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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Differential Equations (FP2.4)

International A Level Mathematics · FP2: Further Pure Mathematics 2 (WFM02)

26 questions · 275 marks · Mark schemes included

Past papers October 2020 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

26 questions · 275 marks · 2 sub-topics

SUB-TOPIC FP2.4a

FIRST-ORDER DIFFERENTIAL EQUATIONS

Sub-topic FP2.4a · 13 questions · 125 marks · lowest tariff first



1.

$$\frac{dy}{dx} - 3y \tan x = \sec^2 x$$

(a) Show that an integrating factor for this differential equation is given by

$$p(x) = \cos^3 x \quad (2)$$

Given that $y = 4$ when $x = \frac{\pi}{4}$

(b) determine the particular solution of the differential equation.

Give your answer in the form $y = f(x)$. (4)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)

4. (a) Determine the general solution of the differential equation

$$\frac{dy}{dx} - 3y \tan x = e^{4x} \sec^3 x$$

giving your answer in the form $y = f(x)$

(5)

- (b) Determine the particular solution for which $y = 4$ at $x = 0$

(2)

Question 4 continued

Lined area for writing the answer to Question 4 continued.

Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

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(Total for Question 4 is 7 marks)



6. Obtain the general solution of the equation

$$x^2 \frac{dy}{dx} + (x \cot x + 2)xy = 4 \sin x \quad 0 < x < \pi$$

Give your answer in the form $y = f(x)$

(8)

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Question 6 continued

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(Total 8 marks)

Q6



Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total for Question 2 is 8 marks)



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4. (a) Show that the substitution $y^2 = \frac{1}{z}$ transforms the differential equation

$$\frac{dy}{dx} + 2y = 3xy^3 \quad y \neq 0 \quad (\text{I})$$

into the differential equation

$$\frac{dz}{dx} - 4z = -6x \quad (\text{II})$$

(3)

- (b) Obtain the general solution of differential equation (II).

(5)

- (c) Hence obtain the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(1)

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Question 4 continued

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(Total 9 marks)

Q4



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4. (a) Determine the general solution of the differential equation

$$(x+1)\frac{dy}{dx} - xy = e^{3x} \quad x > -1$$

giving your answer in the form $y = f(x)$.

(7)

- (b) Determine the particular solution of the differential equation for which $y = 5$ when $x = 0$ (2)

(2)

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Question 4 continued

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(Total 9 marks)

Q4

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Question 3 continued

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(Total for Question 3 is 9 marks)

4. (a) Show that the substitution $y^2 = \frac{1}{t}$ transforms the differential equation

$$\frac{dy}{dx} + y = xy^3 \quad (\text{I})$$

into the differential equation

$$\frac{dt}{dx} - 2t = -2x \quad (\text{II})$$

(3)

(b) Solve differential equation (II) and determine y^2 in terms of x .

(6)

Question 4 continued

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(Total for Question 4 is 9 marks)

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8. (a) Show that the substitution $v = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 6xy = 3xe^{x^2}y^3 \quad x > 0 \quad (I)$$

into the differential equation

$$\frac{dv}{dx} - 12vx = -6xe^{x^2} \quad x > 0 \quad (\text{II})$$

(5)

- (b) Hence find the general solution of the differential equation (I), giving your answer in the form $y^2 = f(x)$.

(6)

Question 8 continued

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Question 8 continued

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Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END



Question 7 continued

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Question 7 continued

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Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 11 marks)



Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 11 marks)



Question 8 continued

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Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

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Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

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Question 8 continued

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Q8

(Total 14 marks)

END

TOTAL FOR PAPER: 75 MARKS



SUB-TOPIC FP2.4b

SECOND-ORDER DIFFERENTIAL EQUATIONS

Sub-topic FP2.4b · 13 questions · 150 marks · lowest tariff first



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2. Determine the general solution of the differential equation

$$2 \frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} - 3y = 2e^{3x}$$

(6)



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Q2

Question 8 continued

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Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 10 marks)

Question 4 continued

Lined area for writing the answer to Question 4 continued.





Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 10 marks)



Question 7 continued

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Q7



Question 4 continued

Lined area for writing the answer to Question 4 continued.





Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 11 marks)



6. (a) Determine the general solution of the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 37y = 6e^{5x}$$

(6)

Given that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(b) determine the particular solution for this differential equation.

(5)



Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 11 marks)



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6. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 6\cos x$$

(7)

- (b) Find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$

(5)

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Question 6 continued

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Q6

Question 7 continued

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Question 7 continued

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(Total for Question 7 is 12 marks)



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Question 6 continued

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Question 6 continued

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(Total 13 marks)

Q6





Question 9 continued

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Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



(Total for Question 7 is 13 marks)



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Question 8 continued

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Q8

(Total 14 marks)

END

TOTAL FOR PAPER: 75 MARKS



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 14 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question	Scheme	Marks
1(a)	$\frac{dy}{dx} - 3y \tan x = \sec^2 x$ with $y = 4$ when $x = \frac{\pi}{4}$	
	Mark parts (a) and (b) together	
	$IF = \exp\left(\int -3 \tan x \, dx\right) = \dots$	M1
	$= \exp(3 \ln \cos x) = \cos^3 x$ *	A1*
		(2)
Alt (a)	$\frac{d}{dx}(y \cos^3 x) = \frac{dy}{dx} \cos^3 x + y \times 3 \cos^2 x \times -\sin x$	M1
	$= \cos^3 x \left(\frac{dy}{dx} - 3y \tan x\right)$ hence integrating factor	A1*
		(2)
(b)	$\Rightarrow y \cos^3 x = \int \sec^2 x \times \cos^3 x \, dx$	M1
	$\Rightarrow y \cos^3 x = \sin x \, (+c)$ o.e. OR $y = \sin x \sec^3 x + (c \sec^3 x)$ OR $y = \tan x \sec^2 x + (c \sec^3 x)$ o.e.	A1
	$\Rightarrow 4 \cos^3 \frac{\pi}{4} = \sin \frac{\pi}{4} + c \Rightarrow c = \frac{\sqrt{2}}{2} \Rightarrow y = \tan x \sec^2 x + \frac{\sqrt{2}}{2} \sec^3 x$ o.e.	M1 A1
		(4)
	(6 marks)	
Notes:		
(a)		
M1: Attempts the integrating factor directly. Allow for an attempt at $\exp\left(\pm \int k \tan x\right)$ but no need to evaluate the integral for this mark. Must be shown, but could be implied by an integrated expression of the correct form e.g. $\exp(-3 \ln \sec x)$ or $\exp(3 \ln \cos x)$. If $(\sec x)^{-3}$ is all that is seen then accept for M1 only A1*: Correct simplified integrating factor. Correct processing seen, so exp and ln inverse applied correctly, and correct integration of $-3 \tan x \rightarrow -3 \ln \sec x$ or $3 \ln \cos x$ seen. No sight of exp and ln being correctly processed, or no evidence of correct integration is max M1A0		

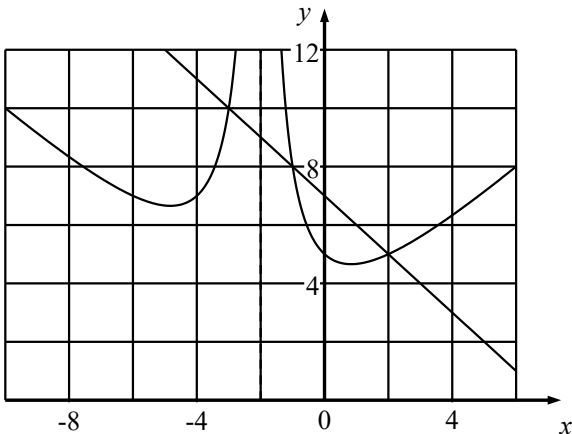
Question Number	Scheme	Marks
2 (a)	E.g. $(x+3)(x-5)=9 \Rightarrow x^2-2x-24=0 \Rightarrow x=...$ OR $(x-5)(x+3)^2-9(x+3)=0 \Rightarrow (x+3)(x-6)(x+4)=0 \Rightarrow x=...$ OR $\frac{(x+3)(x-5)-9}{x+3} < 0 \Rightarrow x^2-2x-24=0 \Rightarrow x=...$	M1
	CVs: 6, -4; -3	A1;B1
	$x < -4, -3 < x < 6$	dM1A1A1
	OR: $x \in (-3, 6) \cup (-\infty, -4)$ or any equivalent notation.	(6)
(b)	$x < 6, x \neq -3$ or any equivalent notation.	B1ftB1 (2)
		[8]
	Notes	
(a) M1	For a correct algebraic method to find the intersection points of $y = x - 5$ and $y = \frac{9}{x+3}$. May set these equal and form a quadratic and solve. May multiply through by $(x+3)^2$ and collect on one side or use any other valid method Eg work from $\frac{(x+3)(x+2)-12}{x+3} > 0$ Answers only from a calculator score M0. Must reach at least a quadratic or cubic before answers given. Do not be concerned with the equality or inequality for this mark.	
A1	For 6, -4 obtained via a valid algebraic method.	
B1	for the CV -3 seen anywhere	
dM1	Obtaining (any) inequalities using all of their critical values and no other numbers.	
A1	For at least one correct interval allowing for ... or ,, used instead of < and >	
A1cso	Both correct ranges and no extras. Use of ... or ,, scores A0. May be written in set notation, and all work should have been correct so penalise if incorrect inequalities method was used at the start. Accept $x < -4$ and/or $-3 < x < 6$ with “and” or “or”	
	For candidates who draw a sketch graph and follow with the cvs without any algebra shown only the B mark is available. Those who use some algebra after their graph may gain marks as earned (possibly all)	
(b) B1ft	For the " $x < 6$ " in some form with the possible exception of the CVs from (a). Allow $x, 6$ if already penalised in (a). It is essentially for realising all the extra (valid) values less than -3 are solutions while retaining all their given solutions. If only the CVs themselves are excluded allow B1. Follow through their answer to (a).	
B1	Fully correct answer. May give as intervals $x < -3, -3 < x < 6$	



Question Number	Scheme	Marks
3	$w = \frac{z}{z + 4i}$	
	$w(z + 4i) = z \Rightarrow z(1 - w) = 4iw \quad \text{or} \quad z = \frac{4iw}{1 - w} \quad \text{oe}$	M1A1
	$ z = 3 \quad \left \frac{4iw}{1 - w} \right = 3$	dM1
	$ 4iw = 3 1 - w $	
	$w = u + iv \quad 16(u^2 + v^2) = 9((1 - u)^2 + v^2)$	ddM1A1
	$16u^2 + 16v^2 = 9(1 - 2u + u^2 + v^2)$	
	$7u^2 + 7v^2 + 18u - 9 = 0$	
	$\left(u + \frac{9}{7}\right)^2 + v^2 = \frac{144}{49}$	dddM1
	Centre $\left(-\frac{9}{7}, 0\right)$ Radius $\frac{12}{7}$	A1A1 (8)
	Notes	
(a)		
M1	re-arrange to $z = \dots$ or an expression $z(\alpha w + \beta) = \gamma w + \delta$	
A1	correct result	
dM1	dep (on first M1) using $ z = 3$ with their previous result	
ddM1	dep (on both previous M marks) use $w = u + iv$ (or $w = x + iy$ or any other pair of letters) and attempts the squares of the moduli. The i's must be dealt with correctly, but allow e.g. $3^2 \rightarrow 3$ for a correct equation quadratic in u and v after squaring (including squaring coefficients).	
A1	for a correct equation quadratic in u and v after squaring (including squaring coefficients).	
dddM1	dep (on all previous M marks) re-arrange to the completed square form of the equation of a circle (same coeffs for the squared terms) or implied by a correct centre or radius following a correct equation with terms gathered.	
A1	either correct and exact.	
A1	both correct and exact.	
	Note: Allow recovery for the last three A's if all that is incorrect in is the wrong sign in their expression for z , ie $z = \frac{-4iw}{1 - w}$	
	If you see alternative methods, e.g. via Apollonian approaches or attempts to use $z = x + iy$ in the original equation, that you feel are worthy of credit please use Review to consult your team leader.	

Question Number	Scheme	Marks
4	$\frac{dy}{dx} - 3y \tan x = e^{4x} \sec^3 x$	
(a)	$e^{-3 \int \tan x dx} = e^{-3 \ln \sec x} = \sec^{-3} x \text{ or } \cos^3 x$	M1A1
	$\cos^3 x \frac{dy}{dx} - 3y \sin x \cos^2 x = e^{4x} \cos^3 x \sec^3 x$	
	$\frac{d}{dx}(y \cos^3 x) = e^{4x} \Rightarrow y \cos^3 x = \int e^{4x} dx$	M1
	$y \cos^3 x = \frac{1}{4} e^{4x} (+c)$	M1
	$y = \left(\frac{1}{4} e^{4x} + c\right) \sec^3 x \text{ or } y = \left(\frac{1}{4} e^{4x} + c\right) \cos^{-3} x \text{ oe}$	A1
		(5)
(b)	$y = 4, \quad x = 0 \quad 4 = \left(\frac{1}{4} + c\right)$	
	$c = \frac{15}{4}$	M1
	$y = \frac{1}{4}(e^{4x} + 15) \sec^3 x \text{ or } \frac{1}{4}(e^{4x} + 15) \cos^{-3} x \text{ oe}$	A1
		(2)
		[7]
(a) M1 A1 M1 M1 A1	Notes Attempt the integrating factor, including integration of $(-3)\tan x$; $\ln \cos$ or $\ln \sec$ seen Correct simplified integrating factor $\sec^{-3} x$ or $\cos^3 x$ Multiply the equation by the integrating factor and integrate the LHS. Look for $y \times$ their IF = $\int (e^{4x} \sec^3 x \times \text{their IF}) dx$ (condone missing dx) Integrate RHS, constant not needed. Must be a function they can integrate and a valid attempt (e.g. allowing coefficient slips). Correct result in the demanded form, including $y = \dots$, constant included	
(b) M1 A1	Use the given initial conditions to obtain a value for c Fully correct final answer. Must include $y = \dots$ but allow A1 if missing and penalised in (a). May be in the form $y \cos^3 x = \dots$ or $4y \cos^3 x = \dots$	

Question Number	Scheme	Marks
2 (a)	$\frac{3r+1}{r(r-1)(r+1)} = \frac{A}{r} + \frac{B}{r-1} + \frac{C}{r+1}$	M1A1 (2)
(b)	$\frac{3r+1}{r(r-1)(r+1)} = -\frac{1}{r} + \frac{2}{r-1} - \frac{1}{r+1}$ $\frac{2}{1} - \frac{1}{2} - \frac{1}{3} \quad \frac{2}{n-3} - \frac{1}{n-2} - \frac{1}{n-1}$ $\frac{2}{2} - \frac{1}{3} - \frac{1}{4} \quad \frac{2}{n-2} - \frac{1}{n-1} - \frac{1}{n}$ $\frac{2}{3} - \frac{1}{4} - \frac{1}{5} \quad \frac{2}{n-1} - \frac{1}{n} - \frac{1}{n+1}$ $\frac{2}{4} - \frac{1}{5} - \frac{1}{6}$	
	$= 2 - \frac{1}{2} + \frac{2}{2} - \frac{1}{n} - \frac{1}{n} - \frac{1}{n+1}$	
(c)	$\frac{5}{2} - \frac{2}{n} - \frac{1}{n+1} = \frac{5n(n+1) - 4(n+1) - 2n}{2n(n+1)}, = \frac{5n^2 - n - 4}{2n(n+1)}$	
	$\sum_2^{20} - \sum_2^{14}$ $= \frac{5 \times 20^2 - 20 - 4}{2 \times 20 \times 21} - \frac{5 \times 14^2 - 14 - 4}{2 \times 14 \times 15}$ $= \frac{13}{210}$	
(a) M1 A1 (b) M1 dM1 A1 M1 A1cso (c) M1 A1	Correct method for obtaining the PFs Correct PFs Show sufficient terms at both ends (eg 3 at start and 2 at end) to demonstrate the cancelling. (This can be implied by correct work at the next line) Must be using PFs of the correct form and start at $r = 2$ unless extra terms are ignored at next stage. Can be split into $\sum \left(\frac{1}{r-1} - \frac{1}{r} \right) + \sum \left(\frac{1}{r-1} - \frac{1}{r+1} \right)$ Extract the non-cancelled terms (min 4 correct terms but 5/2 counts as 3 correct) Depends on first M of (b) Correct terms extracted Write terms using the common denominator, numerator need not be simplified. Must start with a min of 3 terms inc terms with denominators n and $(n+1)$ Correct answer from correct working Form and use the difference of the 2 summations shown using their result from (b) or an earlier form seen in (b) Correct exact answer, as shown or equivalent	dM1A1 M1, A1 cso (5) M1 A1 (2) [9]

Question Number	Scheme	Marks
3	 $\frac{x^2 + 3x + 10}{x + 2} = 7 - x$ $x^2 + 3x + 10 = 14 + 5x - x^2$ $x^2 - x - 2 = 0 \quad (x - 2)(x + 1) = 0$ CVs 2, -1 $\frac{-(x^2 + 3x + 10)}{x + 2} = 7 - x$ $-x^2 - 3x - 10 = 14 + 5x - x^2$ $8x = -24 \quad \text{CV } -3$ $x < -3 \quad -1 < x < 2$	This sketch on its own scores no marks, but it may be seen in the work M1 dM1 A1A1 M1 A1 dddM1A1A1 [9]
NB M1 dM1 A1 A1 M1 A1 dddM1 A1 A1	No algebra implies no marks Form a quadratic equation or inequality, no simplification needed Solve the 3TQ any valid method Depends on the first M mark. Either CV Both CVs Change the sign of LHS or RHS and obtain an equation (quadratic or linear, no simplification needed) Correct CV from solving the linear equation $x <$ their smallest CV and x between their other 2 CVs All M marks above needed Either inequality correct Both inequalities correct “and” between the inequalities is acceptable. If $\cap$ used, deduct an A mark.	

Question Number	Scheme	Notes	Marks
1(a)	$\frac{10x}{x-6} = 2x+12 \Rightarrow 10x = 2x^2 - 72$ $x^2 - 5x - 36 = 0 \Rightarrow (x+4)(x-9) = 0$ $\Rightarrow x = -4, 9$	M1: Cross multiplies, forms 3TQ and solves (usual rules) A1: Both correct values	M1 A1
(2)			
(b)(i)	$-4 < x < 6, \quad x > 9$	$a < x < 6, \quad x > b$ with their $a < 0$ and $b > 6$	B1ft
(b)(ii)	$\frac{10x}{x-6} = -(2x+12) \Rightarrow$ $x^2 + 5x - 36 = 0 \Rightarrow x = -9, 4$	From a correct equation, cross multiplies, forms 3TQ and solves (usual rules)	M1
	$x < -9, \quad -4 < x < 4, \quad x > 9$	M1: Correct regions $x < \alpha, \beta < x < \gamma, \quad x > \delta$ for their values with $\alpha < \beta < 0 < \gamma < \delta$ condoning any non-strict inequality signs and $x \neq 6$ A1: Fully correct range of values in any suitable notation	M1 A1
(4)			
Total 6			

Additional Notes

(a)+(b) Answers only without an algebraic method will score M0, but the roots (product = -36, sum = ±5) are deducible from the simplified (leading coefficient 1) quadratic in this case, so accept correct answers (or allow sign error) following the correct simplified quadratic to imply the M. If an error means a quadratic does not have simple roots, a method must be shown to solve. However, allow answers following an application of the formula even if the simplification would seem to warrant a calculator.

(a)

M1: Alternative to cross multiply is putting over a common denominator of $x-6$ and forming a quadratic in the numerator, or multiplying through by $(x-6)^2$, expanding, gathering and then factoring out the $x-6$ to leave a quadratic. Method must be clear, solutions by calculator are not acceptable.

A1: Both correct values and no spurious extras.

(b) **Note on the grids (b)(ii) will appear as MAA but is being marked as MMA**

B1ft: Correct region following their values as long as they satisfy that one is less than 0 and the other greater than 6 to match the given sketch. Accept set notation or interval notation. Accept with “and” or “or” between but set notion must use union, not intersection.

M1: Must be attempting to solve the correct initial equation. Same notes as the M1 in part (a) apply.

M1: This mark is essentially for demonstrating they understand the shape of the region that is needed. They may draw on the sketch to help. The solutions to their quadratics will have to fit the indicated pattern but if the made sign errors such that they had -9 and 4 in (a) and -4 and 9 in (b) this mark is recoverable (they will have lost the A in (a) and B1ft in (b) already).

A1: Correct answer, allowing the similar recovery as noted above. Allow if scored from calculator solutions to the quadratic equation. (Same criteria.)

Note: Candidates can recover all the marks in (b)(ii) if they start again by squaring both sides, but algebra would have to be complete and non-calculator method.

$$(2x+12)^2 > \left(\frac{10x}{x-6}\right)^2 \Rightarrow 4((x-6)(x+6))^2 > 100x^2 \Rightarrow (x^2-36)^2 - 25x^2 > 0 \Rightarrow x^4 - 72x^2 + 36^2 - 25x^2 > 0$$

$$\Rightarrow x^4 - 97x^2 + 4^2 \times 9^2 > 0 \Rightarrow (x^2-16)(x^2-81) > 0 \Rightarrow CVsx = \pm 4, \pm 9 \Rightarrow -9 < x, -4 < x < 4, x > 9$$

Answers are not likely to be this concise and will probably not succeed in reaching the answers via non-calculator means.

Question Number	Scheme	Marks
2		
(a)	$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{(r+2)^2 - r(r+3)}{r(r+1)(r+2)}$ $= \frac{r^2 + 4r + 4 - r^2 - 3r}{r(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)} \quad *$	M1 A1* (2)
(b)	$r=1 \quad \frac{3}{1 \times 2} - \frac{4}{2 \times 3} \qquad r=n-1 \quad \frac{n+1}{(n-1)n} - \frac{n+2}{n(n+1)}$ $r=2 \quad \frac{4}{2 \times 3} - \frac{5}{3 \times 4} \qquad r=n \quad \frac{n+2}{n(n+1)} - \frac{n+3}{(n+1)(n+2)}$ $r=3 \quad \frac{5}{3 \times 4} - \frac{6}{4 \times 5}$ $\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)}$ $\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{3(n+1)(n+2) - 2n - 6}{2(n+1)(n+2)} = \frac{n(3n+7)}{2(n+1)(n+2)}$	M1 A1 dM1 A1cao (4)
(a) M1 A1* (b) M1 A1 dM1 A1cao	Attempt a single fraction with the correct denominator (or 2 separate fractions with the correct common denominator) Correct result obtained with no errors in the working. Must include LHS as shown in question or LHS = ... Show sufficient terms to demonstrate the cancelling, min 3 at start and 1 at end or 2 at start and 2 at end. Award by implication if the correct 2 remaining terms are seen Extract the correct 2 remaining terms Attempt common denominator of the form $k(n+1)(n+2)$ Correct result obtained. No need to show a , b and c explicitly.	[6]

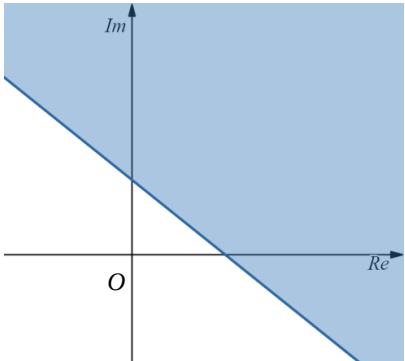
Question Number	Scheme	Notes	Marks
3	$w = \frac{(2+i)z+4}{z-i} \Rightarrow wz - wi = (2+i)z + 4$ $\Rightarrow z = \dots$	Attempts to make z the subject	M1
	$z = \frac{wi+4}{w-2-i}$	Correct equation in any form	A1
	$z = \frac{(u+iv)i+4}{u+iv-2-i}$ $z = \frac{((u+iv)i+4)(u-2-(v-1)i)}{(u-2+(v-1)i)(u-2-(v-1)i)}$	Introduces $w = u + iv$ and multiplies numerator and denominator by the conjugate of the denominator	M1
	$u(v-1) + (4-v)(u-2) = 0$	Sets real part = 0 (with or without denominator) Depends on both M marks above	dM1
		Any correct equation	A1
	$3u + 2v - 8 = 0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 2	$w = \frac{(2+i)z+4}{z-i}, z = yi \Rightarrow w = \frac{(2+i)yi+4}{yi-i}$ $w = \frac{(2+i)yi+4}{yi-i} \times \frac{i}{i}$	Solves simultaneously and multiplies numerator and denominator by i	M1
	$u = \frac{2y}{y-1}, v = \frac{y-4}{y-1}$	Correct real and imaginary parts	A1
	$u = \frac{2y}{y-1} \Rightarrow y = \frac{u}{u-2}$	Attempts y in terms of u or v	M1
	$y = \frac{u}{u-2} \Rightarrow v = \frac{\frac{u}{u-2}-4}{\frac{u}{u-2}-1}$	Obtains an equation connecting u and v	M1
		Any correct equation	A1
	$3u + 2v - 8 = 0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 3	Apply the transformation to any point on the imaginary axis	Eg $(0,0) \rightarrow (0,4)$ $(0,1) \rightarrow (4,-2)$	M1
	Apply the transformation to a second point on the imaginary axis	This is the second M mark on e-PEN	M1
	Both transformations correct	This is the first A mark on e-PEN	A1
	Complete method to obtain an equation for the line thro' their 2 points in the w -plane		M1
	Correct equation in any form		A1
	$3u + 2v - 8 = 0$	Correct equation in the required form (allow any integer multiple)	A1
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$(x+1)\frac{dy}{dx} - xy = e^{3x} \quad x > -1$		
	$\frac{dy}{dx} - \frac{xy}{(x+1)} = \frac{e^{3x}}{(x+1)}$	Correctly rearranged equation	B1
	$I = e^{\int \frac{-x}{x+1} dx} = e^{\int \left(-1 + \frac{1}{x+1}\right) dx}$	Correct strategy for the integrating factor including an attempt at the integration	M1
	$= e^{-x + \ln(x+1)}$	For $-x + \ln(x+1)$	A1
	$= (x+1)e^{-x}$	Correct integrating factor	A1
	$y(x+1)e^{-x} = \int \frac{e^{3x}}{x+1} \times (x+1)e^{-x} dx$	Uses their integrating factor to reach the form $yI = \int QI dx$	M1
	$y(x+1)e^{-x} = \frac{1}{2}e^{2x} + c$	Correct equation (with or without + c)	A1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{ce^x}{(x+1)}$	Correct answer (allow equivalent forms). Must have $y = \dots$	A1
			(7)
(b)	$x = 0, y = 5 \Rightarrow 5 = \frac{1}{2} + c \Rightarrow c = \frac{9}{2}$	Substitutes $x = 0$ and $y = 5$ and attempts to find a value for c .	M1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{9e^x}{2(x+1)}$	Cao (oe) Must have $y = \dots$	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3(a)	$x^2 \frac{dy}{dx} + xy = 2y^2 \quad y = \frac{1}{z}$		
	$\frac{dy}{dx} = -\frac{1}{z^2} \frac{dz}{dx}$	Correct differentiation	B1
	$-\frac{x^2}{z^2} \frac{dz}{dx} + \frac{x}{z} = \frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{dz}{dx} - \frac{z}{x} = -\frac{2}{x^2} *$	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
			(3)
(a) Way 2	$y = \frac{1}{z} \Rightarrow zy = 1 \Rightarrow y \frac{dz}{dx} + z \frac{dy}{dx} = 0$	Correct differentiation	B1
	$-\frac{y}{z} x^2 \frac{dz}{dx} + \frac{x}{z} = \frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{dz}{dx} - \frac{z}{x} = -\frac{2}{x^2} *$	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(a) Way 3	$y = \frac{1}{z} \Rightarrow z = \frac{1}{y} \Rightarrow \frac{dz}{dx} = -\frac{1}{y^2} \frac{dy}{dx}$	Correct differentiation	B1
	$-\frac{1}{y^2} \frac{dy}{dx} - \frac{1}{xy} = -\frac{2}{x^2}$	Substitutes into differential equation (II)	M1
	$x^2 \frac{dy}{dx} + xy = 2y^2$	Obtains differential equation (I) with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(b)	$I = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$	Correct integrating factor of $\frac{1}{x}$	B1
	$\frac{z}{x} = -\int \frac{2}{x^3} dx$	For $Iz = -\int \frac{2I}{x^2} dx$. Condone the “dx” missing.	M1
	$\frac{z}{x} = \frac{1}{x^2} + c$	Correct equation including constant	A1
	$z = \frac{1}{x} + cx$	Correct equation in the required form	A1
			(4)
(c)	$\frac{1}{y} = \frac{1}{x} + cx \Rightarrow -\frac{8}{3} = \frac{1}{3} + 3c \Rightarrow c = -1$	Reverses the substitution and uses the given conditions to find their constant	M1

	$\frac{1}{y} = \frac{1}{x} - x \Rightarrow y = \frac{x}{1-x^2}$	Correct equation for y in terms of x . Allow any correct equivalents e.g. $y = \frac{1}{x^{-1}-x}, y = \frac{1}{\frac{1}{x}-x}$	A1
			(2)
			Total 9



Question Number	Scheme	Marks
1(a)	$ z - 3 - 4i = z + 1 + i \Rightarrow x + iy - 3 - 4i = x + iy + 1 + i $ $(x - 3)^2 + (y - 4)^2 = (x + 1)^2 + (y + 1)^2$	M1
	$x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 + 2x + 1 + y^2 + 2y + 1 \Rightarrow ax + by + c = 0$	dM1
	$8x + 10y - 23 = 0$	A1
		(3)
(a) ALT	Mid-point $\left(\frac{3-1}{2}, \frac{4-1}{2}\right)$ and $m = \frac{4+1}{3+1}$	M1
	$y - \frac{3}{2} = -\frac{4}{5}(x - 1) \Rightarrow ax + by + c = 0$	dM1
	$8x + 10y - 23 = 0$	A1
(b)		B1
		(1)
		Total 4

Notes:

(a)

M1: Introduces $z = x + iy$ (or with other variables) and attempts a correct modulus method – the i's should be dealt with correctly but allow for e.g. minor slips in notation with them before applying Pythagoras. Allow sign slips in the i coordinates, e.g.

$(x - 3)^2 + (y \pm 4)^2 = (x + 1)^2 + (y \pm 1)^2$ Must be + between the brackets.

dM1: Expands and attempts the required form.

A1: This equation or any integer multiple.

Alternative: via perpendicular bisector

M1: Uses the points $(3, \pm 4)$ and $(-1, \pm 1)$ to find the midpoint and gradient between them.

dM1: Correct attempt at the equation of the perpendicular bisector and rearranges to the required form. Must be using negative reciprocal of the gradient between points (but watch for those who go directly to this).

A1: This equation or any integer multiple.

(b) May be scored independently of (a) – using the perpendicular bisector.

B1: Shades the area above an appropriate line, must be in quadrants 1, 2 and 4. Accept with solid, dashed or dotted line. Allow for any line with negative gradient and positive y intercept. Do not be concerned about labelling. Part (a) need not have been correct.



Question Number	Scheme	Marks
2(a)	$x \frac{dy}{dx} - y^3 = 4 \Rightarrow \frac{dy}{dx} + x \frac{d^2y}{dx^2} - 3y^2 \frac{dy}{dx} = 0$ $\text{Alt: } \frac{dy}{dx} = \frac{y^3}{x} + \frac{4}{x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{y^3}{x^2} + \frac{3y^2}{x} \frac{dy}{dx} - \frac{4}{x^2}$	M1A1
	$\Rightarrow \frac{d^2y}{dx^2} + x \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 6y \left(\frac{dy}{dx} \right)^2 - 3y^2 \frac{d^2y}{dx^2} = 0$ $\text{Alt: } \Rightarrow \frac{d^3y}{dx^3} = \frac{2y^3}{x^3} - \frac{3y^2}{x^2} \frac{dy}{dx} - \frac{3y^2}{x^2} \frac{dy}{dx} + \frac{1}{x} \left(6y \left(\frac{dy}{dx} \right)^2 + 3y^2 \frac{d^2y}{dx^2} \right) + \frac{8}{x^3}$	dM1
	$\Rightarrow x \frac{d^3y}{dx^3} = 6y \left(\frac{dy}{dx} \right)^2 + (3y^2 - 2) \frac{d^2y}{dx^2}$	A1
		(4)
(b)	$x = 2, y = 1 \Rightarrow \left(\frac{dy}{dx} \right)_{x=2} = \frac{4+1}{2} = \frac{5}{2}, \left(\frac{d^2y}{dx^2} \right)_{x=2} = \frac{3(1) \left(\frac{5}{2} \right) - \frac{5}{2}}{2} = \frac{5}{2}$ $\left(\frac{d^3y}{dx^3} \right)_{x=2} = \frac{6(1) \left(\frac{25}{4} \right) + \frac{5}{2}}{2} = 20$	M1
	$(y =) f(2) + (x-2)f'(2) + \frac{(x-2)^2}{2} f''(2) + \frac{(x-2)^3}{6} f'''(2) + \dots$ $= 1 + \frac{5}{2}(x-2) + \frac{5}{2} \frac{(x-2)^2}{2} + 20 \frac{(x-2)^3}{6} + \dots$	M1
	$(y =) 1 + \frac{5}{2}(x-2) + \frac{5}{4}(x-2)^2 + \frac{10}{3}(x-2)^3 + \dots$	A1
		(3)
		Total 7

Notes: Accept other derivative notations, e.g. y' , throughout.

(a)

M1: $x \frac{dy}{dx} \rightarrow \frac{dy}{dx} + x \frac{d^2y}{dx^2}$ or $y^3 \rightarrow ky^2 \frac{dy}{dx}$ Alternatively for $\frac{y^3}{x} \rightarrow \pm \alpha \frac{y^3}{x^2} + \frac{ky^2}{x} \frac{dy}{dx}$ (oe via quotient rule), or any correct attempt at implicit differentiation on the equation (coefficient slips condoned).

A1: Any fully correct second derivative expression. The two most common are shown in the scheme.

dM1: Differentiates again using the product rule correctly at least once. Depends on the first method mark. Again, look for correct forms, but allow if coefficients are incorrect.

There may be alternatives where they substitute for $\frac{dy}{dx}$ before differentiating again.

The scheme follows the same principle. If unsure, use the Review system.

A1: Correct answer achieved from correct work. In the Alt they need to substitute the second derivative expression before reaching the final answer.

(b)

M1: Attempts to find values for the first 3 derivatives at $x = 2$. Working need not be seen, in which case accept if all are attempted with at least y' correct providing expressions involving the other derivatives were actually found in part (a).

M1: Correct application of Taylor's theorem with their values up to at least the $(x-2)^2$ term.

A1: Correct simplified expression. May be called y or $f(x)$ or unlabelled for this part.



Question Number	Scheme	Marks
8(a)	$v = y^{-2} \quad \frac{dv}{dy} = -2y^{-3}$ $\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = -\frac{y^3}{2} \frac{dv}{dx}$ $-\frac{y^3}{2} \frac{dv}{dx} + 6xy = 3xe^{x^2} y^3$ $\frac{1}{2} \frac{dv}{dx} - \frac{6xy}{y^3} = -3xe^{x^2}$ $\frac{dv}{dx} - 12vx = -6xe^{x^2} \quad *$	B1 M1A1 dM1A1* (5)
ALT 1	$y = v^{\frac{1}{2}} \quad \frac{dy}{dv} = \frac{1}{2} v^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx} = \frac{1}{2} v^{-\frac{1}{2}} \frac{dv}{dx}$ $-\frac{1}{2} v^{-\frac{1}{2}} \frac{dv}{dx} + 6xv^{\frac{1}{2}} = 3xe^{x^2} v^{\frac{3}{2}}$ $-\frac{1}{2} \frac{dv}{dx} + 6xv = 3xe^{x^2}$ $\frac{dv}{dx} - 12vx = -6xe^{x^2} \quad *$	B1 M1A1 dM1 A1* (5)
ALT 2	$v = y^{-2} \quad \frac{dv}{dy} = -2y^{-3}$ $\frac{dv}{dx} = \frac{dv}{dy} \times \frac{dy}{dx} = -2y^{-3} \frac{dy}{dx}$ $-2y^{-3} \frac{dy}{dx} - 12y^{-2}x = -6xe^{x^2}$ $\frac{dy}{dx} + 6xy = 3xe^{x^2} y^3 \quad x > 0$	B1 M1A1 dM1 A1* (5)
8(a) B1 M1 A1 dM1 A1*	All Methods: Correct derivative Attempt $\frac{dy}{dx}$ or $\frac{dv}{dx}$ using the chain rule Correct derivative Substitute in equation (I) to obtain an equation in v and x only OR in equation (II) to obtain an equation in x and y only (ALT 2) Correct completion with no errors seen	



Question Number	Scheme	Marks
8(b)	IF: $e^{\int -12x dx} = e^{-6x^2}$ $ve^{-6x^2} = \int -6xe^{x^2} \times (e^{-6x^2}) dx = \int -6xe^{-5x^2} dx$ $ve^{-6x^2} = \frac{6}{10}e^{-5x^2} (+c)$ $v(=y^{-2}) = \frac{6}{10}e^{x^2} + ce^{6x^2}$ $y^2 = \frac{1}{\frac{6}{10}e^{x^2} + ce^{6x^2}}$ oe eg $y^2 = \frac{10}{6e^{x^2} + ke^{6x^2}}$	M1A1 dM1 A1 ddM1 A1 (6) [11]
(b) M1 A1 dM1 A1 ddM1 A1	IF of form $e^{\int \pm 12x dx}$ and attempt the integration. Correct IF Multiply through by their IF and integrate the LHS. Depends on first M mark of (b) Correct integration of the complete equation with or without constant Include the constant and multiply through by e^{6x^2} Depends on both previous M marks of (b) Any equivalent to that shown. (No need to change letter used for constant when rearranging)	

Question Number	Scheme	Marks
6(a)	$y = \sec x \Rightarrow \frac{dy}{dx} = \sec x \tan x$	M1
	$\frac{d^2y}{dx^2} = (\sec x \tan x) \tan x + \sec x (\sec^2 x) (= \sec x \tan^2 x + \sec^3 x)$	
	$\frac{d^2y}{dx^2} = \sec x (\sec^2 x - 1) + \sec^3 x = 2\sec^3 x - \sec x$ $\Rightarrow \frac{d^3y}{dx^3} = 6\sec^2 x (\sec x \tan x) - \sec x \tan x$	dM1
	or $\frac{d^3y}{dx^3} = 2\sec x \tan x \sec^2 x + \tan^2 x \sec x \tan x + 3\sec^2 x \sec x \tan x$	
	$\left(\frac{d^3y}{dx^3} = 5\sec^3 x \tan x + \sec x \tan x (\sec^2 x - 1) = 6\sec^3 x \tan x - \sec x \tan x \right)$ $\Rightarrow \frac{d^3y}{dx^3} = \sec x \tan x (6\sec^2 x - 1)$	ddM1 A1
		(4)
(b)	$\sec\left(\frac{\pi}{3}\right) = 2, \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \Rightarrow f\left(\frac{\pi}{3}\right) = 2, f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}, f''\left(\frac{\pi}{3}\right) = 14, f'''\left(\frac{\pi}{3}\right) = 46\sqrt{3}$	M1
	$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \dots$ $\Rightarrow 2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2 + \frac{23\sqrt{3}}{3}\left(x - \frac{\pi}{3}\right)^3$	dM1 A1
		(3)
(c)	$\left(\sec \frac{7\pi}{24} \approx\right) = 2 + 2\sqrt{3}\left(-\frac{\pi}{24}\right) + 7\left(-\frac{\pi}{24}\right)^2 + \frac{23\sqrt{3}}{3}\left(-\frac{\pi}{24}\right)^3 = \dots$	M1
	$\sec \frac{7\pi}{24} \approx 1.636709263... \approx 1.637$	A1
		(2)
		Total 9

Notes**(a)**

M1: Differentiates twice and obtains expression for second derivative allowing for sign errors only

dM1: **Dependent on previous M mark.** Differentiates again and obtains expression for third derivative allowing for sign errors only (which could come from incorrect signs in trig identities)

ddM1: **Dependent on both previous M marks.** Obtains an answer of the correct form or values for p and q .

Allow slips from their third derivative but any trig identities used can only have errors in sign.

A1: Fully correct expression or values for p and q

Note: If they decide not to work in $\sec x$ and $\tan x$ the main scheme still applies – see Alt next page.

Allow “meet in the middle” approaches that determine the values of p and q

(b)

M1: Uses $\sec x$ and their three derivatives to attempt to find values for $f\left(\frac{\pi}{3}\right)$, $f'\left(\frac{\pi}{3}\right)$, $f''\left(\frac{\pi}{3}\right)$ and $f'''\left(\frac{\pi}{3}\right)$

evidenced by values for each with at least 2 correct if no method seen. May be recovered by substitution of values into formula if not all listed.

dM1: **Dependent on previous M mark.** Uses a correct Taylor series with all four of their values. Series must be correct for their values but allow slips if a correct formula is quoted.

A1: Correct series with coefficients in a simplest form e.g., allow $\sqrt{12}$ for $2\sqrt{3}$, $\frac{23}{\sqrt{3}}$ for $\frac{23}{3}\sqrt{3}$ Condone absence of $\sec x = \dots$ or $y = \dots$ Must have had a correct third derivative in (a), though need not have been simplified to the form required (and allow if there was a correct third derivative which was incorrectly simplified if the correct answer is found).

(c)

M1: Shows evidence of substitution of $\frac{7\pi}{24}$ into their series of the right form (powers of $\left(x - \frac{\pi}{3}\right)$). If only a value is given score M0 unless it is the correct 4 s.f. value for their series (1.636709263... allowing awrt 1.637 if (b) is correct). You may need to check the answer.

A1: 1.637 only (not awrt) from a correct series.

Note that $\sec \frac{7\pi}{24} = 1.642679632\dots$

6(a) Alt	$y = \sec x \Rightarrow y = (\cos x)^{-1} \Rightarrow \frac{dy}{dx} = -(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x}$ $\frac{d^2 y}{dx^2} = \frac{\cos^2 x \cos x - 2 \sin x \cos x(-\sin x)}{\cos^4 x}$	M1
	$\frac{d^2 y}{dx^2} = \frac{\cos^3 x + 2 \cos x(1 - \cos^2 x)}{\cos^4 x} = \frac{2 \cos x - \cos^3 x}{\cos^4 x} = 2(\cos x)^{-3} - (\cos x)^{-1}$ $\Rightarrow \frac{d^3 y}{dx^3} = -6(\cos x)^{-4}(-\sin x) + (\cos x)^{-2}(-\sin x)$	ddM1
	$\frac{d^3 y}{dx^3} = \frac{6 \sin x}{\cos^4 x} - \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x} \left(\frac{6}{\cos^2 x} - 1 \right) = \sec x \tan x (6 \sec^2 x - 1)$	ddM1 A1
		(4)
Notes		
Mark as per main scheme allowing sign slips only - forms correct at each stage.		

Question Number	Scheme	Marks
7(a)	$z = y^{-2} \Rightarrow \frac{dz}{dy} = -2y^{-3} \text{ or } 1 = -2y^{-3} \frac{dy}{dz}$ $y^2 = z^{-1} \Rightarrow 2y \frac{dy}{dz} = -z^{-2}$ $y = z^{-\frac{1}{2}} \Rightarrow \frac{dy}{dz} = -\frac{1}{2} z^{-\frac{3}{2}}$	B1
	$\frac{dy}{dx} = \frac{dz}{dx} \cdot \frac{dy}{dz} \text{ (oe)} \Rightarrow \text{e.g., } \frac{dy}{dx} = -\frac{1}{2} y^3 \frac{dz}{dx}, \frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$	M1 A1
	$x \frac{dy}{dx} + y + 4x^2 y^3 \ln x = 0 \Rightarrow \text{e.g.,}$ $-\frac{1}{2} xy^3 \frac{dz}{dx} + y + 4x^2 y^3 \ln x = 0 \quad -\frac{1}{2} xz^{-\frac{3}{2}} \frac{dz}{dx} + z^{-\frac{1}{2}} + 4x^2 z^{-\frac{3}{2}} \ln x = 0$	dM1
	$\frac{dz}{dx} - \frac{2}{xy^2} - 8x \ln x = 0 \quad \frac{dz}{dx} - \frac{2z}{x} - 8x \ln x = 0$ $\Rightarrow \frac{dz}{dx} - \frac{2z}{x} = 8x \ln x \quad *$	A1*
		(5)
(b)	$(\text{IF}) = e^{\int -\frac{2}{x} (dx)}$	M1
	$= e^{-2 \ln x} \quad (= x^{-2})$	A1
	$x^{-2} z = \int x^{-2} (8x \ln x) [dx]$	M1
	E.g. Parts: $\int x^{-1} \ln x dx$: $(u = \ln x, u' = x^{-1}, v' = x^{-1}, v = \ln x)$ $\Rightarrow I = (\ln x)^2 - kI \Rightarrow I = p(\ln x)^2$ Or substitution: $t = \ln x, \frac{dt}{dx} = x^{-1} \Rightarrow I = k \int x^{-1} \ln x \cdot x dt = k \int t dt = pt^2$	M1
	$\int x^{-1} \ln x dx = \frac{1}{2} (\ln x)^2 [+c]$	A1
	$x^{-2} z = 4(\ln x)^2 + k \Rightarrow z = 4x^2 (\ln x)^2 + kx^2$ $\Rightarrow y^2 = \frac{1}{4x^2 (\ln x)^2 + kx^2} \text{ oe}$	A1
		(6)
		Total 11
Notes		
(a) B1: Any correct equation following differentiation of the given substitution. Could be implied. M1: Uses a correct chain rule to obtain an equation linking $\frac{dy}{dx}$ (or $\frac{dx}{dy}$) and $\frac{dz}{dx}$ (or $\frac{dx}{dz}$) A1: Any correct equation Note that the first three marks could be scored by a first step of, e.g.: $\frac{dz}{dx} = -2y^{-3} \frac{dy}{dx} \text{ or } 2y \frac{dy}{dx} = -z^{-2} \frac{dz}{dx} \text{ or } \frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$ dM1: Dependent on previous M mark. Substitutes their $\frac{dy}{dx}$ into differential equation (I). Need not replace fully for this mark.		

Question Number	Scheme	Notes	Marks
2	$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y^2 = e^{x^2-9}$		
	$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 10y\frac{dy}{dx} = 2xe^{x^2-9}$	M1: Differentiates to obtain three correct terms A1: Fully correct differentiated equation	M1 A1
	$y(3) = 2, y'(3) = -1$ $y''(3) - 4(-1) - 5 \times 2^2 = 1 \Rightarrow y''(3) = 17$ $y'''(3) - 4(17) - 10(2)(-1) = 6 \Rightarrow y'''(3) = 54$	Uses the given equation and their equation to obtain values for $y''(3)$ and $y'''(3)$	M1
	$(y =) y(3) + y'(3)(x-3) + \frac{y''(3)}{2}(x-3)^2 + \frac{y'''(3)}{3!}(x-3)^3 \Rightarrow$ $2 - 1(x-3) + \frac{17}{2}(x-3)^2 + \frac{54}{3!}(x-3)^3$	A correct unsimplified Taylor series for their $y''(3)$ and $y'''(3)$	M1
	$(y =) 2 - (x-3) + \frac{17}{2}(x-3)^2 + 9(x-3)^3$	Correct simplified series. "y = ..." is not required	A1
			(5)
			Total 5

Additional Notes

M1: Attempts to differentiate using the chain rule at least once – at least three terms correct, allowing for slips in rearranging before differentiation.

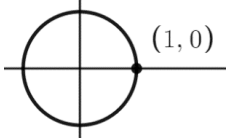
A1: Any correct third derivative expression. They may have rearranged first.

M1: Finds the values of the second and third derivatives using the given equation and their third derivative expression (which must have at least three terms). Condone one slip in substitutions as long as evidence of the correct values used is seen. If substitution is not seen, correct values can imply the method.

M1: Applies the Taylor Series formula correctly with the given values of y and $\frac{dy}{dx}$ and with their values for

the second and third derivatives – the first two terms must be correct. If the values of their second and third derivatives are not explicitly stated but embedded in the series, it must be clear that a correct attempt at the series has been used – the denominators 2 and 3! (or 6) must be seen – but their values need not be correct. Condone a miscopy of one of **their** values if it is clearly a correct attempt to apply their value to the series.

A1: Correct fully simplified series expansion. The $y =$ is not required and condone if $f(x)$ is used.

Question Number	Scheme	Notes	Marks
3(a)		Circle centred at origin with radius of 1 correctly indicated in any way. May use $\pm i$ or ± 1 in appropriate position on imaginary axis.	B1
(1)			
(b)	$w = \frac{9iz - i}{z + 1}$ $\Rightarrow wz + w = 9iz - i \Rightarrow z(w - 9i) = -w - i$ $\Rightarrow z = \frac{-w - i}{w - 9i} = \frac{w + i}{9i - w} \text{ oe}$	M1: Makes z the subject A1: Correct expression oe	M1 A1
	$ z = \left \frac{-w - i}{w - 9i} \right \Rightarrow w - 9i = w - (-i) $	Uses $ z = 1$ and obtains $ \pm w \pm z_1 = \pm w \pm z_2 $ Requires previous M mark.	dm1
	$\Rightarrow v = "4" \quad \text{or} \quad \begin{aligned} u^2 + (v - 9)^2 &= u^2 + (v + 1)^2 \\ -18v + 81 &= 2v + 1 \\ 20v &= 80 \\ v &= 4 \end{aligned}$	Obtains $v = k$ with k correct for their $ w - pi = w - qi $ oe or substitutes $u + iv$ for w , applies Pythagoras correctly and obtains a value for v Allow " $y = \dots$ " for this mark. Condone $v = 4i$ for this mark. Requires both previous M marks.	ddM1
	$v = 4$ only	Correct equation	A1
(5)			
Total 6			
Alt 1: using points on $z = 1$	$z = 1 \Rightarrow w = \frac{9i - i}{1 + 1}, \quad z = i \Rightarrow w = \frac{9i^2 - i}{i + 1}$	M1: Uses two correct values of z in the formula to obtain two expressions for w A1: Both expressions for w correct	M1 A1
	$w = 4i, \quad w = \frac{-i - 9}{1 + i} \times \frac{1 - i}{1 - i} = \frac{-10 + 8i}{2} = -5 + 4i$	Simplifies expressions to $a + bi$ form including correct method to rationalise denominators. Requires previous M mark.	dm1
	$\Rightarrow v = "4"$	Obtains $v = b$ with b correct for their $a + bi$ forms. Allow " $y = \dots$ " for this mark. Condone $v = 4i$ for this mark. Requires both previous M marks.	ddM1
	$v = 4$ only	Correct equation	A1
(5)			
Alt 2: using $u + iv$	$w = \frac{9iz - i}{z + 1}$ $\Rightarrow wz + w = 9iz - i \Rightarrow z(w - 9i) = -w - i$	M1: Makes z the subject A1: Correct expression oe	M1 A1

	$\Rightarrow z = \frac{-w-i}{w-9i} = \frac{w+i}{9i-w}$ oe		
	$\Rightarrow z = \frac{-u-iv-i}{u+iv-9i} \times \frac{u-(v-9)i}{u-(v-9)i}$ $= \frac{-u^2-(v+1)(v-9)-10ui}{u^2+(v-9)^2}$ $\Rightarrow x = -\frac{u^2+(v+1)(v-9)}{u^2+(v-9)^2} \quad y = -\frac{10u}{u^2+(v-9)^2}$	Replaces w with $u + vi$ and rationalises the denominator and extracts real and imaginary parts	dM1
	$x^2 + y^2 = 1 \Rightarrow \frac{(u^2+(v+1)(v-9))^2 + (10u)^2}{(u^2+(v-9)^2)^2} = 1$ $\Rightarrow \dots \Rightarrow (v-4)(u^2+v^2-18v+81) = 0$	Applies $x^2 + y^2 = 1$ and simplifies to identify a factor $v-k=0$ Same conditions as main scheme.	ddM1
	$v = 4$ only	Correct equation	A1
(5)			
Alt 3: using $x+iy$	$w = \frac{9i(x+iy)-i}{x+iy+1} \times \frac{x+1-iy}{x+1-iy} = \dots$ $= \frac{-9y(x+1) + (9x-1)y + ((9x-1)(x+1) + 9y^2)i}{(x+1)^2 + y^2}$	M1: Replaces z by $x + iy$ and rationalises the denominator. A1: Correct expression oe	M1 A1
	$= \frac{-10y + (9x^2 + 8x - 1 + 9y^2)i}{1 + 2x + x^2 + y^2}$ $= \frac{-10y + (8x - 1 + 9)i}{1 + 2x + 1} \left[= \frac{-5y + 4(x+1)i}{x+1} \right]$	Expands and applies $x^2 + y^2 = 1$ to simplify to linear coefficients	dM1
	$= \frac{-5y}{x+1} + 4i \Rightarrow v = 4$	Simplifies to a constant imaginary term and deduces the equation. Same conditions as main scheme.	ddM1
	$v = 4$ only	Correct equation	A1
(5)			

Additional Notes

(a)

B1: Draws the correct circle, allow tolerance with shape, look for the intent, and indicates the radius is 1. Accept ± 1 on the imaginary axis or incorrect notation of coordinates as long as the radius of 1 is implied. May be drawn as the radius. Accept $r=1$ stated next to the diagram, but not $x^2 + y^2 = 1$ without further indication of the radius being 1.

(b)

Note for the main scheme ddM1 first deduction is from using the perpendicular bisector of $9i$ and $-i$.

Question Number	Scheme	Notes	Marks
4	$4\frac{d^2y}{dx^2} - 11\frac{dy}{dx} - 3y = 78e^{3x}$ $y = \frac{9}{2}$, $\frac{dy}{dx} = 0$ at $x = 0$		
(a)	$y = \lambda xe^{3x} \Rightarrow$ $y' = \lambda e^{3x} + 3\lambda xe^{3x}$, $y'' = 3\lambda e^{3x} + 3\lambda e^{3x} + 9\lambda xe^{3x}$	Correct first and second derivatives (need not be simplified).	B1
	$4(6\lambda e^{3x} + 9\lambda xe^{3x}) - 11(\lambda e^{3x} + 3\lambda xe^{3x}) - 3(\lambda xe^{3x}) = 78e^{3x}$ $\Rightarrow 13\lambda e^{3x} = 78e^{3x} \Rightarrow \lambda = 6$	M1: Substitutes their attempts at the derivatives of form $\alpha e^{3x} + \beta xe^{3x}$ into the differential equation and solves for λ A1: Correct value from fully correct	M1 A1cso
(3)			
(b)	$4m^2 - 11m - 3 = 0 \Rightarrow (4m+1)(m-3) = 0$ $\Rightarrow m = -\frac{1}{4}, 3$	Forms auxiliary equation and solves (usual rules)	M1
	CF: $y = Ae^{-\frac{1}{4}x} + Be^{3x}$	For $Ae^{m_1x} + Be^{m_2x}$ with their real and different values of m	M1
	$y = Ae^{-\frac{1}{4}x} + Be^{3x} + "6"xe^{3x}$	Correct GS ft their non-zero λ including “ $y = \dots$ ”	A1ft
(3)			
(c)	$\frac{dy}{dx} = -\frac{1}{4}Ae^{-\frac{1}{4}x} + 3Be^{3x} + 6e^{3x} + 18xe^{3x}$ or ft	M1: Differentiates to obtain an expression of the correct form A1: Correct ft expression	M1 A1ft
	$x = 0, y = \frac{9}{2} \Rightarrow \frac{9}{2} = A + B$ $x = 0, \frac{dy}{dx} = 0 \Rightarrow 0 = -\frac{1}{4}A + 3B + 6$ $\Rightarrow A = \dots, B = \dots$	Substitutes the given values into their equations and solves simultaneously for A and B	M1
	$y = 6e^{-\frac{1}{4}x} - \frac{3}{2}e^{3x} + 6xe^{3x}$	Fully correct particular solution (including “ $y = \dots$ ” unless this was penalised in part (b))	A1
(4)			
Total 10			

Additional Notes

You may mark this question as a whole. Do not be concerned about part labelling as many will answer out of order, e.g. finding CF before PI.

(a) **Note: This appears on ePEN as MMA but we are marking it as BMA**

B1: Correct first and second derivatives.

M1: Full process to find the constant. The derivatives need not have been correct for this mark but must have form $\alpha e^{3x} + \beta xe^{3x}$ ($\alpha, \beta \neq 0$), look for the correct process.

A1cso: Correct value from fully correct work. The xe^{3x} must have cancelled in their equation giving only $\lambda = 6$.

(b)

M1: Correct attempt to solve the auxiliary equation. If no method is shown the correct values can imply the attempt.

M1: Correct CF for their values, which must be real and distinct.

A1: Correct solution following through their value for λ . Must include the $y = \dots$.



(c)

M1: Differentiates to correct form $Pe^{\frac{x}{4}} + Qe^{3x} + Rxe^{3x} + "6"e^{3x}$ ($P, Q, R \neq 0$), condoning at most one slip in powers. Must involve the product rule.

A1ft: Correct expression following through on their value for “6”.

M1: For a full process to set up and solve simultaneous equations in A and B (or their variables) using the given values. Do not be concerned about algebraic slips, as long as two equations are set up then allow for any values following them.

A1: Fully correct, but only penalise a missing $y = \dots$ in the first instance it would otherwise gain a mark. Allow from solutions which gained $\lambda = 6$ in (a) even if from incorrect work.

Question Number	Scheme	Notes	Marks
5(a)	$\frac{2}{r(r+1)(r+2)} = \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$ $2 = A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ $r=0: 2=2A \Rightarrow A=1$ $r=-1: 2=-B \Rightarrow B=-2$ $r=-2: 2=2C \Rightarrow C=1$	Any valid method to obtain at least one constant with partial fractions of the correct form. Could be implied	M1
	$\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}$	Correct partial fractions	A1
(2)			
(b)	$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \begin{cases} \frac{1}{1} - \frac{2}{2} + \frac{1}{3} + \\ \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \\ \frac{1}{3} - \frac{2}{4} + \frac{1}{5} + \\ \dots + \dots \\ \frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1} + \\ \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} \end{cases}$	Attempts at least three rows with their partial fractions of a correct form (see notes). Implied by correct expression	M1
	$= 1 - 1 + \frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$	A1: Correct constant terms or algebraic terms extracted A1: Fully correct expression	A1 A1
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2} = \frac{(n+1)(n+2) - 2(n+2) + 2(n+1)}{2(n+1)(n+2)}$	Combines terms of the form p , $\frac{q}{n+b}$, and $\frac{r}{n+c}$ and no others into a single fraction	M1
	$= \frac{n^2 + 3n + 2 - 2n - 4 + 2n + 2}{2(n+1)(n+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$	Correct expression	A1
(5)			
(c)	$\frac{n(n+3)}{2(n+1)(n+2)} > \frac{7}{15}$ $15n^2 + 45n > 14n^2 + 42n + 28$	Forms inequality or equation, cross-multiplies or puts over common denominator and expands brackets	M1
	$n^2 + 3n - 28 > 0$ $(n+7)(n-4) > 0$ $\Rightarrow n = \dots \text{ or } n > \dots \quad (n > 4)$	Forms and solves 3TQ inequality or equation (usual rules)	dM1
	$n = 5$	5 only	A1
(3)			
Total 10			

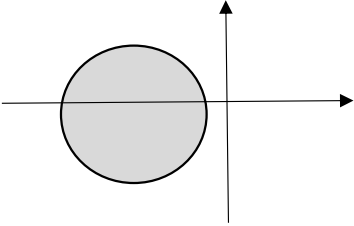
Additional Note

(a)

M1: Must be correct partial fraction form with three fractions. Note that $\frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ is not a complete partial fraction form so is M0, but this form can be used to score full marks in part (b).
A1: Correct form.



Question Number	Scheme	Notes	Marks
7(a) Way 2	$w = \frac{z-3}{2i-z} = \frac{x+iy-3}{2i-x-iy} = \frac{x-3+i(x+3)}{2i-x-i(x+3)}$ [Note that it is possible to replace x with $y-3$]	M1: Uses $z = x + iy$ and $y = x + 3$ in the given transformation A1: Correct expression for w in terms of x	M1 A1
	$\frac{x-3+i(x+3)}{-x-i(x+1)} = u+iv \Rightarrow x-3+i(x+3) = -xu+v(x+1)-iu(x+1)-ivx$	Applies $w = u + iv$ and multiplies	M1
	$x-3 = -ux+vx+v, \quad x+3 = -ux-u-vx$ $x = \frac{3+v}{1+u-v}, \quad x = \frac{-3-u}{1+u+v}$	Equates real and imaginary parts and makes x the subject twice	M1
	$3+3u+3v+v+uv+v^2 = -3-3u+3v-u-u^2+uv$ $\Rightarrow u^2+v^2+7u+v+6=0$	Equates expressions for x to obtain a circle equation with 4 or 5 real unlike terms.	dddM1
	$\Rightarrow \left(u+\frac{7}{2}\right)^2 + \left(v+\frac{1}{2}\right)^2 = \frac{49}{4} + \frac{1}{4} - 6 = \frac{13}{2} \Rightarrow \text{centre: } \left(-\frac{7}{2}, -\frac{1}{2}\right) \text{ radius: } \frac{\sqrt{26}}{2} \text{ or } \sqrt{\frac{13}{2}}$ M1: Applies a correct process to extract the centre and/or radius from a circle equation, however obtained, with 4 or 5 real unlike terms. One correct coordinate (but condone wrong sign) or radius correct for their circle. May use $u^2+v^2+2gu+2fv+c=0 \Rightarrow \text{centre: } (-g, -f), \text{ radius} = \sqrt{g^2+f^2-c}$ A1: For correct centre or radius from a correct circle equation A1: For correct centre and radius from a correct circle equation Centre as coordinates, $x/u=..., y/v=...$ or as $-\frac{7}{2}-\frac{1}{2}i$ and allow $(-\frac{7}{2}, -\frac{1}{2}i)$ (8)		M1 A1 A1
Way 3	e.g., 3 points on line are (0,3), (1,4) and (2,5) or $z_1 = 3i, z_2 = 1+4i, z_3 = 2+5i$	Attempts three points/complex numbers on $y = x + 3$ with 2 correct	M1
	$w = \frac{z-3}{2i-z} \Rightarrow w_1 = \frac{3i-3}{-i} \quad w_2 = \frac{-2+4i}{-1-2i} \quad w_3 = \frac{-1+5i}{-2-3i}$	Correct transformed complex numbers	A1
	$w_1 = \frac{3i-3}{-i} \times \frac{i}{i} \quad w_2 = \frac{-2+4i}{-1-2i} \times \frac{-1+2i}{-1+2i} \quad w_3 = \frac{-1+5i}{-2-3i} \times \frac{-2+3i}{-2+3i}$ At least two correct multipliers to remove “i” from denominator seen or implied (one if $(-1, 2)$ used). Requires 2 correct points/complex numbers on line		M1
	$w_1 = -3-3i \quad w_2 = -\frac{6}{5}-\frac{8}{5}i \quad w_3 = -1-i$	Two correct complex numbers in $a + ib$ form or as points	M1
	$\text{e.g., } x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow \frac{12}{5}g + \frac{16}{5}f - c = 0$ $2g + 2f - c = 0$	Uses a correct general equation of a circle to form three simultaneous equations.	dddM1
	$\Rightarrow g = \frac{7}{2}, f = \frac{1}{2}, c = 6 \Rightarrow \text{centre } (-g, -f): \left(-\frac{7}{2}, -\frac{1}{2}\right) \text{ radius} = \sqrt{g^2+f^2-c} = \frac{\sqrt{26}}{2} \text{ or } \sqrt{\frac{13}{2}}$ M1: Solves and obtains at least one correct coordinate (but condone wrong sign) or radius for their constants A1: Correct centre or radius from correct work A1: Correct centre and radius from correct work (8)		M1 A1 A1

Question Number	Scheme	Notes	Marks
7(b) (i) & (ii)		M1: Any circle with the whole interior indicated. Ignore any inconsistencies with their stated centre, value for radius (which may have been negative) or circle equation. If shaded, consider the shaded area but if not allow any credible indication such as an “R” inside the circle unless they have clearly indicated a segment. A1: Correct circle drawn in the correct position with whole interior shaded. Entirely in quadrants 2 & 3 and centre if marked in Q3 (if not marked then more than half of the circle in Q3). Condone if it appears that the area above the x-axis is greater than the area below provided the centre is indicated in Q3. Must be shaded but does not require a label. Circumference may be dotted/dashed line. Ignore incorrect labelling of centre/axes/intersections but requires full marks in (a).	M1 (B1 on ePen) A1 (B1 on ePen)
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
8(a)	Allow “single fraction” to be implied by sum/difference of fractions with same denominator or a product of fractions. No further fractions in numerator/denominator.		

	$\cot 2x \{ + \tan x \} = \frac{\cos 2x}{\sin 2x} \left\{ + \frac{\sin x}{\cos x} \right\}$	Uses $\cot 2x = \frac{\cos 2x}{\sin 2x}$ or e.g., $\frac{\cos 2x}{2 \sin x \cos x}$	M1
	$\frac{\cos 2x + 2 \sin^2 x}{2 \sin x \cos x} \Rightarrow$ $\text{e.g., } \frac{1 - 2 \sin^2 x + 2 \sin^2 x}{2 \sin x \cos x} \text{ or } \frac{\cos^2 x - \sin^2 x + 2 \sin^2 x}{2 \sin x \cos x}$ $\frac{2 \cos^2 x - 1 + 2 \sin^2 x}{\sin 2x} \text{ or } \frac{\cos 2x + 1 - \cos 2x}{\sin 2x}$ $\text{OR } \frac{\cos 2x + \tan x \sin 2x}{\sin 2x} \Rightarrow$ $\Rightarrow \frac{\cos 2x + \frac{\sin x}{\cos x} \times 2 \sin x \cos x}{\sin 2x} \Rightarrow \text{e.g., } \frac{1 - 2 \sin^2 x + 2 \sin^2 x}{\sin 2x}$ $\text{OR } \frac{\cos 2x \cos x + \sin x \sin 2x}{\sin 2x \cos x} \Rightarrow$ $\frac{\cos x}{\sin 2x \cos x} \text{ or } \frac{\cos^3 x - \sin^2 x \cos x + 2 \sin^2 x \cos x}{\sin 2x \cos x}$	Uses sufficient correct identities e.g., $\cos 2x = 1 - 2 \sin^2 x$ $\cos 2x = \cos^2 x - \sin^2 x$ $\cos 2x = 2 \cos^2 x - 1$ $2 \sin^2 x = 1 - \cos 2x$ $\cos 2x \cos x + \sin x \sin 2x = \cos(2x - x)$ to obtain a correct single fraction with numerator in terms of $\sin x$ and/or $\cos x$ or “$\cos 2x + 1 - \cos 2x$”. A qualifying fraction must be seen before $\frac{1}{2 \sin x \cos x} \text{ or } \frac{1}{\sin 2x}$ Condone poor notation.	A1 (M1 on ePen)
	$= \frac{1}{2 \sin x \cos x} \text{ or } \frac{1}{\sin 2x} = \operatorname{cosec} 2x^*$	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. $\sin x^2$ for this mark.	A1*
(3)			
Alt	$\cot 2x \{ + \tan x \} = \frac{1 - \tan^2 x}{2 \tan x} \{ + \tan x \}$	Uses $\cot 2x = \frac{1 - \tan^2 x}{2 \tan x}$	M1
	$\frac{1 - \tan^2 x + 2 \tan^2 x}{2 \tan x} \Rightarrow$ $\text{e.g., } \frac{\tan^2 x + 1}{2 \tan x} \Rightarrow \frac{\left(\frac{\sin x}{\cos x}\right)^2 + 1}{2 \frac{\sin x}{\cos x}} \Rightarrow \frac{\cos x (\sin^2 x + \cos^2 x)}{2 \cos^2 x \sin x}$ $\text{or } \frac{\tan^2 x + 1}{2 \tan x} \left\{ \times \frac{\cos x}{\cos x} \right\} \Rightarrow \frac{\sin^2 x + \cos^2 x}{2 \sin x \cos x}$ $\text{or } \frac{\sec^2 x}{2 \tan x} \text{ or } \frac{\cos x}{2 \cos^2 x \sin x}$	Uses correct identities e.g., $\tan x = \frac{\sin x}{\cos x}$ or to obtain a correct single fraction in $\sin x$ and $\cos x$ but allow $\frac{\sec^2 x}{2 \tan x}$ following use of $\sec^2 x = 1 + \tan^2 x$ A qualifying fraction must be seen before $\frac{1}{2 \sin x \cos x} \text{ or } \frac{1}{\sin 2x}$ Condone poor notation.	A1 (M1 on ePen)
	$\frac{1}{2 \sin x \cos x} \text{ or } \frac{1}{\sin 2x} = \operatorname{cosec} 2x^*$	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. $\sin x^2$ for this mark.	A1*
(3)			

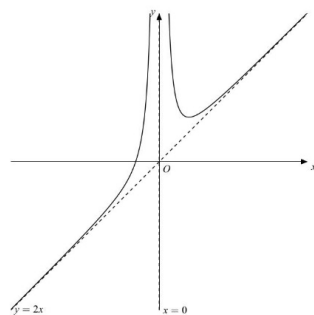
Question Number	Scheme	Notes	Marks
8(b)	Examples:		M1 A1



	$y^2 = w \sin 2x \Rightarrow 2y \frac{dy}{dx} = \frac{dw}{dx} \sin 2x + 2w \cos 2x$ $\text{or } y = w^{\frac{1}{2}} (\sin 2x)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2} w^{-\frac{1}{2}} (\sin 2x)^{-\frac{1}{2}} (2 \cos 2x) + \frac{1}{2} w^{\frac{1}{2}} \frac{dw}{dx} (\sin 2x)^{-\frac{1}{2}}$ $\text{or } w = \frac{y^2}{\sin 2x} \Rightarrow \frac{dw}{dx} = \frac{2y \sin 2x \frac{dy}{dx} - y^2 \cdot 2 \cos 2x}{\sin^2 2x}$ $\text{or } w = y^2 \operatorname{cosec} 2x \Rightarrow 2y \frac{dy}{dx} \operatorname{cosec} 2x - 2y^2 \operatorname{cosec} 2x \cot 2x$ M1: Attempts the differentiation of the given substitution using the product/quotient and chain rules and obtains an equation in $\frac{dy}{dx}$ and $\frac{dw}{dx}$ of the correct form (sign/coefficient errors only and allow sign errors with quotient/product rule). This mark is not available for work in $\frac{dy}{dw}$ or $\frac{dw}{dy}$ unless appropriate work follows to achieve an equation in $\frac{dy}{dx}$ and $\frac{dw}{dx}$ of the correct form. A1: Correct differentiation	
	$y \frac{dy}{dx} + y^2 \tan x = \sin x \rightarrow \text{e.g., } \frac{1}{2} \left(\frac{dw}{dx} \sin 2x + 2w \cos 2x \right) + w \sin 2x \tan x = \sin x$ A recognisable attempt to eliminate y from the original equation to obtain an equation involving $\frac{dw}{dx}$, w and x only. Not dependent.	M1
	$\Rightarrow \frac{dw}{dx} + 2w (\cot 2x + \tan x) = \frac{2 \sin x}{\sin 2x}$ $\Rightarrow \frac{dw}{dx} + 2w \operatorname{cosec} 2x = \sec x *$ Fully correct work leading to the given equation with $2w (\cot 2x + \tan x)$ or e.g., $2w \cot 2x + 2w \tan x$ clearly replaced by $2w \operatorname{cosec} 2x$ but allow $\cot 2x$ written as $\frac{1}{\tan 2x}$ or $\frac{\cos 2x}{\sin 2x}$ and/or $\tan x$ written as $\frac{\sin x}{\cos x}$ If the result in (a) is not clearly used there must be full equivalent work. Allow use of “csc 2x”	A1*
		(4)

Question Number	Scheme	Notes	Marks
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8(c)	$\frac{dw}{dx} + 2w \operatorname{cosec} 2x = \sec x \Rightarrow \text{IF} = e^{2 \int \operatorname{cosec} 2x dx} = \tan x$ or $e^{-\ln(\operatorname{cosec} 2x + \cot 2x)} \Rightarrow \frac{1}{\operatorname{cosec} 2x + \cot 2x} \text{ or } \frac{1}{\cot x} \text{ or } \tan x$	M1: $e^{2 \int \operatorname{cosec} 2x (dx)}$ condoning omission of one or both “2”s A1: $\tan x$ oe Allow $k \tan x$ e.g., $e^{2c} \tan x$ Not just $e^{\ln(\tan x)}$	M1 A1
	$\Rightarrow w \tan x = \int \tan x \sec x \{dx\}$	Correctly applies their integrating factor to the equation, i.e., $\Rightarrow \text{IF} \times w = \int \text{IF} \times \sec x \{dx\}$ Allow equivalents for $\sec x$. Condone “y” used for “w” for this mark.	M1
	$\Rightarrow w \tan x = \sec x (+c)$	Correct equation oe with or without constant.	A1
	Using $\text{IF} = \frac{1}{\operatorname{cosec} 2x + \cot 2x} \Rightarrow \text{RHS of } \int \frac{\sec x}{\operatorname{cosec} 2x + \cot 2x} dx$ which is likely to need rewriting as $\int \tan x \sec x dx$ Note that IBP on $\sec x \tan x$ by writing it as $\sec^2 x \sin x$ can lead to $\sin x \tan x + \cos x (+c)$ Use Review for any attempts at integration you are unsure about.		
	e.g., $y^2 = w \sin 2x$ and $w \tan x = \sec x + c \Rightarrow \frac{y^2}{\sin 2x} \tan x = \sec x + c$ $\Rightarrow y^2 = \dots \left\{ \frac{\sin 2x}{\tan x} (\sec x + c) \right\}$ Substitutes for w correctly and reaches $y^2 = \dots$ Their $y^2 = \dots$ must be consistent with their equation in w and x that immediately followed their integration. This mark requires both previous M marks and an attempt at integration that includes a “+ c” A further example is: $w = \operatorname{cosec} x + \frac{c}{\tan x} \Rightarrow y^2 = \operatorname{cosec} x \sin 2x + \frac{c \sin 2x}{\tan x}$		
	$\left\{ \text{e.g., } y^2 = \frac{2 \sin x \cos^2 x}{\sin x} \left(\frac{1}{\cos x} + c \right) \Rightarrow \right\}$ $y^2 = 2 \cos x + A \cos^2 x$ Any correct $y^2 = \dots$ equation with RHS fully in terms of $\cos x$. E.g. accept $y^2 = 2 \cos x + 2c \cos^2 x$ $y^2 = \cos x (2 + A \cos x)$ $y^2 = 2 \cos^2 x \left(\frac{1}{\cos x} + c \right)$ Ignore any inconsistencies with the constant e.g., $2c$ later written as c		
			(6)
			Total 13

Question	Scheme	Marks	
8(a)	$\frac{dv}{dx} = \frac{dy}{dx} - 2$	B1	
	$\frac{dy}{dx} + 2yx(y - 4x) = 2 - 8x^3 \rightarrow \frac{dv}{dx} + 2 + 2(v + 2x)x(v + 2x - 4x) = 2 - 8x^3$		
	$\rightarrow \frac{dv}{dx} + 2 + 2x(v^2 - 4x^2) = 2 - 8x^3$	M1	
	$\rightarrow \frac{dv}{dx} = -2xv^2 *$	A1*	
		(4)	
(b)	$\frac{1}{v^2} \frac{dv}{dx} = -2x \Rightarrow \int v^{-2} dv = -2 \int x dx$	B1	
	$\Rightarrow \frac{v^{-1}}{-1} = -2 \frac{x^2}{2} (+c)$	M1	
	$\Rightarrow \frac{1}{v} = x^2 + c$	A1	
	$\Rightarrow v = \frac{1}{x^2 + c}$	A1	
		(4)	
(c)	$y = 2x + \frac{1}{x^2 + c}$	B1ft	
		(1)	
(d)	$-1 = 2 \times -1 + \frac{1}{1+c} \Rightarrow c = \dots$	M1	
	$y = 2x + \frac{1}{x^2}$	A1	
		Attempts the sketch for their equation, with at least one of - One branch correct - Vertical asymptote for their equation - Long term behaviour tends to infinity - Minimum in quadrant 1	M1
		Fully correct shape, two branches tending to infinity as x tends to infinity both directions, with minimum in first quadrant No need for oblique asymptote marked.	A1
		y -axis a vertical asymptote labelled	B1ft
			(5)
(14 marks)			
Notes:			
(a)			

B1: Correct differentiation of the given transformation. Allow any correct connecting derivative, e.g.

$$\frac{dy}{dv} = 1 + 2\frac{dx}{dv} \quad \text{or} \quad \frac{dv}{dy} = 1 - 2\frac{dx}{dy}$$

M1: For a complete substitution into the equation (1).

M1: Applies difference of squares, or completely expands brackets of the left hand side. Alternatively, may rearrange and factorise to give $8x^2y - 2xy^2 - 8x^3 = -2x(y^2 - 4xy + 4x^2) = -2x(y - 2x)^2$ before substituting.

A1*: Reaches the given answer with no errors seen.

(b)

B1: Correct separation of the variables.

M1: Attempts the integration, usual rule, power increased by 1 on at least one term. No need for $+c$ for the method.

A1: Correct integration including the $+c$

A1: Correct expression for v .

(c)

B1: Follow through their answer to (b), so $y = 2x +$ their v from (b)

(d)

M1: Uses the point $(-1, -1)$ to find a value for the constant in their equation. Must have had a constant of integration in their equation to score this mark.

A1: Correct equation for y following a correct general solution. Withhold this mark for $y = 2x + \frac{1}{x^2} + c$ leading to the correct equation.

Note: the following three marks may be scored from a correct equation that arose from having no constant in

(b) or from $y = 2x + \frac{1}{x^2} + c$ (which gives the same equation).

M1: Attempts a sketch for their curve. See scheme. Look for at least one of the key features for their equation shown.

A1: Correct shape, two branches tending to infinity as x tends to infinity both directions with a minimum in first quadrant. Not a follow through mark, so must be the correct curve.

B1ft: Correct vertical asymptote at $x = 0$. Need not be labelled if it is clearly the y -axis. Follow through their equation as long as there is at least one vertical asymptote (ie for a negative c they need a pair of asymptotes symmetric about the y -axis).

General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Question Number	Scheme	Marks
8(a)	E.g. $\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{1}{x} \frac{dy}{dt}$ or $t = \ln x \Rightarrow x = e^t \Rightarrow \frac{dx}{dy} = e^t \frac{dt}{dy}$ or $t = \ln x \Rightarrow \frac{dt}{dy} = \frac{1}{x} \frac{dx}{dy}$ etc.	B1
	E.g. $\frac{d^2y}{dx^2} = -\frac{1}{x^2} \frac{dy}{dt} + \frac{1}{x} \frac{d^2y}{dt^2} \frac{dt}{dx}$ or $\frac{dx}{dy} = e^t \frac{dt}{dy} \Rightarrow \frac{dy}{dx} = e^{-t} \frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = -e^{-t} \frac{dt}{dx} \frac{dy}{dt} + e^{-t} \frac{d^2y}{dt^2} \frac{dt}{dx}$ or $x \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{d^2y}{dt^2} \frac{dt}{dx}$ etc	M1
	E.g. $\left(t = \ln x \Rightarrow \frac{dt}{dx} = \frac{1}{x} \Rightarrow \right) \frac{d^2y}{dx^2} = -\frac{1}{x^2} \frac{dy}{dt} + \frac{1}{x} \frac{d^2y}{dt^2} \frac{dt}{dx} = -e^{-2t} \frac{dy}{dt} + e^{-2t} \frac{d^2y}{dt^2}$ or $\left(x = e^t \Rightarrow \frac{dx}{dt} = e^t \Rightarrow \right) \frac{d^2y}{dx^2} = -e^{-t} \frac{dt}{dx} \frac{dy}{dt} + e^{-t} \frac{d^2y}{dt^2} \frac{dt}{dx} = -e^{-2t} \frac{dy}{dt} + e^{-2t} \frac{d^2y}{dt^2}$ or $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{d^2y}{dt^2} \frac{dt}{dx} \Rightarrow e^t \frac{d^2y}{dx^2} + \frac{1}{e^t} \frac{dy}{dt} = \frac{1}{e^t} \frac{d^2y}{dt^2}$ $\Rightarrow \frac{d^2y}{dx^2} = e^{-2t} \left(\frac{d^2y}{dt^2} - \frac{dy}{dt} \right) *$	A1*
		(3)
(b)	$x^2 \frac{d^2y}{dx^2} - 2y = 1 + 4 \ln x - 2(\ln x)^2 \Rightarrow e^{2t} \times e^{-2t} \left(\frac{d^2y}{dt^2} - \frac{dy}{dt} \right) - 2y = 1 + 4t - 2t^2$ $\Rightarrow \frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 1 + 4t - 2t^2 *$	B1*
		(1)
(c)	$m^2 - m - 2 = 0 \Rightarrow m = 2, -1$ $y = Ae^{2t} + Be^{-t}$	M1 A1
	$y = at^2 + bt + c \Rightarrow \frac{dy}{dt} = 2at + b \Rightarrow \frac{d^2y}{dt^2} = 2a$	M1
	$\Rightarrow 2a - 2at - b - 2at^2 - 2bt - 2c = 1 + 4t - 2t^2$ $2a = 2 \Rightarrow a = 1$ $-2a - 2b = 4 \Rightarrow b = -3$ $2a - b - 2c = 1 \Rightarrow c = 2$	dM1
	$y = Ae^{2t} + Be^{-t} + t^2 - 3t + 2$	A1
		(5)
(d)	$y = Ax^2 + Bx^{-1} + (\ln x)^2 - 3 \ln x + 2$	B1ft
		(1)
		Total 10

Notes:

(a)

B1: Any correct equation linking $\frac{dy}{dx}$ and $\frac{dy}{dt}$ (or their reciprocals).M1: Differentiates again wrt x or t with attempts at both the product rule **and** chain rule to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$. Allow slips as long as an attempt at both rules is made.A1*: Eliminates the x 's and $\frac{dx}{dt}$'s from the equation and completes to obtain the printed answer with no errors.

(b)

B1*: Shows a clear substitution into the equation and proceeds to the given answer with no incorrect steps seen. Working may be minimal as long as there is at least one step before the given answer that shows substitution has been made.

(c)

M1: Forms and attempts (need not be a correct attempt) to solve $m^2 - m - 2 = 0$ and attempts to form the appropriate complementary function for their roots. Allow if variables are incorrect for the methods (ie x used instead of t).

A1: Correct complementary function. The $y = \dots$ may be missing. Must be in terms of t .

M1: Starts with the correct form the particular integral and differentiates twice.

dM1: Substitutes into the correct differential equation and solves to find at least two of a and b and c by comparing coefficients. Do not be concerned about the method of solving, look for any attempt reaching values for at least two of the constants.

A1: Correct solution. Must start $y = \dots$. Allow if stated in (d).

(d)

B1ft: Correct equation with exponentials simplified to powers of x (follow through their answer to (c)) and isw if incorrect simplification takes place after a correct answer seen. Should start $y = \dots$ accept if GS $=$, or similar, is used for **both** (c) and (d). Accept $\ln^2 x$ for $(\ln x)^2$ but not $\ln x^2$.

Note: parts (c) and (d) may be merged, but to award the A mark for (d) a correct general solution must be given. Allow the mark in (d) if seen in part (c).

June 2025
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1(a)	$r = 4\sqrt{2}$	Correct r Accept $\pm 4\sqrt{2}$ or $\pm\sqrt{32}$ or other processed equivalents.	B1
	$\theta = -\pi + \arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$	Complete method to find an argument in the correct quadrant for their r	M1
	$\Rightarrow -2\sqrt{2} - (2\sqrt{6})i = 4\sqrt{2}e^{-\frac{2\pi}{3}i}$	Correct answer in the form specified. Accept with $\sqrt{32}$	A1
(3)			
(b)	$z^5 = -2\sqrt{2} - (2\sqrt{6})i$		
	$z = (4\sqrt{2})^{\frac{1}{5}} e^{\frac{\frac{2\pi}{3} + 2k\pi}{5}i}$	Attempts De Moivre correctly with either all roots indicated or simplified r with one argument. See additional notes.	M1
	$\sqrt{2}e^{\frac{2\pi}{15}i}, \sqrt{2}e^{\frac{4\pi}{15}i}, \sqrt{2}e^{\frac{2\pi}{3}i},$ $\sqrt{2}e^{\frac{8\pi}{15}i}, \sqrt{2}e^{\frac{14\pi}{15}i}$	Three correct roots provided coefficients of π for θ are single fractions, allow for $0 < \theta < 2\pi$ Allow $(4\sqrt{2})^{\frac{1}{5}}$ oe for $\sqrt{2}$ and correct roots in trig form	A1
		All five correct roots and no extra. Must have $-\pi < \theta \leq \pi$. Allow e.g., $\sqrt{2}e^{\frac{10\pi}{15}i}$ for $\sqrt{2}e^{\frac{2\pi}{3}i}$	A1
(3)			
Total 6			

Additional Notes:

(a)

B1: Accept positive or negative root. Allow if seen in part (b).

M1: Requires a method to achieve an argument in the correct quadrant for their r . Accept attempts at

$\arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) \pm \pi$ or $\arctan\left(\frac{2\sqrt{2}}{2\sqrt{6}}\right) \pm \pi$ for attempts where $r > 0$, or equivalent methods via arccos or arcsin

(must use r as the denominator). May be implied by sight of any of $-\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{6}$ or $-\frac{5\pi}{6}$ if no method is shown.

In cases where a negative r is given, accept attempts at just $\arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right)$ or $\arctan\left(\frac{2\sqrt{2}}{2\sqrt{6}}\right)$ provided they

do not subsequently adjust to a different quadrant.

A1: Correct answer in the form specified, with $-\pi < \theta < \pi$. Must be given in correct form, but accept

answer with negative r , i.e. $-4\sqrt{2}e^{\frac{\pi}{3}i}$. Accept with exact processed equivalents for r .

Note: Correct answer with no working shown will score B1M1(implied)A1.



(b)

M1: Applies de Moivre correctly with their answer to (a) which could be in trig form, to reach either simplified modulus with one correct argument for their (a), or multiple arguments with unsimplified modulus.

Condone missing “i” for the M.

Note: If $\pm 2k\pi$ is not used or is used incorrectly this mean you can score M1A0A0 for just $\sqrt{2}e^{-\frac{2\pi}{15}i}$

A1: Accept for three correct distinct roots allowing arguments in the range $-\pi < \theta < \pi$ or $0 < \theta < 2\pi$ for this mark. The modulus need not be simplified for this mark.

A1: All 5 correct roots in the form specified.



Question Number	Scheme	Notes	Marks
6(a)	$ z+1-13i =3 z-7-5i \Rightarrow (x+1)^2 + (y-13)^2 = 9\{(x-7)^2 + (y-5)^2\}$ Correct application of Pythagoras Accept 3 or 9 on RHS		M1
	$\Rightarrow x^2 + y^2 - 16x - 8y + 62 = 0$	Correct equation in any form with terms collected	A1
	Centre (8, 4)	Correct centre. i included scores A0	A1
	$r^2 = 64 + 16 - 62 = \dots$	Correct method for r or r^2	M1
	$r = \sqrt{18}$ or $3\sqrt{2}$	Correct radius. Must be exact.	A1
			(5)
(b)	$\arg(z-8-6i) = -\frac{3\pi}{4} \Rightarrow y-6 = x-8$	Converts the given locus to the correct Cartesian form	B1
	$\Rightarrow x^2 + y^2 - 16x - 8y + 62 = 0$ $\Rightarrow x^2 + (x-2)^2 - 16x - 8(x-2) + 62 = 0 \Rightarrow x = \dots$ or $\Rightarrow (y+2)^2 + y^2 - 16x - 8(y+2) + 62 = 0 \Rightarrow y = \dots$	Uses both Cartesian equations to obtain an equation in one variable and attempts to solve	M1
	$x = 7 - 2\sqrt{2}$ or $y = 5 - 2\sqrt{2}$	One correct “coordinate”	A1
	R is $7 - 2\sqrt{2} + (5 - 2\sqrt{2})i$ or $x = 7 - 2\sqrt{2}$ and $y = 5 - 2\sqrt{2}$	Correct complex number or coordinates and no others. Must be exact	A1
			(4)
			Total 9

Question Number	Scheme	Marks
1(a)	$\frac{2}{\sqrt{r} + \sqrt{r-2}} \times \frac{\sqrt{r} - \sqrt{r-2}}{\sqrt{r} - \sqrt{r-2}}$	M1
	$\frac{2(\sqrt{r} - \sqrt{r-2})}{r - (r-2)} = \sqrt{r} - \sqrt{r-2} *$	A1*
		(2)
(b)	$r = 2: \sqrt{2} - \sqrt{2-2} (= \sqrt{2} - 0) \quad \dots r = n-2: \sqrt{n-2} - \sqrt{n-4}$ $r = 3: \sqrt{3} - \sqrt{3-2} (= \sqrt{3} - 1) \quad r = n-1: \sqrt{n-1} - \sqrt{n-3}$ $r = 4: \sqrt{4} - \sqrt{4-2} (= 2 - \sqrt{2}) \quad r = n: \sqrt{n} - \sqrt{n-2}$	M1
	$\left[\sum_{r=2}^n \frac{2}{\sqrt{r} + \sqrt{r-2}} \right] = \sqrt{n} + \sqrt{n-1} - 1$	A1 A1
		(3)
(c)	$\left[\sum_{r=4}^{50} \frac{2}{\sqrt{r} + \sqrt{r-2}} \right] f(50) - f(3) = \sqrt{50} + \sqrt{49} - 1 - (\sqrt{3} + \sqrt{2} - 1)$	M1
	$= 5\sqrt{2} + 7 - 1 - \sqrt{3} - \sqrt{2} + 1 = 7 + 4\sqrt{2} - \sqrt{3}$	A1
		(2)
		Total 7
Notes		
(a) M1: Indicates intention to multiply either side by a correct fraction, may use $\frac{\sqrt{r-2} - \sqrt{r}}{\sqrt{r-2} - \sqrt{r}}$. The “2” may be missing. May work in reverse from right hand side to left, and this is fine. Alternatively, may multiply the initial expression through by $\sqrt{r} + \sqrt{r-2}$ and use a sequence of equivalences (though accept with $\Rightarrow$ or nothing between lines). A1: Fully correct proof. A result from rationalisation that is not the given answer must be seen. If using a sequences of equivalences there must be a minimal conclusion. (b) M1: Correct process of differences evidenced in their work, e.g. attempts any three of the 6 expression shown. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at any of $r = 0$ or 1 if they start earlier than $r = 2$. A1: Correct algebraic terms or correct constant term(s) extracted. Accept unsimplified expressions such as “$-\sqrt{1} - \sqrt{0}$” A1: Fully correct simplified expression. Must have simplified the $\sqrt{1}$ to 1 (c) M1: Attempts $f(50) - f(3)$ using their answer to (b). Must be indication of subtraction. The ‘-1’s may be omitted, and allow if a slip is made. They may subtract the sum of their results from $r = 2$ and $r = 3$ from (b) or by using (a) again to obtain $f(3)$ but their answer to (b) must be used for $f(50)$. Allow from attempts starting at $r = 1$ in their summations. $f(50) - f(4)$ is M0. A1: Correct expression or $A = 7, B = 4, C = -1$ following a correct answer to (b). The decimal answer 10.92480344... without evidence of the M mark is 0/2		

Question	Scheme	Marks
6(a)	$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 37y = 6e^{5x}$	
	Aux. Eqn. $4m^2 - 4m + 37 = 0 \Rightarrow m = \frac{4 \pm \sqrt{16 - 16 \times 37}}{8} = \frac{1}{2} \pm 3i$	M1
	C.F. is $(y =) e^{\frac{1}{2}x} (A \cos 3x + B \sin 3x)$	A1
	PI try $(y =) ke^{5x}$	B1
	$\frac{dy}{dx} = 5ke^{5x}, \frac{d^2y}{dx^2} = 25ke^{5x} \Rightarrow 4(25ke^{5x}) - 4(5ke^{5x}) + 37(ke^{5x}) = 6e^{5x}$	M1
	$\Rightarrow 100k - 20k + 37k = 6 \Rightarrow k = \dots (k = \frac{2}{39} \text{ or } \frac{6}{117} \text{ o.e.})$	M1
	So gen sol. is $y = \frac{2}{39}e^{5x} + e^{\frac{1}{2}x} (A \cos 3x + B \sin 3x)$	A1ft
		(6)
(b)	$x = 0, y = 0 \Rightarrow 0 = \frac{2}{39} + A$	M1
	$\frac{dy}{dx} = \frac{10}{39}e^{5x} + \frac{1}{2}e^{\frac{1}{2}x} (A \cos 3x + B \sin 3x) + e^{\frac{1}{2}x} (-3A \sin 3x + 3B \cos 3x)$	M1A1ft
	$x = 0, \frac{dy}{dx} = 0 \Rightarrow \frac{10}{39} + \frac{1}{2}A + 3B = 0 \Rightarrow B = \dots, A = \dots$	M1
	$(y =) \frac{2}{39}e^{5x} + e^{\frac{1}{2}x} \left(-\frac{2}{39} \cos 3x - \frac{1}{13} \sin 3x \right)$	A1
		(5)

(11 marks)**Notes:****(a)**

M1: Forms correct auxiliary equation and attempts to solve by any valid means. Implied by correct roots. Usual rules for solving a quadratic apply.

A1: Correct complementary function, allow without the “ $y =$ ”. Accept $y = Ae^{\left(\frac{1}{2}+3i\right)x} + Be^{\left(\frac{1}{2}-3i\right)x}$ or from De

Moirve can use e.g. $y = Ce^{\frac{1}{2}x} \cos(3x + \alpha)$ etc. Condone mixed variables for this mark. May not be seen until the end.

B1: Correct form for the particular integral selected. Must include ke^{5x} but accept with any extra terms that will disappear when constants found.

M1: Differentiates their PI twice and substitutes into the equation. Allow sign/coefficient errors only.

M1: Proceeds to find the value of the constant(s) from a valid PI. Note that the constant value need not be simplified, so accept exact equivalents such as $\frac{6}{117}$ etc

A1ft: Correct general solution. **Must have $y = \dots$ and be in terms of x only.** Follow through their CF in terms of x only but the PI must be correct. May appear at the start of (b) instead of the end of (a) which is fine.



(b)

M1: Uses the initial condition for x to find the value for “ A ” (their cosine coefficient). If by error of CF they obtain an equation in A and B then score for the attempt of producing a suitable equation.

M1: Differentiates the GS. Allow sign/coefficient errors only, so a full method for product rule.

A1ft: Correct differentiation following through their PI- but must have been of the correct form. Also allow ft for an incorrect value of A if found before differentiation

M1: Uses the initial conditions to find values for all constants in their equation.

A1: Correct particular solution. Allow exact equivalents for the coefficients. Accept without ‘ $y = \dots$ ’ so e.g. $PS =$ is ok here. **Must be in terms of x only with no mixed variables etc**

NOTE: Alt method – it is possible for candidates to form two simultaneous equations in A and B from the first and second derivatives and solve to find A and B . If you see this then please send to review. Any attempts at differentiating the complex number form for the CF can score full marks, and leads to

$$(y =) \left(-\frac{1}{39} + \frac{1}{26}i\right)e^{\left(\frac{1}{2}+3i\right)x} + \left(-\frac{1}{39} - \frac{1}{26}i\right)e^{\left(\frac{1}{2}-3i\right)x} + \frac{2}{39}e^{5x}$$

Which is mathematically equivalent to the $\frac{2}{39}e^{5x} + e^{\frac{1}{2}x}\left(-\frac{2}{39}\cos 3x - \frac{1}{13}\sin 3x\right)$

This is marked as per the main scheme



Question Number	Scheme	Marks
3	$x^2 + x - 2 < \frac{1}{2}x + \frac{5}{2}$ $2x^2 + x - 9 < 0$ $\text{CVs } x = \frac{-1 \pm \sqrt{73}}{4}$ $-x^2 - x + 2 < \frac{1}{2}x + \frac{5}{2}$ $2x^2 + 3x + 1 > 0 \quad (2x+1)(x+1) > 0$ $\text{CVs } x = -\frac{1}{2}, -1$ $\frac{-1 - \sqrt{73}}{4} < x < -1, \quad -\frac{1}{2} < x < \frac{-1 + \sqrt{73}}{4}$	M1 A1 M1 M1 A1 M1A1 [7]
NB M1 A1 M1 M1 A1 M1 A1	No algebra implies no marks The first 5 marks can all be awarded if equations rather than inequalities are shown Obtain and solve a 3TQ (any valid method including calculator) 2 correct CVs Allow decimal equivalents (1.886..., -2.386...), min 3 sf, rounded or truncated Multiply either side by -1 Obtain and solve a 3TQ (any valid method including calculator) 2 correct CVs Form 2 double inequalities with their CVs. No overlap between these inequalities. Correct inequality signs required here or for final mark Correct inequalities obtained. Values must be exact, but note that 0.5 is exact. Allow “and” but not “∩”. May be written in set language with “∪” and round brackets	

Question Number	Scheme	Marks
4 (a)	$y^2 = z^{-1} \Rightarrow 2y \frac{dy}{dx} = -\frac{1}{z^2} \frac{dz}{dx}$ oe eg $\frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$ $2y \frac{dy}{dx} + 4y^2 = 6xy^4$ $-\frac{1}{z^2} \frac{dz}{dx} + \frac{4}{z} = \frac{6x}{z^2}$ $\frac{dz}{dx} - 4z = -6x$ *	B1 M1 A1 * (3)
(b)	$IF = e^{\int -4dx} = e^{-4x}$ $e^{-4x} \left(\frac{dz}{dx} - 4z \right) = e^{-4x} \times -6x$ $ze^{-4x} = -6 \int xe^{-4x} dx$ $= -6 \left[-\frac{1}{4} xe^{-4x} + \int \frac{1}{4} e^{-4x} dx \right]$ $= -6 \left[-\frac{1}{4} xe^{-4x} - \frac{1}{16} e^{-4x} \right] (+c)$ oe $= \frac{3}{2} xe^{-4x} + \frac{3}{8} e^{-4x} (+c)$ $z = \frac{3}{2} x + \frac{3}{8} + ce^{4x}$ oe	B1 M1 M1 A1 A1 (5)
ALT	$\frac{dz}{dx} - 4z = -6x$ $m - 4 = 0 \Rightarrow m = 4 \Rightarrow$ CF is $z = Ae^{4x}$ PI: $z = \lambda + \mu x$ $\frac{dz}{dx} = \mu \Rightarrow \mu - 4(\lambda + \mu x) = -6x$ $4\mu = 6 \quad 4\lambda = \mu, \Rightarrow \mu = \frac{3}{2}, \lambda = \frac{3}{8}$ $z = \frac{3}{2} x + \frac{3}{8} + Ae^{4x}$	B1 M1 M1,A1 A1
(c)	$y^2 = \frac{1}{\frac{3}{2}x + \frac{3}{8} + ce^{4x}} = \frac{8}{(12x + 3 + Ae^{4x})}$ oe	B1ft (1)
		[9]

Question Number	Scheme	Marks
(a)		
B1	Correct derivative seen explicitly or used	
M1	Substitutions made. Only award when an equation in x and z only is reached (if working equation I to II) or an equation in x and y is reached (if working II to I)	
A1 *	Correct result obtained with no errors in working	
(b)		
B1	Correct IF seen explicitly or used	
M1	Multiply through by their IF and integrate the LHS. Accept I for e^{-4x} on LHS only	
M1	Apply parts in the correct direction to RHS to obtain $Axe^{-4x} + B \int e^{-4x} dx$ with $A = \pm \frac{3}{2}$ and $B = \pm \frac{3}{2}$	
A1	Correct integration of RHS, constant not needed	
A1	Include the constant and treat it correctly. Answer in form $z = \dots$	
ALT		
B1	Correct CF May not be seen until GS is formed	
M1	For a PI of the correct form	
M1	Differentiate their PI, substitute in the equation and extract 2 equations for the unknowns	
A1	Solve the two equations to obtain correct values for the unknowns	
A1	Correct GS obtained	
(c)		
B1ft	Any equivalent to that shown. (no need to change letter for constant if rearranged) Must start $y^2 = \dots$ and must include a constant.	

Question Number	Scheme	Marks
5(a)	$-2x \frac{d^2y}{dx^2} + (2-x^2) \frac{d^3y}{dx^3}$	M1
	$+5\left(\frac{dy}{dx}\right)^2 + 5x \times 2 \frac{dy}{dx} \frac{d^2y}{dx^2}, = 3 \frac{dy}{dx}$	M1A1, B1
	$\frac{d^3y}{dx^3}(2-x^2) + \frac{d^2y}{dx^2}\left(10x \frac{dy}{dx} - 2x\right) + 5\left(\frac{dy}{dx}\right)^2 = 3 \frac{dy}{dx}$	
	$\frac{d^3y}{dx^3} = \frac{1}{(2-x^2)} \left(2x \frac{d^2y}{dx^2} \left(1 - 5 \frac{dy}{dx} \right) - 5 \left(\frac{dy}{dx} \right)^2 + 3 \frac{dy}{dx} \right) *$	A1* (5)
	ALT 1 $\frac{d^2y}{dx^2} = \frac{3y - 5x \left(\frac{dy}{dx} \right)^2}{(2-x^2)}$	
	$\frac{d^3y}{dx^3} = \frac{\left[3 \frac{dy}{dx} - 5 \left(\frac{dy}{dx} \right)^2 - 5x \times 2 \frac{dy}{dx} \frac{d^2y}{dx^2} \right] (2-x^2) - \left[3y - 5x \left(\frac{dy}{dx} \right)^2 \right] (-2x)}{(2-x^2)^2}$	M1M1A1
	$\frac{d^3y}{dx^3} = \frac{\left[3 \frac{dy}{dx} - 5 \left(\frac{dy}{dx} \right)^2 - 10x \frac{dy}{dx} \frac{d^2y}{dx^2} \right] (2-x^2) + 2x(2-x^2) \frac{d^2y}{dx^2}}{(2-x^2)^2}$	M1 (NB: B1 on ePEN)
	$\frac{d^3y}{dx^3} = \frac{1}{(2-x^2)} \left(2x \frac{d^2y}{dx^2} \left(1 - 5 \frac{dy}{dx} \right) - 5 \left(\frac{dy}{dx} \right)^2 + 3 \frac{dy}{dx} \right) *$	A1* (5)

Question Number	Scheme	Marks
ALT 2	$\frac{d^2y}{dx^2} = \frac{3y}{(2-x^2)} - \frac{5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)^2}$	M1M1A1
	$\frac{d^3y}{dx^3} = \frac{3\frac{dy}{dx}(2-x^2) - 3y(-2x)}{(2-x^2)^2}$	
	$- \frac{\left[5\left(\frac{dy}{dx}\right)^2 + 5x \times 2 \frac{dy}{dx} \frac{d^2y}{dx^2}\right](2-x^2) - 5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	
	$\frac{d^3y}{dx^3} = \frac{3\frac{dy}{dx}(2-x^2) - \left((2-x^2)\frac{d^2y}{dx^2} + 5x\frac{dy}{dx}\right)(-2x)}{(2-x^2)^2}$	
	$- \frac{\left[5\left(\frac{dy}{dx}\right)^2 + 5x \times 2 \frac{dy}{dx} \frac{d^2y}{dx^2}\right](2-x^2) - 5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	
	$\frac{d^3y}{dx^3} = \frac{1}{(2-x^2)} \left(2x \frac{d^2y}{dx^2} \left(1 - 5 \frac{dy}{dx} \right) - 5 \left(\frac{dy}{dx} \right)^2 + 3 \frac{dy}{dx} \right) *$	A1*
(b)	$x=0 \Rightarrow 2 \frac{d^2y}{dx^2} = 9 \quad \frac{d^2y}{dx^2} = \frac{9}{2}$	B1
	$\frac{d^3y}{dx^3} = \frac{1}{2} \left(-5 \left(\frac{dy}{dx} \right)^2 + 3 \frac{dy}{dx} \right) = \frac{1}{2} \left(-5 \times \frac{1}{16} + \frac{3}{4} \right) = \frac{7}{32}$	M1
	$y = 3 + \frac{1}{4}x + \frac{9}{2} \frac{x^2}{2!} + \frac{7}{32} \frac{x^3}{3!}$	M1
	$y = 3 + \frac{1}{4}x + \frac{9}{4}x^2 + \frac{7}{192}x^3$	A1 (4)
		[9]

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = v + x \frac{dv}{dx}$ or $\frac{dv}{dx} = x^{-1} \frac{dy}{dx} - x^{-2} y$ (oe)	M1A1
	$\frac{d^2 y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x \frac{d^2 v}{dx^2}$ or $\frac{d^2 v}{dx^2} = -x^{-2} \frac{dy}{dx} + x^{-1} \frac{d^2 y}{dx^2} + 2x^{-3} y - x^{-2} \frac{dy}{dx}$ (oe)	dM1A1
	$3 \left(2 \frac{dv}{dx} + x \frac{d^2 v}{dx^2} \right) - \frac{6}{x} \left(v + x \frac{dv}{dx} \right) + \frac{6xv}{x^2} + 3xv = x^2$ (oe in reverse)	ddM1
	$3x \frac{d^2 v}{dx^2} + 6 \frac{dv}{dx} - 6 \frac{dv}{dx} - \frac{6}{x} v + \frac{6v}{x} + 3xv = x^2$	
	$3 \frac{d^2 v}{dx^2} + 3v = x$ *	A1 * (6)
(b)	$3\lambda^2 + 3 = 0$ so $\lambda = \pm i$	M1
	$(v =) Ae^{ix} + Be^{-ix}$ or $(v =) C \cos x + D \sin x$	A1
	P.I: Try $(v =) kx (+l)$	B1
	$\frac{dv}{dx} = k$ $\frac{d^2 v}{dx^2} = 0$	
	$3 \times 0 + 3(kx (+l)) = x$	M1
	$k = \frac{1}{3}$ ($l = 0$)	
	$v = Ae^{ix} + Be^{-ix} + \frac{1}{3}x$ or $v = C \cos x + D \sin x + \frac{1}{3}x$	A1
	$y = x \left(Ae^{ix} + Be^{-ix} + \frac{1}{3}x \right)$ or $y = x \left(C \cos x + D \sin x + \frac{1}{3}x \right)$	B1ft (6)
		[12]
	Notes	
(a)		
M1	Attempt to find a relevant first derivative from $y = xv$ e.g to get $\frac{dy}{dx}$ or $\frac{dv}{dx}$ - product or quotient rule must be used. Methods via $\frac{d..}{dv}$ would require a chain rule to reach a relevant derivative.	
A1	Correct derivative	
dM1	Attempt to differentiate their $\frac{dy}{dx}$ or $\frac{dv}{dx}$ to obtain an expression for $\frac{d^2 y}{dx^2}$ or $\frac{d^2 v}{dx^2}$ - product rule must be used. Depends on the previous M mark	
A1	Correct expression for $\frac{d^2 y}{dx^2}$ or $\frac{d^2 v}{dx^2}$	

Question Number	Scheme	Marks
5 NB	Question states "Use algebra..." so purely graphical solutions (using calculator?) score 0/7. A sketch and some algebra to find intersection points can score. $2x^2 + x - 3 \geq 0$ $2x^2 + x - 3 = 3(1-x) \Rightarrow 2x^2 + 4x - 6 = 0$ $2x^2 + 4x - 6 \Rightarrow x^2 + 2x - 3 = (x+3)(x-1) = 0$ $x = -3, 1$ $2x^2 + x - 3 \leq 0$ $-2x^2 - x + 3 = 3(1-x) \Rightarrow 2x^2 - 2x = 0$ $2x(x-1) = 0, x = 0, 1$ $x < -3 \quad 0 < x < 1 \quad x > 1$	M1 A1 M1 A1 dM1A1A1 [7]
M1 A1 M1 A1 dM1 A1 A1	The first 4 marks can be awarded with any inequality sign or = Assume $2x^2 + x - 3 \geq 0$ and obtain a 3TQ Correct CVs obtained from a correct equation. Assume $2x^2 + x - 3 \leq 0$ and obtain a 2 or 3TQ Correct CVs obtained from a correct equation. Form 3 distinct inequalities with their 3 CVs. Can have < or $\leq$, > or $\geq$. Must have scored both previous M marks. Accept $x < -3 \quad 0 < x \quad x \neq 1$ All 3 correct CVs used correctly Inequalities fully correct. "and" between the inequalities is acceptable. If $\cap$ is used, award A0 here. Fully correct set language accepted.	
ALT	<i>Squaring both sides</i> $(2x^2 + x - 3)^2 > 9(1-x)^2$ $4x^4 + 4x^3 - 20x^2 + 12x > 0$ $x(x+3)(x-1)(x-1) > 0$ CVs: $x = 0, -3, 1$ Then as main scheme	M1A1 M1 A1
M1 A1 M1 A1	These 4 marks can be awarded with any inequality sign or = Square both sides and collect terms to obtain a quartic with 4 or 5 terms Correct quartic Factorise their quartic 3 correct CVs	



Question Number	Scheme	Marks
6(a)	$m^2 - 6m + 8 = 0$ $(m-2)(m-4) = 0, m = 2, 4$ (CF =) $Ae^{2x} + Be^{4x}$ PI: $y = \lambda x^2 + \mu x + \nu$ $y' = 2\lambda x + \mu \quad y'' = 2\lambda$ $2\lambda - 6(2\lambda x + \mu) + 8(\lambda x^2 + \mu x + \nu) = 2x^2 + x$ $\lambda = \frac{1}{4}, -12\lambda + 8\mu = 1, 2\lambda - 6\mu + 8\nu = 0$ $\lambda = \frac{1}{4}, \mu = \frac{1}{2}, \nu = \frac{5}{16}$ $y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$	M1 A1 B1 M1 M1 A1A1 A1ft (8)
(a)M1 A1 B1 M1 M1 A1 A1 A1ft	Form aux equation and attempt to solve (any valid method). Equation need not be shown if CF is correct or complete solution ($m = 2, 4$) is shown Correct CF $y = ..$ not needed. Correct form for PI Their PI (minimum 2 terms) differentiated twice and substituted in the equation Coefficients equated Any 2 values correct All 3 values correct A complete solution, follow through their CF and PI. All 3 M marks must have been earned. Must start $y = ..$	
6(b)	$y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$ $1 = A + B + \frac{5}{16}$ $\frac{dy}{dx} = 2Ae^{2x} + 4Be^{4x} + \frac{1}{2}x + \frac{1}{2} \quad 0 = 2A + 4B + \frac{1}{2}$ $A = \frac{13}{8} \quad B = -\frac{15}{16} \quad \text{oe}$ $y = \frac{13}{8}e^{2x} - \frac{15}{16}e^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16} \quad \text{oe}$	M1 M1 dM1A1 A1ft (5) [13]
(b) M1 M1 dM1 A1 A1ft	Substitute $y = 1$ and $x = 0$ in their complete solution from (a) Differentiate and substitute $\frac{dy}{dx} = 0, x = 0$ Solve the 2 equations to $A = ...$ or $B = ...$. Depends on the two previous M marks Both values correct Particular solution, follow through their general solution and A and B . Must start $y = ...$	



Question Number	Scheme	Notes	Marks
9(a)(i)	$x = t^{\frac{1}{2}} \Rightarrow \frac{dx}{dy} = \frac{1}{2} t^{-\frac{1}{2}} \frac{dt}{dy} \Rightarrow \frac{dy}{dx} = \dots \text{ or } t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \dots$ Applies the chain rule and proceeds to an expression for $\frac{dy}{dx}$		M1
	$\Rightarrow \frac{dy}{dx} = 2t^{\frac{1}{2}} \frac{dy}{dt}$	Any correct expression for $\frac{dy}{dx}$ in terms of y and t	A1
(a)(ii)	$\frac{dy}{dx} = 2t^{\frac{1}{2}} \frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dt} t^{-\frac{1}{2}} \frac{dt}{dx} + 2t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx}$ dM1: Uses the product rule to differentiate an equation of the form $\frac{dy}{dx} = kt^{\frac{1}{2}} \frac{dy}{dt}$ or equivalent e.g. $\frac{dy}{dx} = kx \frac{dy}{dt}$ to obtain $\frac{d^2y}{dx^2} = \alpha t^{-\frac{1}{2}} \frac{dy}{dt} \frac{dt}{dx} + \dots \text{ or } \frac{d^2y}{dx^2} = \dots + \beta t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx}$ or equivalent expressions where ... is non-zero A1: <u>Any</u> correct expression for $\frac{d^2y}{dx^2}$		dM1A1
	$\frac{dy}{dt} t^{-\frac{1}{2}} \frac{dt}{dx} + 2t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx} = \frac{dy}{dt} t^{-\frac{1}{2}} \times 2t^{\frac{1}{2}} + 2t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx}$ $\frac{d^2y}{dx^2} = 2 \frac{dy}{dt} + 4t \frac{d^2y}{dt^2}$ Correct expression in terms of y and t		A1
			(5)
(b)	$x \frac{d^2y}{dx^2} - (6x^2 + 1) \frac{dy}{dx} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}} \left(2 \frac{dy}{dt} + 4t \frac{d^2y}{dt^2} \right) - (6t + 1) 2t^{\frac{1}{2}} \frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$ Substitutes their expressions from part (a) and replaces x with $t^{\frac{1}{2}}$		M1
	$2t^{\frac{1}{2}} \frac{dy}{dt} + 4t^{\frac{3}{2}} \frac{d^2y}{dt^2} - 12t^{\frac{3}{2}} \frac{dy}{dt} - 2t^{\frac{1}{2}} \frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$ $\Rightarrow 4 \frac{d^2y}{dt^2} - 12 \frac{dy}{dt} + 9y = t^*$ Obtains the given answer with no errors and sufficient working shown – at least one intermediate line after substitution but check working. Must follow full marks in (a) apart from SC below.		A1*
			(2)

Special case in (a) and (b) for those who do not have (a) in terms of y and t only:

$$t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \dots \text{ Scores M1. } \dots = 2x \frac{dy}{dt} \text{ scores A0 in (a)(i)}$$

$$\frac{dy}{dx} = 2x \frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = 2 \frac{dy}{dt} + 2x \frac{d^2y}{dt^2} \frac{dt}{dx} = 2 \frac{dy}{dt} + 4x^2 \frac{d^2y}{dt^2} \text{ Scores dM1A1A0 in (a)(ii)}$$

$$x \frac{d^2y}{dx^2} - (6x^2 + 1) \frac{dy}{dx} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}} \left(2 \frac{dy}{dt} + 4x^2 \frac{d^2y}{dt^2} \right) - (6t + 1) 2x \frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$$

$$\Rightarrow t^{\frac{1}{2}} \left(2 \frac{dy}{dt} + 4t \frac{d^2y}{dt^2} \right) - (6t + 1) 2t^{\frac{1}{2}} \frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}} \Rightarrow 4 \frac{d^2y}{dt^2} - 12 \frac{dy}{dt} + 9y = t^* \text{ Scores M1A1 in (b)}$$

Mark (c) and (d) together

(c)	$4m^2 - 12m + 9 = 0 \Rightarrow m = \frac{3}{2}$	Attempts to solve $4m^2 - 12m + 9 = 0$ Apply general guidance for solving a 3TQ if necessary.	M1
	$(y =) e^{\frac{3}{2}t} (At + B)$	Correct CF. No need for “ $y = \dots$ ” Condone $(y =) e^{\frac{3}{2}x} (Ax + B)$ here but must be in terms of t in the GS. Allow equivalents for the $\frac{3}{2}$.	A1
	$(y =) at + b \Rightarrow \frac{dy}{dt} = a \Rightarrow \frac{d^2y}{dt^2} = 0$ $\Rightarrow -12a + 9(at + b) = t$ Starts with the correct PI form and differentiates to obtain $\frac{dy}{dt} = a$ and $\frac{d^2y}{dt^2} = 0$ and substitutes. NB starting with a PI of $y = at$ is M0		M1
	$9a = 1 \Rightarrow a = \dots$ $9b - 12a = 0 \Rightarrow b = \dots$	Complete method to find a and b by comparing coefficients. Depends on the previous method mark.	dM1
	$y = e^{\frac{3}{2}t} (At + B) + \frac{1}{9}t + \frac{4}{27}$	Correct GS including “ $y = \dots$ ” and must be in terms of t (no x 's). Allow equivalent exact fractions for the constants.	A1
			(5)
(d)	$y = e^{\frac{3}{2}x^2} (Ax^2 + B) + \frac{1}{9}x^2 + \frac{4}{27}$ Correct equation including “ $y = \dots$ ” (follow through their answer to (c)). Allow equivalent exact fractions for the constants. For the ft, the answer to (c) must be in terms of t and the answer to (d) should be the same as (c) with t replaced with x^2 . If there is no final answer to (c) you can award B1ft if the equation is correct in terms of x if it follows the previous work.		B1ft
			(1)
			Total 13

Question Number	Scheme	Notes	Marks
7(a)	$x = e^u \Rightarrow \frac{dx}{du} = e^u = x$ or $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x} \frac{dy}{du}$ or $\frac{dy}{dx} = \frac{dy}{du} \times e^{-u}$ oe	Correct form for equation in $\frac{dy}{dx}$ and $\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = \frac{1}{x} \frac{d^2y}{du^2} \frac{du}{dx} - \frac{1}{x^2} \frac{dy}{du}$ or $\frac{d^2y}{dx^2} = e^{-u} \frac{d^2y}{du^2} \frac{du}{dx} - e^{-2u} \frac{dy}{du}$ oe	Correct form for an equation linking $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{du^2}$	M1
	$2x^2 \left(\frac{1}{x^2} \frac{d^2y}{du^2} - \frac{1}{x^2} \frac{dy}{du} \right) + 3x \left(\frac{1}{x} \frac{dy}{du} \right) - y = 27x^2$	Substitutes to eliminate $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ and $\frac{du}{dx}$	M1
	$2 \frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} *$	Fully correct proof	A1*
(4)			
(b)	$2m^2 + m - 1 = (2m - 1)(m + 1) = 0 \Rightarrow m = \frac{1}{2}, -1$	Forms auxiliary equation and solves (usual rules)	M1
	$(y =) Ae^{\frac{1}{2}u} + Be^{-u}$	Correct CF for their two real values of m	M1
	$y = \lambda e^{2u}, y' = 2\lambda e^{2u}, y'' = 4\lambda e^{2u}$ $\Rightarrow 8\lambda + 2\lambda - \lambda = 9\lambda = 27 \Rightarrow \lambda = 3$	With correct form of PI, differentiates twice and substitutes to find a value for λ	M1
	$\left(y = Ae^{\frac{1}{2}u} + Be^{-u} + 3e^{2u} \right)$ $y = Ax^{\frac{1}{2}} + Bx^{-1} + 3x^2$	Correct GS in terms of x with no exponentials. Must include the $y =$	A1
(4)			
(c)	$\frac{dy}{dx} = \frac{1}{2} Ax^{-\frac{1}{2}} - Bx^{-2} + 6x$	Correct form for $\frac{dy}{dx}$ from GS of form $y = Ax^p + Bx^q + rx^2$	M1
	$x = \frac{1}{4}, y = \frac{11}{16} \Rightarrow \frac{11}{16} = \frac{1}{2} A + 4B + \frac{3}{16}$ and $x = \frac{1}{4}, \frac{dy}{dx} = 1 \Rightarrow 1 = A - 16B + \frac{3}{2}$	Uses $x = \frac{1}{4}$ with $y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$ to obtain two equations in the constants (from CF)	M1
	$\frac{1}{2} A + 4B = \frac{1}{2}, A - 16B = -\frac{1}{2}$ $\Rightarrow A = \frac{1}{2}, B = \frac{1}{16}$	Having found two equations in two CF constants from using the conditions, solves for A and B . Requires previous M mark	dM1
	$y = \frac{1}{2} \left(\frac{1}{8} \right)^{\frac{1}{2}} + \frac{1}{16} \left(\frac{1}{8} \right)^{-1} + 3 \left(\frac{1}{8} \right)^2 = \dots$	Substitutes $x = \frac{1}{8}$ into their PS and obtains a value for y . Requires previous 2 M marks	ddM1
	$= \frac{\sqrt{2}}{8} + \frac{1}{2} + \frac{3}{64} = \frac{\sqrt{2}}{8} + \frac{35}{64} = \frac{1}{64} (8\sqrt{2} + 35)$	$\frac{1}{64} (8\sqrt{2} + 35)$ only	A1
(5)			

Question Number	Scheme	Marks
4		
(a)	$ 18\sqrt{3} - 18i = 18\sqrt{(3+1)} = 36$ $\tan \theta = \frac{-18}{18\sqrt{3}} \quad \theta = -\frac{\pi}{6}, \quad 18\sqrt{3} - 18i = 36 \left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) \right)$	B1 M1, A1cao (3)
(b)	$z^4 = 36 \left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6} \right) = 36 \left(\cos\left(2k\pi - \frac{\pi}{6}\right) + i \sin\left(2k\pi - \frac{\pi}{6}\right) \right)$ $z = \sqrt[4]{36} \left(\cos\left(\frac{12k\pi - \pi}{24}\right) + i \sin\left(\frac{12k\pi - \pi}{24}\right) \right)$ $k = 0 \quad z_0 = \sqrt[4]{36} \left(\cos\left(\frac{-\pi}{24}\right) + i \sin\left(\frac{-\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\left(\frac{-\pi}{24}\right)}$ $k = 1 \quad z_1 = \sqrt[4]{36} \left(\cos\left(\frac{11\pi}{24}\right) + i \sin\left(\frac{11\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\frac{11\pi}{24}}$ $k = 2 \quad z_2 = \sqrt[4]{36} \left(\cos\left(\frac{23\pi}{24}\right) + i \sin\left(\frac{23\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\frac{23\pi}{24}}$ $k = -1 \quad z_3 = \sqrt[4]{36} \left(\cos\left(-\frac{13\pi}{24}\right) + i \sin\left(-\frac{13\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\left(\frac{-13\pi}{24}\right)}$	M1 M1 B1 A1ft A1ft (5) [8]
(a) B1 M1 A1cao (b) M1 M1 B1 A1ft A1ft NB	Correct modulus Attempt argument using $\tan \theta = \frac{\pm 18}{18\sqrt{3}}$ or other valid method. Can be implied by $\theta = \pm \frac{\pi}{6}$ Correct answer in the required form. Valid method for generating at least 2 roots, rotation through $\frac{\pi}{2}$ accepted Apply de Moivre or use the rotation method Any one correct root Second root in required form All 4 roots in the required form Follow through their $\sqrt[4]{36}$ but 36 not acceptable. Argument in degrees – M1M1B1A0A0 (ie treat as mis-read) Incorrect argument: B0A1ftA1ft available Answers in $r(\cos \theta + i \sin \theta)$ form – deduct final A marks	

Question Number	Scheme	Marks
5	$w = \frac{z-3i}{z+2i}$ $w(z+2i) = z-3i \quad z = \frac{i(2w+3)}{1-w}$ $ z =1 \quad \left \frac{i(2w+3)}{1-w} \right = 1$ $ i(2w+3) = 1-w $ $w = u + iv \quad (2u+3)^2 + 4v^2 = (1-u)^2 + v^2$ $4u^2 + 12u + 9 + 4v^2 = 1 - 2u + u^2 + v^2$ $3u^2 + 3v^2 + 14u + 8 = 0$ $u^2 + v^2 + \frac{14}{3}u + \frac{8}{3} = 0$ $\left(u + \frac{7}{3}\right)^2 + v^2 = -\frac{8}{3} + \frac{49}{9} = \frac{25}{9}$	M1 dM1 ddM1 dddM1 A1
(i)	Centre $\left(-\frac{7}{3}, 0\right)$	A1
(ii)	Radius $\frac{5}{3}$	A1 (7)
		[7]
(a) M1 dM1 ddM1 dddM1 A1 A1 A1	re-arrange to $z = \dots$ dep (on first M1) using $ z =1$ with their previous result dep (on both previous M marks) use $w = u + iv$ (or any other pair of letters inc (x, y)) and find the moduli (or square of it) dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms) for a correct equation in u and v with coeffs of u^2 and v^2 both 1 Correct centre, must be in coordinate brackets. Completion of square need not be shown. Correct radius Centre and radius must come from a correct circle equation for the A marks	

Question Number	Scheme	Marks
6.	$\frac{dy}{dx} + \frac{(x \cot x + 2)}{x} y = \frac{4 \sin x}{x^2}$ $\text{IF} = e^{\int \frac{(x \cot x + 2)}{x} dx}$ $= e^{(\ln \sin x + 2 \ln x)}$ $= x^2 \sin x$ $\frac{d}{dx}(\text{their IF} \times y) = \text{their IF} \times \frac{4 \sin x}{x^2}$ $yx^2 \sin x = \int 4 \sin^2 x dx = 4 \int \frac{1 - \cos 2x}{2} dx = 4 \left(\frac{x}{2} - \frac{1}{4} \sin 2x \right) (+C)$ $y = \frac{2x - \sin 2x + C}{x^2 \sin x} \quad \text{oe}$	B1 M1 A1 A1 M1 dM1A1 A1cao [8]
B1 M1 A1 A1 M1 dM1 A1 A1	Divide through by x^2 Attempt an IF of the form $e^{\int \frac{k(x \cot x + 2)}{x} dx}$ $(\ln \sin x + 2 \ln x)$ Correct IF Multiply through by their IF and write LHS in form shown – can be implied by next line. Allow if IF is seen instead of their function provided an IF has been attempted. Allow use of their RHS Attempt to integrate $\sin^2 x$, including using $\sin^2 x = \frac{1}{2}(1 \pm \cos 2x)$ $\cos 2x \rightarrow k \sin 2x$ depends on previous M mark Correct integration, constant not needed Include the constant and treat it correctly. Must have $y = \dots$	



Question Number	Scheme	Marks
7 (a)	$r \sin \theta = 2a \sin \theta + 2a \sin \theta \cos \theta \quad \text{OR} \quad r \sin \theta = 2a \sin \theta + a \sin 2\theta$ $\frac{d(r \sin \theta)}{d\theta} = 2a \cos \theta + 2a \cos^2 \theta - 2a \sin^2 \theta \quad \left \quad \frac{d(r \sin \theta)}{d\theta} = 2a \cos \theta + 2a \cos 2\theta \right.$ $2 \cos^2 \theta + \cos \theta - 1 = 0 \quad \text{terms in any order}$ $(2 \cos \theta - 1)(\cos \theta + 1) = 0$ $\cos \theta = \frac{1}{2} \quad \theta = \frac{\pi}{3} \quad (\theta = \pi \text{ need not be seen})$ $r = 2a \times \frac{3}{2} = 3a$	B1 M1 A1 dM1A1 A1 (6)
(b)	$\text{Area} = \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4a^2 (1 + \cos \theta)^2 d\theta$ $= 2a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 + 2 \cos \theta + \cos^2 \theta) d\theta$ $= 2a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1 + 2 \cos \theta + \frac{1}{2} (\cos 2\theta + 1) \right) d\theta$ $= 2a^2 \left[\theta + 2 \sin \theta + \frac{1}{2} \left(\frac{1}{2} \sin 2\theta + \theta \right) \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= 2a^2 \left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} - \left(\frac{\pi}{6} + 1 + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{12} \right) \right]$ $= 2a^2 \left(\frac{\pi}{4} + \sqrt{3} - 1 \right)$ $\text{Area of } \triangle OAB = \frac{1}{2} \times 3a \times (2 + \sqrt{3})a \times \sin \frac{\pi}{6} \left(= \frac{3}{4} a^2 (2 + \sqrt{3}) \right)$ $\text{Shaded area} = 2a^2 \left(\frac{\pi}{4} + \sqrt{3} - 1 \right) - \frac{3}{4} a^2 (2 + \sqrt{3}) = \frac{a^2}{4} (2\pi - 14 + 5\sqrt{3})$	M1 M1 dM1A1 M1 NB: A1 on e-PEN M1A1cao (7) [13]

Question Number	Scheme	Marks
(a)		
B1	Multiply r by $\sin \theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen	
M1	Differentiate $r \sin \theta$ or $r \cos \theta$ (using product rule or using double angle formula first)	
A1	Correct derivative for $r \sin \theta$	
dM1	Use $\sin^2 \theta + \cos^2 \theta = 1$ to form a 3TQ in $\cos \theta$ and attempt its solution by a valid method	
A1	Correct value for θ	
A1	Correct r	
(b)		
M1	Use area $= \frac{1}{2} \int r^2 d\theta$ with $r = 2a + 2a \cos \theta$, no limits needed,	
M1	Use a double angle formula to obtain a function ready for integrating (Alt method uses integration by parts – may be seen)	
dM1	Attempt the integration $\cos 2\theta \rightarrow \frac{1}{k} \sin 2\theta$ $k = \pm 2$ or ± 1	
A1	Correct integration,	
M1	Substitute the limits (need not be simplified). Limits $\frac{\pi}{6}$ and their θ from (a) provided this is $> \frac{\pi}{6}$	
M1	NB: A1 on e-PEN Obtain the area of ΔOAB and subtract from their previous area	
A1	Correct answer	

Question Number	Scheme	Marks
8 (a)	$x = e^u \quad \frac{dx}{du} = e^u \quad \text{or} \quad \frac{du}{dx} = e^{-u} \quad \text{or} \quad \frac{dx}{du} = x$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u} \frac{dy}{du}$ $\frac{d^2y}{dx^2} = -e^{-u} \frac{du}{dx} \frac{dy}{du} + e^{-u} \frac{d^2y}{du^2} \frac{du}{dx} = e^{-2u} \left(-\frac{dy}{du} + \frac{d^2y}{du^2} \right)$ $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 8y = 4 \ln x$ $e^{2u} \times e^{-2u} \left(-\frac{dy}{du} + \frac{d^2y}{du^2} \right) + 3e^u \times e^{-u} \frac{dy}{du} - 8y = 4 \ln(e^u)$ $\frac{d^2y}{du^2} + 2 \frac{dy}{du} - 8y = 4u \quad *$	B1 M1 M1A1 dM1 A1*cso (6)
B1 M1 M1 A1 dM1 A1*cso	$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used Obtaining $\frac{dy}{dx}$ using chain rule here or seen later Obtaining $\frac{d^2y}{dx^2}$ using product rule (penalise lack of chain rule by the A mark) Correct expression for $\frac{d^2y}{dx^2}$ any equivalent form Substituting in the equation to eliminate x (u and y only). Depends on the 2nd M mark Obtaining the given result from completely correct work	
	ALTERNATIVE 1 $x = e^u \quad \frac{dx}{du} = e^u = x$ $\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du} = x \frac{dy}{dx}$ $\frac{d^2y}{du^2} = 1 \frac{dx}{du} \times \frac{dy}{dx} + x \frac{d^2y}{dx^2} \times \frac{dx}{du} = x \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2}$ $x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}$ $\left(\frac{d^2y}{du^2} - \frac{dy}{du} \right) + 3x \times \frac{1}{x} \frac{dy}{du} - 8y = 4 \ln(e^u)$ $\frac{d^2y}{du^2} + 2 \frac{dy}{du} - 8y = 4u \quad *$	B1 M1 M1A1 dM1A1*cso (6)

Question Number	Scheme	Marks
(b)	$m^2 + 2m - 8 = 0$ $(m + 4)(m - 2) = 0, \quad m = -4, 2$ $CF = Ae^{-4u} + Be^{2u}$ PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, $c = 0$) $\frac{dy}{du} = a \quad \frac{d^2y}{du^2} = 0$ $0 + 2a - 8(au + b) = 4u$ $a = -\frac{1}{2} \quad b = -\frac{1}{8}$ $\therefore y = Ae^{-4u} + Be^{2u} - \frac{1}{2}u - \frac{1}{8}$	M1A1 A1 M1 dM1A1 B1ft (7)
(c)	$y = Ax^{-4} + Bx^2 - \frac{1}{2}\ln x - \frac{1}{8}$	B1 (1) [14]
(b) M1 A1 A1 M1 dM1 A1 B1ft	Writing down the correct aux equation and solving to $m = \dots$ (usual rules) Correct solution ($m = -4, 2$) Correct CF – can use any (single) variable Using an appropriate PI and finding $\frac{dy}{du}$ and $\frac{d^2y}{du^2}$ Use of $y = \lambda u$ scores M0 Substitute in the equation to obtain values for the unknowns. Depends on the second M1 Correct unknowns two or three (with $c = 0$) A complete solution, follow through their CF and a non-zero PI. Must have $y = a$ function of u Allow recovery of incorrect variables.	
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x No ft here x^{-4} or $e^{-4\ln x}$ and x^2 or $e^{2\ln x}$ allowed. Must start $y = \dots$	

Question Number	Scheme	Notes	Marks
6	$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 8e^{-3t}$	$t \dots 0$	
(a)	$m^2 + 6m + 13 = 0 \Rightarrow m = \frac{-6 \pm \sqrt{36 - 52}}{2}$ $\{ = -3 \pm 2i \}$	Forms correct auxiliary equation and obtains a correct numerical expression for at least one root by formula or uses CTS (apply usual CTS rule below). One correct root if no working	M1
	CTS rule: $m^2 + 6m + 13 = 0 \Rightarrow \left(m \pm \frac{6}{2}\right)^2 \pm q \pm 13 = 0, q \neq 0 \Rightarrow m = \dots$		
	CF examples: $(x =) e^{-3t} (A \cos 2t + B \sin 2t)$ or $(x =) Ae^{-3t} \cos(-2t) + Be^{-3t} \sin(-2t)$ or $(x =) Pe^{(-3+2i)t} + Qe^{(-3-2i)t}$ or $(x =) e^{-3t} (Pe^{2it} + Qe^{-2it})$	Correct complementary function in any form, allow if the “x=” is missing or wrong and accept for this mark if the CF is given fully in terms of x instead of t.	A1
	PI: $\{x =\} \lambda e^{-3t}$	Correct form for the particular integral selected. Must include λe^{-3t} but accept with any extra terms that correctly disappear when coefficients found. Accept “PI=”. If λe^{pt} is used $p = -3$ must be seen later.	B1
	$\frac{dx}{dt} = -3\lambda e^{-3t}; \frac{d^2x}{dt^2} = 9\lambda e^{-3t}$ $\Rightarrow 9\lambda e^{-3t} + 6(-3\lambda e^{-3t}) + 13\lambda e^{-3t} = 8e^{-3t}$	Differentiates a PI of any form twice (provided it has at least one constant and is a function of t) and substitutes into the equation. Allow only sign/coefficient errors only in the differentiation. Their PI must lead to non-zero derivatives.	M1
	$\Rightarrow 9\lambda - 18\lambda + 13\lambda = 8 \Rightarrow \lambda = \dots (2)$	Proceeds to find the value of the constant following use of a PI of the correct form . Any unnecessary extra terms in the PI must be found to be zero	dM1
	$x = "e^{-3t} (A \cos 2t + B \sin 2t)" + 2e^{-3t}$	Correct general solution ft on their CF only – any CF provided it has at least one constant and is in terms of t. Must have x = ... Do not allow if their CF is miscopied or mathematically changed	A1ft
	Work with a PI of the form λte^{-3t} is B0M1dM0A0 max even if $2e^{-3t}$ is obtained. Only condone incorrect variables if they are recovered but refer to the note for the first A1.		(6)



Question Number	Scheme	Notes	Marks
6(b)	$x = \frac{1}{2}$ at $t = 0$ $\Rightarrow \frac{1}{2} = A + 2 \left(\Rightarrow A = -\frac{3}{2} \right)$	Uses the initial condition for x in their GS to find a linear equation in one or two constants. Allow for GS = CF or CF + PI and the constant may come from the +PI	M1
	$x = e^{-3t} (A \cos 2t + B \sin 2t) + 2e^{-3t}$ $\frac{dx}{dt} = e^{-3t} (-2A \sin 2t + 2B \cos 2t) - 3e^{-3t} (A \cos 2t + B \sin 2t) - 6e^{-3t}$ Uses the product rule to differentiate their real GS obtaining an expression in terms of t of the correct form for their GS (sign and coefficient errors only – so do not allow e.g., $\dots e^{pt} \rightarrow \dots e^{qt}$). Allow for GS = CF or CF + PI and does not have to include constants. If they work with a complex function e.g., $x = Pe^{(-3+2i)t} + Qe^{(-3-2i)t} + 2e^{-3t}$ progress is unlikely. This mark is not scored for work in (c)		M1
	$t = 0, \frac{dx}{dt} = \frac{1}{2} \Rightarrow \frac{1}{2} = 2B - 3A - 6 \Rightarrow B = \dots (=1)$ Uses both initial conditions to find values for the 2 constants (no others) in their GS = (CF with 2 constants) + PI(no constants) . One constant must be found to be non-zero. Requires both previous M marks.		ddM1
	Examples: $x = e^{-3t} \left(-\frac{3}{2} \cos 2t + \sin 2t \right) + 2e^{-3t}$ or $x = e^{-3t} \left(-\frac{3}{2} \cos 2t + \sin 2t + 2 \right)$ or $x = 2e^{-3t} - \frac{3}{2} e^{-3t} \cos 2t + e^{-3t} \sin 2t$	Correct particular solution in any form in terms of t . Must be $x = \dots$ unless this was the only reason for final A0 in part (a) due to omission or e.g., “ $y = \dots$ ” was used	A1
			(4)
(c)	$\frac{dx}{dt} = e^{-3t} (3 \sin 2t + 2 \cos 2t) - 3e^{-3t} \left(-\frac{3}{2} \cos 2t + \sin 2t \right) - 6e^{-3t} = 0$ Sets an expression for $\frac{dx}{dt} = 0$. Accept with any unfound constants provided $\frac{dx}{dt} = f(t)$		M1
	$(3 \sin 2t + 2 \cos 2t) - 3 \left(-\frac{3}{2} \cos 2t + \sin 2t \right) - 6 = 0$ Achieves an equation of the form $a \sin bt + c \cos bt + d = 0$ or equivalent with <u>terms uncollected</u> . One of a and c non-zero and b and d non-zero. Must follow a GS = CF + PI where two constants were found for the CF and one for the PI. Requires previous M mark.		dM1
	$\cos 2t = \frac{12}{13} \Rightarrow t = 0.1973955598 \dots \Rightarrow x$ or $a = \frac{1}{2} e^{-3(0.1973 \dots)} \left(4 - 3 \times \frac{12}{13} + 2 \sin(2 \times 0.1973 \dots) \right) = \dots$ Finds a value of t from $\cos kt = c$ ($k \neq 1, -1 < c < 1$) and uses their positive (or made positive) value of t to find a value of x (or a) via their PS. Accept a pair of stated values. Requires both previous M marks.		ddM1
	x or $a = 0.553(1164729 \dots)$	awrt 0.553	A1
			(4)
			Total 14