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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Polar Coordinates (FP2.6)

International A Level Mathematics · FP2: Further Pure Mathematics 2 (WFM02)

13 questions · 141 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

13 questions · 141 marks · 1 sub-topic

SUB-TOPIC FP2.6a

POLAR CURVES, AREAS & TANGENTS

Sub-topic FP2.6a · 13 questions · 141 marks · lowest tariff first



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.

Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)

Question 5 continued

Lined area for writing the answer to Question 5 continued.





Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 9 marks)



10.

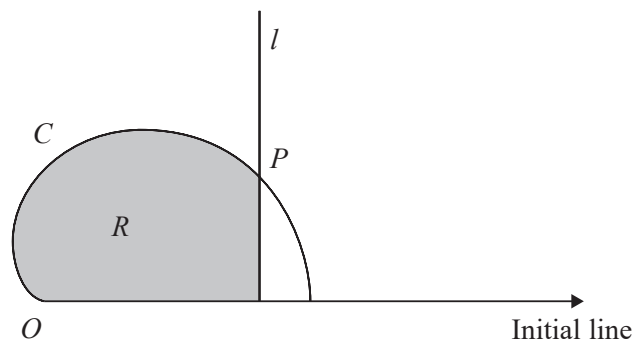


Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 1 + \cos \theta \quad 0 \leq \theta \leq \pi$$

and the line l with polar equation

$$r = k \sec \theta \quad 0 \leq \theta < \frac{\pi}{2}$$

where k is a positive constant.

Given that

- C and l intersect at the point P
- $OP = 1 + \frac{\sqrt{3}}{2}$

(a) determine the exact value of k .

(2)

The finite region R , shown shaded in Figure 1, is bounded by C , the initial line and l .

(b) Use algebraic integration to show that the area of R is

$$p\pi + q\sqrt{3} + r$$

where p , q and r are simplified rational numbers to be determined.

(7)

Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

Lined area for writing the answer to Question 10 continued.



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total 10 marks)

Q3



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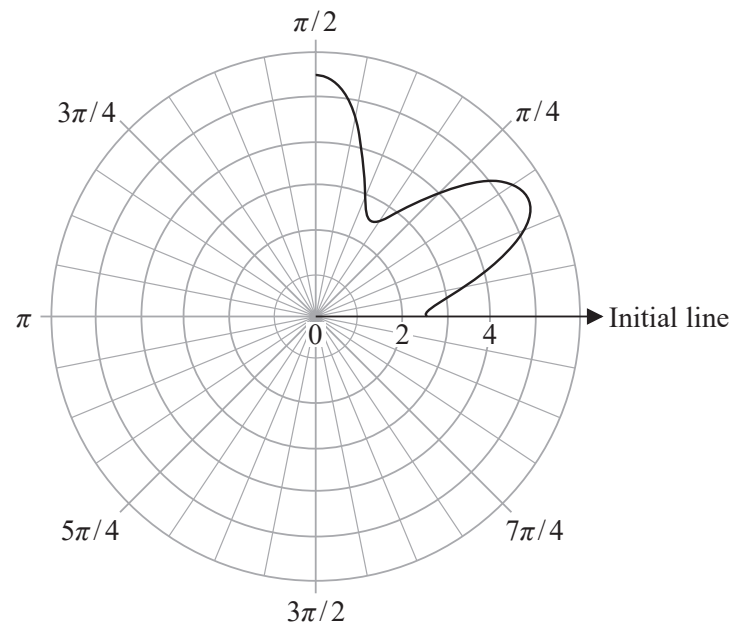


Figure 2

Figure 2 shows part of the curve with polar equation

$$r = 4 - \frac{3}{2} \cos 6\theta \quad 0 \leq \theta < 2\pi$$

(a) Sketch, on the polar grid in Figure 2,

- (i) the rest of the curve with equation $r = 4 - \frac{3}{2} \cos 6\theta \quad 0 \leq \theta < 2\pi$
- (ii) the polar curve with equation $r = 1 \quad 0 \leq \theta < 2\pi$

A spare copy of the grid is given on page 15.

(3)

In part (b) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(b) Determine the exact area enclosed between the two curves defined in part (a).

(7)



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Question 4 continued

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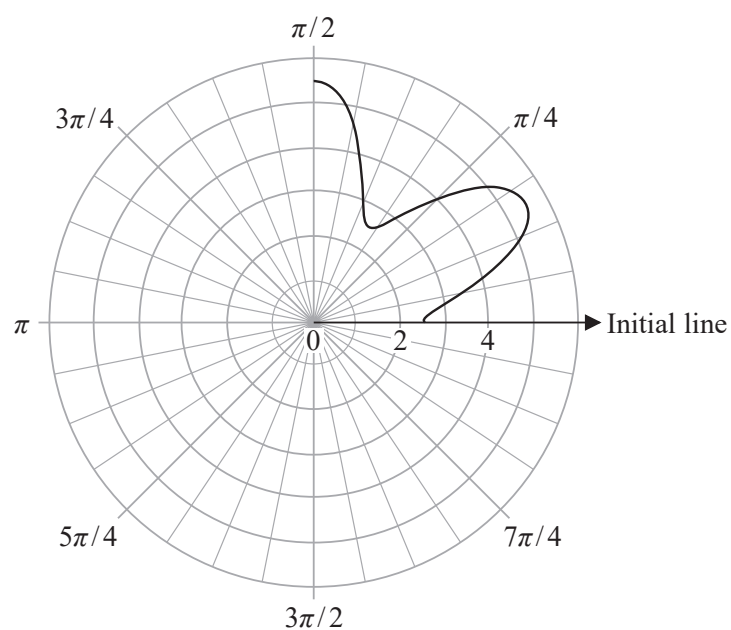
Lined area for writing the answer to Question 4 continued.



Question 4 continued

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Only use this grid if you need to redraw your answer to part (a)



Copy of Figure 2

(Total 10 marks)

Q4



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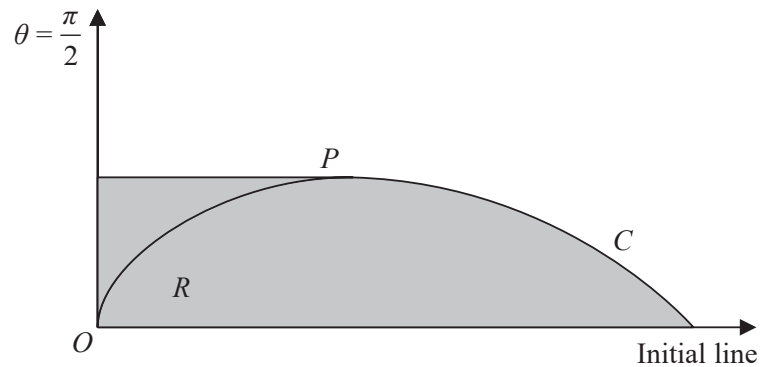


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 1 - \sin \theta \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C , such that the tangent to C at P is parallel to the initial line.

(a) Use calculus to determine the polar coordinates of P

(4)

The finite region R , shown shaded in Figure 1, is bounded by

- the line with equation $\theta = \frac{\pi}{2}$
- the tangent to C at P
- part of the curve C
- the initial line

(b) Use algebraic integration to show that the area of R is

$$\frac{1}{32}(a\pi + b\sqrt{3} + c)$$

where a , b and c are integers to be determined.

(6)

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 10 marks)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 10 marks)



8.

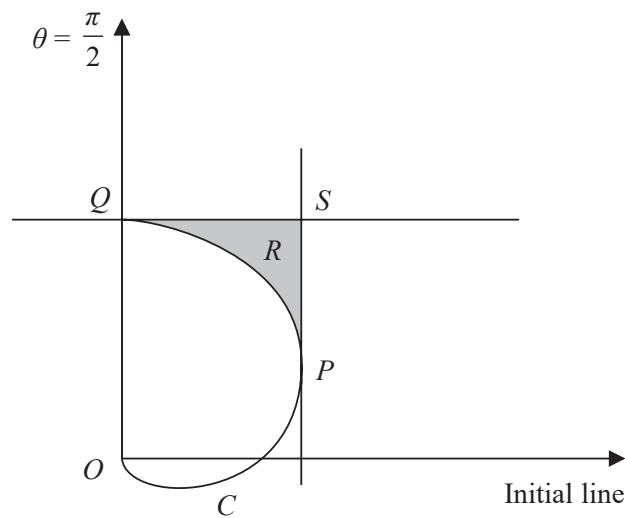
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Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 1 + \sin \theta \quad -\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$$

The point P lies on C such that the tangent to C at P is perpendicular to the initial line.

(a) Use calculus to determine the polar coordinates of P .

(5)

The tangent to C at the point Q where $\theta = \frac{\pi}{2}$ is parallel to the initial line.

The tangent to C at Q meets the tangent to C at P at the point S , as shown in Figure 1.

The finite region R , shown shaded in Figure 1, is bounded by the line segments QS , SP and the curve C .

(b) Use algebraic integration to show that the area of R is

$$\frac{1}{32}(a\sqrt{3} + b\pi)$$

where a and b are integers to be determined.

(6)

Question 8 continued

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Lined area for writing the answer to Question 8 continued.



Question 8 continued

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Lined area for writing the answer to Question 8 continued.



Question 8 continued

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Handwriting practice area with horizontal lines.

(Total 11 marks)

Q8

Grading box with two empty cells.



6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C_1 has equation

$$r = \sqrt{3} + \tan \theta \quad 0 < \theta < \frac{\pi}{2}$$

The tangent to C_1 is perpendicular to the initial line at the point P

- (a) Use calculus to determine, in simplest form, the exact polar coordinates of P

(4)

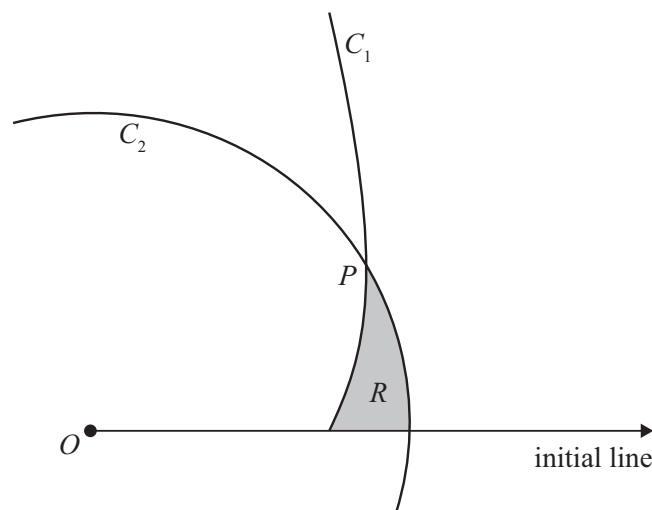


Figure 3

Figure 3 shows a sketch of part of the curve C_1 and part of the curve C_2

The curve C_2 is a circle with centre at the pole O .

The curves C_1 and C_2 intersect at P .

The region R , shown shaded in Figure 3, is bounded by C_1 , C_2 and the initial line.

- (b) Use algebraic integration to determine the area of R , giving the answer in the form

$$a\pi + \frac{\sqrt{3}}{2}(\ln b + c)$$

where a , b and c are rational numbers.

(8)

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

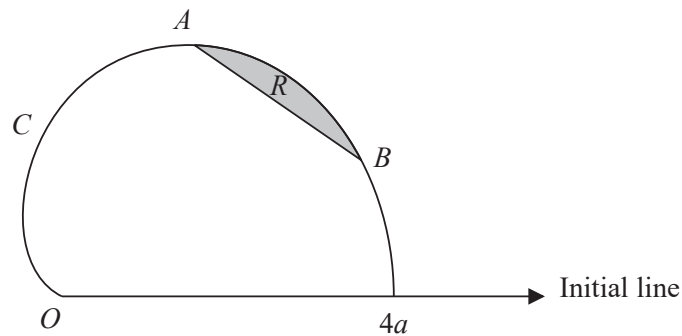
Lined area for writing the answer to Question 6.

(Total for Question 6 is 12 marks)



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**Figure 1**

The curve C , shown in Figure 1, has polar equation

$$r = 2a(1 + \cos \theta) \quad 0 \leq \theta \leq \pi$$

where a is a positive constant.

The tangent to C at the point A is parallel to the initial line.

(a) Determine the polar coordinates of A .

(6)

The point B on the curve has polar coordinates $\left(a(2 + \sqrt{3}), \frac{\pi}{6}\right)$

The finite region R , shown shaded in Figure 1, is bounded by the curve C and the line AB .

(b) Use calculus to determine the exact area of the shaded region R .

Give your answer in the form

$$\frac{a^2}{4}(d\pi - e + f\sqrt{3})$$

where d , e and f are integers.

(7)

Question 7 continued

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Question 7 continued

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(Total 13 marks)

Q7



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7.

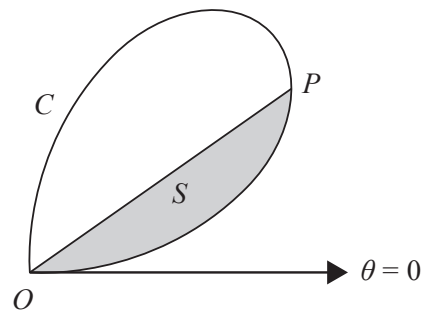


Figure 1

Figure 1 shows a sketch of curve C with polar equation

$$r = 3 \sin 2\theta \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The point P on C has polar coordinates (R, ϕ) . The tangent to C at P is perpendicular to the initial line.

- (a) Show that $\tan \phi = \frac{1}{\sqrt{2}}$ (4)

- (b) Determine the exact value of R . (2)

The region S , shown shaded in Figure 1, is bounded by C and the line OP , where O is the pole.

- (c) Use calculus to show that the exact area of S is

$$p \arctan \frac{1}{\sqrt{2}} + q \sqrt{2}$$

where p and q are constants to be determined.

Solutions relying entirely on calculator technology are not acceptable. (7)

Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total 13 marks)

Q7



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 13 marks)



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Notes	Marks
5(a)	$\frac{2}{r(r+1)(r+2)} = \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$ $2 = A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ $r=0: 2=2A \Rightarrow A=1$ $r=-1: 2=-B \Rightarrow B=-2$ $r=-2: 2=2C \Rightarrow C=1$	Any valid method to obtain at least one constant with partial fractions of the correct form. Could be implied	M1
	$\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}$	Correct partial fractions	A1
(2)			
(b)	$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \begin{cases} \frac{1}{1} - \frac{2}{2} + \frac{1}{3} + \\ \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \\ \frac{1}{3} - \frac{2}{4} + \frac{1}{5} + \\ \dots + \dots \\ \frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1} + \\ \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} \end{cases}$	Attempts at least three rows with their partial fractions of a correct form (see notes). Implied by correct expression	M1
	$= 1 - 1 + \frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$	A1: Correct constant terms or algebraic terms extracted A1: Fully correct expression	A1 A1
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2} = \frac{(n+1)(n+2) - 2(n+2) + 2(n+1)}{2(n+1)(n+2)}$	Combines terms of the form p , $\frac{q}{n+b}$, and $\frac{r}{n+c}$ and no others into a single fraction	M1
	$= \frac{n^2 + 3n + 2 - 2n - 4 + 2n + 2}{2(n+1)(n+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$	Correct expression	A1
(5)			
(c)	$\frac{n(n+3)}{2(n+1)(n+2)} > \frac{7}{15}$ $15n^2 + 45n > 14n^2 + 42n + 28$	Forms inequality or equation, cross-multiplies or puts over common denominator and expands brackets	M1
	$n^2 + 3n - 28 > 0$ $(n+7)(n-4) > 0$ $\Rightarrow n = \dots \text{ or } n > \dots \quad (n > 4)$	Forms and solves 3TQ inequality or equation (usual rules)	dM1
	$n = 5$	5 only	A1
(3)			
Total 10			

Additional Note

(a)

M1: Must be correct partial fraction form with three fractions. Note that $\frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ is not a complete partial fraction form so is M0, but this form can be used to score full marks in part (b).
A1: Correct form.



(b)

M1: For evidence of correctly applying the method of differences to the question. This may be implied by extraction of the correct terms if no/incomplete method is seen. If method is shown they should give at least three lines of fractions to show cancelling.

A1: For either the correct constant terms at the start or the correct algebraic terms at the end extracted from the differences.

A1: All correct terms extracted (and not others)

Note that $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$ will give $\frac{1}{2} - \frac{1}{6} + \frac{1}{6} - \frac{1}{12} + \dots - \frac{1}{(n+1)(n+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$ which can

score M1A1A1, and then the next M for combining these.

M1: Combines the terms to a single fraction provided terms of suitable forms were extracted – as shown in scheme. The denominator must be correct but allow a slip in the numerator.

A1: Correct answer from correct work.

(c)

M1: Sets up and forms the required inequality, or accept as an equality formed, and proceeds to form an unsimplified quadratic equation or inequality by cross multiplying or by **correctly** putting terms over a common denominator and expanding the $n(n+3)$ and $(n+1)(n+2)$ brackets.

dM1: Solves the resulting 3 term quadratic (accept if just one root found). Note: answers which try to solve for the whole solution set for n will carry this out as part of the working, and can score full marks. Solutions to the quadratic by calculator are acceptable – you may need to check and accept if only one root of their quadratic is given.

A1: Correct answer only.

Note: Trial and Improvement or calculator methods which do not form a quadratic inequality or equation will score no marks.

Question Number	Scheme	Notes	Marks
6	$2 \frac{dy}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0$		
(a)	$z = \frac{1}{y^2} \Rightarrow \frac{dz}{dx} = -\frac{2}{y^3} \frac{dy}{dx}$ or $y^2 = \frac{1}{z} \Rightarrow 2y \frac{dy}{dx} = -\frac{1}{z^2} \frac{dz}{dx} \left\{ = -y^4 \frac{dz}{dx} \right\}$ or $y = z^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$	Any correct equation in $\frac{dy}{dx}$ and $\frac{dz}{dx}$ (may be implied)	B1
	$-y^3 \frac{dz}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0$	Substitutes to obtain an equation with $\frac{dz}{dx}$ as the only derivative	M1
	$-\frac{dz}{dx} + \frac{\cot x}{y^2} + \tan x \sec x = 0$ $\frac{dz}{dx} - (\cot x)z = \tan x \sec x$ *	Fully correct proof with intermediate step after substitution	A1*
(3)			
(b)	I.F. = $e^{\int -\cot x dx}$	I.F. = $e^{\int \pm \cot x dx}$	M1
	$= e^{-\ln \sin x} = \frac{1}{\sin x}$	Correct processed I.F. in any form	A1
	$\frac{z}{\sin x} = \int \frac{1}{\sin x} \times \frac{\sin x}{\cos x} \times \frac{1}{\cos x} dx$	Applies I.F. $\times z = \int$ I.F. $\times \tan x \sec x dx$ Condone y for z	M1
	$\frac{z}{\sin x} = \int \sec^2 x dx \Rightarrow \frac{z}{\sin x} = \tan x (+c)$	Correct equation in z and x Condone missing “+c”	A1
	$z = \sin x (\tan x + c) \Rightarrow y^2 = \frac{1}{\sin x (\tan x + c)}$ oe	Fully correct general solution. Accept equivalent forms.	A1
(5)			
(c)	$\frac{4\sqrt{3}}{3} = \frac{1}{\frac{1}{2}(\frac{\sqrt{3}}{3} + c)} \Rightarrow \frac{\sqrt{3}}{3} + c = \frac{\sqrt{3}}{2} \Rightarrow c = \dots \left(\frac{\sqrt{3}}{6} \right)$	Correctly substitutes the given values to find a value for c	M1
	$x = \frac{\pi}{3} \Rightarrow y^2 = \frac{1}{\frac{\sqrt{3}}{2} \left(\sqrt{3} + \frac{\sqrt{3}}{6} \right)} = \frac{4}{7}$ $\Rightarrow y = \pm \frac{2\sqrt{7}}{7}$ oe	dM1: Substitutes $x = \frac{\pi}{3}$ and their c into their equation and reaches $y^2 = \dots$ or $y =$ with trig terms evaluated. A1: Both correct exact values. The negative value must not be rejected.	dM1 A1
(3)			
Total 11			

Additional Notes:

Condone poor notation of $\sin^{-1} x$ for $\operatorname{cosec} x$ as long as it is treated correctly.

(a) Note for A mark you may condone a missing “=0” in intermediate lines as long as it is implied by working.

(b) Note an equivalent form for the answer is $y^2 = \frac{\cos x}{\sin^2 x + K \sin 2x}$ which will lead to $c = \frac{\sqrt{3}}{12}$ in (c)

Question Number	Scheme	Notes	Marks
7	$r = \sqrt{3} \sin \theta \quad (0 \leq \theta \leq \pi) \quad r = 3 \cos \theta \quad \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$		
(a)	$\sqrt{3} \sin \theta = 3 \cos \theta \Rightarrow \tan \theta = \sqrt{3}$ $\Rightarrow \theta = \frac{\pi}{3}, \quad r = \frac{3}{2} \quad \text{or} \quad \left(\frac{3}{2}, \frac{\pi}{3}\right)$	B1: Either correct coordinate B1: Both correct coordinates and no others	B1 B1
(2)			
(b)	$\int (\sqrt{3} \sin \theta)^2 d\theta = K \int \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta d\theta = \dots$ or $\int (3 \cos \theta)^2 d\theta = M \int \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta d\theta = \dots$	Uses one of $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ in an attempt to integrate r^2 for one of the equations.	M1
	$\int (\sqrt{3} \sin \theta)^2 d\theta = K \int \frac{1}{2} - \frac{1}{2} \cos 2\theta d\theta = \dots$ and $\int (3 \cos \theta)^2 d\theta = M \int \frac{1}{2} + \frac{1}{2} \cos 2\theta d\theta = \dots$	Uses both $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ and $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ in an attempt to integrate r^2 for both of the equations to correct form.	M1
	$\int (\sqrt{3} \sin \theta)^2 d\theta = \frac{3}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right)$ and $\int (3 \cos \theta)^2 d\theta = \frac{9}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right)$	Fully correct expression after integration ignoring limits (accept equivalent expressions)	A1
	$\frac{1}{2} \int_0^{\frac{\pi}{3}} (\sqrt{3} \sin \theta)^2 d\theta = \frac{3}{4} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{3}} = \dots$ or $\frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3 \cos \theta)^2 d\theta = \frac{9}{4} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \dots$	Full process for at least one of the two areas required, including correct area formula, with their attempt to apply the double angle formula, and evidence of correct limits applied.	M1
	Area of $R = \frac{1}{2} \int_0^{\frac{\pi}{3}} 3 \sin^2 \theta d\theta + \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9 \cos^2 \theta d\theta$ $= \frac{3}{4} \left[\frac{\pi}{3} - \frac{1}{2} \sin \frac{2\pi}{3} - (0 - 0) \right] + \frac{9}{4} \left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{3} + \frac{1}{2} \sin \frac{2\pi}{3} \right) \right]$	Full and correct process for the area of R . Formulae and limits must all be correct, and evidence of limits applied, with areas added.	DM1
	$\frac{\pi}{4} - \frac{3\sqrt{3}}{16} + \frac{3\pi}{8} - \frac{9\sqrt{3}}{16} = \frac{5\pi}{8} - \frac{3\sqrt{3}}{4}$	Correct answer. Allow $a = \frac{5}{8}, \quad b = -\frac{3}{4}$	A1
(6)			
Total 8			

Additional Notes

(a)

B1: One correct coordinate, do not be concerned about the method for this mark. Allow for $\theta = \frac{\pi}{3}$ seenand isw if they go on to use $\theta = \frac{\pi}{3}$ for this mark.

B1: Both coordinates correct, and no extras. Again, do not be concerned about the method. Condone if given in the wrong order in a coordinate pair.

(b)



Question Number	Scheme	Notes	Marks
5	$r^2 = 100 \cos^2 \theta + 20 \cos \theta \tan \theta + \tan^2 \theta$	Any correct expression for r^2	B1
	$\left\{ \frac{1}{2} \right\} \int_0^{\frac{\pi}{3}} r^2 d\theta = \left\{ \frac{1}{2} \right\} \int_0^{\frac{\pi}{3}} (100 \cos^2 \theta + 20 \sin \theta + \tan^2 \theta) \{d\theta\}$	Attempts formula for the area with their r^2 which may not be expanded Condone missing $\frac{1}{2}$ and limits not required	M1
	$= \frac{1}{2} \int_0^{\frac{\pi}{3}} (50(1 + \cos 2\theta) + 20 \sin \theta + \sec^2 \theta - 1) \{d\theta\}$ M1: Uses $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ or $\tan^2 \theta = \pm \sec^2 \theta \pm 1$ in their r^2 M1: Uses both $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ and $\tan^2 \theta = \pm \sec^2 \theta \pm 1$ in their r^2 Both M marks can be scored without the integral and the $\frac{1}{2}$. Condone mixed variables. A1: Correct integral following $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ and $\tan^2 \theta = \sec^2 \theta - 1$. The $\cos \theta \tan \theta$ must be written as $\sin \theta$ (implied if appropriately integrated later). The $\frac{1}{2}$ is required (it may be seen later) but limits/$d\theta$ are not needed. Allow mixed variables if subsequent work recovers this.		M1 M1 A1
	$= \frac{1}{2} \left[49\theta + 25 \sin 2\theta - 20 \cos \theta + \tan \theta \right]_0^{\frac{\pi}{3}} \text{ or } \left[\frac{49}{2} \theta + \frac{25}{2} \sin 2\theta - 10 \cos \theta + \frac{1}{2} \tan \theta \right]_0^{\frac{\pi}{3}}$ M1: Achieves three of the following four integrated forms: $k \rightarrow k\theta$ (at least once), $\cos 2\theta \rightarrow \dots \sin 2\theta$, $\sin \theta \rightarrow \dots \cos \theta$, $\sec^2 \theta \rightarrow \dots \tan \theta$. Ignore other terms if 3 of the above are satisfied. No $\frac{1}{2}$ or limits required. Condone mixed variables. A1: Correct integration including the $\frac{1}{2}$ (may be seen later). Limits not required. May be unsimplified e.g., 49θ seen as $50\theta - \theta$. Allow mixed variables if subsequent work recovers this.		M1 A1
	$= \frac{1}{2} \left(\frac{49\pi}{3} + 25 \sin \frac{2\pi}{3} - 20 \cos \frac{\pi}{3} + \tan \frac{\pi}{3} - (0 + 0 - 20 + 0) \right)$ $\left\{ = \frac{1}{2} \left(\frac{49\pi}{3} + \frac{25\sqrt{3}}{2} - 10 + \sqrt{3} + 20 \right) \text{ or } \frac{49\pi}{6} + \frac{25\sqrt{3}}{4} - 5 + \frac{\sqrt{3}}{2} + 10 \right\}$ Applies the correct limits to an expression of the form $p\theta + q \sin 2\theta + r \cos \theta + s \tan \theta$ ($p, q, r, s \neq 0$) Allow slips but there must be a clear attempt to substitute, and they must only subtract the value of their r, e.g. if $r = -20$ work must have or imply $\dots - (-20)$ or $+20$. Allow mixed variables if the substitution recovers this.		M1
	$= \frac{1}{12} (98\pi + 81\sqrt{3} + 60)$	Correct answer or values for a, b & c	A1
Note that there are other viable routes through the integration e.g., use of integration by parts			(9)
			Total 9

Question Number	Scheme	Marks
10(a)	$1 + \cos \theta = 1 + \frac{\sqrt{3}}{2} \Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$ $\Rightarrow 1 + \frac{\sqrt{3}}{2} = k \times \frac{2}{\sqrt{3}} \Rightarrow k = \dots$	M1
	$k = \frac{\sqrt{3}}{2} + \frac{3}{4} \text{ oe}$	A1
		(2)
(b)	At P $\theta = \frac{\pi}{6}$	B1
	$\int (1 + \cos \theta)^2 d\theta = \int (1 + 2\cos \theta + \cos^2 \theta) d\theta$ $= \int \left(1 + 2\cos \theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta \right) d\theta$	M1
	$\int (1 + \cos \theta)^2 d\theta = \frac{3}{2}\theta + 2\sin \theta + \frac{1}{4}\sin 2\theta \text{ (oe)}$	A1
	$\frac{1}{2} \left[\frac{3}{2}\theta + 2\sin \theta + \frac{1}{4}\sin 2\theta \right]_{\frac{\pi}{6}}^{\pi}$ $= \frac{1}{2} \left[\left(\frac{3\pi}{2} + 0 + 0 \right) - \left(\frac{\pi}{4} + 1 + \frac{\sqrt{3}}{8} \right) \right] \quad \left(= \frac{5\pi}{8} - \frac{\sqrt{3}}{16} - \frac{1}{2} \right)$	M1
	Triangle: $\frac{1}{2} \times \left(1 + \frac{\sqrt{3}}{2} \right) \sin \frac{\pi}{6} \times \left(1 + \frac{\sqrt{3}}{2} \right) \cos \frac{\pi}{6} = \frac{1}{2} \left(\frac{1}{2} + \frac{\sqrt{3}}{4} \right) \left(\frac{\sqrt{3}}{2} + \frac{3}{4} \right)$ $\left(= \frac{7\sqrt{3}}{32} + \frac{3}{8} \right)$	M1
	Area of R = $\frac{5\pi}{8} - \frac{\sqrt{3}}{16} - \frac{1}{2} + \frac{7\sqrt{3}}{32} + \frac{3}{8}$	dM1
	$\frac{5\pi}{8} + \frac{5\sqrt{3}}{32} - \frac{1}{8}$	A1
		(7)
	Total 9	

Notes:

(a)

M1: Correct method to find the value of k .

A1: Correct value.

(b)

B1: Deduces the correct value of θ at P. May be seen anywhere – in (a) or on Figure 1, or as a limit on their integral.M1: Attempts $\left(\frac{1}{2} \right) \int r^2 d\theta$ achieve (at least) three terms from squaring and applies $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$. The $\frac{1}{2}$ in the area formula may be omitted for this mark.Caution: Many have learned a formula $\int \cos^2 \theta d\theta = \frac{\theta}{2} + \frac{1}{4} \sin 2\theta$ so the use of the double angle formula may be implied by this.

- A1: Correct integration of the r^2
- M1: Applies the limits of their $\frac{\pi}{6}$ and π to their attempt at the integral (there must be some evidence of the attempt to subtract). Alternatively, may use limits 0 to π **and** 0 to their $\frac{\pi}{6}$, then subtract. Not dependent, so allow if incorrect double angle identity was used or only two terms from squaring.
- M1: Uses a correct strategy to find the exact for the area of the triangle. This may be via $\frac{1}{2}ab$ or $\frac{1}{2}ab \sin C$ as shown in scheme (both are equivalent) or by integration
- $$\frac{1}{2} \int_0^{\pi/6} k^2 \sec^2 \theta d\theta = \frac{k^2}{2} \left[A \tan \theta \right]_0^{\pi/6} = \dots \quad \text{Accept } \int_0^{\pi/6} \sec^2 \theta d\theta = \frac{\sqrt{3}}{3} \text{ (oe) without}$$
- further working shown.
- dM1: Fully correct method for the required area. Depends on all previous method marks. The $\frac{1}{2}$ in the area formula must have been included to score this mark.
- A1: Cao. Terms may be in a different order and accept if put over a common denominator.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Summary of changes from Provisional Mark Scheme

A few minor changes were made to the Mark Scheme before marking on the marking service began.

Question Number	Summary of changes
Q01b	Totals for 1(b) and question corrected.
Q06	An alternative method was added. This was seen after standardization and was considered to satisfy the demands of the question and so was worthy of marks
Q08	Final mark should not have been “follow through”



Question	Scheme	Marks
1(a)	$r = \sqrt{(-4)^2 + (-4\sqrt{3})^2} = \dots$	M1
	$\tan \theta = \frac{-4\sqrt{3}}{-4} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) \pm \pi$	M1
	$8 \left(\cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right) \right)$	A1
		(3)
(b)	$z = re^{i\theta} \Rightarrow (re^{i\theta})^3 = -4 - 4\sqrt{3} \Rightarrow r^3 (e^{3i\theta}) = 8e^{-i\frac{2\pi}{3}}$	
	$\Rightarrow r = \sqrt[3]{8} = 2$	M1
	$3\theta = -\frac{2\pi}{3} (+2k\pi) \Rightarrow \theta = -\frac{2\pi}{9} + \left(\frac{2k\pi}{3}\right)$	M1
	So $z = 2e^{-\frac{8\pi}{9}i}, 2e^{-\frac{2\pi}{9}i}, 2e^{\frac{4\pi}{9}i}$	A1ft A1
		(4)
(7 marks)		

Notes:**(a)**

M1: For a correct attempt at the modulus, implied by a correct modulus if no method seen and allow recovery if correct answer follows a minor slip in notation.

M1: For an attempt to find a value of θ in the correct quadrant. Accept $\tan^{-1}(\sqrt{3}) \pm \pi$ or $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \pm \pi$

May be implied by sight of an of $-\frac{2}{3}\pi, \frac{4}{3}\pi, -\frac{5}{6}\pi, \frac{7}{6}\pi$.

A1: cao as in scheme, no other solution.

(b)

M1: Applies De Moivre's Theorem and proceeds to find a value for r ie (their 8) ^{$\frac{1}{3}$}

M1: Proceeds to find at least one value for θ – ie their argument/3.

A1ft: At least two roots correct for their r and θ . (Must come from correct method, watch for correct roots coming from an incorrect angle due to errors.)

A1: All three correct roots and no others. Accept e.g $2e^{i\frac{8\pi}{9}}$ as a slip in notation, so allow marks.



Question Number	Scheme	Notes	Marks
8(a)	$y = r \sin \theta = (1 - \sin \theta) \sin \theta = \sin \theta - \sin^2 \theta$ $\Rightarrow \frac{dy}{d\theta} = \cos \theta - 2 \sin \theta \cos \theta$ or e.g. $\Rightarrow \frac{dy}{d\theta} = \cos \theta - \sin 2\theta$	Differentiates $(1 - \sin \theta) \sin \theta$ to achieve $\pm \cos \theta \pm k \sin \theta \cos \theta$ or equivalent. Use of $y = r \cos \theta$ or $x = r \cos \theta$ scores M0	M1
		Correct derivative in any form.	A1
	$\cos \theta - 2 \sin \theta \cos \theta = 0 \Rightarrow \cos \theta (1 - 2 \sin \theta) = 0 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \dots$ Solves to find a value for θ . Depends on the first M.		dM1
	$\left(\frac{1}{2}, \frac{\pi}{6}\right)$ Correct coordinates and no others. Isw if necessary e.g. if written as $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ after correct values seen or implied award A1. Allow e.g. $\theta = \frac{\pi}{6}, r = \frac{1}{2}$. The value of r must be seen in (a) – i.e. do not allow recovery in (b).		A1
			(4)
(b) Way 1	Note that the $\frac{1}{2}$ in $\frac{1}{2} \int r^2 d\theta$ is not required for the first 4 marks		
	$\int (1 - \sin \theta)^2 d\theta = \int (1 - 2 \sin \theta + \sin^2 \theta) d\theta$ $= \int \left(1 - 2 \sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta\right) d\theta$	Attempts $\left(\frac{1}{2}\right) \int r^2 d\theta$ and applies $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$	M1
	$\int (1 - \sin \theta)^2 d\theta = \frac{3}{2} \theta + 2 \cos \theta - \frac{1}{4} \sin 2\theta (+c)$ Correct integration. Condone mixed variables e.g. $\int (1 - \sin \theta)^2 d\theta = \frac{3}{2} x + 2 \cos \theta - \frac{1}{4} \sin 2\theta (+c)$		A1
	$\left(\frac{1}{2}\right) \left[\frac{3}{2} \theta + 2 \cos \theta - \frac{1}{4} \sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{2}\right) \left[\left(\frac{\pi}{4} + \sqrt{3} - \frac{\sqrt{3}}{8}\right) - (2)\right] \left(\frac{\pi}{8} + \frac{7\sqrt{3}}{16} - 1\right)$ Applies the limits of 0 and their $\frac{\pi}{6}$ to their integration. The $\frac{1}{2}$ is not required. For the integration look for at least $\pm \int \sin \theta d\theta \rightarrow \pm \cos \theta$		M1
	Triangle: $\frac{1}{2} \times \frac{1}{2} \sin \frac{\pi}{6} \times \frac{1}{2} \cos \frac{\pi}{6} \left(= \frac{\sqrt{3}}{32}\right)$ Uses a correct strategy for the area of the triangle		M1
	Area of $R = \frac{\pi}{8} + \frac{7\sqrt{3}}{16} - 1 + \frac{\sqrt{3}}{32}$	Fully correct method for the required area. Depends on all previous method marks.	dM1

	$\frac{1}{32}(4\pi + 15\sqrt{3} - 32)$	Cao	A1
			(6)
			Total 10

	Note that the $\frac{1}{2}$ in $\frac{1}{2} \int r^2 d\theta$ is not required for the first 3 marks		
(b) Way 2	$\int (1 - \sin \theta)^2 d\theta = \int (1 - 2\sin \theta + \sin^2 \theta) d\theta$ $= \int \left(1 - 2\sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta\right) d\theta$	Attempts $\left(\frac{1}{2}\right) \int r^2 d\theta$ and applies $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$.	M1
	$\int (1 - \sin \theta)^2 d\theta = \frac{3}{2}\theta + 2\cos \theta - \frac{1}{4}\sin 2\theta (+c)$ Correct integration. Condone mixed variables e.g. $\int (1 - \sin \theta)^2 d\theta = \frac{3}{2}x + 2\cos \theta - \frac{1}{4}\sin 2\theta (+c)$		A1
	$\left(\frac{1}{2}\right) \left[\frac{3}{2}\theta + 2\cos \theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}} = \left(\frac{1}{2}\right) \left[\left(\frac{3\pi}{4} + 0 - 0\right) - (2)\right] \left(= \frac{3\pi}{8} - 1\right)$ Evidence of use of both limits 0 and $\frac{\pi}{2}$ to their integration. The $\frac{1}{2}$ is not required. For the integration look for at least $\pm \int \sin \theta d\theta \rightarrow \pm \cos \theta$		M1
	Triangle – “Segment”: $\frac{1}{2} \times \frac{1}{2} \sin \frac{\pi}{6} \times \frac{1}{2} \cos \frac{\pi}{6} - \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - \sin \theta)^2 d\theta$ $\frac{\sqrt{3}}{32} - \frac{1}{2} \left[\frac{3}{2}\theta + 2\cos \theta - \frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(= \frac{15\sqrt{3}}{32} - \frac{\pi}{4}\right)$ Uses a fully correct strategy for the area above the curve between O and P. Requires a correct method for the triangle as in Way 1 and a correct method for the “segment” using both their $\frac{\pi}{6}$ and $\frac{\pi}{2}$.		M1
	Area of R = $\frac{3\pi}{8} - 1 + \frac{15\sqrt{3}}{32} - \frac{\pi}{4}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32}(4\pi + 15\sqrt{3} - 32)$	cao	A1
			(6)

Question	Scheme	Marks
5(a)	$y = r \sin \theta = \left(\sqrt{3} + \frac{1 - \cos \theta}{\sin \theta} \right) \sin \theta = \sqrt{3} \sin \theta + 1 - \cos \theta$ $\Rightarrow \frac{dy}{d\theta} = \sqrt{3} \cos \theta + \sin \theta \text{ or } \frac{dy}{d\theta} = \sqrt{3} \cos \theta + \cos \theta \tan \frac{\theta}{2} + \sin \theta \frac{1}{2} \sec^2 \frac{\theta}{2}$ Or equivalent NOTE that $\sin \theta \times \frac{1}{2} \sec^2 \frac{\theta}{2} \equiv \tan \frac{\theta}{2} \equiv \frac{1 - \cos \theta}{\sin \theta}$ so final term may be written this way	M1 A1
	$\frac{dy}{d\theta} = 0 \Rightarrow \sin \theta = -\sqrt{3} \cos \theta \Rightarrow \tan \theta = \dots \Rightarrow \theta = \dots$	M1
	$\theta (= \arctan(-\sqrt{3})) = \frac{2\pi}{3}$	A1
		(4)
Alt (i) For the 2 nd M1A1	$\sqrt{3} \cos \theta + \cos \theta \frac{1 - \cos \theta}{\sin \theta} + \frac{1}{2} \sin \theta \left(1 + \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \right) = 0$ $\Rightarrow 2\sqrt{3} \cos \theta \sin \theta + 2 \cos \theta - 2 \cos^2 \theta + (\sin^2 \theta + 1 - 2 \cos \theta + \cos^2 \theta) = 0$ $\Rightarrow \sqrt{3} \sin 2\theta - \cos 2\theta + 1 = 0$ $\Rightarrow 2 \sin \left(2\theta - \frac{\pi}{6} \right) = -1 \Rightarrow \theta = \dots$	
Alt (ii) For the 2 nd M1A1	$\sqrt{3} \cos \theta + \cos \theta \tan \frac{\theta}{2} + \sin \theta \frac{1}{2} \sec^2 \frac{\theta}{2} = 0 \Rightarrow 2\sqrt{3} + 2 \tan \frac{\theta}{2} + \tan \theta \left(1 + \tan^2 \frac{\theta}{2} \right) = 0$ $\Rightarrow 2\sqrt{3} + 2 \tan \frac{\theta}{2} + \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} \left(1 + \tan^2 \frac{\theta}{2} \right) = 0 \Rightarrow \sqrt{3} + 2 \tan \frac{\theta}{2} - \sqrt{3} \tan^2 \frac{\theta}{2} = 0$ $\Rightarrow \theta = 2 \arctan \sqrt{3} = \dots$	
(b)	$\int r^2 d\theta = \int 3 + 2\sqrt{3} \tan \frac{\theta}{2} + \tan^2 \frac{\theta}{2} d\theta$	M1
	$= \int 3 + 2\sqrt{3} \tan \frac{\theta}{2} + \sec^2 \frac{\theta}{2} - 1 d\theta$	M1
	$= 3\theta + 2\sqrt{3} \times -2 \ln \cos \frac{\theta}{2} + 2 \tan \frac{\theta}{2} - \theta \quad (+c) \text{ o.e.}$	M1 A1
	Substitutes their limits to reach	
	$\text{Area} = \left(\frac{1}{2} \right) \left[2 \frac{2\pi}{3} - 4\sqrt{3} \ln \cos \frac{\pi}{3} + 2 \tan \frac{\pi}{3} - 0 \right] \text{ o.e.}$	M1

	$= 2\sqrt{3} \ln 2 + \frac{2}{3} \pi + \sqrt{3}$	A1
		(6)
(10 marks)		
Notes:		
(a) M1: Applies $y = r \sin \theta$ and attempts the differentiation. The given identity may be applied first, or later, or not at all. Condone sign/coefficient errors only. Use of $y = r \cos \theta$ scores 0 marks in (a) A1: Correct derivative. M1: Sets the derivative equal to zero and solves for θ. If identity was used initially expect as scheme but alternative routes are possible, e.g. see examples above. Allow for any plausible attempt that reaches a value of θ if there are minor errors but attempts at trig identities should be correct up to sign error. A1: Correct value- must be in correct quadrant. If not explicitly written in (a) but used in (b) as their limit then allow this mark to score in (b) (b) M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved. M1: Applies $\tan^2 A = \pm 1 \pm \sec^2 A$ to their integral. Not dependent so may be scored as long as they have a $\tan^2$ term. Note: some candidates may use the identity given in the question, but they must progress to terms which can be integrated (see Alt). M1: Attempts the integration including $\sec^2 \dots \rightarrow k \tan \dots$ and $\tan \dots \rightarrow p \ln \cos \dots$ or equivalent (e.g. $p \ln \sec \dots$). Some candidates may attempt to use the given identity to integrate the $\tan^2 \frac{\theta}{2}$ term (see Alt), but there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate $\tan$ and $\sec^2$ terms, or equivalent terms that can be integrated (see Alt). A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark). Note: equivalent integration is possible using the identity given in (a) (see Alt). M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate $\tan$ and $\sec^2$ terms in their integral A1: Correct answer in the form specified. NOTE: If a mix of variables is seen, and later corrected prior to substitution of limits, then full marks can still be achieved, but if not recovered then deduct the penultimate A mark Note: Omission of the $\frac{1}{2}$ could result in M1M1M1A1M1A0		

(b) Alt for first 5 marks	$\int r^2 d\theta = \int 3 + 2\sqrt{3} \tan \frac{\theta}{2} + \tan^2 \frac{\theta}{2} d\theta$	M1
	$\int 3 + 2\sqrt{3} \tan \frac{\theta}{2} + \frac{1 - 2 \cos \theta + \cos^2 \theta}{\sin^2 \theta} d\theta$ $= \int 3 + 2\sqrt{3} \tan \frac{\theta}{2} + \operatorname{cosec}^2 \theta - 2 \operatorname{cosec} \theta \cot \theta + \operatorname{cosec}^2 \theta - 1 d\theta$	M1
	$= 2\theta + 4\sqrt{3} \ln \sec \frac{\theta}{2} - 2 \cot \theta + 2 \operatorname{cosec} \theta (+c) \text{ o.e.}$	M1 A1
	Substitutes their limits to reach $\text{Area} = \left(\frac{1}{2}\right) \left[2 \frac{2\pi}{3} + 4\sqrt{3} \ln \sec \frac{\pi}{3} - 2 \cot \frac{2\pi}{3} + 2 \operatorname{cosec} \frac{2\pi}{3} - \dots \right] \text{ o.e.}$	M1

Notes:

M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved.

M1: Applies the given identity to their $\tan^2 \frac{\theta}{2}$ term in the integral, separates terms and applies $\cot^2 \theta = \pm \operatorname{cosec}^2 \theta \pm 1$. Not dependent so may be scored as long as they have a $\tan^2$ term.

M1: Attempts the integration including $\operatorname{cosec}^2 \dots \rightarrow k \cot \dots \operatorname{cosec} \dots \cot \dots \rightarrow k \operatorname{cosec} \theta$ and $\tan \dots \rightarrow p \ln \cos \dots$ or equivalent (e.g. $p \ln \sec \dots$). there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate terms that can be integrated.

A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark).

M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate $\operatorname{cosec}^2 \theta$ and $\operatorname{cosec} \theta \cot \theta$ terms in their integral

Note: If this method is used it is unlikely that the final A mark is scored, as substitution of the lower limit leads to an indeterminate form. However, this can be resolved by the appropriate treatment of limits e.g. L'Hopital's rule etc.

Question Number	Scheme	Notes	Marks
6(a)	$ z+1-13i =3 z-7-5i \Rightarrow (x+1)^2 + (y-13)^2 = 9\{(x-7)^2 + (y-5)^2\}$ Correct application of Pythagoras Accept 3 or 9 on RHS		M1
	$\Rightarrow x^2 + y^2 - 16x - 8y + 62 = 0$	Correct equation in any form with terms collected	A1
	Centre (8, 4)	Correct centre. i included scores A0	A1
	$r^2 = 64 + 16 - 62 = \dots$	Correct method for r or r^2	M1
	$r = \sqrt{18}$ or $3\sqrt{2}$	Correct radius. Must be exact.	A1
			(5)
(b)	$\arg(z-8-6i) = -\frac{3\pi}{4} \Rightarrow y-6 = x-8$	Converts the given locus to the correct Cartesian form	B1
	$\Rightarrow x^2 + y^2 - 16x - 8y + 62 = 0$ $\Rightarrow x^2 + (x-2)^2 - 16x - 8(x-2) + 62 = 0 \Rightarrow x = \dots$ or $\Rightarrow (y+2)^2 + y^2 - 16x - 8(y+2) + 62 = 0 \Rightarrow y = \dots$	Uses both Cartesian equations to obtain an equation in one variable and attempts to solve	M1
	$x = 7 - 2\sqrt{2}$ or $y = 5 - 2\sqrt{2}$	One correct “coordinate”	A1
	R is $7 - 2\sqrt{2} + (5 - 2\sqrt{2})i$ or $x = 7 - 2\sqrt{2}$ and $y = 5 - 2\sqrt{2}$	Correct complex number or coordinates and no others. Must be exact	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	$x = t^2 \Rightarrow \frac{dx}{dy} = 2t \frac{dt}{dy}$ oe	Correct application of the chain rule	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2t} \frac{dy}{dt}$ (or e.g. $\frac{1}{2\sqrt{x}} \frac{dy}{dt}$)	Any correct expression for $\frac{dy}{dx}$ or equivalent equation	A1
	$2\sqrt{x} \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x^{-\frac{1}{2}} \frac{dy}{dx} + 2\sqrt{x} \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \frac{dt}{dx}$ (NB $\frac{d^2y}{dt^2} = 2 \frac{dy}{dx} + 4x \frac{d^2y}{dx^2}$)	Fully correct strategy to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$ Chain rule used on at least one term. Depends on the first M mark	dM1
	$4x \frac{d^2y}{dx^2} + 2(1 + 2\sqrt{x}) \frac{dy}{dx} - 15y = 15x \Rightarrow 4x \frac{d^2y}{dx^2} + 4\sqrt{x} \frac{dy}{dx} + 2 \frac{dy}{dx} - 15y = 15x$ $\Rightarrow \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 15y = 15t^2$ * ddM1: Substitutes into the given differential equation. The full substitution must be seen. Depends on both M marks. A1*: Cso		ddM1 A1*
			(5)
(b)	$m^2 + 2m - 15 = 0 \Rightarrow m = 3, -5$	Attempts to solve $m^2 + 2m - 15 = 0$	M1
	$y = Ae^{-5t} + Be^{3t}$	Correct CF	A1
	$y = at^2 + bt + c \Rightarrow \frac{dy}{dt} = 2at + b \Rightarrow \frac{d^2y}{dt^2} = 2a$ $\Rightarrow 2a + 4at + 2b - 15at^2 - 15bt - 15c = 15t^2$ Starts with the correct PI form and differentiates twice and substitutes		M1
	$-15a = 15 \Rightarrow a = \dots$ $4a - 15b = 0 \Rightarrow b = \dots$ $2a + 2b - 15c = 0 \Rightarrow c = \dots$	Complete method to find a , b and c by comparing coefficients. Values for all 3 needed. Depends on the second M mark.	dM1
	$y = Ae^{-5t} + Be^{3t} - t^2 - \frac{4}{15}t - \frac{38}{225}$	Correct GS. Must start $y = \dots$	A1
			(5)
(c)	$y = Ae^{-5\sqrt{x}} + Be^{3\sqrt{x}} - x - \frac{4}{15}\sqrt{x} - \frac{38}{225}$	Correct equation (follow through their answer to (b)) Must start $y = \dots$	B1ft
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$x = r \cos \theta = (1 + \sin \theta) \cos \theta$ $\Rightarrow \frac{dx}{d\theta} = \cos^2 \theta - (1 + \sin \theta) \sin \theta$ or $\Rightarrow \frac{dx}{d\theta} = -\sin \theta + \cos 2\theta$	Differentiates $r \cos \theta$ using product rule or double angle formula	M1
		Correct derivative in any form	A1
	$\cos^2 \theta - (1 + \sin \theta) \sin \theta = 0 \Rightarrow 1 - \sin^2 \theta - \sin \theta - \sin^2 \theta = 0 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ or $-\sin \theta + \cos 2\theta = 0 \Rightarrow -\sin \theta + 1 - 2 \sin^2 \theta = 0 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ Sets $\frac{dx}{d\theta} = 0$ and proceeds to a 3TQ in $\sin \theta$ Depends on the first M mark		dM1
	$\Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ $\Rightarrow \sin \theta = \frac{1}{2}, (-1) \Rightarrow \theta = \dots$	Solves for θ . Depends on both M marks above.	ddM1
	$\left(\frac{3}{2}, \frac{\pi}{6}\right)$	Correct coordinates and no others. Need not be in coordinate brackets.	A1
			(5)
(b)	$\int (1 + \sin \theta)^2 d\theta = \int (1 + 2 \sin \theta + \sin^2 \theta) d\theta$ $= \int \left(1 + 2 \sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta\right) d\theta$	Attempts $\left(\frac{1}{2}\right) \int r^2 d\theta$ and applies $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ Ignore any limits shown	M1
	$\int (1 + \sin \theta)^2 d\theta = \frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta (+c)$	Correct integration (Ignore limits)	A1
	$\frac{1}{2} \left[\frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2} \left[\frac{3\pi}{4} - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8} \right) \right] \left(= \frac{\pi}{4} + \frac{9\sqrt{3}}{16} \right)$	Applies the limits of $\frac{\pi}{2}$ and their $\frac{\pi}{6}$ Substitution must be shown but no simplification needed	M1
	Trapezium: $\frac{1}{2} \left(2 + \left(2 - \frac{3}{2} \sin \frac{\pi}{6} \right) \right) \times \frac{3}{2} \cos \frac{\pi}{6}$ $\left(= \frac{39\sqrt{3}}{32} \right)$	Uses a correct strategy for the area of trapezium $OQSP$	M1
	$\text{Area of } R = \frac{39\sqrt{3}}{32} - \frac{\pi}{4} - \frac{9\sqrt{3}}{16}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32} (21\sqrt{3} - 8\pi)$	Cao	A1
			(6)

Question Number	Scheme	Notes	Marks
6(a)	$x = r \cos \theta = \cos \theta (\sqrt{3} + \tan \theta)$	Uses $x = r \cos \theta$	M1
	$x = \sqrt{3} \cos \theta + \sin \theta \Rightarrow \frac{dx}{d\theta} = \cos \theta - \sqrt{3} \sin \theta = 0$ or $\frac{dx}{d\theta} = \cos \theta \sec^2 \theta - \sin \theta \tan \theta - \sqrt{3} \sin \theta = 0$	Obtains correct form for $\frac{dx}{d\theta}$ and sets equal to 0. May use product rule on $\cos \theta \tan \theta$	dM1
	$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	A1
	$r = \sqrt{3} + \tan \frac{\pi}{6} = \sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$	$r = \frac{4\sqrt{3}}{3}$ or $\frac{4}{\sqrt{3}}$	A1
(4)			
(b)	$\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (\sqrt{3} + \tan \theta)^2 d\theta = \dots$	Uses $\frac{1}{2} \int r^2 d\theta$ and attempts to expands the bracket (accept with just two terms). The $\frac{1}{2}$ may be seen later.	M1
	$\int \dots \tan^2 \theta d\theta \rightarrow \int \dots \pm 1 \pm \sec^2 \theta d\theta$	Applies $\tan^2 \theta = \pm \sec^2 \theta \pm 1$ to the integral (Note: could use parts to integrate – see notes below.)	M1
	$\int \dots \sec^2 \theta d\theta \rightarrow \dots \tan \theta$	Correct attempt to integrate $\sec^2 \theta$ to $k \tan \theta$ (see notes for attempts via parts). Depends on previous M	dM1
	$\int \dots \tan \theta d\theta \rightarrow \dots \ln \cos \theta$ or $\dots \ln \sec \theta$	Correct attempt to integrate $\tan \theta$ to $k \ln \cos \theta$ (oe) within their expression.	M1
	$= \left\{ \frac{1}{2} \right\} \int (3 + 2\sqrt{3} \tan \theta + \tan^2 \theta) d\theta =$ $= \left\{ \frac{1}{2} \right\} \left(2\theta - 2\sqrt{3} \ln \cos \theta + \tan \theta \right)$ or $\theta + \sqrt{3} \ln \sec \theta + \frac{1}{2} \tan \theta$ (oe)	Fully correct integration of $\int r^2 d\theta$ with or without the $\frac{1}{2}$ *see notes for an Alt	A1
	sector area $= \frac{1}{2} \times \left(\frac{4\sqrt{3}}{3} \right)^2 \times \frac{\pi}{6}$	Correct ft expression for sector	B1ft
	$\frac{4\pi}{9} - \left[\left(\frac{\pi}{6} - \sqrt{3} \ln \cos \frac{\pi}{6} + \frac{1}{2} \tan \frac{\pi}{6} \right) - (0 - 0 + 0) \right]$	Their sector – their integration from an attempt at $\frac{1}{2} \int r^2 d\theta$ with their limit applied appropriately	M1
	$= \frac{4\pi}{9} - \frac{\pi}{6} + \sqrt{3} \ln \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}$ $= \frac{5}{18} \pi + \frac{\sqrt{3}}{2} \left(\ln \frac{3}{4} - \frac{1}{3} \right)$	Correct answer in the correct form	A1
(8)			
Total 12			

Additional Notes:

(b)

NB Some candidates have learned $\int \tan^2 \theta \, d\theta = \tan \theta - \theta$ and so do not show the “ $\sec^2 \theta - 1$ ” step. Score

M1 for $\tan^2 \theta \rightarrow \pm \tan \theta \pm \theta$ and dM1 for $\tan^2 \theta \rightarrow \tan \theta - \theta$ in such cases.

Use of parts for the integral:

$$\int \tan^2 \theta \, d\theta = \int \sin \theta \frac{\sin \theta}{\cos^2 \theta} \, d\theta = \sin \theta \frac{1}{\pm \cos \theta} - \int \pm \cos \theta \frac{1}{\pm \cos \theta} \, d\theta \quad \text{M1}$$

$$= \pm \tan \theta \pm \theta \quad \text{dM1}$$

Or

$$\int \tan^2 \theta \, d\theta = \int \sin^2 \theta \sec^2 \theta \, d\theta = \sin^2 \theta \tan \theta - \int 2 \sin \theta \cos \theta \tan \theta \, d\theta \quad \text{M1}$$

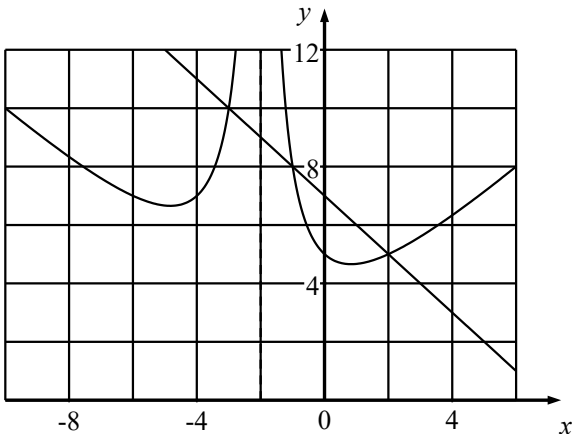
$$= \sin^2 \theta \tan \theta - \int 1 - \cos 2\theta \, d\theta = \sin^2 \theta \tan \theta - \theta + \frac{1}{2} \sin 2\theta \quad \text{dM1}$$

Any variations mark similarly as M1 for first stage of parts applied (allowing for sign errors in trig terms), dM1 for fully integrating as long as the first stage of parts achieves an integral that can be achieved more easily. Watch for the alternate form in the second case.

The B1ft may arise from attempts to use polar areas but must reach a correct expression for the area to

access the mark: $\left\{ \frac{1}{2} \int_0^{\frac{\pi}{6}} \left(\frac{4\sqrt{3}}{2} \right)^2 \, d\theta = \frac{1}{2} \left(\left(\left(\frac{4\sqrt{3}}{2} \right)^2 \times \frac{\pi}{6} \right) - (0) \right) \right\}$

For the M mark for limits, the “ -0 ” for the lower limit may be implied (no need to see it). If no method is shown and only values given from calculator usage, then allow b.o.d. that substitution was correct as long as the correct limits were associated with the integral.

Question Number	Scheme	Marks
3	 $\frac{x^2 + 3x + 10}{x + 2} = 7 - x$ $x^2 + 3x + 10 = 14 + 5x - x^2$ $x^2 - x - 2 = 0 \quad (x - 2)(x + 1) = 0$ CVs 2, -1 $\frac{-(x^2 + 3x + 10)}{x + 2} = 7 - x$ $-x^2 - 3x - 10 = 14 + 5x - x^2$ $8x = -24 \quad \text{CV } -3$ $x < -3 \quad -1 < x < 2$	This sketch on its own scores no marks, but it may be seen in the work M1 dM1 A1A1 M1 A1 dddM1A1A1 [9]
NB M1 dM1 A1 A1 M1 A1 dddM1 A1 A1	No algebra implies no marks Form a quadratic equation or inequality, no simplification needed Solve the 3TQ any valid method Depends on the first M mark. Either CV Both CVs Change the sign of LHS or RHS and obtain an equation (quadratic or linear, no simplification needed) Correct CV from solving the linear equation $x <$ their smallest CV and x between their other 2 CVs All M marks above needed Either inequality correct Both inequalities correct “and” between the inequalities is acceptable. If $\cap$ used, deduct an A mark.	

Question Number	Scheme	Marks
ALT 2	$\frac{d^2y}{dx^2} = \frac{3y}{(2-x^2)} - \frac{5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)^2}$ $\frac{d^3y}{dx^3} = \frac{3\frac{dy}{dx}(2-x^2) - 3y(-2x)}{(2-x^2)^2}$ $- \frac{\left[5\left(\frac{dy}{dx}\right)^2 + 5x \times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2) - 5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$ $\frac{d^3y}{dx^3} = \frac{3\frac{dy}{dx}(2-x^2) - \left((2-x^2)\frac{d^2y}{dx^2} + 5x\frac{dy}{dx}\right)(-2x)}{(2-x^2)^2}$ $- \frac{\left[5\left(\frac{dy}{dx}\right)^2 + 5x \times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2) - 5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$ $\frac{d^3y}{dx^3} = \frac{1}{(2-x^2)} \left(2x\frac{d^2y}{dx^2} \left(1 - 5\frac{dy}{dx} \right) - 5\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx} \right) *$	M1M1A1 M1(B1 on ePEN) A1*
(b)	$x=0 \Rightarrow 2\frac{d^2y}{dx^2} = 9 \quad \frac{d^2y}{dx^2} = \frac{9}{2}$ $\frac{d^3y}{dx^3} = \frac{1}{2} \left(-5\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx} \right) = \frac{1}{2} \left(-5 \times \frac{1}{16} + \frac{3}{4} \right) = \frac{7}{32}$ $y = 3 + \frac{1}{4}x + \frac{9}{2} \frac{x^2}{2!} + \frac{7}{32} \frac{x^3}{3!}$ $y = 3 + \frac{1}{4}x + \frac{9}{4}x^2 + \frac{7}{192}x^3$	B1 M1 M1 A1 (4)

[9]



Question Number	Scheme	Marks
(a)		
M1	Differentiate $(2-x^2)\frac{d^2y}{dx^2}$ using product rule	
M1	Differentiate $5x\left(\frac{dy}{dx}\right)^2$ using product and chain rule	
A1	Correct derivative of $5x\left(\frac{dy}{dx}\right)^2$	
B1	Correct derivative of $3y$	
A1*	Correct result obtained from fully correct working	
ALT 1	<i>Rearrange and use quotient rule</i>	
M1	Use the quotient rule. Denominator must be $(2-x^2)^2$ and numerator to be the difference of 2 terms	
M1	Differentiate $\left[3y - 5x\left(\frac{dy}{dx}\right)^2\right]$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $(2-x^2)\frac{d^2y}{dx^2} + 5x\frac{dy}{dx}$	
A1*	Correct result obtained from fully correct working	
ALT 2	<i>Rearrange, separate into 2 fractions and then use quotient rule</i>	
M1	Use the quotient rule on both fractions. Denominators must be $(2-x^2)^2$ and numerator of each to be the difference of 2 terms	
M1	Differentiate $3y$ using the chain rule and differentiate $5x\left(\frac{dy}{dx}\right)^2$ using product and chain rule	
A1	Fully correct differentiation	
M1	NB: B1 on ePEN Replace $3y$ with $(2-x^2)\frac{d^2y}{dx^2} + 5x\frac{dy}{dx}$	
A1*	Correct result obtained from fully correct working	
(b)		
B1	Correct value of $\frac{d^2y}{dx^2}$	
M1	Use the given result from (a) to obtain a value for $\frac{d^3y}{dx^3}$	
M1	Taylor's series formed using their values for the derivatives (accept 2! or 2 and 3! or 6)	
A1	Correct series, must start (or end) $y = \dots$ but accept $f(x)$ provided $y = f(x)$ defined somewhere	

Question Number	Scheme	Marks
(a)		
M1	Form and solve the auxiliary equation	
A1	Correct CF, either form (Often not seen until GS stated)	
B1	Correct form for the PI	
M1	Differentiate twice and sub in the original equation	
M1	Obtain a pair of simultaneous equations and attempt to solve	
A1	Correct values for both unknowns	
A1ft	Form the GS. Must start $y = \dots$ Follow through their CF (writing CF scores A0) Must have scored a minimum of 2 of the M marks	
(b)		
	For CF $y = e^{-x} (A \cos 2x + B \sin 2x)$	
M1	Sub $x = 0, y = 0$ in their GS and obtain a value for A	
M1	Differentiate their GS Product rule must be used	
A1ft	Correct differentiation of their GS provided this has 4 terms	
dM1	Sub $x = 0, \frac{dy}{dx} = 0$ and their A and obtain a value for B Depends on both previous M marks	
A1	Fully correct PS. Must start $y = \dots$	
ALT(b)		
	For CF $y = e^{-x} (Pe^{i2x} + Qe^{-i2x})$ or $y = Pe^{(-1+2i)x} + Qe^{(-1-2i)x}$	
M1	Sub $x = 0, y = 0$ in their GS and obtain an equation in P and Q	
M1	Differentiate their GS Product rule must be used if $y = e^{-x} (Pe^{i2x} + Qe^{-i2x})$ used	
A1ft	Correct differentiation of their GS	
dM1	Sub $x = 0, \frac{dy}{dx} = 0$ to obtain a second equation and solve the pair of equations The solution must allow for P and Q to be complex	
A1	Fully correct PS. Must start $y = \dots$	

Question Number	Scheme	Marks
7		
(a)	$x = r \cos \theta = 3 \sin 2\theta \cos \theta$ $\frac{dx}{d\theta} = 6 \cos 2\theta \cos \theta - 3 \sin 2\theta \sin \theta = 0$ $2 \cos \theta (\cos^2 \theta - 2 \sin^2 \theta) = 0$	B1 M1 M1
ALT	For the 2 M marks: $x = 6 \sin \theta \cos^2 \theta \Rightarrow \frac{dx}{d\theta} = 6 \cos^3 \theta - 12 \sin^2 \theta \cos \theta = 0$ $\tan \phi = \frac{1}{\sqrt{2}} \quad *$	A1* (4)
(b)	$\tan \phi = \frac{1}{\sqrt{2}} \Rightarrow \sin \phi = \frac{1}{\sqrt{3}}, \cos \phi = \frac{\sqrt{2}}{\sqrt{3}}$ $R = 3 \times 2 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}} = 2\sqrt{2}$	M1 A1 (2)
(c)	Area of sector = $\frac{1}{2} \int r^2 d\theta = \frac{9}{2} \int \sin^2 2\theta d\theta$ $= \frac{9}{2} \int_0^{\arctan(\frac{1}{\sqrt{2}})} \frac{1}{2} (1 - \cos 4\theta) d\theta$ $= \frac{9}{2} \left[\frac{1}{2} \left(\theta - \frac{1}{4} \sin 4\theta \right) \right]_0^{\arctan \frac{1}{\sqrt{2}}}$ $= \frac{9}{4} \left[\arctan \frac{1}{\sqrt{2}} - \frac{1}{4} \sin 4 \left(\arctan \frac{1}{\sqrt{2}} \right) - 0 \right]$ $\sin 4\phi = 2 \sin 2\phi \cos 2\phi = 4 \sin \phi \cos \phi (2 \cos^2 \phi - 1)$ $= 4 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}} \left(2 \times \frac{2}{3} - 1 \right) = \frac{4\sqrt{2}}{9}$ Area of sector = $\frac{9}{4} \left(\arctan \frac{1}{\sqrt{2}} - \frac{1}{4} \times \frac{4\sqrt{2}}{9} \right) = \frac{9}{4} \arctan \frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{4}$	M1 M1 M1A1 dM1 M1 A1 (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	State $x = (r \cos \theta) = 3 \sin 2\theta \cos 2\theta$ May be given by implication	
M1	Attempt to differentiate $x = r \cos \theta$ or $x = r \sin \theta$ Product rule must be used	
M1	Use a correct double angle formula and equate the derivative of $r \cos \theta$ to 0	
ALT	M1 Attempt the differentiation of $x = r \cos \theta$ or $x = r \sin \theta$ using the product rule (after using a double angle formula)	
	M1 Use a correct double angle formula and equate the derivative of $r \cos \theta$ to 0	
A1*	Complete to the given answer and no extras with no errors in the working. Accept θ or ϕ All values seen must be exact	
(b)		
M1	Attempt exact values for $\sin \theta$ and $\cos \theta$ and use these to obtain a value for R . Values for $\sin \theta$ and/or $\cos \theta$ may have been seen in (a)	
A1	A correct, exact value for R , as shown or any equivalent. Award M1A1 for a correct exact answer	
(c)		
M1	Use of Area $= \frac{1}{2} \int r^2 d\theta$ Limits not needed (ignore any shown)	
M1	Use the double angle formula to obtain $k \int \frac{1}{2} (1 \pm \cos 4\theta) d\theta$ Ignore any limits given This is NOT dependent NB: There are other, lengthy, methods of reaching this point	
M1	Attempt the integration $\cos 4\theta \rightarrow \pm \frac{1}{4} \sin 4\theta$ (Not dependent)	
A1	Correct integration of $1 - \cos 4\theta$	
dM1	Correct use of correct limits. Depends on second and third M marks 0 at lower limit need not be shown	
M1	Attempt an exact numerical value for $\sin 4 \left(\arctan \frac{1}{\sqrt{2}} \right)$	
A1	Correct final answer. Award M1A1 for a correct exact final answer	

Question Number	Scheme	Marks
6 (a)	$\frac{d(r \sin \theta)}{d\theta} = 4a \cos \theta + 4a \cos^2 \theta - 4a \sin^2 \theta$ or $4a \cos \theta + 4a \cos 2\theta$ oe (Or allow $\frac{d(r \cos \theta)}{d\theta} = -4a \sin \theta - 8a \cos \theta \sin \theta$ or $-4a \sin \theta - 4a \sin 2\theta$)	M1
	E.g. $4a \cos \theta + 4a \cos^2 \theta - 4a \sin^2 \theta = 0 \Rightarrow \cos \theta + \cos^2 \theta - (1 - \cos^2 \theta) = 0$	M1
	$2 \cos^2 \theta + \cos \theta - 1 = 0$ terms in any order	A1
	$(2 \cos \theta - 1)(\cos \theta + 1) = 0 \Rightarrow \cos \theta = \dots$	ddM1
	$\left(\cos \theta = \frac{1}{2} \Rightarrow \right) \theta = \frac{\pi}{3}$ ($\theta = \pi$ need not be seen)	A1
	$r = 4a \times \frac{3}{2} = 6a$	A1 (6)
(b)	$\text{Area} = \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 16a^2 (1 + \cos \theta)^2 d\theta$	
	$= \frac{16a^2}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 + 2 \cos \theta + \cos^2 \theta) d\theta$	M1
	$= 8a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1 + 2 \cos \theta + \frac{1}{2} (\cos 2\theta + 1) \right) d\theta$	M1
	$= 8a^2 \left[\theta + 2 \sin \theta + \frac{1}{2} \left(\frac{1}{2} \sin 2\theta + \theta \right) \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	dM1A1
	$8a^2 \left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} - \left(\frac{\pi}{6} + 1 + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{12} \right) \right]$	A1
	$8a^2 \left[\frac{\pi}{4} + \sqrt{3} - 1 \right]$	
	$\text{Area } R = 8a^2 \left[\frac{\pi}{4} + \sqrt{3} - 1 \right] - 6a^2 \left(1 + \frac{\sqrt{3}}{2} \right) = a^2 (2\pi + 5\sqrt{3} - 14)$	M1A1 (7)
		[13]
	Notes	
(a)	Attempt the differentiation of $r \sin \theta$ using product rule or $\sin 2\theta = 2 \sin \theta \cos \theta$ OR for this mark only allow differentiation of $r \cos \theta$, inc use of product rule, chain rule or $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$	
M1	Allow errors in coefficients as long as the form is correct.	
M1	Sets their derivative of $r \sin \theta$ equal to zero and achieves a quadratic expression in $\cos \theta$	
A1	Correct 3 term quadratic in $\cos \theta$ (any multiple, including a)	
ddM1	Dep on both M marks. Solve their quadratic (usual rules) giving one or two roots	
A1	Correct quadratic solved to give $\theta = \frac{\pi}{3}$	

	Notes
A1*	Correct r obtained from an intermediate step. Accept as shown in scheme, or $r = 4a \left(1 + \cos \frac{\pi}{3} \right) = 6a$ or equivalent in stages. No need to see coordinates together in brackets
(b)	Note : first 4 marks of (b) do not require limits.
M1	Use of correct area formula, $\frac{1}{2}$ may be seen later, inc squaring the bracket to obtain 3 terms - limits need not be shown.
M1	Use double angle formula (formula to be of form $\cos^2 \theta = \pm \frac{1}{2}(\cos 2\theta \pm 1)$) to obtain an integrable function - limits need not be shown, $\frac{1}{2}$ from area formula may be missing,
dM1	Attempt the integration $\cos \theta \rightarrow \pm k \sin \theta$ and $\cos 2\theta \rightarrow \pm m \sin 2\theta$ - limits not needed – dep on 2 nd M mark but not the first. Note if only two terms arise from squaring allow for $\cos 2\theta \rightarrow \pm m \sin 2\theta$
A1	Correct integration – substitution of limits not required (NB Not follow through)
A1	Include the $\frac{1}{2}$ and substitute the correct limits in a correct integral. Note may be attempted via integral from 0 to $\frac{\pi}{3}$ minus integral from 0 to $\frac{\pi}{6}$ - but attempts at sector formula for the latter is A0.
M1	Attempt the area of the triangle - accept valid attempt even if not subtracted from area. E.g. attempts $\frac{1}{2} OA OB \sin \frac{\pi}{6}$
A1	Correct final answer in the demanded the form.



Question Number	Scheme	Marks
7(a)	$z = y^{-2} \Rightarrow \frac{dz}{dy} = -2y^{-3} \text{ or } 1 = -2y^{-3} \frac{dy}{dz}$ $y^2 = z^{-1} \Rightarrow 2y \frac{dy}{dz} = -z^{-2}$ $y = z^{-\frac{1}{2}} \Rightarrow \frac{dy}{dz} = -\frac{1}{2} z^{-\frac{3}{2}}$	B1
	$\frac{dy}{dx} = \frac{dz}{dx} \cdot \frac{dy}{dz} \text{ (oe)} \Rightarrow \text{e.g., } \frac{dy}{dx} = -\frac{1}{2} y^3 \frac{dz}{dx}, \frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$	M1 A1
	$x \frac{dy}{dx} + y + 4x^2 y^3 \ln x = 0 \Rightarrow \text{e.g.,}$ $-\frac{1}{2} xy^3 \frac{dz}{dx} + y + 4x^2 y^3 \ln x = 0 \quad -\frac{1}{2} xz^{-\frac{3}{2}} \frac{dz}{dx} + z^{-\frac{1}{2}} + 4x^2 z^{-\frac{3}{2}} \ln x = 0$	dM1
	$\frac{dz}{dx} - \frac{2}{xy^2} - 8x \ln x = 0 \quad \frac{dz}{dx} - \frac{2z}{x} - 8x \ln x = 0$ $\Rightarrow \frac{dz}{dx} - \frac{2z}{x} = 8x \ln x \quad *$	A1*
		(5)
(b)	$(\text{IF}) = e^{\int -\frac{2}{x} (dx)}$	M1
	$= e^{-2 \ln x} \quad (= x^{-2})$	A1
	$x^{-2} z = \int x^{-2} (8x \ln x) [dx]$	M1
	E.g. Parts: $\int x^{-1} \ln x \, dx$: $(u = \ln x, u' = x^{-1}, v' = x^{-1}, v = \ln x)$ $\Rightarrow I = (\ln x)^2 - kI \Rightarrow I = p(\ln x)^2$ Or substitution: $t = \ln x, \frac{dt}{dx} = x^{-1} \Rightarrow I = k \int x^{-1} \ln x \cdot x \, dt = k \int t \, dt = pt^2$	M1
	$\int x^{-1} \ln x \, dx = \frac{1}{2} (\ln x)^2 [+c]$	A1
	$x^{-2} z = 4(\ln x)^2 + k \Rightarrow z = 4x^2 (\ln x)^2 + kx^2$ $\Rightarrow y^2 = \frac{1}{4x^2 (\ln x)^2 + kx^2} \text{ oe}$	A1
		(6)
		Total 11
Notes		
(a) B1: Any correct equation following differentiation of the given substitution. Could be implied. M1: Uses a correct chain rule to obtain an equation linking $\frac{dy}{dx}$ (or $\frac{dx}{dy}$) and $\frac{dz}{dx}$ (or $\frac{dx}{dz}$) A1: Any correct equation Note that the first three marks could be scored by a first step of, e.g.: $\frac{dz}{dx} = -2y^{-3} \frac{dy}{dx} \text{ or } 2y \frac{dy}{dx} = -z^{-2} \frac{dz}{dx} \text{ or } \frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$ dM1: Dependent on previous M mark. Substitutes their $\frac{dy}{dx}$ into differential equation (I). Need not replace fully for this mark.		

A1*: Fully correct proof with no errors. There must be an intermediate line of working between the line where they substitute into DE(I) and the given answer.

Note: Use Review for any incorrect but potentially creditworthy attempts that “meet in the middle” or use (II) $\Rightarrow$ (I)

(b)

M1: Correct form for integrating factor. Allow $e^{\int \frac{k}{x}}$ and missing dx

A1: Correct IF in any form

M1: Obtains $IF \cdot z = \int IF \cdot (8x \ln x) [dx]$ Correct for their IF and z not y

M1: Must have achieved an integral of form $k \int x^{-1} (\ln x) [dx]$. Correct form following integration of $x^{-1} \ln x$ (constant not required):

$$\int x^{-1} \ln x dx \Rightarrow p(\ln x)^2 [+c]$$

The work in the scheme above is sufficient but you do not need to scrutinise the details, accept work that leads to the correct form.. Condone “ $\ln x^2$ ” for this mark

A1: Correct integration (constant not required) $\ln x^2$ is A0 unless recovered – must see $(\ln x)^2$ but allow $\ln^2 x$

A1: Any correct equation in $y^2 = \dots$ form e.g., $y^2 = \left[x^2 (4 \ln^2 x + k) \right]^{-1}$ Constant may appear as a multiple, e.g., $8c$



Question Number	Scheme	Marks
8	$r = 6(1 + \cos \theta) \quad 0 \leq \theta \leq \pi$	
(a)	$\theta = 0, r = 6(1 + \cos 0) \Rightarrow 12 \text{ or } (12, 0)$	B1
		(1)
(b)	$\frac{d}{d\theta}(r \sin \theta):$ $= 6 \sin \theta (1 + \cos \theta) \Rightarrow \begin{matrix} = 6 \sin \theta + 6 \sin \theta \cos \theta \Rightarrow \\ 6 \sin \theta (-\sin \theta) + 6 \cos \theta (1 + \cos \theta) \end{matrix} \quad \begin{matrix} = 6 \sin \theta + 6 \sin \theta \cos \theta \Rightarrow \\ 6 \cos \theta + 6(\cos^2 \theta - \sin^2 \theta) \end{matrix} \quad \begin{matrix} = 6 \sin \theta + 3 \sin 2\theta \Rightarrow \\ 6 \cos \theta + 6 \cos 2\theta \end{matrix}$	M1
	$\Rightarrow 2 \cos^2 \theta + \cos \theta - 1 = 0$ $[(2 \cos \theta - 1)(\cos \theta + 1) = 0 \Rightarrow] \quad \cos \theta = \frac{1}{2} \quad [\text{or } -1]$	dM1
	$\theta = \frac{\pi}{3} \quad r = 6\left(1 + \cos \frac{\pi}{3}\right) = 9 \quad \text{or} \quad \left(9, \frac{\pi}{3}\right)$	A1 A1
		(4)
8(c)	$\left[\frac{1}{2}\right] \int r^2 d\theta = \left[\frac{1}{2}\right] \int 36(1 + \cos \theta)^2 [d\theta]$	M1
	$\int r^2 d\theta = 36 \int (1 + 2 \cos \theta + \cos^2 \theta) [d\theta] = 36 \int \left(\frac{3}{2} + 2 \cos \theta + \frac{1}{2} \cos 2\theta\right) [d\theta]$	M1
	$\int (a + b \cos \theta + c \cos 2\theta) [d\theta] = a\theta \pm b \sin \theta \pm \frac{c}{2} \sin 2\theta$	M1
	$18 \left[\frac{3}{2} \theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta\right]_{\dots}$	A1
	$18 \left[\frac{3}{2} \theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta\right]_0^{\frac{\pi}{3}} = 18 \left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right) [-0] \quad \left[= 9\pi + \frac{81}{4} \sqrt{3}\right]$	dM1
	E.g. $OB = 6\left(1 + \cos \frac{\pi}{3}\right) \Rightarrow BQ = 6\left(1 + \cos \frac{\pi}{3}\right) \sin \frac{\pi}{3} \quad \left[= \frac{9\sqrt{3}}{2}\right]$	M1
	$BP = 12 - 9 \cos \frac{\pi}{3} = 12 - \frac{9}{2} = \frac{15}{2}$ $\Rightarrow \text{area } OBPA = \frac{1}{2} \left(12 + \frac{15}{2}\right) \left(\frac{9\sqrt{3}}{2}\right) \quad \text{or} \quad \frac{1}{2} \times \frac{9}{2} \times \frac{9\sqrt{3}}{2} + \frac{9\sqrt{3}}{2} \times \frac{15}{2} \quad \left[= \frac{351}{8} \sqrt{3}\right]$	A1
	$\text{area of region } R = \frac{351}{8} \sqrt{3} - \frac{81}{4} \sqrt{3} - 9\pi = \frac{189}{8} \sqrt{3} - 9\pi$	A1
		(8)
		Total 13

Notes

(a)

B0: Correct values for θ and r or correct coordinates. Condone (0, 12).

(b)

M1: Differentiates $r \sin \theta$. Allow sign errors only. The “6” may be missingdM1: **Dependent on previous M mark.** Uses correct identity/identities to reach a 3TQ in $\cos \theta$ and solves.Apply usual rules or by calculator must obtain at least one real consistent solution where $-1 \leq \cos \theta \leq 1$.Accept $\theta = \frac{\pi}{3}$ following a correct derivative set equal to zero to imply the M if no incorrect working shown

(by calculator or by inspection).

A1: Either r or θ correct.A1: Both coordinates correct. Only accept $\frac{\pi}{3}$ and 9. Withhold last mark if additional answers offered and not rejected, but isw after correct answers seen if only a miscopy is made.

(c)

M1: Attempts $\int r^2 d\theta$ which may include the $\frac{1}{2}$ for the area formula. The multiple 36 may be missing or wrong. Condone poor squaring - allow this mark if there are only two terms.

M1: Uses $\cos^2 \theta = \pm \frac{1}{2} \cos 2\theta \pm \frac{1}{2}$ and obtains an integrand of the form $a + b \cos \theta + c \cos 2\theta$ (constants may be uncollected)

M1: Integrates and obtains a form $a\theta \pm b \sin \theta \pm \frac{c}{2} \sin 2\theta$ (sign errors on trig terms only). May be two terms in θ

A1: Fully correct expression for the area of the sector after integration. Ignore limits but $\frac{1}{2}$ must have been used or appear later.

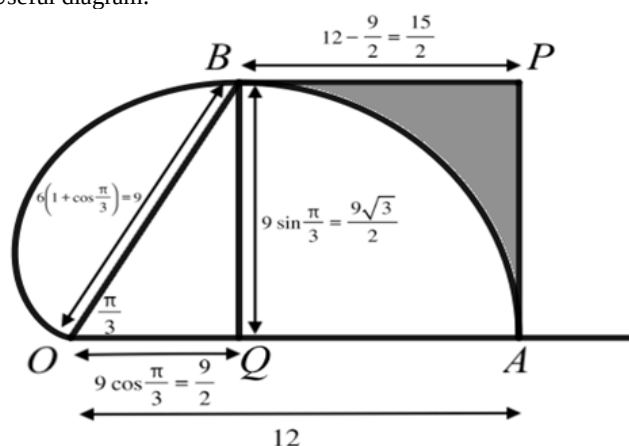
dM1: **Dependent on previous M mark.** Evidence of substitution of “correct” limits into the integral, so their $\frac{\pi}{3}$ (provided $0 < \theta < \frac{\pi}{2}$) and the lower limit must be 0 and lead to zero but this can be implied by omission.

M1: Correct method for the perpendicular distance between l_1 and the initial line with their r and θ (provided $0 < \theta < \frac{\pi}{2}$) for point B. This may only be seen embedded in their attempt at an area, so may be implied if not explicit. E.g. look for “their $r \sin$ their $\frac{\pi}{3}$ ” or may find r again from the equation of curve.

A1: Correct expression for area of trapezium OBPA - may be given as a sum of separate areas, e.g. triangle OBQ + rectangle QBPA. Other formulations are possible.

A1: Correct answer in the correct form or $p = \frac{189}{8}$ or exact equivalent, $q = -9$

Useful diagram:



TOTAL FOR PAPER: 75 MARKS