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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Maclaurin & Taylor Series (FP2.5)

International A Level Mathematics · FP2: Further Pure Mathematics 2 (WFM02)

15 questions · 117 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

15 questions · 117 marks · 1 sub-topic

SUB-TOPIC FP2.5a

MACLAURIN & TAYLOR SERIES

Sub-topic FP2.5a · 15 questions · 117 marks · lowest tariff first



2.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y^2 = e^{x^2-9}$$

Given that $y = 2$ and $\frac{dy}{dx} = -1$ at $x = 3$, determine a Taylor series for y in ascending powers of $(x - 3)$, up to and including the term in $(x - 3)^3$

(5)



(Total for Question 2 is 5 marks)



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1. $\frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} = 2 \cos x$

- (a) Express $\frac{d^3y}{dx^3}$ in terms of x , $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

At $x = 0, y = 2$ and $\frac{dy}{dx} = 5$

- (b) Determine the value of $\frac{d^3y}{dx^3}$ at $x = 0$
- (1)**

- (c) Express y as a series in ascending powers of x , up to and including the term in x^3 (3)

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Q1

$$\frac{dy}{dx} = y^2 - x$$
$$\frac{d^4 y}{dx^4} = Ay \frac{d^3 y}{dx^3} + B \frac{dy}{dx} \frac{d^2 y}{dx^2}$$

(4)

(3)

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 7 marks)



$$x \frac{dy}{dx} - y^3 = 4$$
$$x \frac{d^3 y}{dx^3} = ay \left(\frac{dy}{dx} \right)^2 + (by^2 + c) \frac{d^2 y}{dx^2}$$

(4)

(b) determine the Taylor series expansion for y in ascending powers of $(x - 2)$, up to and including the term in $(x - 2)^3$, giving each coefficient in simplest form.

(3)



(Total for Question 2 is / marks)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



(Total for Question 7 is / marks)



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Question 5 continued

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Q5

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Question 5 continued

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(Total 8 marks)

Q5







(Total for Question 5 is 8 marks)



Question 1 continued

Lined area for writing the answer to Question 1 continued.



Question 1 continued

Lined area for writing the answer to Question 1 continued.



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 8 marks)



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



(Total for Question 9 is 6 marks)





Question 3 continued

Lined area for writing the answer to Question 3 continued.



(Total for Question 3 is 8 marks)

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Q5

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Q4

Question 6 continued

Lined area for writing the answer to Question 6 continued.





Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



(Total for Question 4 is 7 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



June 2025
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1(a)	$r = 4\sqrt{2}$	Correct r Accept $\pm 4\sqrt{2}$ or $\pm\sqrt{32}$ or other processed equivalents.	B1
	$\theta = -\pi + \arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$	Complete method to find an argument in the correct quadrant for their r	M1
	$\Rightarrow -2\sqrt{2} - (2\sqrt{6})i = 4\sqrt{2}e^{-\frac{2\pi}{3}i}$	Correct answer in the form specified. Accept with $\sqrt{32}$	A1
(3)			
(b)	$z^5 = -2\sqrt{2} - (2\sqrt{6})i$		
	$z = (4\sqrt{2})^{\frac{1}{5}} e^{\frac{\frac{2\pi}{3} + 2k\pi}{5}i}$	Attempts De Moivre correctly with either all roots indicated or simplified r with one argument. See additional notes.	M1
	$\sqrt{2}e^{\frac{2\pi}{15}i}, \sqrt{2}e^{\frac{4\pi}{15}i}, \sqrt{2}e^{\frac{2\pi}{3}i},$ $\sqrt{2}e^{\frac{8\pi}{15}i}, \sqrt{2}e^{\frac{14\pi}{15}i}$	Three correct roots provided coefficients of π for θ are single fractions, allow for $0 < \theta < 2\pi$ Allow $(4\sqrt{2})^{\frac{1}{5}}$ oe for $\sqrt{2}$ and correct roots in trig form	A1
		All five correct roots and no extra. Must have $-\pi < \theta \leq \pi$. Allow e.g., $\sqrt{2}e^{\frac{10\pi}{15}i}$ for $\sqrt{2}e^{\frac{2\pi}{3}i}$	A1
(3)			
Total 6			

Additional Notes:

(a)

B1: Accept positive or negative root. Allow if seen in part (b).

M1: Requires a method to achieve an argument in the correct quadrant for their r . Accept attempts at

$\arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) \pm \pi$ or $\arctan\left(\frac{2\sqrt{2}}{2\sqrt{6}}\right) \pm \pi$ for attempts where $r > 0$, or equivalent methods via arccos or arcsin

(must use r as the denominator). May be implied by sight of any of $-\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{6}$ or $-\frac{5\pi}{6}$ if no method is shown.

In cases where a negative r is given, accept attempts at just $\arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right)$ or $\arctan\left(\frac{2\sqrt{2}}{2\sqrt{6}}\right)$ provided they

do not subsequently adjust to a different quadrant.

A1: Correct answer in the form specified, with $-\pi < \theta < \pi$. Must be given in correct form, but accept

answer with negative r , i.e. $-4\sqrt{2}e^{\frac{\pi}{3}i}$. Accept with exact processed equivalents for r .

Note: Correct answer with no working shown will score B1M1(implied)A1.



General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Question Number	Scheme	Notes	Marks
4(a)	$\frac{dy}{dx} = y^2 - x \Rightarrow \frac{d^2y}{dx^2} = 2y \frac{dy}{dx} - 1$	Correct expression for $\frac{d^2y}{dx^2}$	B1
	$\frac{d^3y}{dx^3} = 2y \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2$ M1: Applies the product rule to obtain $\frac{d^3y}{dx^3} = Ay \frac{d^2y}{dx^2} + \dots$ or $\frac{d^3y}{dx^3} = \dots + B \left(\frac{dy}{dx} \right)^2$ where ... is non-zero A1: Correct expression. Apply isw if necessary.		M1 A1
	$\frac{d^3y}{dx^3} = 2y \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 \Rightarrow \frac{d^4y}{dx^4} = 2y \frac{d^3y}{dx^3} + 2 \frac{dy}{dx} \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \frac{d^2y}{dx^2}$ $\frac{d^4y}{dx^4} = 2y \frac{d^3y}{dx^3} + 6 \frac{dy}{dx} \frac{d^2y}{dx^2}$ Correct expression for $\frac{d^4y}{dx^4}$ or correct values for A and B.		A1
	Note: If e.g. $\frac{d^2y}{dx^2} = 2y \frac{dy}{dx}$ is obtained, allow recovery in (a) so B0M1A1A1 is possible.		
			(4)
(b)	$(y)_{-1} = 1, (y')_{-1} = 2, (y'')_{-1} = 3, (y''')_{-1} = 14, (y'''')_{-1} = 64$ Attempts the values up to at least the 3rd derivative using $x = -1$ and $y = 1$ Condone slips provided the intention is clear. May be implied by their values.		M1
	$(y =) 1 + 2(x+1) + \frac{3(x+1)^2}{2} + \frac{14(x+1)^3}{3!} + \frac{64(x+1)^4}{4!} + \dots$ Correct application of the Taylor series in powers of $(x + 1)$ If the expansion is just written down with no formula quoted then it must be correct for their values. E.g. $y = -1 + \dots$ with no evidence $y = f(-1)$ was meant scores M0		M1
	$(y =) 1 + 2(x+1) + \frac{3(x+1)^2}{2} + \frac{7(x+1)^3}{3} + \frac{8(x+1)^4}{3} + \dots$ Correct simplified expansion. The “y =” is not required.		A1
			(3)
			Total 7

Question 5
General Guidance

B1: This mark is for sight of -8 seen as part of their working. It may be seen as e.g. embedded in an inequality, as part of their solution if they consider for example $x > -8$, $x < -8$ or -8 is seen in a sketch etc.

Do not allow for just e.g. $x + 8 > 0$,

M1: Any valid attempt to find at least one critical value other than $x = -8$ (see below).

Condone use of e.g. “=”, “>”, “<” etc as part of their working.

Note these usually come in pairs as $3, -\frac{19}{3}$ or $3, -13$

M1: A valid attempt to find all critical values.

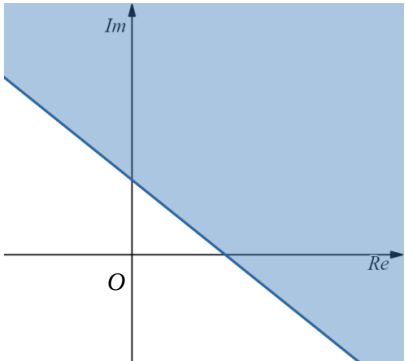
Condone use of e.g. “=”, “>”, “<” etc as part of their working.

A1: Any 2 critical values other than $x = -8$. May be seen embedded in an inequality or on a sketch.

A1: 2 correct regions

A1: All correct with no extra regions



Question Number	Scheme	Marks
1(a)	$ z - 3 - 4i = z + 1 + i \Rightarrow x + iy - 3 - 4i = x + iy + 1 + i $ $(x - 3)^2 + (y - 4)^2 = (x + 1)^2 + (y + 1)^2$	M1
	$x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 + 2x + 1 + y^2 + 2y + 1 \Rightarrow ax + by + c = 0$	dM1
	$8x + 10y - 23 = 0$	A1
		(3)
(a) ALT	Mid-point $\left(\frac{3-1}{2}, \frac{4-1}{2}\right)$ and $m = \frac{4+1}{3+1}$	M1
	$y - \frac{3}{2} = -\frac{4}{5}(x - 1) \Rightarrow ax + by + c = 0$	dM1
	$8x + 10y - 23 = 0$	A1
(b)		B1
		(1)
		Total 4

Notes:

(a)

M1: Introduces $z = x + iy$ (or with other variables) and attempts a correct modulus method – the i's should be dealt with correctly but allow for e.g. minor slips in notation with them before applying Pythagoras. Allow sign slips in the i coordinates, e.g.

$(x - 3)^2 + (y \pm 4)^2 = (x + 1)^2 + (y \pm 1)^2$ Must be + between the brackets.

dM1: Expands and attempts the required form.

A1: This equation or any integer multiple.

Alternative: via perpendicular bisector

M1: Uses the points $(3, \pm 4)$ and $(-1, \pm 1)$ to find the midpoint and gradient between them.

dM1: Correct attempt at the equation of the perpendicular bisector and rearranges to the required form. Must be using negative reciprocal of the gradient between points (but watch for those who go directly to this).

A1: This equation or any integer multiple.

(b) May be scored independently of (a) – using the perpendicular bisector.

B1: Shades the area above an appropriate line, must be in quadrants 1, 2 and 4. Accept with solid, dashed or dotted line. Allow for any line with negative gradient and positive y intercept. Do not be concerned about labelling. Part (a) need not have been correct.



Question Number	Scheme	Marks
3(a)	$\frac{1}{(n+3)(n+5)} \equiv \frac{A}{(n+3)} + \frac{B}{(n+5)}$ $\Rightarrow A = \dots, B = \dots$	M1
	$\frac{1}{2(n+3)} - \frac{1}{2(n+5)} \text{ (oe)}$	A1
		(2)
(b)	$\left(\frac{1}{2}\right) \sum_{r=1}^n \left(\frac{1}{r+3} - \frac{1}{r+5} \right) =$ $\left(\frac{1}{2}\right) \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \dots + \frac{1}{n+2} - \frac{1}{n+4} + \frac{1}{n+3} - \frac{1}{n+5} \right)$	M1
	$= \left(\frac{1}{2}\right) \left(\frac{1}{4} + \frac{1}{5} - \frac{1}{n+4} - \frac{1}{n+5} \right) \text{ (oe)}$	A1
	$= \left(\frac{1}{2}\right) \left(\frac{9(n+4)(n+5) - 20(n+5) - 20(n+4)}{20(n+4)(n+5)} \right) = \dots$	dM1
	$= \frac{n(9n+41)}{40(n+4)(n+5)}$	A1
		(4)
(c)	$\frac{1}{9 \times 11} + \frac{1}{10 \times 12} + \dots + \frac{1}{24 \times 26} = \frac{21(9 \times 21 + 41)}{40 \times 25 \times 26} - \frac{5(9 \times 5 + 41)}{40 \times 9 \times 10}$	M1
	$= \frac{194}{2925}$	A1
		(2)
		Total 8

Notes:

(a)

M1: Correct partial fraction attempt to obtain values for A and B. May be implied by correct values.

A1: Correct expression as shown or any equivalent. May be $\frac{1}{2}$ in each numerator, or the $\frac{1}{2}$ may be factored out. The expression must be given in (a) not just embedded in the summation in (b).

(b)

M1: Uses the method of differences to identify non-cancelling terms. Expect to see at least first two pairs and last pair of terms listed, but this may be implied by the correct first two or last two non-cancelling terms being stated with no cancelling explicitly shown. Allow if the $\frac{1}{2}$ is missing for this mark.A1: Identifies the correct non-cancelling terms. Allow if the $\frac{1}{2}$ is missing for this mark.dM1: Puts over a common denominator $k(n+4)(n+5)$, where $k \neq 1$, and makes progress towards the required form. The $\frac{1}{2}$ may be missing for this mark. Allow slips in the numerator coefficients but should have correct terms in n .

A1: Achieves the correct answer.

(c)

M1: Attempts to use the formula from (b) with $\sum_{r=1}^{21} - \sum_{r=1}^k$ where $k = 5$ or 6 .

Substitution into the formula must be seen at least once.

A1: Correct answer achieved (the M must have been scored). Must be simplified.



Question Number	Scheme	Marks
4(a)	E.g. $2y \frac{dy}{dx} = -\frac{1}{t^2} \frac{dt}{dx}$ or $y^2 = \frac{1}{t} \Rightarrow t = y^{-2} \Rightarrow \frac{dt}{dx} = -2y^{-3} \frac{dy}{dx}$ or $y = t^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{1}{2} t^{\frac{3}{2}} \frac{dt}{dx}$	B1
	$\frac{dy}{dx} + y = xy^3 \Rightarrow -\frac{t^{-\frac{3}{2}}}{2} \frac{dt}{dx} + \frac{1}{\sqrt{t}} = xt^{\frac{3}{2}}$	M1
	$\frac{dt}{dx} - 2t = -2x^*$	A1*
		(3)
(b)	$I = e^{-\int 2 dx} = e^{-2x}$	B1
	$te^{-2x} = -2 \int xe^{-2x} dx$	M1
	$= xe^{-2x} - \int e^{-2x} dx$	M1
	$= xe^{-2x} + \frac{1}{2} e^{-2x} (+c)$	A1
	$t = \frac{1}{y^2} = x + \frac{1}{2} + ce^{2x}$ $\Rightarrow y^2 = \frac{1}{x + \frac{1}{2} + ce^{2x}} \left(\text{or e.g. } \frac{2}{2x+1+Ae^{2x}} \right) \text{ (oe)}$	M1A1
		(6)
		Total 9

Notes:

(a)

B1: Any appropriate correct first derivative statement connecting $\frac{dy}{dx}$ and $\frac{dt}{dx}$. May be implied by use of chain rule when substituting.

M1: Having attempted a first derivative statement, substitutes completely into the given differential equation to obtain an equation in x and t only (which may be the final step).

A1*: Achieves the printed answer with no errors.

(b)

B1: Correct integrating factor. May be implied by working.

M1: For their $IF \times t = -2 \int x \times \text{their } IF (dx)$ provided their IF is a function in x .

M1: Integrates RHS by parts to obtain $\alpha xe^{\pm kx} + \beta \int e^{\pm kx} (dx)$

A1: Correct RHS (not follow through – must be the correct RHS). The $+c$ is not needed for this mark.

M1: Reverses the substitution and makes y^2 the subject. The constant of integration must have been introduced at the right time and been treated correctly, although the algebra in rearranging need not be fully correct.

A1: Correct answer in the required form with the constant correctly placed. Does not need to be fully simplified, and e.g. “ $2c$ ” is fine for the constant and allow for “ $2c$ ” renamed and “ c ” and such. ISW after a correct answer is seen.



Alt (b)	$m - 2 = 0 \Rightarrow m = 2$ so CF is $t = Ae^{2x}$	B1
	For PI try $t = ax + b$	M1
	$\frac{dt}{dx} - 2t = -2x \Rightarrow a - 2(ax + b) = -2x \Rightarrow a = \dots, b = \dots$	M1
	$a = 1, b = \frac{1}{2} \Rightarrow t = Ae^{2x} + x + \frac{1}{2}$	A1
	$t = \frac{1}{y^2} = x + \frac{1}{2} + Ae^{2x}$ $\Rightarrow y^2 = \frac{1}{x + \frac{1}{2} + Ae^{2x}} \left(\text{or e.g. } \frac{2}{2x + 1 + Be^{2x}} \right) \text{ (oe)}$	M1A1
		(6)

Notes:

B1: Solves the auxiliary equation and forms the correct complementary function.

M1: Selects an appropriate form for the particular integral.

M1: Differentiates and substitutes into the differential equation to find values for the constants of the PI.

A1: Correct values and adds the CF and PI (t = may be missing for this mark).

M1: Reverses the substitution and makes y^2 the subject. Must have formed an equation using **both** PI and CF for this mark.

A1: Correct answer in the required form with the constant correctly placed. Does not need to be fully simplified and same conditions for main scheme. ISW after a correct answer is seen.



Question Number	Scheme	Marks
5	$x = -2, 3$	B1
	$\frac{x+1}{(x-3)(x+2)} = 1 - \frac{2}{x-3} \Rightarrow x+1 = x^2 - x - 6 - 2(x+2) \Rightarrow x^2 - 4x - 11 = 0$ or $\frac{x+1}{(x-3)(x+2)} - 1 + \frac{2}{x-3} \leq 0 \Rightarrow \frac{4x - x^2 + 11}{(x-3)(x+2)} = 0$ or $(x+1)(x-3)(x+2) \leq (x-3)^2(x+2)^2 - 2(x-3)(x+2)^2$ $\Rightarrow (x-3)(x+2)[4x - x^2 + 11] = 0$	M1
	$x^2 - 4x - 11 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16+44}}{2} = \dots$	M1
	$x = 2 \pm \sqrt{15}$	A1
	$x < -2, 2 - \sqrt{15} \leq x < 3, x \geq 2 + \sqrt{15}$	M1A1
		(6)
	Total 6	

Notes:

B1: The cv's -2 and 3 stated or used. May be seen on a diagram.

M1: Any complete **algebraic** method to reach at least a expanded **quadratic** for the other two critical values. E.g. equates sides, multiplies through and simplifies, or collects to one side and finds common denominator or multiplies through by a positive expression and collects to one side etc. Do not be concerned about inaccuracies in the inequalities in intermediate steps – score for the overall method leading to a suitable equation being found.

M1: Correct method (usual rules) to solve the resulting **three term quadratic** equation for the remaining cv's. Once a quadratic is reached accept if correct solutions for their quadratic appear without working.

A1: For the cv's $2 \pm \sqrt{15}$ Must be exact.

M1: Uses their cv's form the correct shape for the solution, $x < \alpha, b \leq x < \gamma, x \geq \delta$ allowing for the inequalities to be loose or strict in each case. Number lines, sign tables or graph may be used, or this may be deduced from inequalities work throughout the question.

A1: Fully correct and no extra other region. Must be exact values. Accept interval notation, e.g $(-\infty, -2) \cup [2 - \sqrt{15}, 3) \cup [2 + \sqrt{15}, \infty)$. Note if set notation is used the “union” symbol must be correct, but in language accept “and” or “or”.

Note: Solutions that expand to a quartic and then state the four CVs with no algebraic method shown to solve the quartic score at most SC B1M0M0A0M1A1.



Question Number	Scheme	Notes	Marks
6 Way 1	$w = \frac{z-i}{z+1} \Rightarrow z = \frac{w+i}{1-w}$ (oe)		B1
	$z = \frac{w+i}{1-w} = \frac{u+iv+i}{1-u-iv} \times \frac{1-u+iv}{1-u+iv}$		M1
	$z = \frac{u-u^2+uvi-vi-uv+i-uv+i-uv-v^2-v+iv+i}{(1-u)^2+v^2}$ $\text{Re}(z) = 0 \Rightarrow u-u^2-v^2-v = 0$		M1A1
	$u-u^2-v^2-v = 0 \Rightarrow \left(u-\frac{1}{2}\right)^2 + \left(v+\frac{1}{2}\right)^2 = \frac{1}{2}$		dM1
	Centre: $\left(\frac{1}{2}, -\frac{1}{2}\right)$ Radius: $\frac{1}{\sqrt{2}}$		A1A1
			(7)
Way 2	$w = \frac{z-i}{z+1} \Rightarrow w = \frac{iy-i}{iy+1}$		B1
	$w = \frac{iy-i}{iy+1} \times \frac{1-iy}{1-iy}$		M1
	$u+iv = \frac{iy+y^2-i-y}{1+y^2} = \frac{y^2-y}{1+y^2} + \frac{y-1}{1+y^2}i \Rightarrow \frac{u}{v} = y$ $u = \frac{\frac{u^2}{v^2} - \frac{u}{v}}{1 + \frac{u^2}{v^2}}$ or $v = \frac{\frac{u}{v} - 1}{1 + \frac{u^2}{v^2}} \quad (u-u^2-v^2-v=0)$		M1A1
	$u-u^2-v^2-v = 0 \Rightarrow \left(u-\frac{1}{2}\right)^2 + \left(v+\frac{1}{2}\right)^2 = \frac{1}{2}$		dM1
	Centre: $\left(\frac{1}{2}, -\frac{1}{2}\right)$ Radius: $\frac{1}{\sqrt{2}}$		A1A1

Notes:

B1: Correct rearrangement as shown or equivalent.

M1: Introduces $w = u + iv$ (or may use other variables) and multiplies numerator (and denominator) by the conjugate of the denominator.

M1: Expands the numerator and sets the real part to zero – allow if there are slips in the denominator e.g. wrong sign.

A1: Correct equation in any form.

dM1: Completes the square for u and v . They must have attained an expression on which they need to complete the square on at least one of u or v . Alternatively allow for correct centre up to sign error, or correct radius, deduced from expanded circle equation.

A1: Correct centre – but must be from a circle equation. Accept as coordinates or as

 $\frac{1}{2} - \frac{1}{2}i$ and be tolerant of notation errors, e.g. $\left(\frac{1}{2}, -\frac{1}{2}i\right)$ if the correct point is clear.A1: Correct radius following a correct equation in u and v (allowing sign slips on u and v). Must be exact, but accept equivalents.

Alternative:

B1: Replaces z with iy (oe), or may replace by $x + iy$ and later set $x = 0$ in the equation.

M1: Multiplies numerator and denominator by the conjugate of the denominator.

M1: Rearranges into real and imaginary parts to obtain y in terms of u and v and substitutes into u or v to eliminate y . Allow recovery from slips in the denominator per main scheme.

A1: Correct equation in any form.

dM1: Completes the square for u and v . They must have attained an expression on which they need to complete the square on at least one of u or v . Alternative as per main scheme.

A1: Correct centre. Accept as coordinates or as $\frac{1}{2} - \frac{1}{2}i$ and be tolerant of notation errors,

e.g. $\left(\frac{1}{2}, -\frac{1}{2}i\right)$ if the correct point is clear.

A1: Correct radius from correct work (allowing sign slips on u and v). Must be exact, but accept equivalents.

Note: Some rarer alternatives may be seen, but if you are unsure of a method send to review.

Way 3	$0 \rightarrow -i, i \rightarrow 0, \infty \rightarrow 1$	B1
	E.g. Bisector of 0 and 1 is $u = \frac{1}{2}$	M1
	Bisector of 0 and $-i$ is $v = -\frac{1}{2}$	M1A1
	Bisectors meet at when $u = \frac{1}{2}$ and $v = -\frac{1}{2}$	dM1
	Centre: $\left(\frac{1}{2}, -\frac{1}{2}\right)$ Radius: $\frac{1}{\sqrt{2}}$	A1A1

Notes:

B1: Identifies three correct image points on the circle.

M1: Finds at least two image points on the circle and attempts the perpendicular bisector of at least one pair of points. Alt: attempts to substitute into a general circle equation and makes some progress eliminating one variable.

M1: Finds three image points and attempts two perpendicular bisectors. Alt: uses the third point to form a second equation in the same two variables.

A1: Correct bisectors. Alt: correct equations.

dM1: Finds the intersection of the bisectors. Note MMAM may be implied in this method if the bisectors are obvious (as with the three points shown in the scheme). Alt: valid method for fully solving the simultaneous equations.

A1: Correct centre. Accept as coordinates or as $\frac{1}{2} - \frac{1}{2}i$ and be tolerant of notation errors,

e.g. $\left(\frac{1}{2}, -\frac{1}{2}i\right)$ if the correct point is clear.

A1: Correct radius from correct work (allowing sign slips on u and v). Must be exact, but accept equivalents.

Way 4: Using inverse points of the imaginary axis, e.g. ± 1 is also possible. Here as -1 maps to ∞ so 1 maps to the centre of the circle, and then any other point may be used to work out the radius of the circle. If other inverse points are used it will be more complicated. Send to review if you see any responses like this that you feel deserve merit.



Question Number	Scheme	Marks
7(a)	$y = e^x \sin x \Rightarrow \left(\frac{dy}{dx} = \right) e^x \sin x + e^x \cos x$ $\Rightarrow \left(\frac{d^2y}{dx^2} = \right) e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x = 2e^x \cos x \text{ (oe – see notes)}$	M1
	E.g. (see notes for equivalents) $\Rightarrow \left(\frac{d^3y}{dx^3} = \right) 2e^x \cos x - 2e^x \sin x$ $\Rightarrow \frac{d^4y}{dx^4} = 2e^x \cos x - 2e^x \sin x - 2e^x \cos x - 2e^x \sin x$ $\frac{d^4y}{dx^4} = -4e^x \sin x = -4y \text{ (oe – see notes)}$	dM1 A1
	$\Rightarrow \frac{d^4y}{dx^4} = -4y \Rightarrow \frac{d^5y}{dx^5} = -4 \frac{dy}{dx} \Rightarrow \frac{d^6y}{dx^6} = -4 \frac{d^2y}{dx^2}$	A1
		(4)
		Total 7
(b)	$(y)_0 = 0, \left(\frac{dy}{dx} \right)_0 = 1, \left(\frac{d^2y}{dx^2} \right)_0 = 2, \left(\frac{d^3y}{dx^3} \right)_0 = 2, \left(\frac{d^4y}{dx^4} \right)_0 = 0, \left(\frac{d^5y}{dx^5} \right)_0 = -4, \left(\frac{d^6y}{dx^6} \right)_0 = -8$	M1
	$(y) = 0 + x + \frac{x^2}{2} \times 2 + \frac{x^3}{3!} \times 2 + 0 - \frac{x^5}{5!} \times 4 - \frac{x^6}{6!} \times 8 + \dots$	M1
	$(y) = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + \dots$	A1
		(3)
		Total 7

Notes: Accept other forms of notation for the derivatives, e.g. y' etc.

(a)

M1: Applies the product rule on y and y' to reach at least the second derivative. Condone mislabelling of the derivatives for the Ms. May work in terms of x or y or a combination of both, e.g. $y' = y + e^x \cos x$, $y'' = y' + e^x \cos x - e^x \sin x = y' - y + e^x \cos x = 2y' - 2y$ etc

dM1: Continues the differentiation process to obtain an expression for the 4th derivative. Again, may be working in either x or y or a combination of both.

A1: Any correct fourth derivative expression, may be unsimplified and award when first seen, e.g. may identify $\frac{d^4y}{dx^4} = -4y$ or even $2y''' - 2y''$. Must be correctly identified as the fourth derivative.

A1: Completes the process to obtain a correct expression, or for $k = -4$. May work in x or y or a mixture for the final derivatives.

E.g. $y^{vi} = 2y^v - 2y^{iv} = 4y^{iv} - 4y''' - 2y^{iv} = 2y^{iv} - 4y''' = 4y''' - 4y'' - 4y''' = -4y''$ is one possible variation, but others are possible with earlier eliminations. Most will use main scheme.

(b)

M1: Attempts to find values for all 6 derivatives at $x = 0$, but condone one missing as a slip. Accept values seen stated for each derivative as an attempt.

M1: Applies Maclaurin's theorem correctly with their values, allowing for one slip in the factorials, up to the term in x^6 (or a fifth non-zero term if errors mean the x^6 terms is not needed).

A1: Correct expansion, condone missing the " $y =$ ".



Question Number	Scheme	Notes	Marks
4(a)	$(x+1)\frac{dy}{dx} - xy = e^{3x} \quad x > -1$		
	$\frac{dy}{dx} - \frac{xy}{(x+1)} = \frac{e^{3x}}{(x+1)}$	Correctly rearranged equation	B1
	$I = e^{\int \frac{-x}{x+1} dx} = e^{\int \left(-1 + \frac{1}{x+1}\right) dx}$	Correct strategy for the integrating factor including an attempt at the integration	M1
	$= e^{-x + \ln(x+1)}$	For $-x + \ln(x+1)$	A1
	$= (x+1)e^{-x}$	Correct integrating factor	A1
	$y(x+1)e^{-x} = \int \frac{e^{3x}}{x+1} \times (x+1)e^{-x} dx$	Uses their integrating factor to reach the form $yI = \int QI dx$	M1
	$y(x+1)e^{-x} = \frac{1}{2}e^{2x} + c$	Correct equation (with or without + c)	A1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{ce^x}{(x+1)}$	Correct answer (allow equivalent forms). Must have $y = \dots$	A1
			(7)
(b)	$x = 0, y = 5 \Rightarrow 5 = \frac{1}{2} + c \Rightarrow c = \frac{9}{2}$	Substitutes $x = 0$ and $y = 5$ and attempts to find a value for c .	M1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{9e^x}{2(x+1)}$	Cao (oe) Must have $y = \dots$	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$y = \tan^2 x \Rightarrow \frac{dy}{dx} = 2 \tan x \sec^2 x$	Correct first derivative any correct form	B1
	$\frac{dy}{dx} = 2 \tan x \sec^2 x \Rightarrow \frac{d^2y}{dx^2} = 2 \sec^4 x + 4 \sec^2 x \tan^2 x$ M1: Correct application of the product rule and chain rule A1: Correct expression		M1A1
	$\frac{d^2y}{dx^2} = 2 \sec^4 x + 4 \sec^2 x \tan^2 x \Rightarrow \frac{d^3y}{dx^3} = 8 \sec^4 x \tan x + 8 \sec^2 x \tan^3 x + 8 \sec^4 x \tan x$ Or $\frac{d^2y}{dx^2} = 6 \sec^4 x - 4 \sec^2 x \Rightarrow \frac{d^3y}{dx^3} = 24 \sec^4 x \tan x - 8 \sec^2 x \tan x$ M1: Attempt to differentiate using product and chain rule. At least one term to be correct		M1
	$= 8 \sec^4 x \tan x + 8 \sec^2 x \tan x (\sec^2 x - 1) + 8 \sec^4 x \tan x$ $= 24 \sec^4 x \tan x - 8 \sec^2 x \tan x = 8 \sec^2 x \tan x (3 \sec^2 x - 1)$ Fully correct expression		A1
			(5)
(b)	$(y)_{\frac{\pi}{3}} = 3, (y')_{\frac{\pi}{3}} = 8\sqrt{3}, (y'')_{\frac{\pi}{3}} = 80, (y''')_{\frac{\pi}{3}} = 352\sqrt{3}$	Attempts the values up to the third derivative when $x = \frac{\pi}{3}$	M1
	$y = 3 + 8\sqrt{3}\left(x - \frac{\pi}{3}\right) + \frac{80}{2!}\left(x - \frac{\pi}{3}\right)^2 + \frac{352\sqrt{3}}{3!}\left(x - \frac{\pi}{3}\right)^3 + \dots$ Correct application of the Taylor series 2! or 2, 3! or 6		M1
	$y = 3 + 8\sqrt{3}\left(x - \frac{\pi}{3}\right) + 40\left(x - \frac{\pi}{3}\right)^2 + \frac{176\sqrt{3}}{3}\left(x - \frac{\pi}{3}\right)^3 + \dots$ Correct expansion Must start $y = \dots$ or $\tan^2 x = \dots$ $f(x)$ only accepted if $f(x)$ has been defined to be $\tan^2 x$		A1
			(3)
			Total 8

Question	Scheme	Marks
2	$2m^2 - 5m - 3 = 0 \Rightarrow (2m+1)(m-3) = 0 \Rightarrow m = \dots$	M1
	So C.F. is $(y_{CF} =) Ae^{-\frac{1}{2}x} + Be^{3x}$	A1
	P.I. is $y_{PI} = axe^{3x}$	B1
	$\frac{dy_{PI}}{dx} = 3axe^{3x} + ae^{3x}, \frac{d^2y_{PI}}{dx^2} = 9axe^{3x} + 3ae^{3x} + 3ae^{3x}$ $\Rightarrow 2(9ax + 6a)e^{3x} - 5(3ax + a)e^{3x} - 3axe^{3x} = 2e^{3x} \Rightarrow a = \dots$	M1
	$a = \frac{2}{7}$	A1
	General solution is $y = Ae^{-\frac{1}{2}x} + Be^{3x} + \frac{2}{7}xe^{3x}$	B1ft
		(6)
(6 marks)		
Notes:		
M1: Forms and solves the auxiliary equation. A1: Correct complementary function (no need for $y = \dots$) B1: Correct form for the particular integral. Accept any PI that includes axe^{3x}, so e.g. $(ax+b)e^{3x}$ is fine. M1: Attempts to differentiate their PI twice and substitutes into the left hand side of the equation. The derivatives must be changed functions. There is no need to reach a value for the unknown(s) but their PI must contain an unknown constant. A1: Correct value of a (and any other coefficients as zero). Must have had a suitable PI B1ft: For $y =$ their CF + their PI. Must include the $y =$. The PI must be a function of x that matches their initial choice of PI, with their constants substituted.		

Question Number	Scheme	Marks
5.	$\frac{d^2 y}{dx^2} = -\frac{2}{y} \left(\frac{dy}{dx} \right)^2 + 2$	
(a)	$\left(\frac{dy}{dx} \right) \frac{d^2 y}{dx^2}$ seen	B1
	$\frac{d^3 y}{dx^3} = -\frac{4}{y} \left(\frac{dy}{dx} \right) \frac{d^2 y}{dx^2} + \frac{2}{y^2} \left(\frac{dy}{dx} \right)^3$	M1A1A1
		(4)
ALT:	$\left(\frac{dy}{dx} \right)^2 \rightarrow 2 \left(\frac{dy}{dx} \right) \frac{d^2 y}{dx^2}$ seen $\frac{dy}{dx} \left(\frac{d^2 y}{dx^2} \right) + y \frac{d^3 y}{dx^3} + 4 \left(\frac{dy}{dx} \right) \left(\frac{d^2 y}{dx^2} \right) - 2 \frac{dy}{dx} = 0$ $\frac{d^3 y}{dx^3} = \frac{1}{y} \left(-5 \frac{d^2 y}{dx^2} + 2 \right) \frac{dy}{dx}$	B1 M1A1 A1 (4)
(b)	At $x = 0$ $\frac{d^2 y}{dx^2} = \frac{1}{2} (-2 \times (1)^2 + 4) = 1$	B1
	$\frac{d^3 y}{dx^3} = \frac{1}{2} (-5 \times 1 + 2) \times 1 = \frac{-3}{2}$	M1
	$(y =) 2 + x + (1) \frac{x^2}{2!} + \left(\frac{-3}{2} \right) \frac{x^3}{3!} + \dots$	M1
	$y = 2 + x + \frac{1}{2} x^2 - \frac{1}{4} x^3 + \dots$	A1 (4)
		[8]
	Notes	
(a)		
B1	$\left(\frac{dy}{dx} \right) \frac{d^2 y}{dx^2}$ seen in the differentiation	
M1	Divide equation by y and differentiate wrt x chain and product rules needed. LHS correct	
A1	Either RHS term correct. Need not be simplified.	
A1	Both RHS terms correct. Need not be simplified.	
ALT		
B1	$\left(\frac{dy}{dx} \right)^2 \rightarrow 2 \left(\frac{dy}{dx} \right) \frac{d^2 y}{dx^2}$ correct differentiation of middle term.	
M1	Differentiate before dividing. Product rule must be used.	
A1	Correct differentiation of $y \frac{d^2 y}{dx^2}$ and $-2y$	
A1	Rearrange to a correct expression for $\frac{d^3 y}{dx^3}$ (need not be simplified)	

	Notes
(b)	
B1	Correct value for $\frac{d^2 y}{dx^2}$. May be implied by the term in their expansion.
M1	Use their expression from (a) to obtain a value for $\frac{d^3 y}{dx^3}$ (May be implied - you may need to check if their value follows from their expression in (a).)
M1	Taylor's series formed using their values for the derivatives, accept 2! or 2 and 3! or 6
	Correct series, must start $y = \dots$, or allow $f(x) = \dots$ as long as $y = f(x)$ has been defined in the question.
A1	Must come from a correct expression for $\frac{d^3 y}{dx^3}$

Question Number	Scheme	Notes	Marks
1(a)	$y = \ln(5+3x) \Rightarrow \frac{dy}{dx} = \frac{3}{5+3x}$	Correct first derivative	B1
	$\frac{dy}{dx} = \frac{3}{5+3x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{9}{(5+3x)^2} \Rightarrow \frac{d^3y}{dx^3} = \frac{54}{(5+3x)^3}$ M1: Continues the process of differentiating and reaches $\frac{d^3y}{dx^3} = \frac{k}{(5+3x)^3}$ oe Note this may be achieved via the quotient rule e.g. $\frac{d^3y}{dx^3} = \frac{-9 \times -2 \times 3(5+3x)}{(5+3x)^4}$ A1: Correct simplified third derivative. Allow e.g. $\frac{54}{(5+3x)^3}$ or $54(5+3x)^{-3}$.	M1 A1	
			(3)
(b)	$y_0 = \ln 5, y'_0 = \frac{3}{5}, y''_0 = -\frac{9}{25}, y'''_0 = \frac{54}{125}$ $\Rightarrow \ln(5+3x) \approx \ln 5 + \frac{3}{5}x - \frac{9}{25} \frac{x^2}{2!} + \frac{54}{125} \frac{x^3}{3!} + \dots$ Attempts all values at $x = 0$ and applies Maclaurin's theorem. Evidence for attempting the values can be taken from at least 2 terms. The form of the expansion must be correct including the factorials or their values. Note that this is "Hence" and so do not allow other methods e.g. Formula Book.		M1
	$\ln(5+3x) \approx \ln 5 + \frac{3}{5}x - \frac{9}{50}x^2 + \frac{9}{125}x^3 + \dots$ Correct expansion. The "$\ln(5+3x) =$" is not required.		A1
			(2)
(c)	$\ln(5-3x) \approx \ln 5 - \frac{3}{5}x - \frac{9}{50}x^2 - \frac{9}{125}x^3 + \dots$ Correct expansion even if obtained "from scratch" OR for a correct follow through with signs changed on the coefficients of the odd powers of x only in an expansion of the correct form e.g. a polynomial in ascending powers of x.		B1ft
			(1)
(d)	$\ln \frac{(5+3x)}{(5-3x)} = \ln(5+3x) - \ln(5-3x)$ $\ln 5 + \frac{3}{5}x - \frac{9}{50}x^2 + \frac{9}{125}x^3 + \dots - \left(\ln 5 - \frac{3}{5}x - \frac{9}{50}x^2 - \frac{9}{125}x^3 + \dots \right)$ Subtracts their 2 different series to obtain at least 2 non-zero terms in ascending powers of x.		M1
	$= \frac{6}{5}x + \frac{18}{125}x^3 + \dots$ Correct terms. Allow e.g. $0 + \frac{6}{5}x + 0x^2 + \frac{18}{125}x^3 + \dots$		A1

	Allow both marks to score in (d) provided the correct series have been obtained in (b) and (c) by any means.	
		(2)
		Total 8



Question	Scheme	Marks
3(a)	$2x \frac{d^2y}{dx^2} + y \frac{dy}{dx} - 6xy = 0$ and $\frac{dy}{dx} = 3$, $y = \frac{1}{2}$ when $x = 2$	
	$\frac{d^2y}{dx^2} = \frac{1}{2x} \left(6xy - y \frac{dy}{dx} \right) \Rightarrow \frac{d^2y}{dx^2} \Big _{x=2} = \frac{1}{4} \left(6 - \frac{3}{2} \right) = \frac{9}{8}$	M1A1
	E.g. $2 \frac{d^2y}{dx^2} + 2x \frac{d^3y}{dx^3} + y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 - 6y - 6x \frac{dy}{dx} = 0$ Or $\frac{d^3y}{dx^3} = -\frac{1}{2x^2} \left(6xy - y \frac{dy}{dx} \right) + \frac{1}{2x} \left(6x \frac{dy}{dx} + 6y - \left(\frac{dy}{dx} \right)^2 - y \frac{d^2y}{dx^2} \right)$ Or $2x \frac{d^2y}{dx^2} = -y \frac{dy}{dx} + 6xy \Rightarrow 2 \frac{d^2y}{dx^2} + 2x \frac{d^3y}{dx^3} = -\left(\frac{dy}{dx} \right)^2 - y \frac{d^2y}{dx^2} + 6y + 6x \frac{dy}{dx}$	M1A1 A1
	$\frac{d^3y}{dx^3} \Big _{x=2} = \frac{435}{64}$	A1
		(6)
(b)	$(y) = \frac{1}{2} + 3(x-2) + \frac{9/8}{2}(x-2)^2 + \frac{435/64}{3!}(x-2)^3$	M1
	$y = \frac{1}{2} + 3(x-2) + \frac{9}{16}(x-2)^2 + \frac{145}{128}(x-2)^3$	A1
		(2)
(8 marks)		

Notes:**(a)**

M1: Attempts to find $\frac{d^2y}{dx^2}$ at $x = 2$ (May see after the 3rd derivative has been calculated)

A1: Correct value. Note if a method is used in (a) that does not find this value, score for the attempt at it in (b). A correct value for $\frac{d^2y}{dx^2}$ implies M1A1

M1: Attempts to differentiate the equation (may have been rearranged first) with **at least one** use of **product rule** in evidence (sign/coefficient errors only)

A1: For $y \frac{dy}{dx} \rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2$

A1: Fully correct expression in the third derivative (can be implicit)

A1: Correct value for the third derivative following a correct expression

(b)

M1: Applies the series formula correctly with their values. Must have the correct factorials/implied factorials if not shown. If series is incorrect for their values and no formula shown then M0

A1: Correct answer with simplified coefficients. **Must be** $y = \dots$ so $f(x) = \dots$ is A0

(a) **Alt**

M1: Attempts to differentiate the equation (may have been rearranged first) with at least one use of product rule.

A1: For $y \frac{dy}{dx} \rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$

M1: Attempts to replace $\frac{d^2y}{dx^2}$ in their expression to find $\frac{d^3y}{dx^3}$ in terms of $\frac{dy}{dx}$, x and y only.

A1: Correct replacement at least once

A1: Fully correct expression in the third derivative in terms of $\frac{dy}{dx}$, x and y only.

A1: Correct value for the third derivative at $x = 2$

Note: If in doubt about this method being used, please send to review

Question Number	Scheme	Notes	Marks
2(a)	$(x+2)\frac{dy}{dx} + y = 3x^2 + 6x \Rightarrow$ $\frac{dy}{dx} + \frac{y}{x+2} = \frac{3x(x+2)}{x+2}$ $\Rightarrow \frac{dy}{dx} + \frac{y}{x+2} = 3x$	M1: Reaches $\frac{dy}{dx} + f(x)y = g(x)$ A1: Correct equation in required form. Correct answer implies both marks.	M1 A1
(2)			
(b)	$\text{IF} = \left\{ e^{\int \frac{1}{x+2} (dx)} \right\} = e^{\ln(x+2)} = x+2$	Correct attempt at the IF for their $f(x)$ – must attempt the integration but need not be fully simplified (may be implied by working)	M1
	$"(x+2)" y = \int "3"x"(x+2)"(dx)$	Applies their IF correctly with their kx	M1
	$(x+2)y = x^3 + 3x^2 (+c)$	Correct equation after integration condoning omission of the constant	A1
	$x = 4, y = 18 \Rightarrow c = 108 - 64 - 48 = -4$	Substitutes correct values to find a value for c in their integrated equation	M1
	$y = \frac{x^3 + 3x^2 - 4}{x+2} = \frac{(x+2)(x+2)(x-1)}{x+2}$ $\Rightarrow y = (x+2)(x-1) \text{ or } x^2 + x - 2$	A1: Correct particular solution with $g(x)$ in any form. Condone missing “$y = \dots$” A1: y or $g(x) = (x+2)(x-1)$ or $x^2 + x - 2$	A1 A1
(6)			
Total 8			
Note : The question can be completed via a complementary function and particular integral approach, as below. If the CF is never considered a maximum of 011000 is possible.			
(b) Alt	$(x+2)\frac{dy}{dx} + y = 0 \Rightarrow \frac{1}{y} \frac{dy}{dx} = -\frac{1}{x+2}$ $\Rightarrow \ln y = -\ln(x+2) + c$ $\Rightarrow \ln y = \ln \frac{A}{x+2} \Rightarrow y = \frac{A}{x+2}$	Considers the homogeneous equation and makes progress integrating to an equation $y = \dots$ with constant of integration.	M1
	$y = ax^2 + bx + c \Rightarrow \frac{dy}{dx} = 2ax + b$ $\Rightarrow (x+2)(2ax+b) + ax^2 + bx + c = 3x^2 + 6x$ $x^2 : 3a = 3 \quad x : 4a + 2b = 6 \quad x^0 : 2b + c = 0$ $\Rightarrow a = \dots, b = \dots, c = \dots$	Tries the PI $y = ax^2 + bx + c$, differentiates and substitutes into equation to find a, b and c	M1
	$y = x^2 + x - 2$	Correct particular integral	A1
	$y = x^2 + x - 2 + \frac{A}{x+2} \Rightarrow 18 = 16 + 4 - 2 + \frac{A}{6}$ $\Rightarrow A = \dots ; A = 0$	Uses (4, 18) in their general solution to find the constant ; Correctly shows $A = 0$	M1 A1
	$\Rightarrow y = (x+2)(x-1) \text{ or } x^2 + x - 2$	Correct answer following attempt to combine the CF and PI. Accept y or $g(x)$ as before.	A1

Additional Notes:

No need to see method for solving the cubic – if the correct answer follows directly from the unfactorized cubic that is acceptable for the final A.



Question Number	Scheme	Marks
2		
(a)	$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{(r+2)^2 - r(r+3)}{r(r+1)(r+2)}$ $= \frac{r^2 + 4r + 4 - r^2 - 3r}{r(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)} \quad *$	M1 A1* (2)
(b)	$r=1 \quad \frac{3}{1 \times 2} - \frac{4}{2 \times 3} \qquad r=n-1 \quad \frac{n+1}{(n-1)n} - \frac{n+2}{n(n+1)}$ $r=2 \quad \frac{4}{2 \times 3} - \frac{5}{3 \times 4} \qquad r=n \quad \frac{n+2}{n(n+1)} - \frac{n+3}{(n+1)(n+2)}$ $r=3 \quad \frac{5}{3 \times 4} - \frac{6}{4 \times 5}$ $\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)}$ $\sum_{r=1}^n \frac{r+4}{r(r+1)(r+2)} = \frac{3(n+1)(n+2) - 2n - 6}{2(n+1)(n+2)} = \frac{n(3n+7)}{2(n+1)(n+2)}$	M1 A1 dM1 A1cao (4)
(a) M1 A1*	Attempt a single fraction with the correct denominator (or 2 separate fractions with the correct common denominator) Correct result obtained with no errors in the working. Must include LHS as shown in question or LHS = ...	
(b) M1 A1 dM1 A1cao	Show sufficient terms to demonstrate the cancelling, min 3 at start and 1 at end or 2 at start and 2 at end. Award by implication if the correct 2 remaining terms are seen Extract the correct 2 remaining terms Attempt common denominator of the form $k(n+1)(n+2)$ Correct result obtained. No need to show a , b and c explicitly.	

[6]

Question Number	Scheme	Marks
5(a)	$w = \frac{z+1}{z-3} \Rightarrow wz - 3w = z + 1 \Rightarrow wz - z = 3w + 1 \Rightarrow z = \frac{3w+1}{w-1}$	M1 A1
	$(z =) \frac{(3u+1)+3iv}{(u-1)+iv} \times \frac{(u-1)-iv}{(u-1)-iv} = \dots$	dM1
	$\Rightarrow x = \frac{3u^2 - 2u + 3v^2 - 1}{(u-1)^2 + v^2} \quad y = \frac{-4v}{(u-1)^2 + v^2}$ $(y = 4x \Rightarrow) -4v = 12u^2 - 8u + 12v^2 - 4$	ddM1
	$\Rightarrow 3u^2 + 3v^2 - 2u + v - 1 = 0^*$	A1cso*
		(5)
(b)	$u^2 - \frac{2}{3}u + v^2 + \frac{1}{3}v - \frac{1}{3} = 0 \Rightarrow \left(u - \frac{1}{3}\right)^2 - \frac{1}{9} + \left(v + \frac{1}{6}\right)^2 - \frac{1}{36} - \frac{1}{3} = 0$ $\Rightarrow \left(u - \frac{1}{3}\right)^2 + \left(v + \frac{1}{6}\right)^2 = \frac{17}{36}$ $\Rightarrow \text{centre: } \left(\frac{1}{3}, -\frac{1}{6}\right) \left[\text{allow } x = \frac{1}{3}, y = -\frac{1}{6} \right] \quad \text{radius: } \sqrt{\frac{17}{36}} \text{ or } \frac{\sqrt{17}}{6}$	B1 B1
		(2)
		Total 7

Notes

(a)

M1: Completes an attempt to make z the subject.

A1: Correct expression.

dM1: **Dependent on previous M mark.** Replaces w with $u + iv$ and indicates an appropriate attempt to rationalise the denominator. Accept if $x + iy$ is used instead for this mark.ddM1: **Dependent on both previous M marks.** Equates real and imaginary parts to obtain expressions for x and y in u and/or v only and uses $y = 4x$ to form an equation in u and v only. Condone intermediate incorrect statements that are recovered for this mark, e.g. condone, missing or incorrect (real) denominator or slips with initial inclusion of i in the y if recovered. E.g. any of

$$x = 3u^2 - 2u + 3v^2 - 1, \quad x = \frac{3u^2 - 2u + 3v^2 - 1}{(u-1)^2 - v^2}, \quad y = \frac{-4vi}{(u-1)^2 + v^2} \quad \text{or similar may be recovered for the M's}$$

A1cso*: obtains the equation (must be equal to 0) or states $k = -1$ following correct work. Do not allow recovery from incorrect statements as shown above for the final mark - all may lead to the correct answer but are A0. But if there are no incorrect statements e.g. cancelling of denominators may be implied for the A. Watch outfor an incorrect $z = \frac{3w+1}{1-w}$ which also leads to a correct expression but loses both A marks.**Note:** Alternatively methods are possible. Two are shown on the next page, but are not exhaustive.

(b)

Allow both marks from cases where $k = -1$ is guessed or achieved from fortuitous work.B1: Correct centre **or** radius. Accept unsimplified fractions for this mark.B1: Correct centre **and** radius - fractions must be simplified.

5(a) Alt	$z = x + iy, y = 4x \Rightarrow w = \frac{z+1}{z-3} = \frac{x+4xi+1}{x+4xi-3} = \frac{(x+1)+4ix}{(x-3)+4ix} \times \frac{(x-3)-4ix}{(x-3)-4ix} = \dots$	M1
	$(u+iv) = \frac{(17x^2-2x-3)-16ix}{(x-3)^2+16x^2}$ oe	A1
	$\Rightarrow u = \frac{17x^2-2x-3}{(x-3)^2+16x^2} \quad v = \frac{-16x}{(x-3)^2+16x^2}$ $\Rightarrow 3u^2+3v^2-2u+v = \dots$	dm1
	$\Rightarrow \dots \Rightarrow 3u^2+3v^2-2u+v-1=0^*$ or states $k=-1$	ddM1A1cso*
		(5)
5(a) Alt II	$z = x + iy, y = 4x \Rightarrow w = \frac{z+1}{z-3} = \frac{x+4xi+1}{x+4xi-3}$	
	$\Rightarrow (u+iv)((x-3)+4ix) = (x+1)+4ix \Rightarrow \dots + \dots i = \dots$	M1
	$u(x-3)-4vx+i(v(x-3)+4ux) = (x+1)+4ix$	A1
	$\Rightarrow u(x-3)-4vx = (x+1) \quad v(x-3)+4ux = 4x$ $\Rightarrow x = \frac{3u+1}{u-4v-1} \quad x = \frac{3v}{v+4u-4} \Rightarrow \frac{3u+1}{u-4v-1} = \frac{3v}{v+4u-4}$	dm1
	$(3u+1)(v+4u-4) = 3v(u-4v-1) \Rightarrow 12u^2+12v^2-8u+4v-4=0$ $\Rightarrow 3u^2+3v^2-2u+v-1=0^*$	ddM1A1cso*
		(5)

Notes

(a) Alt I

M1: Uses $y = 4x$ and $z = x + iy$ in the expression for w and multiplies through by complex conjugate of denominatorA1: Correct expression in terms of x (oe)dm1: Replaces w with $u + iv$ and equates real and imaginary parts and substitutes into the LHS of given equation.**Dependent on previous M mark.**ddM1: Proceeds simplify the expression putting over common denominator. They are unlikely to make much progress at this stage in this method. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression equates to 0 and gives minimal conclusion.

(a) Alt II

M1: Uses $y = 4x$ and $z = x + iy$ in the expression for w and equates to $u + iv$, cross multiplies and expands.A1: Correct expression with the i^2 terms simplified.dm1: Equates real and imaginary parts and proceeds to eliminate x solving simultaneously. **Dependent on previous M mark.**ddM1: Proceeds to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression required holds. As main scheme.



Question Number	Scheme	Marks
6(a)	$y = \sec x \Rightarrow \frac{dy}{dx} = \sec x \tan x$	M1
	$\frac{d^2y}{dx^2} = (\sec x \tan x) \tan x + \sec x (\sec^2 x) (= \sec x \tan^2 x + \sec^3 x)$	
	$\frac{d^2y}{dx^2} = \sec x (\sec^2 x - 1) + \sec^3 x = 2\sec^3 x - \sec x$ $\Rightarrow \frac{d^3y}{dx^3} = 6\sec^2 x (\sec x \tan x) - \sec x \tan x$ or $\frac{d^3y}{dx^3} = 2\sec x \tan x \sec^2 x + \tan^2 x \sec x \tan x + 3\sec^2 x \sec x \tan x$	dM1
	$\left(\frac{d^3y}{dx^3} = 5\sec^3 x \tan x + \sec x \tan x (\sec^2 x - 1) = 6\sec^3 x \tan x - \sec x \tan x \right)$ $\Rightarrow \frac{d^3y}{dx^3} = \sec x \tan x (6\sec^2 x - 1)$	ddM1 A1
		(4)
(b)	$\sec\left(\frac{\pi}{3}\right) = 2, \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \Rightarrow f\left(\frac{\pi}{3}\right) = 2, f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}, f''\left(\frac{\pi}{3}\right) = 14, f'''\left(\frac{\pi}{3}\right) = 46\sqrt{3}$	M1
	$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \dots$ $\Rightarrow 2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2 + \frac{23\sqrt{3}}{3}\left(x - \frac{\pi}{3}\right)^3$	dM1 A1
		(3)
(c)	$\left(\sec \frac{7\pi}{24} \approx 2 + 2\sqrt{3}\left(-\frac{\pi}{24}\right) + 7\left(-\frac{\pi}{24}\right)^2 + \frac{23\sqrt{3}}{3}\left(-\frac{\pi}{24}\right)^3 = \dots\right)$	M1
	$\sec \frac{7\pi}{24} \approx 1.636709263... \approx 1.637$	A1
		(2)
		Total 9

Notes**(a)**

M1: Differentiates twice and obtains expression for second derivative allowing for sign errors only

dM1: **Dependent on previous M mark.** Differentiates again and obtains expression for third derivative allowing for sign errors only (which could come from incorrect signs in trig identities)

ddM1: **Dependent on both previous M marks.** Obtains an answer of the correct form or values for p and q .

Allow slips from their third derivative but any trig identities used can only have errors in sign.

A1: Fully correct expression or values for p and q

Note: If they decide not to work in $\sec x$ and $\tan x$ the main scheme still applies – see Alt next page.

Allow “meet in the middle” approaches that determine the values of p and q

(b)

M1: Uses $\sec x$ and their three derivatives to attempt to find values for $f\left(\frac{\pi}{3}\right)$, $f'\left(\frac{\pi}{3}\right)$, $f''\left(\frac{\pi}{3}\right)$ and $f'''\left(\frac{\pi}{3}\right)$

evidenced by values for each with at least 2 correct if no method seen. May be recovered by substitution of values into formula if not all listed.

dM1: **Dependent on previous M mark.** Uses a correct Taylor series with all four of their values. Series must be correct for their values but allow slips if a correct formula is quoted.



Question Number	Scheme	Notes	Marks
2(a)	(i) $z = 6 - 6\sqrt{3}i \Rightarrow z = \sqrt{6^2 + (6\sqrt{3})^2} = 12$	+12 only. Accept if just stated	B1
	(ii) e.g., $\arg z = -\arctan \frac{6\sqrt{3}}{6}$ Attempts an expression for a relevant angle. Look for $\pm \arctan \left(\pm \frac{6\sqrt{3}}{6} \right)$ or e.g., $\pm \tan^{-1} \left(\pm \frac{1}{\sqrt{3}} \right)$ If arctan is not seen allow e.g., $\tan \alpha = \frac{6\sqrt{3}}{6} \Rightarrow \alpha = \frac{\pi}{3}$ with α correct for their $\tan \alpha$ If using sin or cos the hypotenuse must be their 12		M1
	$\arg z$ or $\arg$ or $\text{argument (of } z) = -\frac{\pi}{3}$ * A correct proof with no incorrect work/statements. LHS required. Allow " $\theta =$ " if consistent , e.g., $\theta = -\frac{\pi}{3}$ cannot follow " $\tan \theta = +\sqrt{3}$ "		A1*
(ii) Way 2	$z = 12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 12\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ or $12e^{-\frac{\pi}{3}i}$ or $\cos \theta = \frac{1}{2}$ or $\sin \theta = -\frac{\sqrt{3}}{2}$ [M1] $\Rightarrow \arg z = -\frac{\pi}{3}$ [A1*] M1: Factorises out 12 and writes in trig or exp form or identifies $\cos \theta = \frac{1}{2}$ and $\sin \theta = -\frac{\sqrt{3}}{2}$ A1: Acceptable statement with all work correct		
(ii) Way 3	$z = 12\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ or $12e^{-\frac{\pi}{3}i}$ or $12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 6 - 6\sqrt{3}i$ [M1] $\Rightarrow \arg z = -\frac{\pi}{3}$ [A1*] M1: Assumes result, writes correctly for their 12 and attempts $a + ib$ form A1: Obtains $6 - 6\sqrt{3}i$ and makes acceptable statement with all work correct		
			(3)
(b)	$z = "12" \left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right) \right)$ or $"12" e^{-\frac{\pi}{3}i}$ [no missing "i" unless recovered] Correct trig or exp. form with their 12. Could be implied by their z^4 in trig or exp. form e.g., $("12" e^{-\frac{\pi}{3}i})^4$ Allow equivalent values of θ e.g. $\frac{5\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$. Condone poor bracketing. Allow this mark if $+2k\pi, -2k\pi, \pm 2k\pi$ appears with argument		M1
	$z^4 = 20736 \left(\cos\left(-\frac{4\pi}{3}\right) + i\sin\left(-\frac{4\pi}{3}\right) \right)$ or $20736 \left(\cos -\frac{4\pi}{3} + i\sin -\frac{4\pi}{3} \right)$ or $20736 e^{-\frac{4\pi}{3}i}$ Correct z^4 in any form. 12^4 evaluated and arg. of $-\frac{4\pi}{3}$ (not just $4 \times -\frac{\pi}{3}$) or $\frac{2\pi}{3}$ only although may use e.g., $\sin\left(-\frac{4\pi}{3}\right) = -\sin\left(\frac{4\pi}{3}\right)$. No "k"s. Condone an "unclosed" bracket. Only accept $-10368 + 10368\sqrt{3}i$ or $20736\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ provided evidence of de Moivre.		A1
			(2)



Question Number	Scheme	Notes	Marks
2(c)	$w = z^{\frac{1}{2}} = (\pm)\sqrt[4]{12}\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right) \text{ or e.g., } (\pm)2\sqrt{3}e^{-\frac{\pi}{6}i}$ [no missing “i” unless recovered] Correct use of de Moivre’s theorem with $-\frac{\pi}{3}$ and their 12 to attempt one square root. Allow work with argument of $\frac{5\pi}{3}$ for $-\frac{\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$. Condone poor bracketing. M0 if z^4 used for z. Allow this mark if $+2k\pi, -2k\pi, \pm 2k\pi$ appears with argument		M1
	$w = 3 - \sqrt{3}i, -3 + \sqrt{3}i \text{ oe}$ A1ft: One correct exact root in $a + ib$ or $c(a + ib)$ form (a, b, c may be unsimplified but not numerical trig expressions) ft their 12 only i.e. $(\pm)\sqrt[4]{12}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ A1: Both exact roots (no others) correct in $a + ib$ form – a and b may be unsimplified (but not numerical trig expressions) e.g. accept $a = (\pm)\sqrt{12}\frac{\sqrt{3}}{2}, (\pm)\frac{\sqrt{36}}{2} \quad b = (\mp)\frac{\sqrt{12}}{2}, (\mp)\frac{2\sqrt{3}}{2}$ Accept $\pm(3 - \sqrt{3}i)$ but just $\pm 3 - \sqrt{3}i$ is A1 A0. Just $\pm\sqrt{3}(\sqrt{3} - i)$ is A1 A0		A1ft A1
	Note: $w^2 = r^2(\cos 2\theta + i\sin 2\theta) = z \Rightarrow r, \theta, w = \dots$ is an acceptable approach		(3)
Alt	$w^2 = z \Rightarrow (a + ib)^2 = a^2 - b^2 + 2abi = 6 - 6\sqrt{3}i \Rightarrow a^2 - b^2 = 6, 2ab = -6\sqrt{3}$ $b = -\frac{3\sqrt{3}}{a} \Rightarrow a^2 - \frac{27}{a^2} = 6 \Rightarrow a^4 - 6a^2 - 27 = (a^2 - 9)(a^2 + 3) = 0 \Rightarrow a^2 = 9, a = \pm 3, b = \mp\sqrt{3}$ M1: From a correct starting point, expands and equates real and imaginary parts to form two equations in a and b and obtains at least one value for both a and b $w = 3 - \sqrt{3}i, -3 + \sqrt{3}i$ A1: One correct exact root in $a + ib$ or $c(a + ib)$ form (a, b, c may be unsimplified) A1: Both exact roots (no others) correct in $a + ib$ form – a and b may be unsimplified		
			Total 8

Question Number	Scheme	Notes	Marks
3(a)	$\frac{r}{\sqrt{r(r+1)} + \sqrt{r(r-1)}} \times \frac{\sqrt{r(r+1)} - \sqrt{r(r-1)}}{\sqrt{r(r+1)} - \sqrt{r(r-1)}}$	A correct multiplier to rationalise the denominator seen or implied by correct work	M1
	$= \frac{r(\sqrt{r(r+1)} - \sqrt{r(r-1)})}{r(r+1) - r(r-1)} = \frac{\sqrt{r(r+1)} - \sqrt{r(r-1)}}{2} \text{ or } A = \frac{1}{2}$ Correct expression or correct value for A. Condone poor notation if intention clear. There must be (minimal) correct supporting working.		A1
	Alternative: $A = \frac{r}{(\sqrt{r(r+1)} + \sqrt{r(r-1)})(\sqrt{r(r+1)} - \sqrt{r(r-1)})} = \frac{r}{r(r+1) - r(r-1)} \text{ or } \frac{r}{r^2 + r - r^2 + r} \text{ or } \frac{r}{2r} \Rightarrow A = \frac{1}{2}$ M1: Correctly makes A the subject A1: Correct completion with one intermediate fraction		
			(2)
(b)	$\sum_{r=1}^n \frac{r}{\sqrt{r(r+1)} + \sqrt{r(r-1)}} = \frac{1}{2} \left(\begin{array}{l} \sqrt{1 \times 2} - \sqrt{1 \times 0} (= \sqrt{2} - 0) \\ + \sqrt{2 \times 3} - \sqrt{2 \times 1} (= \sqrt{6} - \sqrt{2}) + \dots \\ \dots + \sqrt{(n-1)(n-1+1)} - \sqrt{(n-1)(n-1-1)} (= \sqrt{n(n-1)} - \sqrt{(n-1)(n-2)}) \\ + \sqrt{n(n+1)} - \sqrt{n(n-1)} \end{array} \right)$ M1: Applies the method of differences for $r=1$ and $r=n$ in the given expression with or without their A and obtains one correct row of these 2. M1: Applies the method of differences for $r=1$, $r=n$ and either $r=2$ or $r=n-1$ in the given expression with/without their A and obtains 2 correct rows of these 4. When considering how many rows are correct, if A has been clearly applied to any term then assess all rows as if A has been applied throughout. Condone missing bracket if their A is applied to a row e.g., "$\frac{1}{2} \times \sqrt{6} - \sqrt{2}$" <u>if it is recovered</u> but e.g., $\frac{\sqrt{6}}{2} - \sqrt{2}$ is an incorrect row. Ignore a row for $r=0$. Condone equivalent work with r or e.g., k used for n. Both marks can be implied by a correct final expression with or without their A provided there are at least any two correct rows of differences i.e., "$\frac{1}{2}(\sqrt{n(n+1)} - 0)$" or $\sqrt{n(n+1)} - 0$ Note: row 3 is "$\frac{1}{2}(\sqrt{12} \text{ (or } 2\sqrt{3}) - \sqrt{6})$", row 4 is "$\frac{1}{2}(\sqrt{20} \text{ (or } 2\sqrt{5}) - \sqrt{12} \text{ (or } 2\sqrt{3}))$" If $\frac{1}{2}$ is fully applied the rows are: $\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}, \frac{\sqrt{12}}{2} \text{ (or } \sqrt{3}) - \frac{\sqrt{6}}{2}, \frac{\sqrt{20}}{2} \text{ (or } \sqrt{5}) - \frac{\sqrt{12}}{2} \text{ (or } \sqrt{3}), \dots$ $\dots, \frac{\sqrt{(n-2)(n-1)}}{2} - \frac{\sqrt{(n-2)(n-3)}}{2}, \frac{\sqrt{n(n-1)}}{2} - \frac{\sqrt{(n-1)(n-2)}}{2}, \frac{\sqrt{n(n+1)}}{2} - \frac{\sqrt{n(n-1)}}{2}$		M1 M1
	$= \frac{1}{2} \sqrt{n(n+1)} \text{ oe e.g., } \frac{\sqrt{n^2 + n}}{2}$	Correct expression in terms of n . No incorrect terms seen in differences work even if cancelled but condone the occasional poor bracket. There should be no "0" so e.g., $\frac{1}{2}(\sqrt{n(n+1)} - 0)$ is A0 Does not require marks in (a)	A1
			(3)

Question Number	Scheme	Notes	Marks
3(c)	$\sum r = \frac{1}{2}n(n+1)$ e.g., sight of $k \times \dots = \sqrt{\frac{1}{2}n(n+1)}$	States or uses the correct summation formula for integers	M1
	$\frac{k}{2}\sqrt{n(n+1)} = \sqrt{\frac{1}{2}n(n+1)} \Rightarrow \frac{k}{2} = \sqrt{\frac{1}{2}} \Rightarrow k = \sqrt{2}$	$\sqrt{2}$ only (Not $\pm$). $k = \sqrt{2}$ must not come from a clearly incorrect equation.	A1
			(2)
			Total 7



Question Number	Scheme	Notes	Marks	
4(a)	$y = \tan\left(\frac{3x}{2}\right) \Rightarrow y' = \frac{3}{2} \sec^2\left(\frac{3x}{2}\right)$	Any correct first derivative. Not implied by $y'\left(\frac{\pi}{6}\right) = 3$	B1	
	$\Rightarrow y'' = 2 \times \frac{3}{2} \sec\left(\frac{3x}{2}\right) \times \sec\left(\frac{3x}{2}\right) \tan\left(\frac{3x}{2}\right) \times \frac{3}{2}$ $\left[= \frac{9}{2} \sec^2\left(\frac{3x}{2}\right) \tan\left(\frac{3x}{2}\right) \right]$	Attempts the second derivative achieving $k \sec^2\left(\frac{3x}{2}\right) \tan\left(\frac{3x}{2}\right)$ or unsimplified equivalent. Not implied by $y''\left(\frac{\pi}{6}\right) = 9$	M1	
	$\Rightarrow y''' = \frac{9}{2} \sec^2\left(\frac{3x}{2}\right) \sec^2\left(\frac{3x}{2}\right) \times \frac{3}{2} + \frac{9}{2} \tan\left(\frac{3x}{2}\right) \times 2 \times \frac{3}{2} \sec^2\left(\frac{3x}{2}\right) \tan\left(\frac{3x}{2}\right)$ $\left[= \frac{27}{4} \sec^4\left(\frac{3x}{2}\right) + \frac{27}{2} \sec^2\left(\frac{3x}{2}\right) \tan^2\left(\frac{3x}{2}\right) \right]$	dM1: Attempts third derivative using the product rule, achieving $P \sec^4\left(\frac{3x}{2}\right) + Q \sec^2\left(\frac{3x}{2}\right) \tan^2\left(\frac{3x}{2}\right)$ or unsimplified equivalent. Requires previous M mark. A1: Correct differentiation. Accept unsimplified. Not implied by $y'''\left(\frac{\pi}{6}\right) = 54$	dM1 A1	
	If $\sec^2\left(\frac{3x}{2}\right) = \tan^2\left(\frac{3x}{2}\right) + 1$ is used the identity must be used correctly and to score M marks expressions of consistent form should be achieved. Note that replacing $\sec^2\left(\frac{3x}{2}\right)$ in $y'' \Rightarrow y''' = \frac{27}{4} \sec^2\left(\frac{3x}{2}\right) + \frac{81}{4} \sec^2\left(\frac{3x}{2}\right) \tan^2\left(\frac{3x}{2}\right)$			
	$y\left(\frac{\pi}{6}\right) = 1, y'\left(\frac{\pi}{6}\right) = 3, y''\left(\frac{\pi}{6}\right) = 9, y'''\left(\frac{\pi}{6}\right) = 54$ Attempts values (but allow numerical trig expressions) for y and their 3 derivatives at $\frac{\pi}{6}$ - accept stated values or insertion into a series of the correct form			M1
	$(y =) 1 + 3\left(x - \frac{\pi}{6}\right) + \frac{9}{2!}\left(x - \frac{\pi}{6}\right)^2 + \frac{54}{3!}\left(x - \frac{\pi}{6}\right)^3 + \dots$ Applies Taylor's correctly about $\frac{\pi}{6}$ with their values/numerical trig expressions. If values are not seen separately the work should imply a correct formula but allow a recognisable attempt at the series following the correct general formula stated. Requires previous M mark.			dM1
	$(y =) 1 + 3\left(x - \frac{\pi}{6}\right) + \frac{9}{2}\left(x - \frac{\pi}{6}\right)^2 + 9\left(x - \frac{\pi}{6}\right)^3 + \dots$	Correct expression with coeffs. in simplest form. "y = ..." not required. Requires all previous marks. Score A0 if clear evidence of use of any wrong derivative expression.	A1	
If e.g. $y'''\left(\frac{\pi}{6}\right)$ is found by calculator but $y'(x)$ and $y''(x)$ were seen award 1100110 max			(7)	
Note: With responses that work in sin and cos throughout, to score M marks there must be no loss of form when differentiating (sign and coefficient errors only, also allowing sign errors with product/quotient formulae). Any use of identities must be correct. E.g: $y = \tan\left(\frac{3x}{2}\right) = \frac{\sin\left(\frac{3x}{2}\right)}{\cos\left(\frac{3x}{2}\right)} \Rightarrow y' = \frac{\frac{3}{2} \cos^2\left(\frac{3x}{2}\right) + \frac{3}{2} \sin^2\left(\frac{3x}{2}\right)}{\cos^2\left(\frac{3x}{2}\right)}$ $y'' = \frac{\frac{9}{2} \cos^3\left(\frac{3x}{2}\right) \sin\left(\frac{3x}{2}\right) + \frac{9}{2} \cos\left(\frac{3x}{2}\right) \sin^3\left(\frac{3x}{2}\right)}{\cos^4\left(\frac{3x}{2}\right)} \text{ or } \frac{\frac{9}{2} \cos\left(\frac{3x}{2}\right) \sin\left(\frac{3x}{2}\right)}{\cos^4\left(\frac{3x}{2}\right)} \text{ or } \frac{9 \sin\left(\frac{3x}{2}\right)}{2 \cos^3\left(\frac{3x}{2}\right)}$ $y''' = \frac{\frac{27}{4} \cos^8\left(\frac{3x}{2}\right) + 27 \cos^6\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) + \frac{81}{4} \cos^4\left(\frac{3x}{2}\right) \sin^4\left(\frac{3x}{2}\right)}{\cos^8\left(\frac{3x}{2}\right)} = \frac{27}{4} + 27 \tan^2\left(\frac{3x}{2}\right) + \frac{81}{4} \tan^4\left(\frac{3x}{2}\right)$				

Question Number	Scheme	Notes	Marks
4(b)	$\left\{ y\left(\frac{\pi}{4}\right) = \right\} 1 + 3\left(\frac{\pi}{4} - \frac{\pi}{6}\right) + \frac{9}{2}\left(\frac{\pi}{4} - \frac{\pi}{6}\right)^2 + 9\left(\frac{\pi}{4} - \frac{\pi}{6}\right)^3$ $\text{or } 1 + 3\left(\frac{\pi}{12}\right) + \frac{9}{2}\left(\frac{\pi}{12}\right)^2 + 9\left(\frac{\pi}{12}\right)^3$ Substitutes $\frac{\pi}{4}$ into their expression for y of the correct form with at least the first three terms (series about $\frac{\pi}{6}$). Must have values (not unevaluated trig expressions). If only a decimal value is given then it must be the correct awrt 2.26 to score M1 (2.255314325). If there is no working they must obtain an expression with at least $a + b\pi + c\pi^2$ and correct exact ft a, b and c for their series or $1 + \frac{\pi}{4} + c\pi^2$ with correct exact ft c		M1
	$= 1 + \frac{\pi}{4} + \frac{\pi^2}{32} + \frac{\pi^3}{192} \text{ or } 1 + \frac{1}{4}\pi + \frac{1}{32}\pi^2 + \frac{1}{192}\pi^3$	Correct answer or values for A (32) and B (192). Can be awarded if full marks were not scored in (a).	A1
			(2)
			Total 9

