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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Differentiation (FP3.3)

International A Level Mathematics · FP3: Further Pure Mathematics 3 (WFM03)

10 questions · 74 marks · Mark schemes included

Past papers January 2021 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

10 questions · 74 marks · 1 sub-topic

SUB-TOPIC FP3.3a

DIFFERENTIATING INVERSE TRIGONOMETRIC & HYPERBOLIC FUNCTIONS

Sub-topic FP3.3a · 10 questions · 74 marks · lowest tariff first



1. Given that

$$y = 3x \arcsin 2x \quad 0 \leq x \leq \frac{1}{2}$$

(a) determine an expression for $\frac{dy}{dx}$ (2)

(b) Hence determine the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{4}$, giving your answer in the form $a\pi + b$ where a and b are fully simplified constants to be found. (1)

Question 1 continued

(Total for Question 1 is 3 marks)



4.

$$y = \operatorname{artanh}\left(\frac{\cos x + a}{\cos x - a}\right)$$

where a is a non-zero constant.

Show that

$$\frac{dy}{dx} = k \tan x$$

where k is a constant to be determined.

(4)

Question 4 continued

(Total for Question 4 is 4 marks)



4.

$$y = \operatorname{arsinh} x + \operatorname{arsinh} \left(\frac{1}{x} \right) \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{x-1}{x\sqrt{1+x^2}} \quad (2)$$

(b) Hence determine the exact value of y for which $\frac{dy}{dx} = 0$, giving your answer as a simplified natural logarithm.

(3)

Question 4 continued

(Total for Question 4 is 5 marks)



2.

$$y = \ln(\tanh 2x) \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = p \operatorname{cosech} 4x$$

where p is a constant to be determined.

(4)

(b) Hence determine, in simplest form, the exact value of x for which $\frac{dy}{dx} = 1$

(2)

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Question 2 continued

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Q2

(Total 6 marks)



$x > 1$

(3)

$$x > 1$$

(4)



Question 3 continued

(Total for Question 3 is 7 marks)



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4. (i) $f(x) = x \arccos x \quad -1 \leq x \leq 1$

Find the exact value of $f'(0.5)$.

(3)

(ii) $g(x) = \arctan(e^{2x})$

Show that

$$g''(x) = k \operatorname{sech}(2x) \tanh(2x)$$

where k is a constant to be found.

(5)



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 8 marks)

Q4



Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total 9 marks)

Q3



Question 7 continued



Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)





Question 8 continued



Question 8 continued

TOTAL FOR PAPER IS 75 MARKS



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8.

$$y = \arccos(2\sqrt{x})$$

(a) Determine $\frac{dy}{dx}$

(3)

(b) Show that

$$\int y \, dx = x \arccos(2\sqrt{x}) + \int \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx$$

(2)

(c) Use the substitution $\sqrt{x} = \frac{1}{2} \cos \theta$ to show that

$$\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx = \frac{1}{4} \int_a^b \cos^2 \theta \, d\theta$$

where a and b are limits to be determined.

(4)

(d) Hence, determine the exact value of

$$\int_0^{\frac{1}{8}} \arccos(2\sqrt{x}) \, dx$$

(4)

Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Further Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Notes	Marks
1(a)	$(\cosh A \cosh B + \sinh A \sinh B) = \left(\frac{e^A + e^{-A}}{2} \right) \left(\frac{e^B + e^{-B}}{2} \right) + \left(\frac{e^A - e^{-A}}{2} \right) \left(\frac{e^B - e^{-B}}{2} \right)$ $= \frac{e^{A+B} + e^{A-B} + e^{B-A} + e^{-A-B} + e^{A+B} - e^{A-B} - e^{B-A} + e^{-A-B}}{4}$ Expresses the lhs in terms of exponentials correctly, combines terms and combines fractions with common denominator (Brackets not needed due to fraction lines)		M1
	$= \frac{2e^{A+B} + 2e^{-(A+B)}}{4} = \frac{e^{A+B} + e^{-(A+B)}}{2} = \cosh(A+B)^*$ Fully correct proof with no errors		A1*
			(2)
(b)	$\cosh(x + \ln 2) = \cosh x \cosh(\ln 2) + \sinh x \sinh(\ln 2)$ $= \left(\frac{2 + \frac{1}{2}}{2} \right) \cosh x + \left(\frac{2 - \frac{1}{2}}{2} \right) \sinh x$ Applies the result from part (a) and evaluates both $\cosh(\ln 2)$ and $\sinh(\ln 2)$ Use of (a) must be seen		M1
	$\frac{5}{4} \cosh x + \frac{3}{4} \sinh x = 5 \sinh x$ $\Rightarrow \frac{5}{4} \cosh x = \frac{17}{4} \sinh x$	Collects terms and reaches $a \cosh x = b \sinh x$ oe Depends on the first M mark	dM1
	$5 \cosh x = 17 \sinh x \text{ oe}$	Correct equation	A1
	$x = \frac{1}{2} \ln \left(\frac{1 + \frac{5}{17}}{1 - \frac{5}{17}} \right)$ Or $\frac{e^{2x} - 1}{e^{2x} + 1} = \frac{5}{17} \Rightarrow x = \dots$	Moves to $\tanh x$ and uses the correct logarithmic form for $\operatorname{artanh} x$ or reverts to exponential forms and solves for x Depends on both M marks	ddM1
	$x = \frac{1}{2} \ln \left(\frac{11}{6} \right)$	Cao $\left(\text{Accept integer multiples of } \frac{11}{6} \right)$	A1
			(5)
			Total 7

Notes:

(i)

B1: Correct completion of the square. May be implied by an integral of the form $\frac{1}{2} \int \frac{1}{\sqrt{(x+1)^2 + \frac{5}{4}}} dx$

M1: Obtains $k \operatorname{arsinh} f(x)$ ($k \in \mathbb{R}$) or other equivalent form e.g.

$$\frac{1}{2} \ln \left[(x+1) + \sqrt{(x+1)^2 + \frac{5}{4}} \right] \text{ or } \frac{1}{2} \ln \left[(2x+2) + \sqrt{4x^2 + 8x + 9} \right] \text{ etc}$$

A1: Correct integration (condone omission of $+c$). Missing ' h ' is A0 so arsin is A0 but condone arcsinh x .
(ii)

M1: Applies integration by parts to obtain an expression of the form $a x \operatorname{arcosh} 3x - b \int \frac{x}{\sqrt{cx^2 - 1}} dx$

A1: Correct expression (condone missing ' dx ' etc)

M1: Integrates $b \int \frac{x}{\sqrt{\pm(cx^2 - 1)}} \rightarrow k \sqrt{\pm(cx^2 - 1)}$ or $b \int \frac{x}{\sqrt{cx^2 - 1}} \rightarrow k \sinh(\operatorname{arcosh} 3x)$

A1: Fully correct integration (condone omission of $+c$)

ALT

M1: Applies integration by parts (after substitution of $u = \operatorname{arcosh} 3x$) to obtain an expression of the form $au \cosh u - \int b \cosh u du$

A1: Correct expression (condone missing ' du ' etc)

M1: Integrates $\int b \cosh u du \rightarrow c \sinh u$

A1: Replaces x and obtains a fully correct integration (condone omission of $+c$)

Note: attempts that re-write $\operatorname{arcosh} 3x$ in $\ln$ form then attempt to integrate score zero.

Question Number	Scheme	Marks
Accept vectors in i, j, k form or column vector form throughout		
3(a)	$\lambda = (1 \times 1) + (-1 \times 2) + (4 \times 1) = \dots$	M1
	$\lambda = 3$	A1
		(2)
(b)	$\begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} " \lambda " \\ " 2 \lambda " \\ " \lambda " \end{pmatrix} \Rightarrow \begin{matrix} 3 + 2a + b = " 2 \lambda " \\ a + 2 + b = " \lambda " \end{matrix} \Rightarrow a = \dots, b = \dots$	M1
	$a = 2, b = -1$	A1
		(2)
(c)(i)	$ \mathbf{M} - \lambda \mathbf{I} = \begin{vmatrix} 1 - \lambda & -1 & 4 \\ 3 & 2 - \lambda & b \\ a & 1 & -1 - \lambda \end{vmatrix}$ $= (1 - \lambda)[(\lambda - 2)(\lambda + 1) + 1] + 3(-1 - \lambda) + 2 + 4[3 - 2(2 - \lambda)] \quad (\text{via first row})$ $= (1 - \lambda)[(\lambda - 2)(\lambda + 1) + 1] - 3[(1 + \lambda) - 4] + 2[1 - 4(2 - \lambda)] \quad (\text{via first column})$ $= (1 - \lambda)(2 - \lambda)(-1 - \lambda) + 2 + 12 - 8(2 - \lambda) + (1 - \lambda) + 3(-1 - \lambda) \quad (\text{Sarrus})$	M1
	$\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0 \Rightarrow (\lambda - 3)(\lambda^2 + \lambda - 2) = 0 \Rightarrow \lambda = \dots$	dM1
	$\lambda = 1, -2, (3)$	A1
		(3)
(ii)	$\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} x - y + 4z = x \\ 3x + 2y - z = y \\ 2x + y - z = z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} x - y + 4z = -2x \\ 3x + 2y - z = -2y \\ 2x + y - z = -2z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	M1
	$\lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \quad \text{or} \quad \lambda = -2 \rightarrow \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	A1
	$\lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \lambda = -2 \rightarrow \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	A1
		(3)
		Total 10

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Question Number	Scheme	Notes	Marks
2(a)	$\det(\mathbf{M} - \lambda \mathbf{I}) = \begin{vmatrix} 2 - \lambda & 0 & 3 \\ 0 & -4 - \lambda & -3 \\ 0 & -4 & -\lambda \end{vmatrix}$ $= \text{e.g., } (2 - \lambda)[(-4 - \lambda)(-\lambda) - (-4)(-3)] - 0 + 3(0)$ $\text{or } (2 - \lambda)[(-4 - \lambda)(-\lambda) - (-4)(-3)] - 0 + 0$ $\text{Sarrus} \Rightarrow (2 - \lambda)(-4 - \lambda)(-\lambda) - (2 - \lambda)(-3)(-4)$	Obtains an unsimplified cubic expression for $\det(\mathbf{M} - \lambda \mathbf{I})$ condoning sign/copying slips only. Allow poor bracketing if intention clear.	M1
	Note: It is possible to just use $\mathbf{M}\mathbf{x} = \lambda\mathbf{x}$ e.g., $-4y = \lambda z \Rightarrow y = -\frac{\lambda z}{4} \text{ and } -4y - 3z = \lambda y \Rightarrow \lambda z - 3z = -\frac{\lambda^2 z}{4} \Rightarrow \lambda^2 + 4\lambda - 12 = 0 \Rightarrow \dots$ Score the M1 for achieving a 3TQ in λ from appropriate work condoning copying/sign slips only		
	$(2 - \lambda)(\lambda^2 + 4\lambda - 12) = 0 \text{ or } \lambda^3 + 2\lambda^2 - 20\lambda + 24 = 0 \text{ or } -\lambda^3 - 2\lambda^2 + 20\lambda - 24 = 0$ $(2 - \lambda)(\lambda - 2)(\lambda + 6) = 0 \text{ or } (\lambda + 6)(\lambda - 2)(\lambda - 2) = 0$ $\lambda_1 = -6 \quad (\lambda_2 = 2)$ dM1: Solves $\det(\mathbf{M} - \lambda \mathbf{I}) = 0$ to obtain any value for λ including 2. Not usual rules – award for any value seen that is consistent with their equation. The “=0” can be implied by a solution. Note that they may disregard the $(2 - \lambda)$ and solve a quadratic. A1: –6 from a correct equation. Accept both solutions e.g., “–6, 2” and allow if mislabelled and/or –6 rejected. No incorrect solutions.		dM1 A1
	$2x + 3z = -6x$ $\mathbf{M}\mathbf{x} = -6\mathbf{x} \Rightarrow -4y - 3z = -6y \Rightarrow x = \dots, y = \dots, z = \dots$ $-4y = -6z$ $8x + 3z = 0$ $(\mathbf{M} + 6\mathbf{I})\mathbf{x} = \mathbf{0} \Rightarrow 2y - 3z = 0 \Rightarrow x = \dots, y = \dots, z = \dots$ $-4y + 6z = 0$	Uses $\mathbf{M}\mathbf{x} = \lambda\mathbf{x}$ or $(\mathbf{M} - \lambda \mathbf{I})\mathbf{x} = \mathbf{0}$ with any of their non-zero eigenvalues (however obtained) to form simultaneous equations and solves. No requirement for a vector for this mark. There is no need to check their values but award M0 for a zero solution.	M1
	Note: Could find vector product of first 2 rows of $\mathbf{M} - \lambda \mathbf{I}$ i.e., $(8\mathbf{i} + 3\mathbf{k}) \times (2\mathbf{j} - 3\mathbf{k}) = (-6\mathbf{i} + 24\mathbf{j} + 16\mathbf{k}) \quad (\text{two correct components})$		
	$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 12 \\ 8 \end{pmatrix} \Rightarrow \frac{1}{\sqrt{3^2 + 12^2 + 8^2}} \begin{pmatrix} -3 \\ 12 \\ 8 \end{pmatrix}$	Correct method to normalise their eigenvector no matter how this vector is obtained provided it has at least 2 non-zero components. Only allow slips if there is working.	M1
	$\text{e.g., } \frac{1}{\sqrt{217}} \begin{pmatrix} -3 \\ 12 \\ 8 \end{pmatrix} \text{ or } \begin{pmatrix} -\frac{3\sqrt{217}}{217} \\ \frac{12\sqrt{217}}{217} \\ \frac{8\sqrt{217}}{217} \end{pmatrix} \text{ or } \begin{pmatrix} -\frac{3}{\sqrt{217}} \\ \frac{12}{\sqrt{217}} \\ \frac{8}{\sqrt{217}} \end{pmatrix} \text{ or } \frac{1}{2\sqrt{217}} \begin{pmatrix} -6 \\ 24 \\ 16 \end{pmatrix}$	A correct normalised eigenvector in any form. Note direction may be reversed. May use i, j, k notation	A1

(6)



Summary of changes from Provisional Mark Scheme

A few minor changes were made to the Mark Scheme before marking on the marking service began.

Question Number	Summary of changes
Q03b	The B1 for a correct matrix of minors was changed to an M1 to be fairer to the candidates so that a small slip in the matrix of minors was not penalised as heavily.
Q04ii	The method marks were made dependent on each other as candidates were unlikely to make significant progress unless the first derivative was of the correct form and some first derivatives were trivial expressions.
Q06c	Alternative 3 added
Q07ii	Alternative added



Question Number	Scheme	Notes	Marks
1(a)	$1 - \tanh^2 x \equiv \operatorname{sech}^2 x$		
	$1 - \tanh^2 x = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^2$	Replaces the $\tanh x$ on the lhs with a correct expression in terms of exponentials.	B1
	$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} = \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2}$ or e.g. $\frac{2e^{2x} \times 2e^{-2x}}{(e^x + e^{-x})^2}$	Attempts to find common denominator and expand numerator	M1
	$= \left(\frac{4}{(e^x + e^{-x})^2} \right) = \operatorname{sech}^2 x^*$	Obtains the rhs with no errors.	A1cso
			(3)
ALT 1	$1 - \tanh^2 x = (1 - \tanh x)(1 + \tanh x)$		
	$= \left(1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) \right) \left(1 + \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) \right)$	Uses the difference of 2 squares on the lhs and replaces the $\tanh x$ with a correct expression in terms of exponentials.	B1
	$= \left(\frac{2e^{-x}}{e^x + e^{-x}} \right) \left(\frac{2e^x}{e^x + e^{-x}} \right)$	Attempt to find common denominators and simplify numerators.	M1
	$= \left(\frac{4}{(e^x + e^{-x})^2} \right) = \operatorname{sech}^2 x^*$	Obtains the rhs with no errors.	A1cso
ALT 2	$\operatorname{sech}^2 x = \frac{4}{(e^x + e^{-x})^2}$	Replaces the $\operatorname{sech} x$ on the rhs with a correct expression in terms of exponentials.	B1
	$= \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2} = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$	Attempts to express the “4” in terms of the denominator.	M1
	$= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^2 = 1 - \tanh^2 x^*$	Obtains the lhs with no errors.	A1cso

(b)	$2 \operatorname{sech}^2 x + 3 \tanh x = 3 \Rightarrow 2(1 - \tanh^2 x) + 3 \tanh x = 3$ $\Rightarrow 2 \tanh^2 x - 3 \tanh x + 1 = 0$ Uses $\operatorname{sech}^2 x = 1 - \tanh^2 x$ and forms a 3 term quadratic in $\tanh x$		M1
	$(2 \tanh x - 1)(\tanh x - 1) = 0 \Rightarrow \tanh x = \dots$	Solves 3TQ by any valid method including calculator.	M1
	$\tanh x = \frac{1}{2} \rightarrow x = \ln \sqrt{3}$	$\ln \sqrt{3}$. Accept $\frac{1}{2} \ln 3, -\frac{1}{2} \ln \frac{1}{3}$ And no other answers.	A1
			(3)
ALT	$2 \operatorname{sech}^2 x + 3 \tanh x = 3 \Rightarrow 2 \left(\frac{4}{(e^x + e^{-x})^2} \right) + 3 \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) = 3$ $\Rightarrow 8 + 3(e^{2x} - e^{-2x}) = 3(e^{2x} + 2 + e^{-2x}) \Rightarrow \dots$ Substitutes the correct exponential forms, attempts to eliminate fractions and collect terms		M1
	$6e^{-2x} = 2 \Rightarrow e^{-2x} = \frac{1}{3}$	Rearranges to reach $e^{-2x} = \dots$	M1
	$x = \ln \sqrt{3}$	$\ln \sqrt{3}$. Accept $\frac{1}{2} \ln 3, -\frac{1}{2} \ln \frac{1}{3}$ And no other answers.	A1
			Total 6

Question Number	Scheme	Notes	Marks
2.	$y = \sqrt{9-x^2}, 0 \leq x \leq 3$		
(a)	$\frac{dy}{dx} = -\frac{x}{\sqrt{9-x^2}}$	Correct derivative in any form.	B1
	Note that the derivative may be obtained implicitly after squaring e.g. $y = \sqrt{9-x^2} \Rightarrow y^2 = 9-x^2 \Rightarrow 2y \frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{\sqrt{9-x^2}}$		
	Length of $C = \int \sqrt{1 + \frac{x^2}{9-x^2}} dx$	Uses $\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ with their $\frac{dy}{dx}$	M1
	Note that the above may be obtained via the implicit route as e.g. $\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int \sqrt{1 + \frac{x^2}{y^2}} dx = \int \sqrt{1 + \frac{x^2}{9-x^2}} dx$ In which case the B1 is implied.		
	$= \int \sqrt{\frac{9}{9-x^2}} dx = 3 \arcsin \frac{x}{3} (+c) \left(\text{or } -3 \arccos \frac{x}{3} (+c) \right)$ $\int_0^3 \sqrt{\frac{9}{9-x^2}} dx = 3 \arcsin(1) - 3 \arcsin(0) \left(\text{or } -3 \arccos(1) + 3 \arccos(0) \right)$ Finds common denominator, integrates to obtain arcsin... or arccos... and applies the limits 0 and 3.		M1
	$= \frac{3\pi}{2} *$	Obtains the printed answer with no errors. This mark should be withheld if there is no evidence at all of the limits being applied.	A1
	Special case: If $+\frac{x}{\sqrt{9-x^2}}$ is obtained for $\frac{dy}{dx}$ score B0M1M1A1 if otherwise correct but allow full recovery in (b)		
			(4)
(b)	Surface Area $= \int 2\pi\sqrt{9-x^2} \left(\sqrt{\frac{9}{9-x^2}} \right) dx$	Uses $\int 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ with their $\frac{dy}{dx}$	M1
	$= \int_0^3 6\pi dx = 6\pi [x]_0^3 = \dots$	Integrates to obtain kx and applies the limits 0 and 3. Condone omission of the lower limit.	M1
	$= 18\pi$	18π cao	A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
3.	$\mathbf{M} = \begin{pmatrix} 3 & 1 & p \\ 1 & 1 & 2 \\ -1 & p & 2 \end{pmatrix}$		
(a)	$\det \mathbf{M} = \begin{vmatrix} 3 & 1 & p \\ 1 & 1 & 2 \\ -1 & p & 2 \end{vmatrix}$ $= 3(2-2p) - 1(2+2) + p(p+1)$	Attempts determinant. Requires at least 2 correct "terms". May use other rows/columns or rule of Sarrus.	M1
	$= p^2 - 5p + 2$	Correct simplified determinant.	A1
	$p^2 - 5p + 2 = 0 \Rightarrow p = \dots$	Solves 3TQ	M1
	$\frac{5 \pm \sqrt{17}}{2}$	Correct values.	A1
			(4)
(b)	Minors $\begin{pmatrix} 2-2p & 4 & p+1 \\ (2-p^2) & 6+p & (3p+1) \\ 2-p & (6-p) & 2 \end{pmatrix}$	Attempts the matrix of minors. If there is any doubt look for at least 6 correct elements. May be implied by their matrix of cofactors.	M1 (B1 on EPEN)
	Cofactors $\begin{pmatrix} 2-2p & -4 & p+1 \\ -(2-p^2) & 6+p & -(3p+1) \\ 2-p & -(6-p) & 2 \end{pmatrix}$	Attempts cofactors.	M1
		Correct matrix	A1
	$\mathbf{M}^{-1} = \frac{1}{p^2 - 5p + 2} \begin{pmatrix} 2-2p & p^2-2 & 2-p \\ -4 & 6+p & p-6 \\ p+1 & -3p-1 & 2 \end{pmatrix}$	Transposes matrix of cofactors and divides by determinant.	M1
		Follow though their det $\mathbf{M}$ from part (a) but the adjoint matrix must be correct.	A1ft
			(5)
			Total 9

Question Number	Scheme	Notes	Marks
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Question Number	Scheme	Notes	Marks
3(a)	$y = \operatorname{arsech}\left(\frac{x}{2}\right) \Rightarrow \operatorname{sech} y = \frac{x}{2}$ $\Rightarrow \frac{dx}{dy} = -2 \operatorname{sech} y \tanh y$	Takes “sech” of both sides and differentiates to obtain $\frac{dx}{dy} = k \operatorname{sech} y \tanh y$ or equivalent.	M1
	$\Rightarrow \frac{dx}{dy} = -2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2}$ M1: Replaces $\operatorname{sech} y$ with $\frac{x}{2}$ and $\tanh y$ with $\sqrt{1-\left(\frac{x}{2}\right)^2}$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q .	A1
			(4)
(a) Way 2	$y = \operatorname{arsech}\left(\frac{x}{2}\right) \Rightarrow \operatorname{sech} y = \frac{x}{2}$ $\Rightarrow \cosh y = \frac{2}{x} \Rightarrow \sinh y \frac{dy}{dx} = -\frac{2}{x^2}$	Takes “sech” of both sides, changes to “cosh” and differentiates to obtain $\sinh y \frac{dy}{dx} = \frac{k}{x^2}$ or equivalent.	M1
	$\Rightarrow \frac{dy}{dx} = -\frac{2}{x^2 \sinh y} = -\frac{2}{x^2 \sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ M1: Replaces $\sinh y$ with $\sqrt{\left(\frac{2}{x}\right)^2 - 1}$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q .	A1
(a) Way 3	$y = \operatorname{arsech}\left(\frac{x}{2}\right) \Rightarrow y = \operatorname{arcosh}\left(\frac{2}{x}\right)$ Changes to “arcosh” correctly. Score this as the second M mark on EPEN.		M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{2}{x}\right)^2 - 1}} \times -\frac{2}{x^2}$ M1: Differentiates to the form $\frac{k}{x^2 \sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ oe A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only. Score this as the first M mark and first A mark on EPEN.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q .	A1

(a) Way 4	$y = \operatorname{arsech}\left(\frac{x}{2}\right) \Rightarrow \operatorname{sech} y = \frac{x}{2} \Rightarrow \left(\frac{x}{2}\right)^2 = \operatorname{sech}^2 y \Rightarrow \tanh y = \sqrt{1 - \left(\frac{x}{2}\right)^2}$ $\Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = -x \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}}$ $\text{Differentiates to } \operatorname{sech}^2 y \frac{dy}{dx} = kx \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}} \text{ or equivalent}$	M1
	$\Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = -x \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}} \Rightarrow \frac{x^2}{4} \frac{dy}{dx} = -x \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -\frac{4}{x} \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}}$ M1: Replaces $\operatorname{sech}^2 y$ with $\left(\frac{2}{x}\right)^2$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.	M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$ Correct derivative in the required form or correct values for p and q.	A1
(a) Way 5	$y = \operatorname{arsech}\left(\frac{x}{2}\right) \Rightarrow \operatorname{sech} y = \frac{x}{2} \Rightarrow y = \operatorname{artanh}\left(\sqrt{1 - \left(\frac{x}{2}\right)^2}\right)$ Changes to “artanh” correctly. Score this as the second M mark on EPEN.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{2} \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}}}{1 - \left(1 - \frac{x^2}{4}\right)} \times -\frac{x}{2}$ $kx \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}} \text{ oe } \frac{kx \left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}}}{1 - \left(1 - \frac{x^2}{4}\right)}$ M1: Differentiates to the form A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only. Score this as the first M mark and first A mark on EPEN.	M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$ Correct derivative in the required form or correct values for p and q.	A1

There may be other methods used.
If you are in any doubt if the method deserves any marks use Review.

(b)	$f(x) = \tanh^{-1}(x) + \operatorname{sech}^{-1}\left(\frac{x}{2}\right) \Rightarrow f'(x) = \frac{1}{1-x^2} - \frac{2}{x\sqrt{4-x^2}}$ Correct $f'(x)$ following through their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ Also allow with “made up” p and q or the letters p and q.	B1ft
	$\frac{1}{1-x^2} - \frac{2}{x\sqrt{4-x^2}} = 0 \Rightarrow 2(1-x^2) = x\sqrt{4-x^2} \Rightarrow 4(1-x^2)^2 = x^2(4-x^2)$ Sets $\frac{dy}{dx} = 0$ with their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ and squares both sides to reach a quartic equation	M1
	$5x^4 - 12x^2 + 4 = 0$	Correct quartic A1
	$5x^4 - 12x^2 + 4 = 0 \Rightarrow x^2 = 2, \quad 0.4$ $\Rightarrow x = \dots$	Solves their quartic equation to obtain a value for x^2 and proceeds to a value for x . Apply usual rules for solving and check if necessary. Allow complex roots. M1
	$x = \sqrt{\frac{2}{5}}$	Correct exact answer (allow equivalents e.g. $\frac{\sqrt{10}}{5}$). If any extra answers given score A0 e.g. $x = \pm\sqrt{\frac{2}{5}}$ A1
		(5)
		Total 9

Special case:

It is possible for a correct solution in (b) following a sign error in (a) e.g.

$$\frac{dy}{dx} = \frac{2}{x\sqrt{4-x^2}}$$

$$f(x) = \tanh^{-1}(x) + \operatorname{sech}^{-1}\left(\frac{x}{2}\right) \Rightarrow f'(x) = \frac{1}{1-x^2} + \frac{2}{x\sqrt{4-x^2}}$$

$$\frac{1}{1-x^2} + \frac{2}{x\sqrt{4-x^2}} = 0 \Rightarrow 2(1-x^2) = -x\sqrt{4-x^2} \Rightarrow 4(1-x^2)^2 = x^2(4-x^2) \text{ etc.}$$

This is likely to score M1M1A0A0 in (a) but allow full recovery in (b) if it leads to the correct answer.

Question Number	Scheme/Notes		Marks	
7(a)	$y = \arccos(\operatorname{sech} x)$			
	e.g.:	$\cos y = \operatorname{sech} x \Rightarrow$	M1	
	$\frac{dy}{dx} = -\frac{(-\operatorname{sech} x \tanh x)}{\sqrt{1-\operatorname{sech}^2 x}}$	$-\sin y \frac{dy}{dx} = -\operatorname{sech} x \tanh x$ or, e.g., $-\sin y = -\operatorname{sech} x \tanh x \frac{dx}{dy}$		
	$\cos y = (\cosh x)^{-1} \Rightarrow$ $-\sin y \frac{dy}{dx} = -(\cosh x)^{-2} \sinh x$			
	Differentiates to obtain an equation in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ of the correct form e.g. condone coefficient sign errors only.			
	$\frac{dy}{dx} = \frac{\operatorname{sech} x \tanh x}{\tanh x}$	$\sqrt{1-\operatorname{sech}^2 x} \frac{dy}{dx} = \operatorname{sech} x \tanh x$ $\Rightarrow \tanh x \frac{dy}{dx} = \operatorname{sech} x \tanh x$	$\sqrt{1-\operatorname{sech}^2 x} \frac{dy}{dx} = \frac{\sinh x}{\cosh^2 x}$ $\Rightarrow \tanh x \frac{dy}{dx} = \frac{\sinh x}{\cosh^2 x}$	dM1
	Uses correct identities to obtain an equation in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x only with no roots but accept $\sqrt{\tanh^2 x}$ as “no roots”			
	$\Rightarrow \frac{dy}{dx} = \operatorname{sech} x$	$\Rightarrow \frac{dy}{dx} = \operatorname{sech} x$	$\frac{dy}{dx} = \frac{\cosh x}{\sinh x} \cdot \frac{\sinh x}{\cosh^2 x}$ $\Rightarrow \frac{dy}{dx} = \operatorname{sech} x$	A1*
	Fully correct proof. An equation in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ and exactly two different hyperbolic functions with no roots must be seen before the given answer but accept $\sqrt{\tanh^2 x}$ as “no roots” Withhold this mark for any mathematical error e.g., clear use of $\frac{d}{dx}(\arccos x) = +\frac{1}{\sqrt{1-x^2}}$ and $\frac{d}{dx}(\operatorname{sech} x) = +\operatorname{sech} x \tanh x$ or e.g. hyperbolic functions written as trig functions or vice versa. Allow slips if they are recovered but clear and consistent errors score A0			
	Note: There may be other methods seen, e.g., using exponentials and “meeting in the middle”			
				(3)



(b)	e.g. $\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$ or $\frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$ or $\frac{-\operatorname{sech}^2 x}{\tanh^2 x}$ or $1 - \coth^2 x$ etc. or e.g. $\frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$ or $\frac{2e^{2x}(e^{2x} - 1) - 2e^{2x}(e^{2x} + 1)}{(e^{2x} - 1)^2}$ or $\frac{-4}{(e^x - e^{-x})^2}$ etc. Correct derivative of $\coth x$ in any form. Allow recovery if they write e.g. $-\operatorname{cosec}^2 x$ when $-\operatorname{cosech}^2 x$ is clearly implied by subsequent work.	B1
	e.g., $\operatorname{sech} x - \operatorname{cosech}^2 x = 0 \Rightarrow \operatorname{sech} x = \operatorname{cosech}^2 x \Rightarrow \frac{1}{\cosh x} = \frac{1}{\sinh^2 x} \Rightarrow$ $a \cosh^2 x + b \cosh x + c = 0$ or $a \operatorname{sech}^2 x + b \operatorname{sech} x + c = 0$ or $\operatorname{sech} x - \operatorname{cosech}^2 x = 0 \Rightarrow \frac{2}{e^x + e^{-x}} - \left(\frac{2}{e^x - e^{-x}}\right)^2 = 0 \Rightarrow$ $\Rightarrow Ae^{4x} + Be^{3x} + Ce^{2x} + De^x + E = 0$ Sets $f'(x) = 0$ and uses correct identities to obtain a 3TQ in $\cosh x$ or $\operatorname{sech} x$ or substitutes the correct exponential forms and obtains a 5 term quartic in e^x	M1
	$\cosh^2 x - \cosh x - 1 = 0$ or $\operatorname{sech}^2 x + \operatorname{sech} x - 1 = 0$ oe or $\Rightarrow e^{4x} - 2e^{3x} - 2e^{2x} - 2e^x + 1 = 0$ oe Correct quadratic equation or correct quartic equation.	A1
	$\cosh x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \left(= \frac{1 + \sqrt{5}}{2} \right)$ or e.g., $\left(\operatorname{sech} x + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 = 0 \Rightarrow \operatorname{sech} x = \left(\frac{-1 + \sqrt{5}}{2}\right)$ Solves quadratic resulting from $\operatorname{sech} x +$ their derivative of $\coth x = 0$ Must obtain a real and exact value > 1 (or between 0 and 1 if sech used). Apply usual rules. (No need to reject invalid values) If no solving method seen one solution must be consistent with their equation. For the 5 term quartic in e^x progress is unlikely unless they proceed via e.g. $(e^{2x} - (1 + \sqrt{5})e^x + 1)^2 = 0$	dM1
	$x = \operatorname{arcosh}\left(\frac{1 + \sqrt{5}}{2}\right) = \ln\left(\frac{1 + \sqrt{5}}{2} + \sqrt{\left(\frac{1 + \sqrt{5}}{2}\right)^2 - 1}\right)$ or $\frac{e^x + e^{-x}}{2} = \frac{1 + \sqrt{5}}{2} \Rightarrow e^{2x} - (1 + \sqrt{5})e^x + 1 = 0 \Rightarrow e^x = \frac{1 + \sqrt{5} + \sqrt{(1 + \sqrt{5})^2 - 4}}{2} \Rightarrow x = \dots$ Uses correct logarithmic form or exponentials to find x as a $\ln$ of an exact value. Exponential definition must be correct and quadratic solving subject to usual rules or consistent with their equation leading to a value of $e^x > 0$	ddM1
	$\Rightarrow x = \ln\left(\frac{1}{2}(1 + \sqrt{5}) + \sqrt{\frac{1}{2}(1 + \sqrt{5})}\right)$ or accept $x = \ln\left(\frac{1 + \sqrt{5}}{2} + \sqrt{\frac{1 + \sqrt{5}}{2}}\right)$ Note that $x = \ln\frac{1}{2}(1 + \sqrt{5}) + \sqrt{\frac{1}{2}(1 + \sqrt{5})}$ scores A0	A1
		(6)
		Total 9

Correct work in (b) leading to:

$$\cosh^2 x - \cosh x - 1 = 0 \Rightarrow \cosh x = \frac{1 + \sqrt{5}}{2}$$

$$x = \operatorname{arcosh}\left(\frac{1 + \sqrt{5}}{2}\right) = \ln\left(\frac{1 + \sqrt{5}}{2} + \sqrt{\frac{1 + \sqrt{5}}{2}}\right)$$

With no evidence where the $\sqrt{\frac{1 + \sqrt{5}}{2}}$ comes from, scores: B1M1A1dM1ddM0A0



Question Number	Scheme	Notes	Marks
2	Condone poor notation e.g., determinant lines used for matrix bracketing		
(a)	$\det \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & -2 & -3 \end{pmatrix} \left\{ = 2 \times (-3 + 8) \right\} = 10$	Correct value for determinant, seen or stated and not just in a final answer	B1
	$\left\{ \text{Minors: } \begin{pmatrix} 5 & -12 & -3 \\ 0 & -6 & -4 \\ 0 & 8 & 2 \end{pmatrix} \Rightarrow \right\} \text{Cofactors: } \begin{pmatrix} 5 & 12 & -3 \\ 0 & -6 & 4 \\ 0 & -8 & 2 \end{pmatrix}$	Attempts the cofactor matrix with at least 6 correct elements	M1
	Inverse is $\frac{1}{\text{"10"}} \begin{pmatrix} 5 & 0 & 0 \\ 12 & -6 & -8 \\ -3 & 4 & 2 \end{pmatrix} \text{ or e.g., } \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{6}{5} & -\frac{3}{5} & -\frac{4}{5} \\ -\frac{3}{10} & \frac{2}{5} & \frac{1}{5} \end{pmatrix}$	Correct inverse but allow ft on their "10". Allow equivalent fractions/decimals. A0 if clearly obtained incorrectly	A1ft
	Work to obtain $\text{Adj}(\mathbf{M})$ must be seen but it may be minimal, e.g., sight of the matrix of minors followed by the correct answer is acceptable. Note that B0 M1 A1 is possible.		(3)
(b)	$\frac{1}{10} \begin{pmatrix} 5 & 0 & 0 \\ 12 & -6 & -8 \\ -3 & 4 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \dots$	Multiplies their $\mathbf{M}^{-1}$ by $\begin{pmatrix} u \\ v \\ w \end{pmatrix}$ Must use a matrix other than $\mathbf{M}$ – not just changed by application of determinant. Condone sight of $\mathbf{vM}^{-1} = \dots$ but must not be a clearly incorrect multiplication method	M1
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 5u \\ 12u - 6v - 8w \\ -3u + 4v + 2w \end{pmatrix} \text{ or } \begin{pmatrix} \frac{6}{5}u - \frac{3}{5}v - \frac{4}{5}w \\ -\frac{3}{10}u + \frac{2}{5}v + \frac{1}{5}w \end{pmatrix} \text{ or } \frac{1}{d} \begin{pmatrix} 5u \\ 12u - 6v - 8w \\ -3u + 4v + 2w \end{pmatrix} \text{ or } \begin{pmatrix} \frac{5}{d}u \\ \frac{12}{d}u - \frac{6}{d}v - \frac{8}{d}w \\ -\frac{3}{d}u + \frac{4}{d}v + \frac{2}{d}w \end{pmatrix}$ A1ft: Two correct vector components, coordinates or equations, ft their $d \neq 0$ A1ft: All three correct ft their non-zero $d \neq 0$ Must be exact (and not rounded decimals for ft) These ft marks are not available for an incorrect $\text{Adj}(\mathbf{M})$		A1ft A1ft
			(3)
Alt Using $\mathbf{M}$	$\begin{aligned} 2x &= u & x &= \dots \\ y + 4z &= v & \Rightarrow y &= \dots \\ 3x - 2y - 3z &= w & z &= \dots \end{aligned}$	Uses $\mathbf{M} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$ and finds x, y and z as functions of u, v and w Condone sight of $\mathbf{vM} = \dots$ but must not be a clearly incorrect multiplication method	M1
	$\begin{aligned} x &= \frac{1}{2}u \\ y &= \frac{6}{5}u - \frac{3}{5}v - \frac{4}{5}w \\ z &= -\frac{3}{10}u + \frac{2}{5}v + \frac{1}{5}w \end{aligned}$	A1: Two correct equations A1: All three correct Any form with terms collected	A1 A1
			(3)

Question Number	Scheme	Notes	Marks
2(c)	$3x - 7y + 2z = -3 \Rightarrow 3\left(\frac{1}{2}u\right) - 7\left(\frac{6}{5}u - \frac{3}{5}v - \frac{4}{5}w\right) + 2\left(-\frac{3}{10}u + \frac{3}{5}v + \frac{1}{5}w\right) = -3$	Substitutes their expressions into the equation for Π_1	M1
	$-15u + 10v + 12w = -6$	Correct equation . Terms in any order but constant isolated. Accept any integer multiples.	A1
			(2)
			Total 8
Alts	To gain any marks by an alternative approach, a complete attempt at a Cartesian equation for Π_2 must be made by a viable strategy e.g.,		
	general point on $3x - 7y + 2z = -3$ is $\left(s, t, -\frac{3}{2}s + \frac{7}{2}t - \frac{3}{2}\right)$ $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & -2 & -3 \end{pmatrix} \begin{pmatrix} s \\ t \\ -\frac{3}{2}s + \frac{7}{2}t - \frac{3}{2} \end{pmatrix} \Rightarrow \begin{matrix} u = 2s \\ v = -6s + 15t - 6 \\ w = \frac{15}{2}s - \frac{25}{2}t + \frac{9}{2} \end{matrix} \Rightarrow \begin{matrix} v = -3u + 15t - 6 \\ t = -\frac{2}{25}\left(w - \frac{15}{2}\left(\frac{u}{2}\right) - \frac{9}{2}\right) \\ \Rightarrow v = -3u - \frac{6}{5}w + \frac{9}{2}u + \frac{27}{5} - 6 \end{matrix}$ Obtains a plane equation in any Cartesian form		M1
	$\left\{v = \frac{3}{2}u - \frac{6}{5}w - \frac{3}{5}\right\}$ $-15u + 10v + 12w = -6$	Correct equation . Terms in any order but constant isolated. Accept any integer multiples.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
3(a) Way 1 Identities first then squares	$y = \frac{1}{2}(\tan x + \cot x) \Rightarrow \frac{dy}{dx} = \frac{1}{2}(\sec^2 x - \operatorname{cosec}^2 x)$ oe	Correct derivative. Any equivalent.	B1
	$= \frac{1}{2}(1 + \tan^2 x - (1 + \cot^2 x)) \quad \left\{ = \frac{1}{2}(\tan^2 x - \cot^2 x) \right\}$	Applies $\sec^2 x = \pm \tan^2 x \pm 1$ and $\operatorname{cosec}^2 x = \pm \cot^2 x \pm 1$ to their derivative	M1
	$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(\tan^4 x + \cot^4 x - 2 \tan^2 x \cot^2 x)$	Squares to a 3 term expression (or 4 if middle terms uncollected) $2 \tan^2 x \cot^2 x$ can be seen as 2 Requires previous M mark.	dM1
	$\left\{ 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{4}(\tan^4 x + \cot^4 x - 2) \right\}$ $\Rightarrow \frac{1}{4}(\tan^4 x + \cot^4 x + 2)$ or $\frac{1}{4} \tan^4 x + \frac{1}{4} \cot^4 x + \frac{1}{2}$ Not implied. Must be seen	Adds the 1 and achieves either expression shown but allow the constant to be multiplied by $\tan^2 x \cot^2 x$ May be seen as e.g., $\frac{1}{2} \sqrt{\tan^4 x + \cot^4 x + 2 \tan^2 x \cot^2 x}$	A1
	$s = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan^2 x + \cot^2 x) dx$ * Allow $\int \frac{1}{2}(\tan^2 x + \cot^2 x)$ or $\frac{1}{2} \int \tan^2 x + \cot^2 x$	M1: Applies the arc length formula with their $\frac{dy}{dx}$ A1: Correct result achieved with no clear mathematical errors seen. Condone omission of “dx” and/or limits and occasional missing arguments.	M1 A1*
	Converting to sin & cos: likely to score max of 100010 unless tan & cot are convincingly recovered		(6)
Way 2 Squares first then identities	$y = \frac{1}{2}(\tan x + \cot x) \Rightarrow \frac{dy}{dx} = \frac{1}{2}(\sec^2 x - \operatorname{cosec}^2 x)$ oe	Correct derivative. Any equivalent.	B1
	$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(\sec^4 x + \operatorname{cosec}^4 x - 2 \sec^2 x \operatorname{cosec}^2 x)$	Squares a derivative of the correct form to obtain a 3 (or 4 if middle terms uncollected) term expression.	M1
	$= \frac{1}{4}((1 + \tan^2 x)^2 + (1 + \cot^2 x)^2 - 2(1 + \tan^2 x)(1 + \cot^2 x))$ $\left\{ = \frac{1}{4}(1 + 2 \tan^2 x + \tan^4 x + 1 + 2 \cot^2 x + \cot^4 x - 2 - 2 \tan^2 x - 2 \cot^2 x - 2 \tan^2 x \cot^2 x) \right\}$	Applies $\sec^2 x = \pm \tan^2 x \pm 1$ twice and $\operatorname{cosec}^2 x = \pm \cot^2 x \pm 1$ twice . Requires previous M mark.	dM1
	$\left\{ 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{4}(\tan^4 x + \cot^4 x - 2) \right\}$ $\Rightarrow \frac{1}{4}(\tan^4 x + \cot^4 x + 2)$ or $\frac{1}{4} \tan^4 x + \frac{1}{4} \cot^4 x + \frac{1}{2}$ Not implied. Must be seen	Adds the 1 and achieves either expression shown but allow the constant to be multiplied by $\tan^2 x \cot^2 x$ May be seen as e.g., $\frac{1}{2} \sqrt{\tan^4 x + \cot^4 x + 2 \tan^2 x \cot^2 x}$	A1
	$s = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^2 x + \cot^2 x dx$ * Allow $\int \frac{1}{2}(\tan^2 x + \cot^2 x)$ or $\frac{1}{2} \int \tan^2 x + \cot^2 x$	M1: Applies the arc length formula with their $\frac{dy}{dx}$ A1: Correct result achieved with no clear mathematical errors seen. Condone omission of “dx” and/or limits and occasional missing arguments.	M1 A1*
	Converting to sin & cos: likely to score max of 100010 unless tan & cot are convincingly recovered		(6)

Question Number	Scheme	Notes	Marks
3(b)	$\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan^2 x + \cot^2 x) dx = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2 x - 1 + \operatorname{cosec}^2 x - 1) dx$	Applies $\tan^2 x = \pm \sec^2 x \pm 1$ and $\cot^2 x = \pm \operatorname{cosec}^2 x \pm 1$ to the integral	M1
	Work in sin and cos must use identities (sign errors only) and lead to a result of the form below after integration condoning the absence of a term in x but allow the last M to be available following a completed attempt at integration.		
	$= \frac{1}{2} \left[\tan x - \cot x - 2x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1: For $\pm \sec^2 x \rightarrow \pm \tan x$ and $\pm \operatorname{cosec}^2 x \rightarrow \pm \cot x$ Requires previous M mark. A1: Correct integration. Limits not required.	dM1 A1
	$\frac{1}{2} \left(\tan \frac{\pi}{3} - \cot \frac{\pi}{3} - \frac{2\pi}{3} - \left(\tan \frac{\pi}{6} - \cot \frac{\pi}{6} - \frac{2\pi}{6} \right) \right)$ $\left\{ \frac{1}{2} \left(\sqrt{3} - \frac{2\pi}{3} - \frac{\sqrt{3}}{3} - \left(\frac{\sqrt{3}}{3} - \frac{\pi}{3} - \sqrt{3} \right) \right) \right\}$	Applies the limits (see note below) following any completed attempt at integration. Allow slips provided it is a clear attempt at $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$	M1
	Correct answer in any exact simplified form with 2 terms e.g. $\frac{1}{2} \left(\frac{4\sqrt{3}}{3} - \frac{\pi}{3} \right), \frac{2\sqrt{3}}{3} - \frac{\pi}{6}, \frac{2}{\sqrt{3}} - \frac{\pi}{6}, \frac{1}{3} \left(2\sqrt{3} - \frac{\pi}{2} \right), \frac{4\sqrt{3} - \pi}{6}$		A1
	Note they may apply the limits $\frac{\pi}{4} \& \frac{\pi}{6}$ or $\frac{\pi}{3} \& \frac{\pi}{4}$ and then double the result.		(5)
	Just the answer or decimal answer (0.6311017628) is 0/5		Total 11

Question Number	Scheme	Notes	Marks
4	Allow any suitable vector notation throughout this question.		
(a)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} \Rightarrow \dots \text{ or } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} \Rightarrow \dots$ $-x + 3y + 3z = -5 \quad \text{and} \quad 2x - 5z = 16$	M1: Uses $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$ at least once to obtain a plane equation A1: Both correct equations. Accept in $\mathbf{r} \cdot \mathbf{n} = p$ form	M1 A1
	e.g., $x = \frac{16 + 5z}{2}$	Obtains one variable (may be written as parameter for all marks) in terms of one of the other variables	M1
	$z = \frac{2x - 16}{5} \Rightarrow x = 5 + 3y + 3\left(\frac{2x - 16}{5}\right)$ $\Rightarrow 5x = 25 + 15y + 6x - 48 \Rightarrow x = -15y + 23$ $\left\{ x = -15y + 23 = \frac{16 + 5z}{2} \right\}$	M1: Obtains the variable/parameter in terms of the third variable (or the two other variables in terms of the parameter) A1: Both correct equations	M1 A1 (M1 on epen)
	Alternatively, $y = \frac{-x + 23}{15} = \frac{6 - z}{6} \quad \text{or} \quad z = \frac{2x - 16}{5} = 6 - 6y$		
	$\left\{ \frac{x - 0}{1} = \frac{y - \frac{23}{15}}{-\frac{1}{15}} = \frac{z + \frac{16}{5}}{\frac{2}{5}} \Rightarrow \right\} \quad \mathbf{r} = \begin{pmatrix} 0 \\ \frac{23}{15} \\ -\frac{16}{5} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -\frac{1}{15} \\ \frac{2}{5} \end{pmatrix}$ M1: Attempts vector equation of line but “$\mathbf{r} =$” may be missing. Requires all previous M marks. Allow numerical slips but it must be a correct method i.e., an attempt at $\Rightarrow \frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} \Rightarrow \mathbf{r} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$ A1: Any correct equation including “$\mathbf{r} =$”		dM1 A1
	Or $\left\{ \frac{x - 23}{-15} = \frac{y - 0}{1} = \frac{z - 6}{-6} \Rightarrow \right\} \quad \mathbf{r} = \begin{pmatrix} 23 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \quad \text{or} \quad \left\{ \frac{x - 8}{\frac{5}{2}} = \frac{y - 1}{-\frac{1}{6}} = \frac{z - 0}{1} \Rightarrow \right\} \quad \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{6} \\ 1 \end{pmatrix}$		
	Note that the line may be given in $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$ or $\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$ form		(7)



Question Number	Scheme	Notes	Marks
4(a) Alt Finds point and vector product of normals	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} \Rightarrow \dots \quad \text{or} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} \Rightarrow \dots$ $-x + 3y + 3z = -5 \quad \text{and} \quad 2x - 5z = 16$	M1: Uses $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$ at least once to obtain a plane equation A1: Both correct equations Accept in $\mathbf{r} \cdot \mathbf{n} = p$ form	M1 A1
	e.g., $x = 0 \Rightarrow z = -\frac{16}{5}$	Sets one variable equal to a value and finds a value for another variable. Correct for their equations if no working.	M1
	$3y = -5 - 3\left(-\frac{16}{5}\right) \Rightarrow y = \frac{23}{15} \Rightarrow \left(0, \frac{23}{15}, -\frac{16}{5}\right)$ Or e.g., $(23, 0, 6), (8, 1, 0)$ Points will have the form $(23 - 15\alpha, \alpha, 6 - 6\alpha)$	M1: Proceeds to find a value for the remaining variable. Correct for their equations if no working. A1: Correct values	M1 A1 (M1 on epen)
	$\begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} = \dots \Rightarrow \mathbf{r} = \begin{pmatrix} 0 \\ \frac{23}{15} \\ -\frac{16}{5} \end{pmatrix} + \lambda \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix}$ $\left\{ \mathbf{r} = \begin{pmatrix} 23 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \right\}$	dM1: Attempts vector product of normals (two correct components if method unclear) and forms vector equation with point and direction in correct places but allow for a copying error or mix up with components. Note that they could obtain the direction from 2 points on the line. Requires all previous M marks. “ $\mathbf{r} =$ ” may be missing. A1: Any correct equation including “ $\mathbf{r} =$ ”	dM1 A1
			(7)

Question Number	Scheme	Notes	Marks
4(b)	Note: If 0/5 allow SC 00010 for a correct volume formula seen for tetrahedron ABCD e.g., $\frac{1}{6} \left \overrightarrow{CD} \cdot (\overrightarrow{CA} \times \overrightarrow{CB}) \right $ Allow with missing modulus but not vector arrows unless implied by further work.		
Way 1 STP inc. $\overrightarrow{CD}$	$\left \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \right = \sqrt{262} \Rightarrow \overrightarrow{CD} = \frac{5}{\sqrt{262}} \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix}$	Attempts magnitude (allow numerical slip) of their direction vector and scales correctly to length 5	M1
	Let C be the point $(8, 1, 0)$ $\overrightarrow{CA} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} = \dots \left\{ \begin{pmatrix} -6 \\ 3 \\ -5 \end{pmatrix} \right\}$ and $\overrightarrow{CB} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} = \dots \left\{ \begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix} \right\}$	Finds vectors for any two edges other than CD . Could be implied by a distance calculation if C and/or D defined . This mark is not scored if either vector is in terms of a parameter unless it is assigned a value (or is eliminated appropriately) later.	M1
	$\overrightarrow{CD} \cdot (\overrightarrow{CA} \times \overrightarrow{CB}) = \frac{5}{\sqrt{262}} \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 3 \\ -5 \end{pmatrix} \times \begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix} = \dots \left\{ = -\frac{910}{\sqrt{262}} \right\}$	Uses an appropriate scalar triple product with their vectors and finds a value. Must not include position vectors . Could be inexact. M0 if clear evidence of an inappropriate method	M1
	$V = \frac{1}{6} \left \overrightarrow{CD} \cdot (\overrightarrow{CA} \times \overrightarrow{CB}) \right = \dots = \frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	dM1 : Divides their STP result by 6 and obtains a positive value. Could be inexact. Modulus might not be seen. Requires previous M mark. A1: A correct exact value	dM1 A1
			(5)
Way 2 STP not inc. $\overrightarrow{CD}$	$\left \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \right = \sqrt{262} \Rightarrow \overrightarrow{CD} = \frac{5}{\sqrt{262}} \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix}$	Attempts magnitude (allow numerical slip) of their direction vector and scales correctly to length 5	M1
	Let C be the point $(8, 1, 0)$ $\overrightarrow{AC} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} = \dots \left\{ \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} \right\}$ and $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} = \dots \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$	Finds vectors for any two edges other than CD . Could be implied by a distance calculation if C and/or D defined . (See also comment for second M1 in Way 1 re use of a parameter)	M1
	$\overrightarrow{OD} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + \frac{5}{\sqrt{262}} \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \Rightarrow \overrightarrow{AD} = \begin{pmatrix} \frac{-75}{\sqrt{262}} + 8 \\ \frac{5}{\sqrt{262}} + 1 \\ \frac{-30}{\sqrt{262}} \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} \frac{-75}{\sqrt{262}} + 6 \\ \frac{5}{\sqrt{262}} - 3 \\ \frac{-30}{\sqrt{262}} + 5 \end{pmatrix}$ $\Rightarrow \overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = \begin{pmatrix} \frac{-75}{\sqrt{262}} + 6 \\ \frac{5}{\sqrt{262}} - 3 \\ \frac{-30}{\sqrt{262}} + 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} = \dots \left\{ = -\frac{910}{\sqrt{262}} \right\}$	Uses an appropriate scalar triple product with their vectors and finds a value. Must not include position vectors . Could be inexact. M0 if clear evidence of an inappropriate method	M1
	$V = \frac{1}{6} \left \overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) \right = \dots = \frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	dM1 : Divides their STP result by 6 and obtains a positive value. Could be inexact. Modulus might not be seen. Requires previous M mark. A1: A correct exact value	dM1 A1
			(5)

Question Number	Scheme	Notes	Marks
4(b) Way 3 Triangle area + perp. distance to plane & vol. of pyramid	$\begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} = \sqrt{262} \Rightarrow \overline{CD} = \frac{5}{\sqrt{262}} \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix}$	Attempts magnitude of their direction vector and scales to length 5. See note after next M below.	M1
	Let C be the point $(8, 1, 0)$ $Area \Delta ACD = \frac{1}{2} \overline{CD} \times \overline{CA} = \frac{1}{2} \left \frac{5}{\sqrt{262}} \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix} \times \begin{pmatrix} -6 \\ 3 \\ -5 \end{pmatrix} \right = \dots \left\{ = \frac{65\sqrt{19}}{2\sqrt{262}} \right\}$ Uses formula to find a value for the area of one of the faces. Must be a full method (vector product and modulus). Condone missing $\frac{1}{2}$ Any attempts by trig/Pythagoras must be complete and credible		M1
	Note: It is possible to obtain the area of a relevant triangle such as ACD by e.g., finding the length of the perpendicular distance of point A to the line and multiplying this by $\frac{1}{2} \times 5$ – in such cases allow the first M for completing a viable attempt at the height of the triangle and the second for the area (Condone missing $\frac{1}{2}$)		
	ΔACD is in Π_1 so perp. height of tetrahedron is shortest dist. of $B(3, 6, -2)$ to $-x + 3y + 3z = -5$: $\left \frac{-1 \times 3 + 3 \times 6 + 3 \times (-2) + 5}{\sqrt{(-1)^2 + 3^2 + 3^2}} \right = \dots \left\{ \frac{14}{\sqrt{19}} \right\}$	Obtains a value for the perpendicular height via formula or any credible method (examples below)	M1
	Parallel planes: $\begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = 9$, $\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = -5 \Rightarrow \left \frac{-5 - 9}{\sqrt{(-1)^2 + 3^2 + 3^2}} \right = \frac{14}{\sqrt{19}}$ Projection/Resolving: $\overline{BA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow \frac{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}}{\sqrt{(-1)^2 + 3^2 + 3^2}} = \frac{14}{\sqrt{19}}$		
	$V = \frac{1}{3} \times \frac{65\sqrt{19}}{2\sqrt{262}} \times \frac{14}{\sqrt{19}} = \dots = \frac{455}{3\sqrt{262}} \text{ or } \frac{455\sqrt{262}}{786}$	M1: Uses $\frac{1}{3} \times \text{area } \Delta \times \text{perp. height}$ and obtains a positive value. $\frac{1}{2}$ must have been used for triangle area earlier unless they now use $\frac{1}{6} \times \dots$ Requires previous M mark. A1: Either correct exact value	dM1 A1
			(5)
			Total 12

Question Number	Scheme	Notes	Marks
5	$\mathbf{M} = \begin{pmatrix} 1 & 2 & k \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{pmatrix}$		
(i) & (ii) Mark the parts together	$\det \begin{pmatrix} 1-\lambda & 2 & k \\ -1 & -3-\lambda & 4 \\ 2 & 6 & -8-\lambda \end{pmatrix}$ $= \pm[(1-\lambda)((-3-\lambda)(-8-\lambda)-24)-2((-1)(-8-\lambda)-8)+k((-1)(6)-2(-3-\lambda))]$	Recognisable complete attempt at $\det(\mathbf{M}-\lambda\mathbf{I})$. May use other rows/columns. Allow $\pm$ and slips including +2 for first -2	M1
	Sarrus $\Rightarrow \pm[(1-\lambda)(-3-\lambda)(-8-\lambda)+(2)(4)(2)+(k)(-1)(6)-(k)(-3-\lambda)(2)-(1-\lambda)(4)(6)-(2)(-1)(-8-\lambda)]$		
	$= (1-\lambda)(\lambda^2+11\lambda)-2\lambda+2k\lambda$ $= -\lambda^3-10\lambda^2+9\lambda+2k\lambda$ $= \lambda(-\lambda^2-10\lambda+9+2k)$	M1: Obtains $\{\lambda\}(a\lambda^2+b\lambda+c+dk \text{ oe})$ $a, b, c, d \neq 0$ A1: Correct expression – allow: $\pm\{\lambda\}(-\lambda^2-10\lambda+9+2k \text{ oe})$ or $\pm\{\lambda\}(\lambda^2+10\lambda-9-2k \text{ oe})$ Allow quadratic to be unsimplified and the marks can be implied if the initial λ has been removed	M1 A1
	{One eigenvalue is zero, if repeated then} $9+2k=0 \Rightarrow k=...$ or $\{\pm(-\lambda^2-10\lambda+9+2k) \text{ has repeated roots so}\}$ $b^2-4ac=0 \Rightarrow \begin{cases} 100-4(-1)(9+2k)=0 \\ 100-4(1)(-9-2k)=0 \end{cases} \Rightarrow k=...$	Attempts to set their $c+dk=0$ and solves for k or Considers the case of their quadratic $a\lambda^2+b\lambda+c+dk=0$ having a repeated root and uses a valid strategy to find k	M1
	Alternative approaches with $\lambda^2+10\lambda-9-2k=0$: $(\lambda+a)^2=\lambda^2+2a\lambda+a^2 \Rightarrow 2a=10 \Rightarrow -9-2k=5^2 \Rightarrow k=...$ sum of roots $=-10 \Rightarrow \text{root} = -5 \Rightarrow \text{product of roots} = (-5)^2 = -9-2k \Rightarrow k=...$		
	$k = -\frac{9}{2}$ or $k = -17$	One correct value for k	A1
	{One eigenvalue is zero, if repeated then} $9+2k=0 \Rightarrow k=...$ and $\{\pm(-\lambda^2-10\lambda+9+2k) \text{ has repeated roots so}\}$ $b^2-4ac=0 \Rightarrow \begin{cases} 100-4(-1)(9+2k)=0 \\ 100-4(1)(-9-2k)=0 \end{cases} \Rightarrow k=...$	Attempts to set their $c+dk=0$ and solves for k and Considers the case of their quadratic $a\lambda^2+b\lambda+c+dk=0$ having a repeated root and uses a valid strategy to find k	M1
	$k = -\frac{9}{2}$ with eigenvalue -10 {and 0 repeated} $k = -17$ with eigenvalue -5 {repeated and 0}	Both correct values of k and the associated non-zero eigenvalues clearly assigned. No additional eigenvalues or values for k	A1
			Total 7

Question Number	Scheme	Notes	Marks
	$\frac{x^2}{16} + \frac{y^2}{9} = 1 \quad P(4 \cos \theta, 3 \sin \theta)$		
6(a)	$\frac{dy}{dx} = -\frac{3 \cos \theta}{4 \sin \theta}$ or $\frac{2x}{16} + \frac{2y}{9} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{18x}{32y}$ or $\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow y = 3 \left(1 - \frac{x^2}{16}\right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{3}{2} \left(1 - \frac{x^2}{16}\right)^{-\frac{1}{2}} \times -\frac{2x}{16}$	Uses a correct method and finds an expression for $\frac{dy}{dx}$ of the correct form (sign and coefficient slips only)	M1
	$\frac{dy}{dx} = -\frac{3 \cos \theta}{4 \sin \theta}$ oe e.g. $-\frac{3}{4} \cot \theta$ oe	Any correct derivative in terms of θ only.	A1
	$y - 3 \sin \theta = -\frac{3 \cos \theta}{4 \sin \theta} (x - 4 \cos \theta)$ or or $y = -\frac{3 \cos \theta}{4 \sin \theta} x + c \Rightarrow 3 \sin \theta = -\frac{3 \cos \theta}{4 \sin \theta} 4 \cos \theta + c$ $\Rightarrow c = \dots \left\{ \frac{12 \sin^2 \theta + 12 \cos^2 \theta}{4 \sin \theta} \right\}$	Applies correct straight line method using any gradient in terms of θ . If they use $y = mx + c$ they must substitute coordinates correctly and reach $c = \dots$ M0 if use normal gradient	M1
	$\Rightarrow 4y \sin \theta - 12 \sin^2 \theta = -3x \cos \theta + 12 \cos^2 \theta$ or using $y = mx + c: y = -\frac{3 \cos \theta}{4 \sin \theta} x + \frac{3}{\sin \theta} \Rightarrow 4y \sin \theta = -3x \cos \theta + 12$ $\Rightarrow 3x \cos \theta + 4y \sin \theta \{ = 12(\cos^2 \theta + \sin^2 \theta) \} = 12$ M1: Multiplies through to remove fraction to obtain an equation with trig expressions in sin and cos only. Allow this mark if they go straight to the given answer from a correct equation. Can score from use of a normal gradient and/or with coordinates wrongly placed but there must have been an attempt at a line. A1*: Correct equation from correct work. $\sin^2 \theta$ and $\cos^2 \theta$ must be seen somewhere in the working. Accept e.g., $\sin^2 \theta + \cos^2 \theta = 1$ seen in side-working		M1 A1*
			(5)
(b)	$y - 3 \sin \theta = \frac{4 \sin \theta}{3 \cos \theta} (x - 4 \cos \theta)$ oe e.g., $4x \sin \theta - 3y \cos \theta = 7 \sin \theta \cos \theta$ or $y = \frac{4 \sin \theta}{3 \cos \theta} x + c$ $\Rightarrow 3 \sin \theta = \frac{4 \sin \theta}{3 \cos \theta} 4 \cos \theta + c \Rightarrow c = \dots \left\{ \frac{-7 \sin \theta \cos \theta}{3 \cos \theta} \right\}$	M1: Applies correct straight line method with the negative reciprocal of their tangent gradient. If $y = mx + c$ is used coordinates must be substituted correctly and $c = \dots$ reached A1: Any correct equation	M1 A1
			(2)



Question Number	Scheme	Notes	Marks
6(c)	$A \text{ is } \left(\frac{4}{\cos \theta}, 0 \right)$	Any correct x -axis intercept of the tangent. Allow e.g., $\{x = \frac{12}{3 \cos \theta}, 4 \sec \theta$ Could be on a diagram or implied by midpoint	B1
	$x = 0 \Rightarrow y - 3 \sin \theta = -\frac{16}{3} \sin \theta \Rightarrow B \text{ is } \left(0, -\frac{7}{3} \sin \theta \right)$	Sets $x = 0$ in their normal equation (changed gradient) and finds y . Could be implied. Allow just $-\frac{7}{3} \sin \theta$ oe	M1
	So midpoint M of AB is $\left(\frac{2}{\cos \theta}, -\frac{7}{6} \sin \theta \right)$	Any correct midpoint. Accept any equivalents and as $x = \dots, y = \dots$	A1
	$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \left(-\frac{6}{7} y \right)^2 + \left(\frac{2}{x} \right)^2 = 1$	Uses $\sin^2 \theta + \cos^2 \theta = 1$ to obtain an equation in x and y only. May follow incorrect or no attempt at midpoint	M1
	$\Rightarrow \frac{36}{49} y^2 + \frac{4}{x^2} = 1 \Rightarrow 36x^2 y^2 + 49 \times 4 = 49x^2$ $\Rightarrow x^2 (49 - 36y^2) = 196$	dM1: Rearranges to the form $x^2 (p \pm qy^2) = r, p, q, r \in \mathbb{Z}$ Requires all previous M marks. A1: Correct equation	dM1 A1
			(6)
	Note that is possible to use e.g., $1 + \tan^2 \theta = \sec^2 \theta$, for example: $M \left(2 \sec \theta, \frac{-7 \tan \theta}{6 \sec \theta} \right) \Rightarrow \sec \theta = \frac{x}{2}, y = \frac{-7 \tan \theta}{3x} \Rightarrow \tan \theta = \frac{-3xy}{7} \Rightarrow 1 + \frac{9x^2 y^2}{49} = \frac{x^2}{4} \text{ (2nd M1)}$ $\Rightarrow 1 + \frac{9x^2 y^2}{49} = \frac{x^2}{4} \Rightarrow 196 + 36x^2 y^2 = 49x^2 \Rightarrow x^2 (49 - 36y^2) = 196 \text{ (3rd M1, A1)}$		Total 13

Question Number	Scheme	Notes	Marks
7(a) Way 1	$I_n = \int \cosh^n 2x \, dx = \int \cosh 2x \cosh^{n-1} 2x \, dx$ $= \frac{1}{2} \sinh 2x \cosh^{n-1} 2x - \int \frac{1}{2} \sinh 2x \times (n-1) \cosh^{n-2} 2x \times 2 \sinh 2x \, dx$	M1: Correct split and attempts to apply parts to obtain an expression of the correct form (sign and coefficient errors only). A1: Any correct expression	M1 A1
	$\left\{ = \frac{1}{2} \sinh 2x \cosh^{n-1} 2x - (n-1) \int \sinh^2 2x \cosh^{n-2} 2x \, dx \right\}$ $= \frac{1}{2} \sinh 2x \cosh^{n-1} 2x - (n-1) \int (\cosh^2 2x - 1) \cosh^{n-2} 2x \, dx$	Applies $\sinh^2 2x = \pm \cosh^2 2x \pm 1$ Requires previous M mark.	dM1
	$\Rightarrow I_n = \frac{1}{2} \sinh 2x \cosh^{n-1} 2x - (n-1) (I_n - I_{n-2})$	Introduces I_n and I_{n-2} – not implied by given answer. Requires previous M mark.	ddM1
	$\left\{ \Rightarrow nI_n = \frac{1}{2} \sinh 2x \cosh^{n-1} 2x + (n-1) I_{n-2} \right\}$ $I_n = \frac{\sinh 2x \cosh^{n-1} 2x}{2n} + \frac{n-1}{n} I_{n-2}^*$	Fully correct proof. Condone missing 'dx's. Poor bracketing must be recovered before given answer but no other errors e.g., sin for sinh, or wrong or missing arguments	A1*
	Accept e.g., $I_n = \frac{(n-1)I_{n-2}}{n} + \frac{1}{2n} \sinh 2x \cosh^{n-1} 2x$		(5)
Way 2	$I_n = \int \cosh^n 2x \, dx = \int \cosh^2 2x \cosh^{n-2} 2x \, dx$ $= \int (\sinh^2 2x + 1) \cosh^{n-2} 2x \, dx$	M1: Correct split and applies $\sinh^2 2x = \pm \cosh^2 2x \pm 1$ to obtain an expression of the correct form (sign and coefficient errors only). A1: Correct expression	M1 A1
	$\left\{ = \int \cosh^{n-2} 2x \, dx + \int \sinh^2 2x \cosh^{n-2} 2x \, dx \right\}$ $\int \sinh^2 2x \cosh^{n-2} 2x \, dx \left\{ = \int \sinh 2x \cosh^{n-2} 2x \sinh 2x \, dx \right\}$ $= \frac{1}{2(n-1)} \sinh 2x \cosh^{n-1} 2x - \frac{1}{n-1} \int \cosh^n 2x \, dx$	Attempts to apply parts to obtain an expression of the correct form for $\int \sinh^2 2x \cosh^{n-2} 2x \, dx$ Requires previous M mark.	dM1
	$\Rightarrow I_n = I_{n-2} + \frac{1}{2(n-1)} \sinh 2x \cosh^{n-1} 2x - \frac{1}{n-1} I_n$	Introduces I_n and I_{n-2} – not implied by given answer. Requires previous M mark.	ddM1
	$\left\{ \Rightarrow (n-1)I_n = \frac{1}{2} \sinh 2x \cosh^{n-1} 2x + (n-1)I_{n-2} - I_n \right\}$ $I_n = \frac{\sinh 2x \cosh^{n-1} 2x}{2n} + \frac{n-1}{n} I_{n-2}^*$	Fully correct proof. Condone missing 'dx's. Poor bracketing must be recovered before given answer but no other errors e.g., sin for sinh, or wrong or missing arguments	A1*
	Accept e.g., $I_n = \frac{(n-1)I_{n-2}}{n} + \frac{1}{2n} \sinh 2x \cosh^{n-1} 2x$		(5)

Question Number	Scheme	Notes	Marks
7(b)	$(1 + \cosh 2x)^3 = 1 + 3 \cosh 2x + 3 \cosh^2 2x + \cosh^3 2x$ Correct expansion. Could be implied e.g. by $x + 3I_1 + 3I_2 + I_3$ and allow if correct but terms are not collected. Condone if partially or completely in “x” provided terms <u>are</u> collected		B1
	$\int \cosh^2 2x \, dx \text{ or } I_2 = \frac{1}{4} \sinh 2x \cosh 2x + \frac{1}{2} I_0 \text{ or}$ $\int \cosh^3 2x \, dx \text{ or } I_3 = \frac{1}{6} \sinh 2x \cosh^2 2x + \frac{2}{3} I_1$	Completes an attempt to apply the reduction formula for I_2 or I_3. May be slips but must get two terms. May be seen with I_0 / I_1 attempted and/or embedded in expression for $\int (1 + \cosh 2x)^3 \, dx$	M1
	$I_0 = x \quad I_1 = \frac{1}{2} \sinh 2x$ $\int (1 + \cosh 2x)^3 \, dx = \int (1 + 3 \cosh 2x) \, dx + 3I_2 + I_3 =$ $x + \frac{3}{2} \sinh 2x + \frac{3}{4} \sinh 2x \cosh 2x + \frac{3}{2} x + \frac{1}{6} \sinh 2x \cosh^2 2x + \frac{1}{3} \sinh 2x (+c)$	$I_0 = x$ and $I_1 = \pm k \sinh 2x$ (condone I_1 from formula) and $\int (1 + 3 \cosh 2x) \, dx \rightarrow x \pm q \sinh 2x$ and uses the above to obtain an expression for $\int (1 + \cosh 2x)^3 \, dx$ Requires previous M mark.	dM1
	Note: One of I_2 and I_3 may be attempted directly – if so, correct identities must be used and an expression of a correct form obtained. Examples: $I_2 = \int \cosh^2 2x \, dx = \int \left(\frac{1}{2} \cosh 4x + \frac{1}{2} \right) dx = \frac{1}{8} \sinh 4x + \frac{x}{2}$ $\Rightarrow x + \frac{3}{2} \sinh 2x + \frac{3}{8} \sinh 4x + \frac{3}{2} x + \frac{1}{6} \sinh 2x \cosh^2 2x + \frac{1}{3} \sinh 2x (+c)$ $I_3 = \int \cosh^3 2x \, dx = \int \cosh 2x (\sinh^2 2x + 1) dx = \frac{1}{6} \sinh^3 2x + \frac{1}{2} \sinh 2x$ $\Rightarrow x + \frac{3}{2} \sinh 2x + \frac{3}{4} \sinh 2x \cosh 2x + \frac{3}{2} x + \frac{1}{6} \sinh^3 2x + \frac{1}{2} \sinh 2x (+c)$ If exponential definitions are used they must be correct.		
	$= \frac{5}{2} x + \frac{11}{6} \sinh 2x + \frac{3}{4} \sinh 2x \cosh 2x + \frac{1}{6} \sinh 2x \cosh^2 2x (+c)$	Correct answer. Award when a correct expression with collected like terms is seen.	A1
	$I_2 \text{ attempted directly } \Rightarrow \frac{5}{2} x + \frac{11}{6} \sinh 2x + \frac{3}{8} \sinh 4x + \frac{1}{6} \sinh 2x \cosh^2 2x (+c)$ $I_3 \text{ attempted directly } \Rightarrow \frac{5}{2} x + 2 \sinh 2x + \frac{3}{4} \sinh 2x \cosh 2x + \frac{1}{6} \sinh^3 2x (+c)$		(4)
	If identities are used before a correct answer is seen with like terms collected then the work must be correct		Total 9

Question Number	Scheme	Notes	Marks
8(a)	$\left\{ \frac{dy}{dx} = \right\} \operatorname{arcosh} 5x + \frac{ax}{\sqrt{bx^2-1}} \text{ or } \operatorname{arcosh} 5x + \frac{cx}{\sqrt{x^2-d}} \text{ (M1)} \Rightarrow \operatorname{arcosh}(5x) + \frac{5x}{\sqrt{25x^2-1}} \text{ (A1)}$ M1: Differentiates to obtain expression of the correct form $a, b, c, d \neq 0$ A1: Correct differentiation. Any equivalent form.		M1 A1
			(2)
(b)	$\frac{d}{dx} (x \operatorname{arcosh}(5x)) = \operatorname{arcosh}(5x) + " \frac{5x}{\sqrt{25x^2-1}} " \Rightarrow \int \operatorname{arcosh}(5x) dx = x \operatorname{arcosh}(5x) - \int " \frac{5x}{\sqrt{25x^2-1}} " dx$ M1: Rearranges their answer to (a) correctly and integrates or uses the correct formula to apply parts to $1 \times \operatorname{arcosh} 5x$ to obtain the above.		M1
	$\int \operatorname{arcosh}(5x) dx = x \operatorname{arcosh}(5x) - \int \frac{5x}{\sqrt{25x^2-1}} dx$ A1: Correct expression – but see note below on limited ft		A1 (limited ft)
	$= x \operatorname{arcosh}(5x) - \frac{1}{5} (25x^2 - 1)^{\frac{1}{2}} (+c)$	M1: $\int \frac{Ax}{\sqrt{Bx^2-1}} dx \rightarrow C(Bx^2-1)^{\frac{1}{2}}$ A1: Fully correct expression with $x \operatorname{arcosh}(5x)$ - see note below for limited ft	M1 A1 (limited ft)
	Note: Substitutions: $u = 5x \Rightarrow (u^2 - 1)^{\frac{1}{2}} \Rightarrow \left[\frac{1}{5} \sqrt{u^2 - 1} \right]_{\frac{5}{4}}^3$ $u = 25x^2 - 1 \Rightarrow \left[\frac{1}{5} \sqrt{u} \right]_{\frac{9}{16}}^8$ M1: Correct form A1: Fully correct expression with $x \operatorname{arcosh}(5x)$		
	A limited ft for one of the errors in (a) shown below applies for the first two A marks. However also allow the following if this error occurs in part (b) which is most likely to come from not rearranging and effectively restarting by using parts. Note that substitutions could be used. $a = 1 \Rightarrow x \operatorname{arcosh}(5x) - \int \frac{x}{\sqrt{25x^2-1}} dx \Rightarrow x \operatorname{arcosh}(5x) - \frac{1}{25} (25x^2 - 1)^{\frac{1}{2}} (+c)$ $b = 5 \Rightarrow x \operatorname{arcosh}(5x) - \int \frac{5x}{\sqrt{5x^2-1}} dx \Rightarrow x \operatorname{arcosh}(5x) - (5x^2 - 1)^{\frac{1}{2}} (+c)$ $a = -5 \Rightarrow x \operatorname{arcosh}(5x) + \int \frac{5x}{\sqrt{25x^2-1}} dx \Rightarrow x \operatorname{arcosh}(5x) + \frac{1}{5} (25x^2 - 1)^{\frac{1}{2}} (+c)$		
	$\int_{\frac{1}{4}}^{\frac{3}{5}} \operatorname{arcosh} 5x dx = \frac{3}{5} \operatorname{arcosh}(3) - \frac{1}{5} \sqrt{25 \times \frac{9}{25} - 1} - \left(\frac{1}{4} \operatorname{arcosh}\left(\frac{5}{4}\right) - \frac{1}{5} \sqrt{25 \times \frac{1}{16} - 1} \right)$ Applies appropriate limits (note substitutions above) with subtraction the right way round seen to obtain an expression of the form $x \operatorname{arcosh}(5x) \pm f(x)$ where $f(x)$ has come from integration		M1
	$= \frac{3}{5} \operatorname{arcosh}(3) - \frac{2\sqrt{2}}{5} - \frac{1}{4} \operatorname{arcosh}\left(\frac{5}{4}\right) + \frac{3}{20}$	Correct answer seen in any form. Must not follow clearly incorrect work.	A1
	$\operatorname{arcosh} 3 = \ln(3 + \sqrt{3^2 - 1^2}) \text{ or } \operatorname{arcosh}\left(\frac{5}{4}\right) = \ln\left(\frac{5}{4} + \sqrt{\left(\frac{5}{4}\right)^2 - 1^2}\right)$ $\left\{ \Rightarrow \frac{3}{5} \ln(3 + \sqrt{8}) - \frac{2\sqrt{2}}{5} - \frac{1}{4} \ln 2 + \frac{3}{20} \right\}$	Converts $\operatorname{arcosh}(3)$ or $\operatorname{arcosh}\left(\frac{5}{4}\right)$ to any correct log form. Independent mark but must have obtained $x \operatorname{arcosh}(5x) \pm f(x)$ where $f(x)$ has come from integration	M1
	$= \frac{3}{20} - \frac{2\sqrt{2}}{5} + \ln(3 + 2\sqrt{2})^{\frac{3}{5}} - \frac{1}{4} \ln 2$ Must not follow clearly incorrect work.	Correct answer. Terms in any order but otherwise written as shown. Allow values for p, q, r & k	A1
			(8)
			Total 10

Question	Scheme	Marks
8(a)	$\frac{dy}{dx} = \pm \frac{1}{\sqrt{1-k\sqrt{x}^2}} \times \dots x^{-\frac{1}{2}}$ or $\cos y = 2x^{\frac{1}{2}} \Rightarrow \pm \sin y \frac{dy}{dx} = \dots x^{-\frac{1}{2}}$	M1
	$\frac{dy}{dx} = \pm \frac{1}{\sqrt{1-4x}} \times \left(Kx^{\frac{1}{2}} \right)$ or $\frac{dy}{dx} = \pm \frac{Kx^{\frac{1}{2}}}{\sqrt{1-(2\sqrt{x})^2}}$	dM1
	$\frac{dy}{dx} = -\frac{1}{\sqrt{x}\sqrt{1-4x}}$ oe e.g. $\frac{dy}{dx} = -\frac{1}{\sqrt{x-4x^2}}$	A1
		(3)
(b) Way 1	$\int y \, dx = \int 1 \times \arccos(2\sqrt{x}) \, dx = x \arccos(2\sqrt{x}) - \int x \frac{-1}{\sqrt{x}\sqrt{1-4x}} \, dx$	M1
	$= x \arccos(2\sqrt{x}) + \int \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx^*$	A1*
		(2)
	Way 2	
	$\frac{d}{dx} (x \arccos(2\sqrt{x})) = 1 \cdot \arccos(2\sqrt{x}) + x \cdot \frac{-1}{\sqrt{x}\sqrt{1-4x}}$	M1
	$\Rightarrow \int \arccos(2\sqrt{x}) \, dx = x \arccos(2\sqrt{x}) + \int \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx^*$	A1*
		(2)
(c)	$\frac{1}{2\sqrt{x}} \frac{dx}{d\theta} = -\frac{1}{2} \sin \theta, \, dx = -\sqrt{x} \sin \theta \, d\theta, \, \frac{dx}{d\theta} = -\frac{1}{2} \sin \theta \cos \theta$ $\frac{dx}{d\theta} = -\frac{1}{4} \sin 2\theta$	B1
	$\int \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx = \int \frac{-(\frac{1}{2} \cos \theta)^2 \sin \theta}{\sqrt{1-4(\frac{1}{2} \cos \theta)^2}} \, d\theta$	M1
	$= -\frac{1}{4} \int \frac{\cos^2 \theta \sin \theta}{\sqrt{1-\cos^2 \theta}} \, d\theta = -\frac{1}{4} \int \cos^2 \theta \, d\theta$	A1
	$x=0 \Rightarrow \theta = \frac{\pi}{2}$ $x=\frac{1}{8} \Rightarrow \theta = \frac{\pi}{4}$ So $\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx = \frac{1}{4} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$	A1
		(4)

(d)	$\frac{1}{4} \int \frac{1}{2} (1 + \cos 2\theta) d\theta = K \left(\theta \pm \frac{1}{2} \sin 2\theta \right)$	M1
	$\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}} dx = \frac{1}{8} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \dots \left(= \frac{\pi}{32} - \frac{1}{16} \right)$ or e.g. $\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}} dx = \frac{1}{8} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\frac{1}{8} \left[\arccos 2\sqrt{x} + \frac{1}{2} \sin 2 \arccos 2\sqrt{x} \right]_0^{\frac{1}{8}}$ $= \dots \left(= -\frac{1}{8} \left(\frac{\pi}{4} + \frac{1}{2} - \frac{\pi}{2} \right) \right)$	dM1
	$\Rightarrow \int_0^{\frac{1}{8}} \arccos(2\sqrt{x}) dx = \left[x \arccos 2\sqrt{x} \right]_0^{\frac{1}{8}} + \frac{\pi}{32} - \frac{1}{16} = \frac{1}{8} \arccos \frac{1}{\sqrt{2}} - 0 + \frac{\pi}{32} - \frac{1}{16}$	dM1
	$= \frac{\pi}{16} - \frac{1}{16} \text{ oe}$	A1
		(4)

(13 marks)

Notes:

(a)

M1: Attempts to apply the arccos derivative formula together with chain rule. Look for

$$\frac{dy}{dx} = \pm \frac{1}{\sqrt{1-k\sqrt{x}^2}} \times f(x) \text{ where } f(x) \text{ is an attempt at differentiating } 2\sqrt{x} \text{ where } f(x) \neq \alpha\sqrt{x}$$

 Note that k may be 1 for this mark.

Alternatively, takes cosine of both sides and differentiates to the form shown in the scheme.

dM1: Correct form for the overall derivative achieved, may be errors in sign or constants with $k \neq 1$

 Alternatively, divides through by $\sin y$ and applies Pythagorean identity to achieve derivative in terms of x .

A1: Correct derivative, but need not be simplified. Award when first seen and isw.

(b) Way 1

M1: Attempts to apply integration by parts to $1 \times \arccos(2\sqrt{x})$.

 Look for $x \arccos(2\sqrt{x}) - \int x'' \text{ their (a)'' } dx$ or $u = \arccos(2\sqrt{x}) \Rightarrow \frac{du}{dx} = \text{part(a)}, \frac{dv}{dx} = 1 \Rightarrow v = x$
A1*: Correct work leading to the printed answer. There must be a clear statement for the integration by parts before the given answer is stated.

 So e.g. $u = \arccos(2\sqrt{x}) \Rightarrow \frac{du}{dx} = \text{part(a)}, \frac{dv}{dx} = 1 \Rightarrow v = x$

$$\Rightarrow \int \arccos(2\sqrt{x}) dx = x \arccos(2\sqrt{x}) + \int \frac{\sqrt{x}}{\sqrt{1-4x}} dx \text{ * scores M1A0}$$

$$\text{You can condone } \int \arccos(2\sqrt{x}) dx = x \arccos(2\sqrt{x}) + \int \frac{x^{\frac{1}{2}}}{\sqrt{1-4x}} dx \text{ *}$$



Way 2

M1: Applies the product rule to $x \arccos(2\sqrt{x})$, look for $1 \cdot \arccos(2\sqrt{x}) + x \cdot \text{"their (a)"}$.

A1*: Rearranges and integrates to achieve the given result, with no errors seen.

(c)

B1: Any correct expression involving dx and $d\theta$, see examples in scheme.

M1: Makes a complete substitution in the integral $\int \frac{\sqrt{x}}{\sqrt{1-4x}} dx$ to achieve an integral in θ only.

Ignore attempts at substitution into the $x \arccos(2\sqrt{x})$.

A1: A correct simplified integral aside from limits. May be implied by e.g. $\frac{1}{4} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 \theta d\theta$

Note that this mark depends on the B mark.

A1: Finds correct limits for θ and applies to the integral by reversing the sign – i.e. correct answer with limits and sign all correct. Accept equivalent limits e.g. $-\frac{\pi}{4}$ to $-\frac{\pi}{4}$ or $\frac{\pi}{2}$ to $\frac{3\pi}{4}$

Note that this mark depends on the B mark.

(d)

M1: Applies double angle identity to get the integral in a suitable form and attempts to integrate.

Accept $\cos^2 \theta = \frac{1}{2}(\pm 1 \pm \cos 2\theta)$ used as identity and look for $1 \rightarrow \theta$ and $\cos 2\theta \rightarrow \pm \frac{1}{2} \sin 2\theta$

dM1: Applies their limits (either way round) to their integral in θ or reverse substitution and applies limits 0 and $\frac{1}{8}$.

Depends on the previous method mark.

dM1: Applies limits of 0 and $\frac{1}{8}$ to the $x \arccos(2\sqrt{x})$ to obtain a value (or their limits either way round if they applied the substitution to this to obtain a value) and combines with the result of the other integral.

Depends on both previous method marks.

A1: Correct final answer.

