right)}{x^2(4 + \ln x)}$ or $\frac{d^2y}{dx^2} = -\frac{1}{4x}(4 + \ln x)^{-\frac{3}{2}} \times \frac{1}{x} - \frac{1}{x^2} \times \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}}$ oe	M1
	$= \frac{\dots}{4x^2(4 + \ln x)^{\frac{3}{2}}} = -\frac{9 + 2 \ln x}{4x^2(4 + \ln x)^{\frac{3}{2}}}$ *	M1 A1*
		(5)
Alt(a)	$y^2 = 4 + \ln x \Rightarrow 2y \frac{dy}{dx} = \frac{1}{x}$	M1 A1
	$\Rightarrow 2y \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = -\frac{1}{x^2}$	M1
	$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2yx^2} - \frac{2}{8x^2y^3} = \frac{-2y^2 - 1}{4x^2y^3}$	M1
	$= -\frac{9 + 2 \ln x}{4x^2(4 + \ln x)^{\frac{3}{2}}}$ *	A1*
(b)		(5)
	$y_{x=1} = 2, \frac{dy}{dx}\bigg _{x=1} = \frac{1}{4}, \frac{d^2y}{dx^2}\bigg _{x=1} = -\frac{9}{32}$	M1
	So $y = 2 + \frac{1}{4}(x-1) - \frac{1}{2!} \times \frac{9}{32}(x-1)^2 + \dots$	M1
	$= 2 + \frac{1}{4}(x-1) - \frac{9}{64}(x-1)^2 + \dots$	A1
		(3)
(8 marks)		

Notes:
(a)

M1: Attempts the derivative of y using the chain rule, look for $\frac{K}{x}(4 + \ln x)^{-\frac{1}{2}}$ oe

A1: Correct derivative.

M1: Attempts the second derivative of y using the product or quotient rule and chain rule. Look for the correct form for their $\frac{dy}{dx}$ for the answer up to slips in coefficients.

M1: Attempts to simplify to get correct denominator. Must be correct work for their second derivative, but may have been errors in differentiating.

A1*: For a correct unsimplified second derivative, with no errors before reaching the given answer.

Note it is a given answer so needs a suitable intermediate line with at least the formation of the correct common denominator between two fractions before reaching the answer.



Alt:

M1: Squares and uses implicit differentiation to achieve $\alpha y \frac{dy}{dx} = \frac{\beta}{x}$

A1: Correct derivative.

M1: Differentiates again using implicit differentiation and product rule. Look for $\gamma y \frac{d^2y}{dx^2} + \delta \left(\frac{dy}{dx} \right)^2 = \frac{\nu}{x^2}$

M1: Makes $\frac{d^2y}{dx^2}$ the subject and forms single fraction with denominator kx^2y^3

A1*: Obtains the correct second derivative, with no errors seen in working.

(b)

M1: Evaluates y , $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1$, if substitution is not seen, accept stated values for all three following attempts at the first and second derivatives as an attempt to find these.

M1: Applies Taylor's theorem with their values.

A1: Correct expression (don't be concerned if the $y =$ is missing.)

5(b) Alt	$y = \sqrt{4 + \ln(1 + (x-1))} = \sqrt{4 + \left((x-1) - \frac{(x-1)^2}{2} + \dots \right)}$	M1
	$= 4^{\frac{1}{2}} + \frac{1}{2} \times 4^{-\frac{1}{2}} \times \left((x-1) - \frac{(x-1)^2}{2} \right) + \frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times 4^{-\frac{3}{2}} \times ((x-1) - \dots)^2 + \dots$	M1
	$= 2 + \frac{1}{4}(x-1) - \frac{1}{8}(x-1)^2 - \frac{1}{64}(x-1)^2 + \dots = 2 + \frac{1}{4}(x-1) - \frac{9}{64}(x-1)^2 + \dots$	A1
		(3)

Notes:

M1: Writes the x as $1 + (x-1)$ and attempts to expand using the Maclaurin series for $\ln(1+x)$ with correct expansion of $\ln(1+(x-1))$.

M1: Attempts a binomial expansion using their $\ln$ expansion. Alternatively, may gain this before the first M

if they expand using $\ln$'s, e.g. $4^{\frac{1}{2}} + \frac{1}{2} 4^{-\frac{1}{2}} \ln x + \frac{\frac{1}{2} \times -1}{2!} (\ln x)^2$

A1: Fully correct expression (don't be concerned if the $y =$ is missing.)



Question	Scheme	Marks
6(a)	Let $x = \arctan A$ and $y = \arctan B$ then $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$	B1
	Or $\tan(\arctan A - \arctan B) = \frac{\tan \arctan A - \tan \arctan B}{1 + \tan \arctan A \tan \arctan B}$	
	$\tan(x - y) = \frac{A - B}{1 + AB} \Rightarrow x - y = \arctan\left(\frac{A - B}{1 + AB}\right)$	M1
	So $\arctan A - \arctan B = x - y = \arctan\left(\frac{A - B}{1 + AB}\right)^*$	A1*
		(3)
(b)	$A = r + 2, B = r \Rightarrow \left(\frac{A - B}{1 + AB}\right) = \frac{r + 2 - r}{1 + (r + 2)r} = \frac{2}{...}$	M1
	$= \frac{2}{r^2 + 2r + 1} = \frac{2}{(1 + r)^2}^*$	A1*
		(2)
(c)	$\sum_{r=1}^n \arctan\left(\frac{2}{(1+r)^2}\right) = \sum_{r=1}^n (\arctan(r+2) - \arctan(r)) = ...$	M1
	$= (\cancel{\arctan 3} - \arctan 1) + (\cancel{\arctan 4} - \arctan 2) + (\cancel{\arctan 5} - \cancel{\arctan 3}) + ...$ $+ (\arctan(n+1) - \cancel{\arctan(n-1)}) + (\arctan(n+2) - \cancel{\arctan n})$	A1
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \arctan 1$	M1
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \frac{\pi}{4}$	A1
		(4)
(d)	As $n \rightarrow \infty$, $\arctan n \rightarrow \frac{\pi}{2}$	M1
	So $\sum_{r=1}^{\infty} \arctan\left(\frac{2}{(1+r)^2}\right) = \frac{\pi}{2} + \frac{\pi}{2} - \arctan 2 - \frac{\pi}{4} = \frac{3\pi}{4} - \arctan 2$	A1
		(2)
(11 marks)		

Notes:**(a)****B1:** For any correct statement or use of the compound angle formula with **consistent variables** of x and y or $\arctan A$ and $\arctan B$. Can be either way round (may be working in reverse).**M1:** Attempts to apply $\tan$ or $\arctan$ on an appropriate identity with either x and y or $\arctan A$ and $\arctan B$.Should have $\frac{\tan x \pm \tan y}{1 \pm \tan x \tan y}$ (oe with arctans or A 's and B 's) as part of the identity, and allow if they changebetween x, y and $\arctan$'s during the step.**A1*:** Must have scored the B and M marks. Replaces $\tan x$ and $\tan y$ by A and B respectively if appropriate with fully correct work leading to the given result and conclusion made and no erroneous statements.**Note: for working in reverse e.g.**Let $x = \arctan A$ and $y = \arctan B$ then

$$\arctan A - \arctan B = \arctan\left(\frac{A-B}{1+AB}\right) \Leftrightarrow x - y = \arctan\left(\frac{A-B}{1+AB}\right) \Leftrightarrow \tan(x-y) = \frac{A-B}{1+AB} \quad \text{Scores M1}$$

$$\Leftrightarrow \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \quad \text{Scores B1 – but enter as the first mark.}$$

Which is the correct identity for $\tan(x-y)$ hence the result is true.

Score A1

The conclusion here must include reference to the identity being true, e.g. with a tick, or statement, before deducing the final result.

(b)**M1:** Substitutes in $A = r + 2$ and $B = r$ and simplifies the numerator to 2 (may be implied)**A1*:** Expands the denominator (must be seen) and then factorises to the given result, no errors seen.**(c)****M1:** Applies the result of (a) to the series – allow if they have a different A and B due to error.**A1:** At least first three and final two brackets of terms correctly written out – must be clear enough to show cancelling.**M1:** Extracts the non-cancelling terms.**A1:** Correct result with no errors seen – must see the $\arctan 1$ before reaching $\frac{\pi}{4}$.**Note:** Insufficient terms to gain the first A is not an error, so M1A0M1A1 is possible if e.g. only the first two terms are shown. Condone missing brackets on $\arctan n + 1$ etc.**(d)****M1:** Identifies the value $\arctan$ tends towards as n increase. Need not see limits, as long as the value is identified.**A1:** Correct answer.