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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Further Matrix Algebra (FP3.6)

International A Level Mathematics · FP3: Further Pure Mathematics 3 (WFM03)

18 questions · 160 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

18 questions · 160 marks · 1 sub-topic

SUB-TOPIC FP3.6a

EIGENVALUES, EIGENVECTORS & DIAGONALISATION

Sub-topic FP3.6a · 18 questions · 160 marks · lowest tariff first



3.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(b) find $\mathbf{A}^{-1}$, giving your answer in terms of k .

(4)

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Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total 6 marks)

Q3

5.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & k \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{pmatrix} \text{ where } k \text{ is a constant}$$

Given that $\mathbf{M}$ has a repeated eigenvalue, determine

- (i) the possible values of k ,
- (ii) all corresponding eigenvalues of $\mathbf{M}$ for each value of k .

(7)





Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

(Total for Question 5 is 7 marks)



$$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

where k is a constant

(3)

(b) determine $\mathbf{A}^{-1}$, giving your answer in simplest form in terms of k .

(4)



Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 7 marks)



Question 6 continued

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Question 6 continued

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Question 6 continued

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Q6

(Total 8 marks)



Question 3 continued



Question 3 continued

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Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 8 marks)



$$\mathbf{A} = \begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$$

- (4)

- (4)



Question 6 continued



Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 8 marks)





Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)







Question 2 continued

(Total for Question 2 is 8 marks)







Question 4 continued

(Total for Question 4 is 8 marks)



Question 3 continued

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Question 3 continued

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(Total 9 marks)

Q3

Grading box for Q3.



5.

$$\mathbf{M} = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

Given that 8 is an eigenvalue of $\mathbf{M}$

- (a) determine an eigenvector corresponding to the eigenvalue 8 (2)
- (b) Determine the other two eigenvalues of $\mathbf{M}$. (3)
- (c) Hence find an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^T \mathbf{M} \mathbf{P} = \mathbf{D}$ (4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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Q5

(Total 9 marks)



3.

$$\mathbf{M} = \begin{pmatrix} 3 & 1 & p \\ 1 & 1 & 2 \\ -1 & p & 2 \end{pmatrix} \text{ where } p \text{ is a real constant}$$

(a) Find the exact values of p for which $\mathbf{M}$ has no inverse.

(4)

Given that $\mathbf{M}$ does have an inverse,

(b) find $\mathbf{M}^{-1}$ in terms of p .

(5)

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Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total 9 marks)

Q3

Box for marking the answer to Question 3.



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 9 marks)

Q4





Question 2 continued



Question 3 continued



Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)



Question 4 continued

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Question 4 continued

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(Total 11 marks)

Q4

Grading boxes for Q4.



Question 5 continued

Question 5 continued



(Total for Question 3 is 12 marks)







Question 4 continued

(Total for Question 4 is 14 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Question Number	Scheme	Notes	Marks
1	$7 \cosh x + 3 \sinh x = 2e^x + 7 \Rightarrow$ $7 \left(\frac{e^x + e^{-x}}{2} \right) + 3 \left(\frac{e^x - e^{-x}}{2} \right) = 2e^x + 7$ $\left\{ \frac{7}{2}e^x + \frac{7}{2}e^{-x} + \frac{3}{2}e^x - \frac{3}{2}e^{-x} = 2e^x + 7 \right\}$	Substitutes at least one correct exponential form for either of the hyperbolic terms and achieves an equation in exponentials and constants alone	M1
	$\Rightarrow 7(e^{2x} + 1) + 3(e^{2x} - 1) = 4e^{2x} + 14e^x$ $\left\{ \Rightarrow 5e^{2x} + 2 = 2e^{2x} + 7e^x \right\}$	Multiplies through by e^x to obtain any equation that would form a 3TQ in e^x if like terms were collected	M1
	$\Rightarrow 6e^{2x} - 14e^x + 4 = 0 \quad \left\{ 3e^{2x} - 7e^x + 2 = 0 \right\}$	A correct three term quadratic in e^x . Could be implied by a correct root even if terms have not been collected.	A1
	$\Rightarrow (3e^x - 1)(e^x - 2) = 0 \Rightarrow e^x = \dots$	Solves their 3TQ - usual rules. One correct root for their quadratic if no working. Ignore labelling of the roots even if e.g., " x " is used.	M1
	$x = \ln 2, \ln \frac{1}{3}$	Both correct and simplified but do not isw if there are other answers . Allow $-\ln \frac{1}{2}$ for $\ln 2$ and $-\ln 3$ or $\ln 3^{-1}$ for $\ln \frac{1}{3}$	A1
	Answer only is 0/5		Total 5
	Note that it is possible to multiply through by e^{-x} to form an equation in e^{-2x} , e^{-x} and constants. Score as main scheme, e.g., $\frac{7}{2}e^x + \frac{7}{2}e^{-x} + \frac{3}{2}e^x - \frac{3}{2}e^{-x} = 2e^x + 7$ $\Rightarrow \frac{7}{2} + \frac{7}{2}e^{-2x} + \frac{3}{2} - \frac{3}{2}e^{-2x} = 2 + 7e^{-x} \quad (\text{M1})$ $\Rightarrow 2e^{-2x} - 7e^{-x} + 3 = 0 \quad (\text{A1})$ $(2e^{-x} - 1)(e^{-x} - 3) = 0 \Rightarrow e^{-x} = \frac{1}{2}, 3 \quad (\text{M1})$ $\Rightarrow e^x = 2, \frac{1}{3} \Rightarrow x = \ln 2, \ln \frac{1}{3} \quad (\text{A1})$		

Question Number	Scheme	Marks
5(a)	$\int_1^6 x^n (3x-2)^{-\frac{1}{2}} dx = \left[\frac{2}{3} x^n (3x-2)^{\frac{1}{2}} \right]_{(1)}^{(6)} - \frac{2}{3} \int_{(1)}^{(6)} nx^{n-1} (3x-2)^{\frac{1}{2}} dx$	M1A1
	$= \dots - \frac{2}{3} \int_{(1)}^{(6)} nx^{n-1} (3x-2)(3x-2)^{-\frac{1}{2}} dx$	dM1
	$= \frac{2}{3} \times 6^n \times 4 - \frac{2}{3} \times 1 - \frac{2}{3} n \times 3I_n + \frac{4}{3} nI_{n-1}$ $\left(I_n = \frac{8}{3} \times 6^n - \frac{2}{3} - 2nI_n + \frac{4}{3} nI_{n-1} \right)$	ddM1
	$3I_n = 8 \times 6^n - 2 - 6nI_n + 4nI_{n-1}$ $\Rightarrow (3+6n)I_n = 4nI_{n-1} + 8 \times 6^n - 2 *$	A1*
		(5)
(b)	$I_0 = \int_1^6 (3x-2)^{-\frac{1}{2}} dx = \left[\frac{2}{3} (3x-2)^{\frac{1}{2}} \right]_1^6 = \frac{8}{3} - \frac{2}{3} = 2$	B1
	$21I_3 = 12I_2 + 1728 - 2 \left(\text{Implied by } 21I_3 = \frac{9966}{5} \text{ or } I_3 = \frac{3322}{35} \right)$ Or $15I_2 = 8I_1 + 288 - 2 \left(\text{Implied by } 15I_2 = 334 \text{ or } I_2 = \frac{334}{15} \right)$ Or $9I_1 = 4I_0 + 48 - 2 \left(\text{Implied by } 9I_1 = 54 \right)$	M1
	$21I_3 = 12 \left(\frac{8}{15} I_1 + \frac{286}{15} \right) + 1728 - 2 = 12 \left(\frac{8}{15} \left(\frac{4}{9} I_0 + \frac{46}{9} \right) + \frac{286}{15} \right) + 1726 = \dots \left(\frac{9966}{5} \right)$ or $= \frac{12}{15} (8I_1 + 288 - 2) + 1726 = \frac{32}{45} (4I_0 + 48 - 2) + \frac{9774}{5} = \dots \left(\frac{9966}{5} \right)$	M1
	$I_3 = \frac{3322}{35}$	A1
		(4)
		Total 9

Notes

(a)

M1: Integrates by parts to obtain $\left[\alpha x^n (3x-2)^{\frac{1}{2}} \right]_{(1)}^{(6)} - \beta \int_{(1)}^{(6)} x^{n-1} (3x-2)^{\frac{1}{2}} dx$ with or without limits

A1: Correct expression (limits not required on either part, and 'dx' may be missing)

dm1: Writes $(3x-2)^{\frac{1}{2}}$ as $(3x-2)(3x-2)^{-\frac{1}{2}}$ **Requires previous M mark.**

ddM1: Splits the integral, applies the given limits to the first expression and introduces I_n and I_{n-1}

Requires both previous M marks

A1*: Completes the proof with no mathematical errors seen to obtain the printed answer. Allow recovery of poor bracketing. Limits need not be shown on the integrals. Loss and subsequent recovery of the occasional 'n' and/or 'dx' loses this mark.

(b)

B1: Correct value for I_0 This may be implied by a correct value for I_1 , I_2 or I_3

M1: Applies the reduction formula **at least once** to obtain I_3 in terms of I_2 or I_2 in terms of I_1 or I_1 in terms of I_0 . Allow slips but must have the correct number of terms. May be embedded in another expression for I_1 , I_2 or I_3 . This could be implied by the exact numerical values seen coupled with an obvious attempt. Allow a multiple of I_3 , I_2 , I_1 e.g. $21I_3 = \dots$

M1: Completes the process to obtain I_3 in terms of I_0 (or I_1 if I_1 is found by integration) and substitutes for I_0 (or I_1) to find a value for kI_3 or I_3

A1: For $\frac{3322}{35}$ or exact equivalent

Note: It is possible for I_1 to be found using integration, and a correct value would imply B1, but method marks are for using the reduction formula.

Question Number	Scheme	Marks
6(a)	$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix}$	
	$ \mathbf{A} = 12 - 2 - k(20 + 12) + 2(-5 - 18)$ (via first row) $ \mathbf{A} = 12 - 2 - 5(4k + 2) + 6(-2k - 6)$ (via first column) $ \mathbf{A} = 12 - 12k - 10 - 36 - 2 - 20k$ (Sarrus)	M1
	$ \mathbf{A} = 0 \Rightarrow 10 - 32k - 46 = 0 \Rightarrow k = \dots$	M1
	$k = -\frac{9}{8}$	A1
		(3)
(b)	$\begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & 32 & -23 \\ 4k+2 & -8 & -1-6k \\ -2k-6 & -12 & 3-5k \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -32 & -23 \\ -4k-2 & -8 & 6k+1 \\ -2k-6 & 12 & 3-5k \end{pmatrix}$	M1A1
	$\begin{pmatrix} 10 & -32 & -23 \\ -4k-2 & -8 & 6k+1 \\ -2k-6 & 12 & 3-5k \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -4k-2 & -2k-6 \\ -32 & -8 & 12 \\ -23 & 6k+1 & 3-5k \end{pmatrix}$ $\mathbf{A}^{-1} = \frac{-1}{32k+36} \begin{pmatrix} 10 & -4k-2 & -2k-6 \\ -32 & -8 & 12 \\ -23 & 6k+1 & 3-5k \end{pmatrix}$	dM1A1
		(4)
		Total 7

Notes

(a)

M1: Correct determinant attempt. This may be along any row or down any column, or via a “shoelace” approach (rule of Sarrus). Condone sign slips, but the overall structure should be correct.**M1:** Sets their linear expression for the determinant = 0 and solves for k **A1:** Correct value

(b)

M1: Applies the correct method to reach at least a matrix of cofactors. Two correct rows or two correct columns needed. This may be implied by the transpose of their cofactor matrix, if the cofactor matrix is not seen, with at least two correct rows or two correct columns.**A1:** Correct cofactor matrix**dM1:** Transposes their cofactor matrix and divides by their determinant. **Depends on previous method mark.** If the candidate miscopies one element from their cofactor matrix, then allow this mark.**A1:** Correct matrix. Allow any equivalent correct simplified matrix.**Note:** a correct matrix of minors followed by a fully correct answer scores M1A1dM1A1

Question Number	Scheme	Notes	Marks
2(i)	$3x^2 + 12x + 24 = 3(x^2 + 4x + 8)$ $= 3((x+2)^2 + 4)$	Obtains $3((x+2)^2 + \dots)$ or $3(x+2)^2 + \dots$ Must include 3 now or later	M1
	$3((x+2)^2 + 4)$ or $3(x+2)^2 + 12$		A1
	$\int \frac{1}{3x^2 + 12x + 24} dx = \frac{1}{3} \int \frac{1}{(x+2)^2 + 4} dx = \frac{1}{6} \arctan \frac{x+2}{2} (+c)$ M1: Use of arctan A1: Fully correct expression (condone omission of + c)		M1A1
			(4)
(ii)	$27 - 6x - x^2 = -(x^2 + 6x - 27)$ $= -((x+3)^2 - 36)$	Obtains $-((x+3)^2 + \dots)$ or $-(x+3)^2 + \dots$	M1
	$-((x+3)^2 - 36)$ or $36 - (x+3)^2$		A1
	$\int \frac{1}{\sqrt{27 - 6x - x^2}} dx = \int \frac{1}{\sqrt{36 - (x+3)^2}} dx = \arcsin\left(\frac{x+3}{6}\right) (+c)$ (Or $= -\arccos\left(\frac{x+3}{6}\right) (+c)$) M1: Use of arcsin (or $-\arccos$) A1: Fully correct expression (condone omission of + c)		M1A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
3	$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix}$		
(a)	$ \mathbf{M} - \lambda \mathbf{I} = \begin{vmatrix} 3-\lambda & -4 & k \\ 1 & -2-\lambda & k \\ 1 & -5 & 5-\lambda \end{vmatrix} = \begin{vmatrix} 0 & -4 & k \\ 1 & -5 & k \\ 1 & -5 & 2 \end{vmatrix}$ $(0) + 4[2-k] + k[-5+5]$ Attempts $ \mathbf{M} - \lambda \mathbf{I} $ using $\lambda = 3$		M1
	$(0) + 4[2-k] + k[-5+5] = 0 \Rightarrow k = \dots$ Uses $ \mathbf{M} - \lambda \mathbf{I} = 0$ and solves for k		M1
	$k = 2$	Cao	A1
			(3)
(b)	$(3-\lambda)[(\lambda+2)(\lambda-5)+10] + 4(5-\lambda-2) + 2(-5+2+\lambda) = 0$ Attempts $ \mathbf{M} - \lambda \mathbf{I} = 0$ using their value of k		M1
	$\Rightarrow (3-\lambda)[(\lambda+2)(\lambda-5)+12] = 0$ $(\lambda+2)(\lambda-5)+12 \Rightarrow \lambda^2 - 3\lambda + 2 = 0 \Rightarrow (\lambda-2)(\lambda-1) = 0 \Rightarrow \lambda = \dots$ Uses $\lambda = 3$ as a factor to obtain and solve a 3TQ to find the other eigenvalues (Alternatively may use calculator to solve $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$)		M1
	$\lambda = 1, 2$	Correct values	A1
			(3)
(c)	$\begin{pmatrix} 3 & -4 & 2 \\ 1 & -2 & 2 \\ 1 & -5 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{cases} 3x - 4y + 2z = 3x \\ x - 2y + 2z = 3y \\ x - 5y + 5z = 3z \end{cases}$	Uses the eigenvalue 3 and their k to form at least 2 equations in x , y and z	M1
	$\alpha \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ (α a constant)	Any correct eigenvector. Allow any constant multiple of $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	A1
	$\frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	Correct normalised vector	A1
			(3)
			Total 9



Question Number	Scheme	Notes	Marks
4.	$I_n = \int x^n \cos x \, dx$		
(a)	$\int x^n \cos x \, dx = x^n \sin x - \int nx^{n-1} \sin x \, dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$= x^n \sin x - \left\{ -nx^{n-1} \cos x + \int n(n-1)x^{n-2} \cos x \, dx \right\}$ Uses integration by parts again (dependent on the first M)		dM1
	$= x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}^*$ Fully correct proof with no errors		A1*
			(4)
ALT			
	$I_n = \int x^n \cos x \, dx = \int x^{n-1} (x \cos x) \, dx$		
	$= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-2} (x \sin x + \cos x) \, dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-1} \sin x \, dx - (n-1)I_{n-2}$		
	$= x^n \sin x + x^{n-1} \cos x - (n-1) \left\{ -x^{n-1} \cos x + (n-1)I_{n-2} \right\} - (n-1)I_{n-2}$ Uses integration by parts again (dependent on the first M)		dM1
	$= x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}^*$ Fully correct proof with no errors		A1*
(b)	$I_0 = \sin x \quad (+k)$		B1
	$I_4 = x^4 \sin x + 4x^3 \cos x - 12I_2$	Applies the reduction formula once for I_4 or I_2	M1
	$= x^4 \sin x + 4x^3 \cos x - 12(x^2 \sin x + 2x \cos x - 2I_0)$ Applies the reduction formula again and obtains an expression for I_4 which can include I_0 but not I_2		M1
	$= (x^4 - 12x^2 + 24) \sin x + (4x^3 - 24x) \cos x + c$ Award A1 for either bracket and A1 for the other If the answer is not factorised but is otherwise correct, award A1A0		A1A1
			(5)
			Total 9

Question Number	Scheme	Notes	Marks
5	$\frac{x^2}{25} - \frac{y^2}{4} = 1 \quad y = mx + c$		
(a)	$\frac{x^2}{25} - \frac{(mx+c)^2}{4} = 1 \Rightarrow 4x^2 - 25(m^2x^2 + 2cmx + c^2) = 100$ Substitutes to obtain a quadratic in x and eliminates fractions		M1
	$4x^2 - 25(m^2x^2 + 2cmx + c^2) = 100$ $(\Rightarrow (25m^2 - 4)x^2 + 50cmx + 25c^2 + 100 = 0)$ Correct 3TQ		A1
	" $b^2 = 4ac$ " $\Rightarrow (50cm)^2 = 4(25m^2 - 4)(25c^2 + 100)$ Uses ' $b^2 = 4ac$ ' or equivalent		M1
	$2500c^2m^2 = 2500c^2m^2 + 10000m^2 - 400c^2 - 1600$ $10000m^2 = 400c^2 + 1600$ $25m^2 = c^2 + 4$ Fully correct proof with no errors		A1*
			(4)
ALT 1	Using hyperbolic parameters:		
	$x = 5 \cosh t, y = 2 \sinh t \Rightarrow \frac{dy}{dx} = \frac{2 \cosh t}{5 \sinh t}$		
	$\frac{2 \cosh t}{5 \sinh t}(x - 5 \cosh t) = y - 2 \sinh t$ M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed)		M1A1
	$y = \frac{2 \cosh t}{5 \sinh t}x - \frac{2 \cosh^2 t - 25 \sinh^2 t}{\sinh t}$		
	$25m^2 = \frac{4 \cosh^2 t}{\sinh^2 t}, 4 + c^2 = 4 + \frac{4}{\sinh^2 t} = \frac{4(\sinh^2 t + 1)}{\sinh^2 t} = \frac{4 \cosh^2 t}{\sinh^2 t}$ Extracts $25m^2$ and $4 + c^2$ from their equation		M1
	$\therefore 25m^2 = 4 + c^2$ * Fully correct proof with no errors		A1*
			(4)
ALT 2	Using trigonometric parameters:		
	$x = 5 \sec t, y = 2 \tan t \Rightarrow \frac{dy}{dx} = \frac{2 \sec t}{5 \tan t}$		
	$\frac{2 \sec t}{5 \tan t}(x - 5 \sec t) = y - 2 \tan t$ M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed)		M1A1
	$y = \frac{2 \sec t}{5 \tan t}x + \frac{2 \tan^2 t - 2 \sec^2 t}{\tan t}$		
	$25m^2 = \frac{4 \sec^2 t}{\tan^2 t} = \frac{4}{\sin^2 t} \quad 4 + c^2 = 4 \left(1 + \frac{1}{\tan^2 t}\right) = 4 \left(\frac{\sin^2 t + \cos^2 t}{\sin^2 t}\right) = \frac{4}{\sin^2 t}$ Extracts $25m^2$ and $4 + c^2$ from their equation		M1
	$\therefore 25m^2 = 4 + c^2$ * Fully correct proof with no errors		A1*
			(4)

(b)	$25m^2 = c^2 + 4$ and $2 = m + c$ $25m^2 = (2 - m)^2 + 4$ or $25(2 - c)^2 = c^2 + 4$ Uses the given hyperbola and the straight line with the result from (a) to obtain an equation in m or c	M1
	$24m^2 + 4m - 8 = 0$ or $24c^2 - 100c + 96 = 0$	Correct 3TQ in m or c A1
	$24m^2 + 4m - 8 = 0 \Rightarrow m = \frac{1}{2}, -\frac{2}{3}$ Or $24c^2 - 100c + 96 = 0 \Rightarrow c = \frac{3}{2}, \frac{8}{3}$	Solves their 3TQ in m or c M1
	$y = \frac{1}{2}x + \frac{3}{2}$ or $y = -\frac{2}{3}x + \frac{8}{3}$	One correct tangent A1
	$y = \frac{1}{2}x + \frac{3}{2}$ and $y = -\frac{2}{3}x + \frac{8}{3}$	Both correct tangents A1
		(5)
(c)	$m = \frac{1}{2}, c = \frac{3}{2} \Rightarrow \frac{9}{4}x^2 + \frac{75}{2}x + \frac{625}{4} = 0 \Rightarrow x = \dots$ or $m = -\frac{2}{3}, c = \frac{8}{3} \Rightarrow \frac{64}{9}x^2 - \frac{800}{9}x + \frac{2500}{9} = 0 \Rightarrow x = \dots$ Uses one of their m and c pairs and solves for x	M1
	$x = -\frac{25}{3}, y = -\frac{8}{3}$ or $x = \frac{25}{4}, y = -\frac{3}{2}$	One correct point A1
	$x = -\frac{25}{3}, y = -\frac{8}{3}$ and $x = \frac{25}{4}, y = -\frac{3}{2}$	Both correct points A1
		(3)
		Total 12

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix}$		
	$ \mathbf{A} = a - 2 + a - 1 + 2 - 1 (= 2a - 2)$	Correct determinant in any form	B1
	$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \rightarrow \begin{pmatrix} a-2 & a-1 & 1 \\ -a-2 & a-1 & 3 \\ -2 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} a-2 & 1-a & 1 \\ a+2 & a-1 & -3 \\ -2 & 0 & 2 \end{pmatrix}$ Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed		M1
	$\begin{pmatrix} a-2 & 1-a & 1 \\ a+2 & a-1 & -3 \\ -2 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} a-2 & a+2 & -2 \\ 1-a & a-1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ Correct transpose of cofactors		A1
	$\mathbf{A}^{-1} = \frac{1}{2a-2} \begin{pmatrix} a-2 & a+2 & -2 \\ 1-a & a-1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse	A1
			(4)
(b)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$= \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 12-6\lambda \\ 4+2\lambda \\ 6+3\lambda \end{pmatrix} = \dots$	Attempt to multiply the parametric form of l_2 by their inverse	M1
	$= \begin{pmatrix} 6-\lambda \\ -4+4\lambda \\ 2-\lambda \end{pmatrix}$	Correct parametric form	A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l = \dots$ award A0	A1
			(4)
			Total 8

	Alternatives for (b)		
(i)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$	Attempt $\mathbf{A}^{-1}$ (point on l_2) and $\mathbf{A}^{-1}$ (direction of l_2)	M1
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} -6 \\ 24 \\ -6 \end{pmatrix}$	Both correct (NB No ft)	A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l = \dots$ award A0	A1
			(4)
(ii)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$	Attempt $\mathbf{A}^{-1}$ (point on l_2) for 2 points	M1
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ 9 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 30 \\ 0 \\ 6 \end{pmatrix}$	Both correct (NB No ft)	A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Obtain the direction vector and deduce correct equation (allow equivalent forms) but if given as $l = \dots$ award A0	A1
			(4)

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Notes	Marks
1(a)	$(\cosh A \cosh B + \sinh A \sinh B) = \left(\frac{e^A + e^{-A}}{2} \right) \left(\frac{e^B + e^{-B}}{2} \right) + \left(\frac{e^A - e^{-A}}{2} \right) \left(\frac{e^B - e^{-B}}{2} \right)$ $= \frac{e^{A+B} + e^{A-B} + e^{B-A} + e^{-A-B} + e^{A+B} - e^{A-B} - e^{B-A} + e^{-A-B}}{4}$ Expresses the lhs in terms of exponentials correctly, combines terms and combines fractions with common denominator (Brackets not needed due to fraction lines)		M1
	$= \frac{2e^{A+B} + 2e^{-(A+B)}}{4} = \frac{e^{A+B} + e^{-(A+B)}}{2} = \cosh(A+B)^*$ Fully correct proof with no errors		A1*
			(2)
(b)	$\cosh(x + \ln 2) = \cosh x \cosh(\ln 2) + \sinh x \sinh(\ln 2)$ $= \left(\frac{2 + \frac{1}{2}}{2} \right) \cosh x + \left(\frac{2 - \frac{1}{2}}{2} \right) \sinh x$ Applies the result from part (a) and evaluates both $\cosh(\ln 2)$ and $\sinh(\ln 2)$ Use of (a) must be seen		M1
	$\frac{5}{4} \cosh x + \frac{3}{4} \sinh x = 5 \sinh x$ $\Rightarrow \frac{5}{4} \cosh x = \frac{17}{4} \sinh x$	Collects terms and reaches $a \cosh x = b \sinh x$ oe Depends on the first M mark	dM1
	$5 \cosh x = 17 \sinh x \text{ oe}$	Correct equation	A1
	$x = \frac{1}{2} \ln \left(\frac{1 + \frac{5}{17}}{1 - \frac{5}{17}} \right)$ Or $\frac{e^{2x} - 1}{e^{2x} + 1} = \frac{5}{17} \Rightarrow x = \dots$	Moves to $\tanh x$ and uses the correct logarithmic form for $\operatorname{artanh} x$ or reverts to exponential forms and solves for x Depends on both M marks	ddM1
	$x = \frac{1}{2} \ln \left(\frac{11}{6} \right)$	Cao (Accept integer multiples of $\frac{11}{6}$)	A1
			(5)
			Total 7

Way 2			
(b)	$\cosh(x + \ln 2) = \cosh x \cosh(\ln 2) + \sinh x \sinh(\ln 2)$ $= \left(\frac{2 + \frac{1}{2}}{2}\right) \cosh x + \left(\frac{2 - \frac{1}{2}}{2}\right) \sinh x$ Applies the result from part (a) and evaluates both $\cosh(\ln 2)$ and $\sinh(\ln 2)$ Use of (a) must be seen	M1	
	$\Rightarrow 5 \cosh x = 17 \sinh x$ dM1: Collects terms and reaches an equation of form $A \cosh x = B \sinh x$ A1: Correct equation		dM1A1
	$5 \left(\frac{e^x + e^{-x}}{2} \right) = 17 \left(\frac{e^x - e^{-x}}{2} \right)$		
	$12e^x = 22e^{-x} \Rightarrow e^{2x} = \frac{22}{6} \Rightarrow x = \dots$	Changes to exponentials (correct forms) And solves for x	ddM1
	$x = \frac{1}{2} \ln \left(\frac{11}{6} \right)$	Cao (Accept integer multiples of $\frac{11}{6}$)	A1
Way 3			
	$\cosh(x + \ln 2) = \cosh x \cosh(\ln 2) + \sinh x \sinh(\ln 2)$ $\left(\frac{e^x + e^{-x}}{2} \right) \left(\frac{e^{\ln 2} + e^{-\ln 2}}{2} \right) + \left(\frac{e^x - e^{-x}}{2} \right) \left(\frac{e^{\ln 2} - e^{-\ln 2}}{2} \right) = 5 \left(\frac{e^x - e^{-x}}{2} \right)$ Applies the result from part (a) and uses the exponential forms of the hyperbolic functions. Use of (a) must be seen		M1
	eg $5e^x + 5e^{-x} = 17e^x - 17e^{-x}$ oe	Evaluates $e^{\ln 2}$ and $e^{-\ln 2}$ and starts to collect terms	dM1
	$12e^{2x} = 22 \Rightarrow e^{2x} = \frac{11}{6}$	Correct value for e^{2x}	A1
	$x = \dots$	Solves for x	ddM1
	$x = \frac{1}{2} \ln \left(\frac{11}{6} \right)$	Cao (Accept integer multiples of $\frac{11}{6}$)	A1

NB: Squaring and obtaining a value for $\sinh x$ or $\cosh x$ introduces extra answers. If these extra answers are then eliminated M1A1 is available but if no attempt at elimination is made award M0A0

Question Number	Scheme	Notes	Marks
2(i)	Throughout both parts of this question do not penalise omission of dx or $d\theta$		
	$5 + 4x - x^2 = 9 - (x - 2)^2$ oe	Correct completion of the square Any correct result	B1
	$\int \frac{1}{\sqrt{5 + 4x - x^2}} dx = \int \frac{1}{\sqrt{9 - (x - 2)^2}} dx = \sin^{-1}\left(\frac{x - 2}{3}\right)(+c)$ M1: Obtains $k \sin^{-1} f(x)$ A1: Correct integration (+ c not needed)		M1A1
			(3)
(ii)	$x = 6 \Rightarrow \theta = \frac{\pi}{3}$ $x = 2\sqrt{3} \Rightarrow \theta = \frac{\pi}{6}$	Correct θ limits in radians	B1
	$\int \frac{18}{(x^2 - 9)^{\frac{3}{2}}} dx = \int \frac{18 \times 3 \sec \theta \tan \theta}{(9 \sec^2 \theta - 9)^{\frac{3}{2}}} d\theta$ M1: For $\int \frac{18}{((3 \sec \theta)^2 - 9)^{\frac{3}{2}}} \times \left(\text{their } \frac{dx}{d\theta}\right) d\theta$		M1
	$\int \frac{54 \sec \theta \tan \theta}{(9 \sec^2 \theta - 9)^{\frac{3}{2}}} d\theta = 54 \int \frac{\sec \theta \tan \theta}{27 \tan^3 \theta} d\theta = 2 \int \frac{\sin \theta \cos^3 \theta}{\cos^2 \theta \sin^3 \theta} d\theta$ $2 \int \frac{\cos \theta}{\sin^2 \theta} d\theta \quad \text{oe} \quad \text{eg } 2 \int \frac{\sec \theta}{\tan^2 \theta} d\theta$ Correct simplified integral		A1
	$2 \int \frac{\cos \theta}{\sin^2 \theta} d\theta = 2 \int \operatorname{cosec} \theta \cot \theta d\theta = -2 \operatorname{cosec} \theta (+c)$ Obtains $k \operatorname{cosec} \theta (+c)$		M1
	$[-2 \operatorname{cosec} \theta]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -2 \operatorname{cosec} \frac{\pi}{3} + 2 \operatorname{cosec} \frac{\pi}{6}$	Uses changed limits correctly. Depends on all previous method marks.	dM1
	$= 4 - \frac{4}{3}\sqrt{3}$	Cao Allow these 2 marks if limits have been given in degrees	A1
			(6)
			Total 9



ALT	For B1 and final dM1A1 of (ii)	
	dM1: Reverse the substitution A1: Correct reversed result A1: enter as B1 on e-PEN Correct final answer	



Question Number	Scheme	Notes	Marks
3(a)	3	Correct value seen in (a)	B1
			(1)
(b)	$\begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8x \\ 8y \\ 8z \end{pmatrix} \Rightarrow \begin{matrix} -2x+5y=8x \\ 5x+y-3z=8y \\ -3y+6z=8z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots$ Correct method for the eigenvector (making a variable equal to 0 is not a correct method)		M1
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$	Any correct eigenvector	A1
			(2)
(c)	$ \mathbf{M} - \lambda \mathbf{I} = \begin{vmatrix} -2-\lambda & 5 & 0 \\ 5 & 1-\lambda & -3 \\ 0 & -3 & 6-\lambda \end{vmatrix} = 0$ $\Rightarrow (-2-\lambda)[(1-\lambda)(6-\lambda)-9]-5[5(6-\lambda)]=0 \Rightarrow \lambda = \dots$ NB CE is $\lambda^3 - 5\lambda^2 - 42\lambda + 144 = 0$ but may only find the constant term		M1
	$\lambda = -6$	Correct third eigenvalue The work for these 2 marks may be seen in (a) – award them Correct third eigenvalue by a different method – send to review	A1
	$\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -6 \end{pmatrix}$	Correct D following through their third eigenvalue	A1ft
	$\begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6x \\ -6y \\ -6z \end{pmatrix} \Rightarrow \begin{matrix} -2x+5y=-6x \\ 5x+y-3z=-6y \\ -3y+6z=-6z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ 1 \end{pmatrix}$ Correct strategy for 3rd eigenvector		M1
	$\mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}} \\ \frac{1}{\sqrt{3}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{42}} \end{pmatrix}$	Fully correct matrix consistent with their D May have $\frac{\sqrt{3}}{3}$ etc	A1
			(5)
			Total 8



Question Number	Scheme	Notes	Marks
4.	$y = \operatorname{artanh}\left(\frac{\cos x + a}{\cos x - a}\right)$		
	$\frac{dy}{dx} = \frac{1}{1 - \left(\frac{\cos x + a}{\cos x - a}\right)^2} \times \frac{(\cos x - a) \times -\sin x - (\cos x + a) \times -\sin x}{(\cos x - a)^2}$ or $\frac{dy}{dx} = \frac{1}{1 - \left(\frac{\cos x + a}{\cos x - a}\right)^2} \times \left(-\sin x \times (\cos x - a)^{-1} + (\cos x + a) \times \sin x (\cos x - a)^{-2}\right)$ M1: Correct method for the derivative. This requires $\frac{1}{1 - \left(\frac{\cos x + a}{\cos x - a}\right)^2} \times$ An attempt at the quotient (or product) rule. A1: Correct derivative in any form	M1A1	
	$= \frac{(\cos x - a)^2}{(\cos x - a)^2 - (\cos x + a)^2} \times \frac{2a \sin x}{(\cos x - a)^2} = \frac{2a \sin x}{-4a \cos x} = \dots$ Uses correct processing to reach $\lambda \frac{\sin x}{\cos x}$ or $\lambda \tan x$ Depends on the first method mark.		dM1
	$= -\frac{1}{2} \tan x$	CSO	A1 (4)
Way 2	$y = \operatorname{artanh}\left(\frac{\cos x + a}{\cos x - a}\right) \Rightarrow \tanh y = \frac{\cos x + a}{\cos x - a} \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = \frac{2a \sin x}{(\cos x - a)^2}$ Takes tanh of both sides, obtains $\operatorname{sech}^2 y \frac{dy}{dx} =$ an attempt at the quotient or product rule		M1
	$\frac{dy}{dx} = \frac{1}{1 - \left(\frac{\cos x + a}{\cos x - a}\right)^2} \times \frac{2a \sin x}{(\cos x - a)^2}$ Correct derivative in any form		A1
	$= \frac{(\cos x - a)^2}{(\cos x - a)^2 - (\cos x + a)^2} \times \frac{2a \sin x}{(\cos x - a)^2} = \frac{2a \sin x}{-4a \cos x} = \dots$ Uses correct processing to reach $\lambda \frac{\sin x}{\cos x}$ or $\lambda \tan x$ Depends on the first method mark.		dM1
	$= -\frac{1}{2} \tan x$	CSO	A1 (4)

Way 3	Uses substitution $u = \frac{\cos x + a}{\cos x - a}$, obtains $\frac{du}{dx} \left(= \frac{2a \sin x}{(\cos x - a)^2} \right)$ by quotient rule and $\frac{dy}{du} \left(= \frac{1}{1-u^2} \right)$ followed by chain rule to obtain $\frac{dy}{dx} = \frac{1}{1 - \left(\frac{\cos x + a}{\cos x - a} \right)^2} \times \frac{2a \sin x}{(\cos x - a)^2}$		M1
	Correct derivative in any form		A1
	Uses correct processing to reach $\lambda \frac{\sin x}{\cos x}$ or $\lambda \tan x$		dM1
	Depends on the first method mark.		
	$= -\frac{1}{2} \tan x$	CSO	A1 (4)
			Total 4
Way 4	$y = \frac{1}{2} \ln \left(\frac{1 + \frac{\cos x + a}{\cos x - a}}{1 - \frac{\cos x + a}{\cos x - a}} \right) = \frac{1}{2} \ln \left(-\frac{\cos x}{a} \right)$ $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{-\frac{\cos x}{a}} \times \left(\frac{\sin x}{a} \right)$		M1A1
	M1: Converts to correct ln form and uses chain rule to differentiate A1: Correct derivative in any form		
	Uses correct processing to reach $\lambda \frac{\sin x}{\cos x}$ or $\lambda \tan x$		dM1
	Depends on the first method mark.		
	$= -\frac{1}{2} \tan x$	CSO	A1
			(4)

Question Number	Scheme	Notes	Marks
5	$x = 4e^{\frac{1}{2}t}, \quad y = e^t - t \quad 0 \leq t \leq 4$		
	$\frac{dx}{dt} = 2e^{\frac{1}{2}t}, \quad \frac{dy}{dt} = e^t - 1$	Correct derivatives	B1
	NB: Allow missing dt in the following integration work		
	$S = (2\pi) \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} (dt) = (2\pi) \int (e^t - t) \sqrt{\left(4e^{\frac{1}{2}t}\right)^2 + (e^t - t)^2} (dt)$ $\left(= (2\pi) \int (e^t - t) \sqrt{4e^t + e^{2t} - 2e^t + 1} (dt) \right)$		M1
	Applies the surface area formula with or w/o the 2π		
	$= (2\pi) \int (e^t - t)(e^t + 1)(dt)$	Correct simplified integral Brackets must be present unless implied by subsequent work but award by implication if $(2\pi) \int (e^{2t} + e^t - te^t - t)(dt)$ is seen	A1
	$= (2\pi) \int (e^t - t)(e^t + 1)(dt) = (2\pi) \int (e^{2t} + e^t - te^t - t)(dt)$ $= (2\pi) \left[\frac{1}{2} e^{2t} + e^t - te^t + e^t - \frac{1}{2} t^2 \right]$ B1: For $\int te^t dt = te^t - e^t (+c)$ A1: Fully correct integration (the integration may be shown as 2 separate parts and score B1A1 if both parts correct)		B1A1
	$= 2\pi \left[\frac{1}{2} e^{2t} + 2e^t - te^t - \frac{1}{2} t^2 \right]_0^4 = 2\pi \left\{ \left(\frac{1}{2} e^8 + 2e^4 - 4e^4 - 8 \right) - \left(\frac{1}{2} + 2 \right) \right\}$ Applies the limits 0 and 4 Must include 2π now. If 2 integrals have been used limits must be applied to both and the results added Depends on the first M mark (and some valid integration)		dM1
	$\pi(e^8 - 4e^4 - 21)$	Cao	A1
			(7)
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A} = \begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$		
	NB: Work for (a) can only be awarded in (a)		
	$ \mathbf{A} = x(-4+2x) - (-2+4x) + 3(-4+16)$	Correct determinant attempt (expand by any row or column) or use the Rule of Sarrus (send to review if unsure) Sign errors allowed only within the brackets	M1
	$= 2x^2 - 8x + 38$	Correct simplified determinant	A1
	$2x^2 - 8x + 38 = 2(x-2)^2 + 30$ or $\frac{d}{dx}(2x^2 - 8x + 38) = 4x - 8 = 0 \Rightarrow x = 2$ $\Rightarrow 2x^2 - 8x + 38 = \dots$ or $b^2 - 4ac = 64 - 4 \times 2 \times 38 = \dots$	Starts the process of showing $\det \mathbf{A} \neq 0$ E.g. Completes the square, finds the minimum point or finds discriminant May find discriminant of $x^2 - 4x + 19 = \dots$	M1
	$2x^2 - 8x + 38 \geq 30$ or $b^2 - 4ac < 0$ Therefore $\det \mathbf{A} \neq 0$ which means $\mathbf{A}$ is non-singular	Appropriate reasoning for their chosen method and a conclusion stating that $\mathbf{A}$ is non-singular. All 3 previous marks needed (No need to evaluate a discriminant, so ISW slips in calculation provided $64 - 4 \times 2 \times 38 = \dots$ or $16 - 4 \times 19 = \dots$ seen	A1cso
			(4)
(b)	$\begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} -4+2x & -2+4x & -4+16 \\ -1+6 & -x+12 & -2x+4 \\ x-12 & x^2-6 & 4x-2 \end{pmatrix} \rightarrow \begin{pmatrix} -4+2x & 2-4x & 12 \\ -5 & -x+12 & 2x-4 \\ x-12 & -x^2+6 & 4x-2 \end{pmatrix}$ M1: Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed A1: Correct cofactor matrix		M1A1
	$\begin{pmatrix} -4+2x & 2-4x & 12 \\ -5 & -x+12 & 2x-4 \\ x-12 & -x^2+6 & 4x-2 \end{pmatrix} \rightarrow \begin{pmatrix} -4+2x & -5 & x-12 \\ 2-4x & -x+12 & -x^2+6 \\ 12 & 2x-4 & 4x-2 \end{pmatrix}$ $\mathbf{A}^{-1} = \frac{1}{2x^2 - 8x + 38} \begin{pmatrix} -4+2x & -5 & x-12 \\ 2-4x & -x+12 & -x^2+6 \\ 12 & 2x-4 & 4x-2 \end{pmatrix}$ dM1: Transposes and divides by their determinant.		dM1A1

	If their original determinant has been divided by 2 (acceptable for (a)) and then used here it is not their determinant and so scores dM0 2 correct rows or 2 correct columns needed from their previous matrix Depends on previous method mark. A1: Correct matrix		
			(4)
			Total 8



Question Number	Scheme	Notes	Marks
1(i)	$(8) \int \frac{1}{16+x^2} dx = (8) \left(\frac{1}{4} \arctan \left(\frac{x}{4} \right) \right)$	Obtains ...arctan (kx) Allow $k = 1$	M1
	$2 \left[\arctan \left(\frac{x}{4} \right) \right]_4^{4\sqrt{3}} = 2(\arctan \sqrt{3} - \arctan 1) = \dots$	Substitutes the given limits, subtracts either way round and obtains a value (could be a decimal). The substitution does not need to be seen explicitly and may be implied by their value.	dM1
	$\frac{\pi}{6}$ or $p = \frac{1}{6}$ Correct exact value (or value for p) Accept equivalent exact expressions e.g. $\frac{2\pi}{12}$ or $p = \frac{2}{12}$ and isw if necessary.		A1
			(3)
(ii)	$2 \int \frac{1}{\sqrt{9-4x^2}} dx = 2 \left(\frac{1}{2} \arcsin \frac{2x}{3} \right) \left(\text{or e.g. } \arcsin \frac{x}{\frac{3}{2}} \right)$ M1: Obtains ...arcsin (kx). Allow $k = 1$ so allow just arcsin x. A1: Fully correct integration but allow unsimplified as above		M1 A1
	$\left[\arcsin \left(\frac{2x}{3} \right) \right]_3^k = \arcsin \left(\frac{2k}{3} \right) - \arcsin \left(\frac{1}{2} \right) = \frac{\pi}{12}$ $\Rightarrow \arcsin \left(\frac{2k}{3} \right) = \frac{\pi}{12} + \frac{\pi}{6} \Rightarrow \frac{2k}{3} = \sin \left(\frac{\pi}{4} \right) \Rightarrow \frac{2k}{3} = \frac{\sqrt{2}}{2} \Rightarrow k = \dots$ Substitutes the given limits, subtracts either way round, sets $= \frac{\pi}{12}$, uses $\arcsin \left(\frac{1}{2} \right) = \frac{\pi}{6}$ and the correct order of operations condoning sign errors only to reach a value for k e.g. $\pm \alpha \left(\arcsin \left(\frac{2k}{3} \right) - \frac{\pi}{6} \right) = \frac{\pi}{12} \Rightarrow \arcsin \left(\frac{2k}{3} \right) = \frac{\pi}{12\alpha} \pm \frac{\pi}{6} \Rightarrow k = \frac{3 \sin \left(\frac{\pi}{12\alpha} \pm \frac{\pi}{6} \right)}{2}$ Note that k may be inexact (decimal) or may be in terms of “sin” but must have a simplified argument e.g. $k = \frac{3 \sin \left(\frac{\pi}{4} \right)}{2}$		dM1
	$k = \frac{3\sqrt{2}}{4}$ or exact equivalent e.g., $\frac{3}{2\sqrt{2}}$ Note that a common incorrect answer is $k = \frac{3}{2} \sin \left(\frac{5\pi}{24} \right) (= 0.913\dots)$ which comes from an incorrect integral of $2 \arcsin \left(\frac{2x}{3} \right)$ (generally scoring 1010) Condone $x = \frac{3\sqrt{2}}{4}$		A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
2(a) Way 1 TU = I	$\mathbf{TU} = \mathbf{I} \Rightarrow \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix} \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\Rightarrow \text{e.g., } \begin{matrix} 6a + 60 - 8b = 0 & -2 + 3c + 7a = 0 \\ -4a - 36 + 5b = 1 & -3 + 2c + 6a = 1 \end{matrix}$ Obtains at least 2 equations with at least one correct. (condone column $\times$ row multiplication leading to the way 2 equations – see below). Ignore errors in unused elements or equations.		M1
	$\begin{matrix} 6a - 8b = -60 \\ -4a + 5b = 37 \end{matrix} \Rightarrow a = \dots, b = \dots \quad \text{or} \quad \begin{matrix} 7a + 3c = 2 \\ 6a + 2c = 4 \end{matrix} \Rightarrow a = \dots, c = \dots$ Obtains values for two of a , b and c . You do not need to check their values. As long as the previous M mark was scored, it is sufficient to just write down values.		dM1
	$a = 2, b = 9, c = -4$	A1: Two correct values A1: All three correct values and no extra values unless they are rejected.	A1 A1
			(4)
Way 2 UT = I For first 2 marks	$\mathbf{UT} = \mathbf{I} \Rightarrow \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix} \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $12 - 3 - 4a = 1$ $\Rightarrow \text{e.g., } 42 - 6 - 4b = 0$ $[45 + 2c - 36 = 1]$ Obtains at least 2 equations with at least one correct. (condone column $\times$ row multiplication leading to the way 1 equations – see above). Ignore errors in unused elements or equations.		M1
	$\text{e.g., } -4a = -8, -4b = -36 \quad [2c = -8]$ $\Rightarrow a = \dots, b = \dots$ Obtains values for two of a , b and c . You do not need to check their values. As long as the previous M mark was scored, it is sufficient to just write down values.		dM1

Way 3 Inverses For first mark	$\mathbf{T}^{-1} = \mathbf{U} \Rightarrow \frac{1}{4a-5b+36} \begin{pmatrix} 2b-24 & -3b+28 & 4 \\ 6a-3b & -7a+2b & 9 \\ -2a+12 & 3a-8 & -5 \end{pmatrix} = \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix}$ $\Rightarrow \text{e.g., } \frac{4}{4a-5b+36} = -4, \frac{2b-24}{4a-5b+36} = 6 \left[\frac{-7a+2b}{4a-5b+36} = c \right]$ For $\mathbf{T}^{-1} = \frac{1}{f(a,b)} \mathbf{M}$ where $\mathbf{M}$ has at least 1 correct element and obtains 2 equations. Note that there is no requirement to find all the elements of $\mathbf{M}$. OR $\mathbf{U}^{-1} = \mathbf{T} \Rightarrow \frac{1}{-6a-2c+3} \begin{pmatrix} 9a+5c & -4a+5 & 4c+9 \\ -3 & -2 & -6 \\ 15a+8c & -6a+8 & 6c+15 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix}$ $\Rightarrow \text{e.g., } \frac{-3}{-6a-2c+3} = 3, \frac{4c+9}{-6a-2c+3} = 7 \left[\frac{6c+15}{-6a-2c+3} = b \right]$ For $\mathbf{U}^{-1} = \frac{1}{f(a,c)} \mathbf{M}$ where $\mathbf{M}$ has at least 1 correct element and obtains 2 equations Note that there is no requirement to find all the elements of $\mathbf{M}$.	M1
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2(b)	$\frac{x-1}{3} = \frac{y}{-4} = z+2 \Rightarrow [l_2 : \mathbf{r}] = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \pm \lambda \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} \text{ or } \mathbf{r} - \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = \mathbf{0}$ Obtains parametric/vector form (allow one slip only) or clearly identifies position and direction vectors. May be implied by an attempt to transform both.	M1
	$\begin{pmatrix} 6 & -1 & -4 \\ 15 & -4 & -9 \\ -8 & 2 & 5 \end{pmatrix} \begin{pmatrix} 1+3\lambda \\ -4\lambda \\ -2+\lambda \end{pmatrix} = \begin{pmatrix} 6+18\lambda+4\lambda+8-4\lambda \\ 15+45\lambda+16\lambda+18-9\lambda \\ -8-24\lambda-8\lambda-10+5\lambda \end{pmatrix}$ or their $\mathbf{U} \times$ their $\begin{pmatrix} 1 & 3 \\ 0 & -4 \\ -2 & 1 \end{pmatrix}$ or $\times$ their $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\times$ their $\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ or their $\mathbf{U} \times$ their $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\mathbf{U} \times$ e.g. $\begin{pmatrix} 4 \\ -4 \\ -1 \end{pmatrix}$ then $\text{dir} = \begin{pmatrix} 32 \\ 85 \\ -45 \end{pmatrix} - \begin{pmatrix} 14 \\ 33 \\ -18 \end{pmatrix}$ Complete and correct method with their b and c for their $\mathbf{U} \times$ their parametric form or $\mathbf{U} \times$ both vectors or $\mathbf{U} \times 2$ points on the line and attempts direction. Must be an attempt to multiply correctly i.e. clearly not row $\times$ row but allow attempts that use $\mathbf{T}^{-1}$ for $\mathbf{U}$ using their a and b provided all elements are constants and it is a "changed" $\mathbf{T}$ OR $\begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ "2" & 4 & "9" \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+3\lambda \\ -4\lambda \\ -2+\lambda \end{pmatrix} \Rightarrow \begin{matrix} 2x+3y+7z=1+3\lambda \\ 3x+2y+6z=-4\lambda \\ 2x+4y+9z=-2+\lambda \end{matrix}$ $x = 18\lambda + 14$ $\Rightarrow y = 52\lambda + 33$ $z = -18 - 27\lambda$ A complete method using their parametric form and their $\mathbf{T}$ to produce and solve 3 simultaneous equations to find x, y and z in terms of λ Alternatively solves $\mathbf{T}\mathbf{x} = ("i - 2k")$ and $\mathbf{T}\mathbf{x} = ("3i - 4k + k")$ to find position and direction	M1
	$[l_1 : \mathbf{r}] = \begin{pmatrix} 14+18\lambda \\ 33+52\lambda \\ -18-27\lambda \end{pmatrix}$ $\Rightarrow \frac{x-14}{18} = \frac{y-33}{52} = \frac{z+18}{-27}$	dM1: Correctly converts their result into Cartesian equation. Requires previous method mark A1: Correct Cartesian equation - allow equivalents e.g., $\dots = \frac{z-(-18)}{-27}, \dots = \frac{-z-18}{27}$
		(4)
		Total 8

2(b) Alternative

$$x = t \Rightarrow y = \frac{4}{3} - \frac{4}{3}t, \quad z = \frac{1}{3}t - \frac{7}{3}$$

M1: Obtains parametric form (allow one slip only)

$$\begin{pmatrix} 6 & -1 & -4 \\ 15 & '-4' & -9 \\ -8 & '2' & 5 \end{pmatrix} \begin{pmatrix} t \\ \frac{4}{3} - \frac{4}{3}t \\ \frac{1}{3}t - \frac{7}{3} \end{pmatrix} = \begin{pmatrix} 6t - \frac{4}{3} + \frac{4}{3}t - \frac{4}{3}t + \frac{28}{3} \\ 15t - \frac{16}{3} + \frac{16}{3}t - 3t + 21 \\ -8t + \frac{8}{3} - \frac{8}{3}t + \frac{5}{3}t - \frac{35}{5} \end{pmatrix}$$

M1: As above

$$[l_1 : \mathbf{r} =] \begin{pmatrix} 8 + 6t \\ \frac{47}{3} + \frac{52}{3}t \\ -9 - 9t \end{pmatrix}$$

$$\Rightarrow \frac{x-8}{6} = \frac{y - \frac{47}{3}}{\frac{52}{3}} = \frac{z+9}{-9}$$

dM1A1: As above



Question Number	Scheme	Notes	Marks
4(a)	$\mathbf{M}\mathbf{x} = \lambda \mathbf{x} \Rightarrow \begin{pmatrix} 0 & -1 & 3 \\ -1 & 4 & -1 \\ 3 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda \\ -2\lambda \\ \lambda \end{pmatrix} \Rightarrow \text{e.g., } 2+3=\lambda \Rightarrow \lambda=5$ or $(\mathbf{M} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \begin{pmatrix} -\lambda & -1 & 3 \\ -1 & 4-\lambda & -1 \\ 3 & -1 & -\lambda \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \text{e.g., } -\lambda + 2 + 3 = 0 \Rightarrow \lambda = 5$ M1: Correct method leading to a value for λ A1: Correct value Note that the working may be minimal so e.g. $2+3=\lambda \Rightarrow \lambda=5$ is sufficient. Correct answer only scores both marks.		M1 A1
			(2)
(b)	$\begin{pmatrix} 0 & -1 & 3 \\ -1 & 4 & -1 \\ 3 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ or } \begin{pmatrix} 3 & -1 & 3 \\ -1 & 7 & -1 \\ 3 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ or e.g., } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 7 \\ -1 \end{pmatrix}$ $\Rightarrow x = \dots, y = \dots, z = \dots$ Uses $\mathbf{M}\mathbf{x} = -3\mathbf{x}$ or $(\mathbf{M} - (-3)\mathbf{I})\mathbf{x} = \mathbf{0}$ to produce simultaneous equations and obtains values for x, y and z (not all 0) or uses a suitable vector product (with two correct components if method unclear)		M1
	$k \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$	Any correct eigenvector (allow $x = \dots, y = \dots, z = \dots$ and apply isw if a vector is subsequently formed incorrectly)	A1
			(2)
(c)	$\mathbf{M}\mathbf{x} = \lambda \mathbf{x} \Rightarrow \text{e.g., } -1(1) + 3(1) = \lambda \quad \text{or} \quad \lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0$ $(\mathbf{M} - \lambda \mathbf{I})\mathbf{x} = 0 \Rightarrow \text{e.g., } -\lambda - 1 + 3 = 0 \quad \text{or} \quad \det \mathbf{M} = -30 = \lambda_1 \lambda_2 \lambda_3 = -15\lambda$ $\lambda = 2$ Correct value. May be seen in their D which may come from an attempt at $\mathbf{P}^T \mathbf{M} \mathbf{P}$.		B1
	$(\mathbf{D}) = \begin{pmatrix} -3 & 0 & 0 \\ 0 & '2' & 0 \\ 0 & 0 & '5' \end{pmatrix}$	Diagonal matrix with -3 and their eigenvalues anywhere on the leading diagonal and 0's elsewhere. Ignore labelling.	B1ft
	$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{pmatrix}$ Correct method seen to normalise at least one eigenvector of the two given eigenvectors or their eigenvector from part (b). May be seen in their P.		M1
	$\mathbf{D} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{3} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} \end{pmatrix}$ Both fully correct, consistent and labelled matrices. Elements may not have had denominators rationalised. (Any columns of P could be in opposite direction)		A1
			(4)
			Total 8

Note that some candidates go straight into solving $|\mathbf{M} - \lambda\mathbf{I}| = 0$ e.g.

$$\begin{vmatrix} -\lambda & -1 & 3 \\ -1 & 4-\lambda & -1 \\ 3 & -1 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(\lambda(\lambda-4)-1) + 3 + \lambda + 3(1-3(4-\lambda)) = 0$$

$$\Rightarrow \lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0 \Rightarrow \lambda = -3, 5, 2$$

If this is all they do then the B mark in (c) can be awarded for $\lambda = 2$

The other marks in the question are available for the appropriate work.

General Principles for Further Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)



Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
	Mark (a) and (b) together but do not credit work for (a) that is seen in (c)		
5(a)	$\begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8x \\ 8y \\ 8z \end{pmatrix} \text{ or } \begin{pmatrix} -2 & -2 & -1 \\ -2 & -2 & -1 \\ -1 & -1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots$ Correct method for obtaining the eigenvector i – j	Any multiple of this vector	M1 A1
			(2)
(b)	$ \mathbf{M} - \lambda \mathbf{I} = \begin{vmatrix} 6-\lambda & -2 & -1 \\ -2 & 6-\lambda & -1 \\ -1 & -1 & 5-\lambda \end{vmatrix}$ $\Rightarrow \underline{(6-\lambda)}((6-\lambda)(5-\lambda)-1) + \underline{2(2(\lambda-5)-1)} - \underline{1(2+6-\lambda)}$ Correct attempt at the determinant of $\mathbf{M} - \lambda \mathbf{I}$. The terms with single underlining should be correct with correct signs but allow minor slips in the brackets with double underlining. Note that the rule of Sarrus gives $(6-\lambda)(6-\lambda)(5-\lambda) - 2 - 2 - (6-\lambda) - (6-\lambda) - 4(5-\lambda)$		M1
	$\Rightarrow \lambda^3 - 17\lambda^2 + 90\lambda - 144 = 0 \Rightarrow \lambda = \dots$	Solves $\mathbf{M} - \lambda \mathbf{I} = 0$ to obtain 2 different distinct real eigenvalues excluding 8	M1
	$\Rightarrow \lambda = 3, 6, (8)$	For 3 and 6	A1
			(3)



General Principles for Further Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Summary of changes from Provisional Mark Scheme

A few minor changes were made to the Mark Scheme before marking on the marking service began.

Question Number	Summary of changes
Q03b	The B1 for a correct matrix of minors was changed to an M1 to be fairer to the candidates so that a small slip in the matrix of minors was not penalised as heavily.
Q04ii	The method marks were made dependent on each other as candidates were unlikely to make significant progress unless the first derivative was of the correct form and some first derivatives were trivial expressions.
Q06c	Alternative 3 added
Q07ii	Alternative added



Question Number	Scheme	Notes	Marks
4(a)	$\lambda = 3 \Rightarrow \mathbf{M} - 3\mathbf{I} = \begin{vmatrix} 3 & k & 2 \\ k & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3(8) - k(4k) + 2(-4) = 0$ or e.g. $ \mathbf{M} - \lambda\mathbf{I} = \begin{vmatrix} 6 - \lambda & k & 2 \\ k & 5 - \lambda & 0 \\ 2 & 0 & 7 - \lambda \end{vmatrix} = 0$ $\Rightarrow (6 - \lambda)(5 - \lambda)(7 - \lambda) - k(k(7 - \lambda)) + 2(0 - 2(5 - \lambda)) = 0 \Rightarrow 24 - k(4k) - 8 = 0$ Correct interpretation of 3 being an eigenvalue leading to the formation of a quadratic equation in k only. If the method for forming the determinant is not clear then look for at least 2 correct “components”. NB rule of Sarrus gives $24 - 8 - 4k^2 = 0$		M1
	$\Rightarrow 4k^2 = 16 \Rightarrow k = \dots$	Solves quadratic. Depends on the first M.	dM1
	$k = \pm 2$	Correct values	A1
			(3)
(a) Way 2	$\begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{cases} 6x + ky + 2z = 3x \\ kx + 5y = 3y \\ 2x + 7z = 3z \end{cases}$ $z = -\frac{1}{2}x, y = -\frac{1}{2}kx \Rightarrow 6x - \frac{k^2x}{2} - x = 3x \Rightarrow \frac{k^2}{2} = 2$ Eliminates z and y and reaches a quadratic equation in k only		M1
	$\frac{k^2}{2} = 2 \Rightarrow k = \dots$	Solves quadratic. Depends on the first M.	dM1
	$k = \pm 2$	Correct values	A1
(b)	$k = -2 \Rightarrow \mathbf{M} - \lambda\mathbf{I} = \begin{vmatrix} 6 - \lambda & -2 & 2 \\ -2 & 5 - \lambda & 0 \\ 2 & 0 & 7 - \lambda \end{vmatrix}$ $\Rightarrow (6 - \lambda)(7 - \lambda)(5 - \lambda) + 2(2\lambda - 14) + 2(2\lambda - 10) = 0$ Applies a value of k from (a) and a recognisable attempt at the characteristic equation (the “= 0” is not needed here). If the method is not clear then look for at least 2 correct “components”.		M1
	$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0 \Rightarrow \lambda = \dots$	Solves cubic. May use $\lambda = 3$ as a factor or calculator to solve. Depends on the first mark. Allow complex roots.	dM1
	$\lambda = 6, 9, (3)$	Correct values. Allow to come from $k = 2$	A1
			(3)

(c)	$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} 6x - 2y + 2z = 3x \\ -2x + 5y = 3y \\ 2x + 7z = 3z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots$		M1
	or $\begin{pmatrix} 3 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} 6x - 2y + 2z = 0 \\ -2x + 5y = 0 \\ 2x + 7z = 0 \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots$		
	Correct strategy for finding the eigenvector using a value of k from (a) Note that the cross product of any 2 rows or columns of $\mathbf{M} - 3\mathbf{I}$ gives an eigenvector		
	$p \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct eigenvector	A1
$\frac{1}{3} \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct normalised eigenvector	A1	
		(3)	
		Total 9	

Question Number	Scheme	Notes	Marks
1(a)	$ae = \frac{13}{2}$ or $\frac{a}{e} = \frac{72}{13}$	One correct equation in a and e . Allow equivalent correct equations. Could include – or $\pm$ signs	B1
	e.g., $a = \frac{72}{13}e \Rightarrow \frac{72}{13}e^2 = \frac{13}{2} \Rightarrow e^2 = \dots \left(\frac{169}{144}\right)$ or $a = \frac{13}{2e} \Rightarrow \frac{13}{2e^2} = \frac{72}{13} \Rightarrow e^2 = \dots \left(\frac{169}{144}\right)$	Having obtained two equations in a and e of the correct form i.e., $ae = p$ and $\frac{a}{e} = q$ $p, q \neq 0$, solves simultaneously to find a <u>positive</u> value for e^2 (no requirement for $e > 1$) or e . Condone poor algebra provided a value is obtained. May find a first.	M1
	$e = \frac{13}{12}$ or $1\frac{1}{12}$ or 1.08 $\dot{3}$. Not $\pm \frac{13}{12}$ unless negative value clearly rejected in this part.		A1
(3)			
(b)	$\left\{ a = \frac{72}{13} \times \frac{13}{12} = 6 \text{ or } a = \frac{13}{2\left(\frac{13}{12}\right)} = 6 \right\}$ $b^2 = a^2(e^2 - 1) = \dots$ $\left\{ b^2 = 6^2 \left(\left(\frac{13}{12} \right)^2 - 1 \right) = \frac{25}{4} \text{ or } b = \frac{5}{2} \right\}$	With any value for a , which might be seen in part (a), and their e , uses a correct eccentricity formula with correct substitution to find a value for b^2 or b . Could be implied. May see $b = a\sqrt{e^2 - 1}$ or use of e.g., $e = \sqrt{1 + \frac{b^2}{a^2}}$ or $e = \frac{c}{a}$ with $c = \sqrt{a^2 + b^2}$	M1
	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$	Applies $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ correctly for their values. Not dependent. Could use e.g., $b^2x^2 - a^2y^2 = a^2b^2$	M1
	e.g., $25x^2 - 144y^2 = 900$ A correct equation in correct form. Requires all previous 5 marks but allow if 4 marks with A0 in (a) for $e = \pm \frac{13}{12}$ and negative value not rejected in part (a). Any positive integer multiple. Allow equivalents provided variables on one side and constant on the other and y^2 term has negative coefficient. Just $p = 25, q = 144, r = 900$ requires $px^2 - qy^2 = r$ to be seen. Ignore wrong values for p, q, r following a correct equation (e.g., “ $q = -144$ ”)		A1
Alt Using $PS^2 = e^2PM^2$	$\left(x - \frac{13}{2}\right)^2 + y^2 = \left(\frac{13}{12}\right)^2 \left(x - \frac{72}{13}\right)^2$ M1: Forms equation correct for their ae, e and $\frac{a}{e}$ $x^2 - 13x + \frac{169}{4} + y^2 = \frac{169}{144}x^2 - 13x + 36 \Rightarrow \frac{25}{144}x^2 - y^2 = \frac{25}{4}$ M1: Expands and reaches $rx^2 - sy^2 = t, r, s, t \neq 0$ A1: e.g., $25x^2 - 144y^2 = 900$ as main scheme		
(3)			
Total 6			

Question Number	Scheme	Marks
In all parts, accept alternative notation for arsinh and arcosh such as $\sinh^{-1}$ and $\cosh^{-1}$ etc		
2(i)	$4x^2 + 8x + 9 = 4(x+1)^2 + 5$ or $4x^2 + 8x + 9 = (2x+2)^2 + 5$	B1
	$\int \frac{1}{\sqrt{4x^2 + 8x + 9}} dx = \int \frac{1}{\sqrt{4(x+1)^2 + 5}} dx = \frac{1}{2} \operatorname{ar sinh} \left(\frac{2(x+1)}{\sqrt{5}} \right) (+c) \text{ o.e.}$	M1A1
		(3)
(ii)	$\int \operatorname{arcosh} 3x dx = x \operatorname{arcosh} 3x - \int \frac{3x}{\sqrt{9x^2 - 1}} dx$	M1 A1
	$= \dots - \frac{1}{3} (9x^2 - 1)^{\frac{1}{2}}$ or $\dots - \frac{1}{3} \sinh(\operatorname{arcosh} 3x)$	M1
	$\int \operatorname{arcosh} 3x dx = x \operatorname{arcosh} 3x - \frac{1}{3} (9x^2 - 1)^{\frac{1}{2}} (+c)$ or $x \operatorname{arcosh} 3x - \left(x^2 - \frac{1}{9} \right)^{\frac{1}{2}} (+c)$ or $\int \operatorname{arcosh} 3x dx = x \operatorname{arcosh} 3x - \frac{1}{3} \sinh(\operatorname{arcosh} 3x) (+c)$	A1
		(4)
ALT	$u = \operatorname{ar cosh} 3x \Rightarrow x = \frac{1}{3} \cosh u \Rightarrow \frac{dx}{du} = \frac{1}{3} \sinh u$ $\int \operatorname{arcosh} 3x dx = \int \frac{1}{3} u \sinh u du = \frac{u}{3} \cosh u - \int \frac{1}{3} \cosh u du$	M1A1
	$\frac{u}{3} \cosh u - \frac{1}{3} \sinh u (+c)$	M1
	$x \operatorname{ar cosh} 3x - \frac{1}{3} \sinh(\operatorname{ar cosh} 3x) (+c)$ or $x \operatorname{arcosh} 3x - \frac{1}{3} (9x^2 - 1)^{\frac{1}{2}} (+c)$ or $x \operatorname{arcosh} 3x - \left(x^2 - \frac{1}{9} \right)^{\frac{1}{2}} (+c)$	A1
		(4)
		Total 7

Question	Scheme	Marks
1	$\frac{dy}{dx} = \frac{1}{2} \times \frac{2}{\sqrt{(2x)^2 - 1}}$	M1
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{4x^2 - 1} = \frac{4x^2}{4x^2 - 1}$	M1
	$\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int \sqrt{\frac{4x^2}{4x^2 - 1}} dx = 2 \int \frac{x}{\sqrt{4x^2 - 1}} dx$	A1
	$= \frac{2(4x^2 - 1)^{\frac{1}{2}}}{8 \times \frac{1}{2}}$	M1
	$s = \left[\frac{(4x^2 - 1)^{\frac{1}{2}}}{2} \right]_{\frac{7}{2}}^{13} = \frac{1}{2} \left(\sqrt{4 \times 169 - 1} - \sqrt{4 \times \frac{49}{4} - 1} \right) = \dots$	dM1
	$= \frac{1}{2} (15\sqrt{3} - 4\sqrt{3}) = \frac{11}{2} \sqrt{3}$	A1
		(6)
(6 marks)		

Notes:

M1: Attempts $\frac{dy}{dx}$, accept the form $\frac{A}{\sqrt{(2x)^2 - 1}}$. Allow $\frac{A}{\sqrt{2x^2 - 1}}$ (condone missing brackets)

Alternative 1:

Writes $\frac{1}{2} \operatorname{ar} \cosh 2x$ as $\frac{1}{2} \ln(2x + \sqrt{4x^2 - 1})$ leading to

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2x + \sqrt{4x^2 - 1}} \times \left(2 + \frac{4x}{\sqrt{4x^2 - 1}} \right) = \frac{2x + \sqrt{4x^2 - 1}}{\sqrt{4x^2 - 1} (2x + \sqrt{4x^2 - 1})} = \frac{1}{\sqrt{4x^2 - 1}}$$

Alternative 2:

$$y = \frac{1}{2} \operatorname{ar} \cosh 2x \Rightarrow 2y = \operatorname{ar} \cosh 2x \Rightarrow \cosh 2y = 2x \rightarrow 4 \sinh 2y \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{1}{\sinh 2y} = \frac{1}{\sqrt{4x^2 - 1}}$$

If either approach is taken then the same condition for the form of the derivative applies.

Note that this differentiation may be seen in an attempt by parts of $\int y dx$

M1: Attempts to find $1 + \left(\frac{dy}{dx}\right)^2$ using their $\frac{dy}{dx}$ and attempts common denominator.

A1: Reaches a correct simplified integral with $\sqrt{x^2}$ replaced with x as shown in the scheme.

$$\text{Allow equivalent forms e.g. } 2 \int x \sqrt{\frac{1}{4x^2 - 1}} dx, \frac{1}{2} \int \frac{4x}{\sqrt{(2x)^2 - 1}} dx$$

This may be implied by subsequent work.



M1: Attempts the integration and reaches the form $\alpha(\beta x^2 - 1)^{\frac{1}{2}}$. α and/or β may be 1

This may be implied by e.g.

$$u = 4x^2 - 1 \rightarrow k \int \frac{1}{\sqrt{u}} du = \alpha \sqrt{u} \text{ or } u = x^2 \rightarrow k \int \frac{1}{\sqrt{4u-1}} du = \alpha \sqrt{4u-1}$$

dM1: Applies the limits to their integral. **Depends on the previous 2 method marks.**

Any attempts at substitution requires use of changed limits e.g.

$$u = 4x^2 - 1 \rightarrow \frac{1}{4} \int \frac{1}{\sqrt{u}} du \rightarrow \frac{1}{2} [\sqrt{u}]_{48}^{675} = \dots$$

A1: cao Accept equivalents in the correct form, such as $\frac{1}{2}\sqrt{363}$

Examples of alternative for the final 3 marks:

$$\begin{aligned} x = \frac{1}{2} \cosh u &\Rightarrow 2 \int \frac{x}{\sqrt{4x^2 - 1}} dx = \int \frac{\cosh u}{\sqrt{\cosh^2 u - 1}} \frac{1}{2} \sinh u du \\ \int \frac{1}{2} \cosh u du &= \frac{1}{2} [\sinh u]_{\operatorname{arcosh} 7}^{\operatorname{arcosh} 26} = \frac{1}{2} \left(\frac{e^{\ln(26+15\sqrt{3})} - e^{-\ln(26+15\sqrt{3})}}{2} - \frac{e^{\ln(7+4\sqrt{3})} - e^{-\ln(7+4\sqrt{3})}}{2} \right) \\ &= \frac{1}{2} (15\sqrt{3} - 4\sqrt{3}) = \frac{11}{2} \sqrt{3} \end{aligned}$$

Score M1 for a complete method for the substitution leading to $k \sinh u$ and then dM1 for applying changed limits (or reverts back to x) and A1 as above

$$\begin{aligned} x = \frac{1}{2} \sec u &\Rightarrow 2 \int \frac{x}{\sqrt{4x^2 - 1}} dx = \int \frac{\sec u}{\sqrt{\sec^2 u - 1}} \frac{1}{2} \sec u \tan u du \\ \int \frac{1}{2} \sec^2 u du &= \frac{1}{2} [\tan u]_{\operatorname{arcosh} \frac{1}{7}}^{\operatorname{arcosh} \frac{1}{26}} \\ &= \frac{1}{2} (15\sqrt{3} - 4\sqrt{3}) = \frac{11}{2} \sqrt{3} \end{aligned}$$

Score M1 for a complete method for the substitution leading to $k \tan u$ and then dM1 for applying changed limits (or reverts back to x) and A1 as above

Special Case if no integration is attempted:

Note that if candidates do not attempt the integration but obtain the correct exact answer then a special case of **M1M1A1M0A0A1** (4/6) should be awarded.



Question	Scheme	Marks
2.	$\cosh y = x, y < 0 \Rightarrow y = \ln \left[x - \sqrt{x^2 - 1} \right]$	
	$\cosh y = x \Rightarrow x = \frac{e^y + e^{-y}}{2}$	B1
	$\Rightarrow 2xe^y = e^{2y} + 1$	M1
	$\Rightarrow e^{2y} - 2xe^y + 1 = 0 \Rightarrow e^y = \frac{2x \pm \sqrt{(2x)^2 - 4 \times 1 \times 1}}{2}$ or $\Rightarrow e^{2y} - 2xe^y + 1 = 0 \Rightarrow (e^y - x)^2 + 1 - x^2 = 0 \Rightarrow e^y = \dots$	M1
	$= x \pm \sqrt{x^2 - 1}$	A1
	So $y = \ln \left[x - \sqrt{x^2 - 1} \right]^*$	A1*
	since $y < 0 \Rightarrow e^y < 1$ so need $x - \sqrt{x^2 - 1}$ (as $x > 1$ so must subtract)	B1
		(6)

(6 marks)**Notes:**

B1: Correct statement for x in terms of exponentials. $\cosh y = \frac{e^y + e^{-y}}{2}$ scores B0.

M1: Multiplies through by e^y to achieve a quadratic in e^y . (Terms need not be gathered.)

M1: Uses the quadratic formula or other valid method (e.g. completing the square) to solve for e^y .

A1: Correct solution(s) for e^y . Accept if only the negative one is given. Accept $\frac{2x \pm \sqrt{4x^2 - 4}}{2}$

A1*: Completely correct work leading to the given answer regardless of the justification why the negative root is taken (correct or incorrect). Must be no errors seen.

B1: Suitable justification for taking the negative root given.

E.g. $y < 0$ so $y = \ln \left[x - \sqrt{x^2 - 1} \right]$. Condone $x \pm \sqrt{x^2 - 1} < 1$ so $y = \ln \left[x - \sqrt{x^2 - 1} \right]$.

Note that the B1 can only be awarded if all previous marks have been awarded.

But the reason may be given before or after $\ln$ has been taken.

E.g. $(e^y - x)^2 + 1 - x^2 = 0 \Rightarrow e^y - x = \pm \sqrt{x^2 - 1}$ but $y < 0$ so $e^y - x = -\sqrt{x^2 - 1}$

Working backwards:

$$y = \ln \left[x - \sqrt{x^2 - 1} \right] \Rightarrow e^y = x - \sqrt{x^2 - 1} \text{ (B1)} \Rightarrow e^y + e^{-y} = x - \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} \text{ (M1)}$$

$$x - \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = \frac{2x(x - \sqrt{x^2 - 1})}{x - \sqrt{x^2 - 1}} \text{ (M1)} = 2x \text{ (A1)} \Rightarrow x = \frac{e^y + e^{-y}}{2} = \cosh y \text{ (A1)}$$

Final B1 unlikely to be available.



Question	Scheme	Marks
3(a)	$\frac{dy}{dx} = \frac{6 \cos \theta}{-8 \sin \theta}$ or $\frac{2x}{64} + \frac{2y}{36} \frac{dy}{dx} = 0$ or $\frac{dy}{dx} = \frac{1}{4} \times \frac{1}{2} (576 - 9x^2)^{-\frac{1}{2}} \times -18x$	B1
	$m_T = -\frac{3 \cos \theta}{4 \sin \theta} \Rightarrow m_N = -\frac{1}{m_T} = \frac{4 \sin \theta}{3 \cos \theta}$	M1
	So normal is $y - 6 \sin \theta = \frac{4 \sin \theta}{3 \cos \theta} (x - 8 \cos \theta)$ or $y = \frac{4 \sin \theta}{3 \cos \theta} x + c, c = 6 \sin \theta - \frac{4 \sin \theta}{3 \cos \theta} \times 8 \cos \theta$	dM1
	$\Rightarrow 3y \cos \theta - 18 \sin \theta \cos \theta = 4x \sin \theta - 32 \sin \theta \cos \theta$ $\Rightarrow 4x \sin \theta - 3y \cos \theta = 14 \sin \theta \cos \theta^*$	A1*
		(4)
(b)	A is $\left(\frac{7}{2} \cos \theta, 0\right)$ and B is $\left(0, -\frac{14}{3} \sin \theta\right)$	B1
	M is $\left(\frac{\frac{7}{2} \cos \theta}{2}, -\frac{\frac{14}{3} \sin \theta}{2}\right) = \left(\frac{7}{4} \cos \theta, -\frac{7}{3} \sin \theta\right)$	M1
	$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \left(-\frac{3}{7} y\right)^2 + \left(\frac{4}{7} x\right)^2 = 1$	dM1 A1
	$\Rightarrow 16x^2 + 9y^2 = 49$	A1
		(5)
(9 marks)		

Notes:**(a)**

B1: A correct statement for, or involving, $\frac{dy}{dx}$. See examples in scheme for parametric, implicit and direct forms.

M1: Finds $\frac{dy}{dx}$ in terms of θ and applies the perpendicular condition to find gradient of the normal.

dM1: Uses their normal gradient and P to find the equation of the normal

A1*: Correct answer from correct work with at least one intermediate step and no errors seen.

(b)

B1: Correct coordinates for A and B or correct intercepts of l seen or implied by working. Allow in any form simplified or unsimplified.

M1: Uses their A and B to attempt the midpoint, M . May be implied by at least one correct coordinate.

dM1: Uses $\sin^2 \theta + \cos^2 \theta = 1$ with their M to form an equation in x and y only.

Depends on the previous mark.

A1: A correct unsimplified equation.

A1: Correct equation in the required form. Allow any integer multiple.



Special Case: If M is found as e.g. $\left(\frac{7}{4}\cos\theta, \frac{7}{3}\sin\theta\right)$ withhold the final mark only if otherwise correct.



Question	Scheme	Marks
4(a)	$\begin{vmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{vmatrix} = 2 \begin{vmatrix} 3 & 2 \\ 1 & k \end{vmatrix} - 0 \begin{vmatrix} k & 2 \\ -2 & k \end{vmatrix} + (-1) \begin{vmatrix} k & 3 \\ -2 & 1 \end{vmatrix} = 2(3k-2) - (k+6) = \dots$	M1
	$= 6k - 4 - k - 6 = 5k - 10^*$	A1*
		(2)
(b)	$\mathbf{M}^T = \begin{pmatrix} 2 & k & -2 \\ 0 & 3 & 1 \\ -1 & 2 & k \end{pmatrix} \text{ or minors } \begin{pmatrix} 3k-2 & k^2+4 & k+6 \\ 1 & 2k-2 & 2 \\ 3 & 4+k & 6 \end{pmatrix} \text{ or}$ $\text{cofactors } \begin{pmatrix} 3k-2 & -k^2-4 & k+6 \\ -1 & 2k-2 & -2 \\ 3 & -4-k & 6 \end{pmatrix}$	M1
	$\text{Adjugate matrix is } \begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix} (\geq 6 \text{ entries correct})$	M1
	$\text{Hence } \mathbf{M}^{-1} = \frac{1}{5k-10} \begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$	dM1A1
		(4)
(c)	Images of A, B and C are $(5, 4k-18, 3k-16)$, $(0, 7-2k, 9-4k)$ and $(0, 4k-2, 8k-14)$	M1 A1
	$(\pm) 50 = \frac{1}{6} \begin{vmatrix} 5 & 4k-18 & 3k-16 \\ 0 & 7-2k & 9-4k \\ 0 & 4k-2 & 8k-14 \end{vmatrix} \Rightarrow (\pm) 300 = 5(\dots) (= 200k - 400) \Rightarrow k = \dots$	M1
	$(300 = 200k - 400 \Rightarrow) k = \frac{7}{2} \quad \text{or} \quad (-300 = 200k - 400 \Rightarrow) k = \frac{1}{2}$	A1
	$k = \frac{1}{2} \text{ and } k = \frac{7}{2}$	A1
		(5)
	Using volume scale factor. Attempts $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 4 & -8 & 3 \\ -2 & 5 & -4 \\ 4 & -6 & 8 \end{vmatrix} = 4(40-24) + 8(-16+16) + 3(12-20) = \dots$	M1
	$\text{Volume of } T \text{ is } \frac{1}{6} \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \left \frac{1}{6} \begin{vmatrix} 4 & -8 & 3 \\ -2 & 5 & -4 \\ 4 & -6 & 8 \end{vmatrix} \right = \dots \frac{20}{3}$	A1
	$\text{Volume image of } T = \det \mathbf{M} \times \frac{20}{3} \Rightarrow \frac{20}{3} 5k-10 = 50 \Rightarrow k = \dots$	M1



Question Number	Scheme	Notes	Marks
2(a)	$x = -\frac{4}{3}$	$x = -\frac{4}{3}$ or any equivalent equation . Allow $x = \pm \frac{4}{3}$	B1
			(1)
(b)(i) Way 1	$\frac{a}{e} = \frac{4}{3}$ $b^2 = a^2(e^2 - 1) \Rightarrow 5 = a^2\left(\frac{9a^2}{16} - 1\right)$	Uses $\frac{a}{e} = \pm \frac{4}{3}$ oe and a correct eccentricity formula and obtains an equation in a . Condone replacing b^2 with 25 if equation is otherwise correct	M1
	$9a^4 - 16a^2 - 80 = 0$ $\Rightarrow (9a^2 + 20)(a^2 - 4) = 0 \Rightarrow a^2 = \dots$	Solves a 3TQ in a^2 (or equation that would lead to a 3TQ) to find a positive real root (usual rules – but if no working seen they must obtain one positive real value of a^2 or a correct to 3 sf which is consistent with their equation). Do not award if confusion with variable e.g., " $(9a^2 + 20)(a^2 - 4) = 0 \Rightarrow a = 4$ " Requires previous M mark.	dM1
	$a = 2$	Not $a = \pm 2$ unless negative rejected	A1
			(3)
Way 2	$\frac{a}{e} = \frac{4}{3}$ $b^2 = a^2(e^2 - 1) \Rightarrow 5 = \left(\frac{4e}{3}\right)^2(e^2 - 1)$	Uses $\frac{a}{e} = \pm \frac{4}{3}$ oe and a correct eccentricity formula and obtains an equation in e . Condone replacing b^2 with 25 if equation is otherwise correct	M1
	$16e^4 - 16e^2 - 45 = 0$ $\Rightarrow (4e^2 - 9)(4e^2 + 5) = 0 \Rightarrow e^2 = \dots$	Solves a 3TQ in e^2 (or equation that would lead to a 3TQ) to find a positive real root (usual rules – but if no working seen they must obtain one positive real value of e^2 or e correct to 3 sf which is consistent with their equation). Do not award if confusion with variable e.g., " $(4e^2 - 9)(4e^2 + 5) = 0 \Rightarrow e = \frac{9}{4}$ " Requires previous M mark.	dM1
	$\left(e = \frac{3}{2} \Rightarrow\right) a = 2$	Not $a = \pm 2$ unless negative rejected but condone sight of " $e = \pm \frac{3}{2}$ " or " $e = -\frac{3}{2}$ "	A1
			(3)

Question Number	Scheme	Notes	Marks
2(b)(ii)	$e = \frac{3}{2} \Rightarrow ae = \frac{3}{2} \times 2 \text{ or } ae = \frac{3a^2}{4} = \frac{3}{4} \times 4$ $\text{or } ae = c = \sqrt{a^2 + b^2} = \sqrt{2^2 + 5}$	Uses a correct method to obtain a numerical expression for ae or with their values of a , e , a^2 , b^2 etc. however obtained. Condone use of a negative e or a	M1
	Foci are $(\pm 3, 0)$	Both correct foci as coordinates	A1
	Allow " $\frac{a}{e} = \frac{4}{3} \Rightarrow a = 4, e = 3$ " to access the last M mark only in (b) for $(\pm 12, 0)$ provided the values of both a and e are clearly seen beforehand		(2)
			Total 6
	Note that it is possible to answer (ii) before (i) – e.g., Let foci be $(\pm c, 0)$ $a^2 e^2 = c^2 = b^2 + a^2 = 5 + a^2 \text{ and}$ $\frac{a}{e} = \frac{a^2}{ae} = \frac{a^2}{c} = \frac{4}{3} \Rightarrow a^2 = \frac{4}{3}c$ $\Rightarrow c^2 = 5 + \frac{4}{3}c \quad \text{(i) M1: Uses correct formulae to form an equation in } c \text{ – condone } b^2$ replaced with 25 as with main scheme) $\Rightarrow 3c^2 - 4c - 15 = 0 \Rightarrow (3c + 5)(c - 3) = 0 \Rightarrow c = 3$ (i) dM1: Solves 3TQ to find positive real root) $\Rightarrow (\pm 3, 0) \quad \text{(i) A1: Correct foci as coordinates)}$ $a = \sqrt{\frac{4}{3}} \times 3 \quad \text{(ii) M1: Correct method for } a)$ $a = 2 \quad \text{(ii) A1: Correct value)}$		

Question Number	Scheme	Notes	Marks
3 Way 1 Converts to sinh and cosh	$4 \tanh x - \operatorname{sech} x = 1$ $4 \frac{\sinh x}{\cosh x} - \frac{1}{\cosh x} = 1$ $4 \sinh x - 1 - \cosh x = 0$ $4 \frac{e^x - e^{-x}}{2} - 1 - \frac{e^x + e^{-x}}{2} = 0$	Replaces one hyperbolic function with its correct exponential equivalent. Allow for correct replacement of just e.g., $\sinh x$ after using $\tanh x = \frac{\sinh x}{\cosh x}$. May follow errors but do not allow any further marks if the original equation was reduced to one in a single hyperbolic function.	M1
	$3e^{2x} - 2e^x - 5 = 0$	M1: Obtains an equation which if terms are collected is a 3TQ (or 2TQ with no constant) in e^x A1: Correct 3TQ	M1 A1
	$e^x = \frac{2 \pm \sqrt{4+60}}{6} \left(\Rightarrow \frac{2+8}{6} = \frac{5}{3} \right)$	M1: Solves 3TQ (or 2TQ with no constant) in e^x . Apply usual rules. If no working seen they must achieve one correct root of their equation to 3sf which may be complex. If 2TQ must get a correct non-zero root of their equation. A1: Any correct unsimplified expression for e^x that includes the positive root. Must be exact	M1 A1
	$x = \ln \frac{5}{3}$	$\ln \frac{5}{3}, \ln 1\frac{2}{3}, \ln 1.\dot{6}$ only but allow $k = \dots$ No unrejected extra solutions	A1
			Total 6
Way 2 Straight to e^x	$4 \frac{e^x - e^{-x}}{e^x + e^{-x}} - \frac{2}{e^x + e^{-x}} = 1$	Replaces one hyperbolic function with its correct exponential equivalent	M1
	$3e^{2x} - 2e^x - 5 = 0$	M1: Obtains an equation which if terms are collected is a 3TQ (or 2TQ with no constant) in e^x A1: Correct 3TQ	M1 A1
	$e^x = \frac{2 \pm \sqrt{4+60}}{6} \left(\Rightarrow \frac{2+8}{6} = \frac{5}{3} \right)$	M1: Solves 3TQ (or 2TQ with no constant) in e^x . Apply usual rules. If no working seen they must achieve one correct root of their equation to 3sf which may be complex. If 2TQ must get a correct non-zero root of their equation. A1: Any correct unsimplified expression for e^x that includes the positive root. Must be exact	M1 A1
	$x = \ln \frac{5}{3}$	$\ln \frac{5}{3}, \ln 1\frac{2}{3}, \ln 1.\dot{6}$ only but allow $k = \dots$ No unrejected extra solutions	A1
			Total 6
	In Ways 1 & 2, if they form an equation which is not a quadratic in e^x they must achieve the correct exact root of $\frac{5}{3}$ to access the middle four marks		

(i)

M1: Factors out the 4 and completes the square achieving $4\left(x + \frac{3}{2}\right)^2 + \dots$ or without factoring the 4 achieves $(2x + 3)^2 + \dots$ where $\dots$ is anything other than 25. Condone for this mark

$4 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 + \dots} dx$ but this is A0 if no independent correct expression is shown.

A1: Correct completed square form (constant must be gathered).

M1: Attempts the integration with arctan used. Look for $k \arctan(l \times \text{their linear term})$. Allow with any k and l , including 1 or $\frac{1}{4}$. (May be via a substitution but must return to integral in x of the correct form for the mark.)

If an incorrect completion of the square results in $(ax + b)^2 - c^2$ then this mark may be awarded

for $\dots \ln\left(\frac{ax + b - c}{ax + b + c}\right)$

A1: Correct answer, need not be fully simplified but must have at least combined multiple fractions, e.g. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ must have been combined. Accept alternative forms of arctan, e.g. $\tan^{-1}$ or atan.

(ii)

B1: Correct completed square form for the quadratic (need not be fully simplified).

M1: Attempts the integration to either arcosh or ln form. Form should be correct for their $x + 2$ and a . Allow with $\ln| |$ and give benefit of doubt for the M if mod is required. They may use a substitution for this mark, $u = x + 2$, and may be scored for the integral in terms of u . If a substitution such as $x + 2 = \cosh(u)$ or $x + 2 = \sqrt{21} \sec u$ is used they must carry out the full process – with supporting working shown – to reach an integral in u **with correct limits identified** as part of the method, or reversal of the substitution must be carried out.

A1: Correct integral (may be in terms of u if the substitution $u = x + 2$ is used), or correct integral **and** correct limits if a hyperbolic substitution is used. Ignore spurious statements of equality if they give both forms, and award for either correct form. Allow with moduli in the logs.

Note that $x + 2 = \sqrt{21} \sec u$ will lead to an integral $\left[\ln|\tan u + \sec u| \right]_{\arcsin \frac{5}{\sqrt{21}}}^{\arcsin \frac{11}{\sqrt{21}}}$

M1: Applies the limits 3 and 9 to their integral of a correct form (allowing a minor slip, e.g. a sign error inside a ln), or correct limits for their variable if they use a substitution, in an integral of the correct form (may be scored if M0A0 was awarded for insufficient evidence). Evidence of the application of limits must be seen – do not accept solutions which go directly to the answer. All steps of working are asked for in the question.

dM1: Simplifies to a single ln term, applying the arcosh ln (allowing a minor slip, e.g. a sign error inside a ln) formula first if necessary. Again, there must be sufficient evidence after substitution before the final simplified answer.

A1: cso

Question	Scheme	Marks
4(a)	$\begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & a \\ 2 & a & 20/3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ \dots \\ \dots \end{pmatrix}$	M1
	$\lambda_1 = 10$	A1
		(2)
(b)	$\begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & a \\ 2 & a & 20/3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix} \Rightarrow 8 + 3a = 20 \text{ or } 2 + 2a + \frac{20}{3} \times 3 = 30$	M1
	$\Rightarrow a = 4^*$	A1*
		(2)
(c)	$\begin{vmatrix} 4-\lambda & 0 & 2 \\ 0 & 4-\lambda & 4 \\ 2 & 4 & \frac{20}{3}-\lambda \end{vmatrix} = (4-\lambda) \left((4-\lambda) \left(\frac{20}{3}-\lambda \right) - 16 \right) + 2(0-2(4-\lambda))$	M1
	$\left[-\lambda^3 + \frac{44}{3}\lambda^2 - \frac{148}{3}\lambda + \frac{80}{3} = 0 \right] \Rightarrow \frac{1}{3}(4-\lambda)(3\lambda^2 - 32\lambda + 20) = 0 \Rightarrow \lambda = \dots, \dots$	dM1
	$\lambda = 4, \frac{2}{3}, 10$	A1
		(3)
(d)	$\begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & 4 \\ 2 & 4 & 20/3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{cases} (4-\lambda)x + 2z = 0 \\ (4-\lambda)y + 4z = 0 \\ 2x + 4y + \left(\frac{20}{3}-\lambda\right)z = 0 \end{cases} \Rightarrow \dots$	M1
	$\lambda = \frac{2}{3} \Rightarrow k \begin{pmatrix} 3 \\ 6 \\ -5 \end{pmatrix} \text{ or } \lambda = 4 \Rightarrow m \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$	A1
	Normalised	
	$\Rightarrow \frac{v}{\ v\ } = \dots \lambda = \frac{2}{3} \Rightarrow \frac{1}{\sqrt{70}} \begin{pmatrix} 3 \\ 6 \\ -5 \end{pmatrix} \text{ and } \lambda = 4 \Rightarrow \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}; \lambda = 10 \Rightarrow \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$	M1 A1;B1
(e)		(5)
	$\mathbf{P} = \begin{pmatrix} 1/\sqrt{14} & 2/\sqrt{5} & 3/\sqrt{70} \\ 2/\sqrt{14} & -1/\sqrt{5} & 6/\sqrt{70} \\ 3/\sqrt{14} & 0 & -5/\sqrt{70} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2/3 \end{pmatrix}$	B1ft B1ft
		(2)
(14 marks)		

Notes:**(a)**

M1: Correct attempt to find the eigenvalue. Need only see top row calculated. May come from attempts at using $(\mathbf{M} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}$

A1: Correct value.

(b)

M1: Sets up a correct equation in a for their eigenvalue in (a). Must be extracted from the matrix equation. May come from attempts at using $(\mathbf{M} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}$. Condone any unused incorrect equations.

A1*: Shows that $a = 4$ from correct work.

(c)

M1: Attempts to expand $\det(\mathbf{M} - \lambda \mathbf{I})$. This may be along any row or down any column, or via a “shoelace” approach. Condone sign slips, but the overall structure should be correct. Working must be shown – do not accept solutions that just state the factorised form. The expansion must be seen in some form. Accept alternative approaches.

dM1: Sets the characteristic polynomial equal to zero (which may be implied) and solves by a valid method (may be by calculator) – dependent, so as a minimum, an attempt at the expansion must have been seen but the cubic need not have been fully simplified.

A1: Correct remaining eigenvalues.

(d) Note – you may allow recovery of the final 3 marks in this part if the relevant normalisation work is completed in part (e).

M1: Sets up the eigenvector equations in general terms or for one of their eigenvalues and makes some attempt to use them (allow even for the given eigenvector for this mark). Working must be shown, M0 if correct vectors are stated with no supporting working.

A1: Either of the two remaining eigenvectors found, need not be normalised for this mark. Must be given as a vector, but accept row vectors rather than column vectors.

M1: Applies a correct process to normalise at least one of their vectors. This may be done on the given eigenvector (working need not be shown for this one).

A1: Both normalised eigenvectors for $\lambda = 4, \frac{2}{3}$ correct. (Row or column vectors acceptable.)

B1: For correct normalised vector for eigenvalue 10. (Row or column vector acceptable.)

You may see $\lambda = \frac{2}{3} \Rightarrow \begin{pmatrix} 3\sqrt{70}/70 \\ 3\sqrt{70}/35 \\ -\sqrt{70}/14 \end{pmatrix}$ and $\lambda = 4 \Rightarrow \begin{pmatrix} 2\sqrt{5}/5 \\ -\sqrt{5}/5 \\ 0 \end{pmatrix}$; $\lambda = 10 \Rightarrow \begin{pmatrix} \sqrt{14}/14 \\ \sqrt{14}/7 \\ 3\sqrt{14}/14 \end{pmatrix}$ or other

equivalent versions of these vectors, e.g. $\lambda = \frac{2}{3} \Rightarrow \frac{\sqrt{70}}{14} \begin{pmatrix} 3/5 \\ 6/5 \\ -1 \end{pmatrix}$

(e)

B1ft: For either a correct **P** for their **normalised** eigenvectors providing they are non-zero **or** for a correct **D** for their eigenvalues.

B1ft: For both a correct matching **P** and **D** for their **normalised** eigenvectors and their eigenvalues, providing there is no zero column or row.

Note: If **D** and **P** are not labelled then treat their diagonal matrix as **D** and other matrix as **P**. If they are incorrectly labelled (ie **D** as **P**) then withhold the second B (if earned).

