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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Logarithmic Functions and Indices (1)

International GCSE Further Pure Mathematics Topic 1

35 questions · 251 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

35 questions · 251 marks · lowest tariff first



3-MARK QUESTIONS

3 questions in this booklet



- 2 Given that $\frac{4 + 2\sqrt{3}}{5 - 2\sqrt{3}}$ can be written in the form $\frac{a + b\sqrt{3}}{c}$ where a and b are integers and c is prime, find the value of a , the value of b and the value of c .

Show your working clearly.

(3)



- 1 Given that $\frac{2\sqrt{3} - 4}{3\sqrt{3} + 5}$ can be written in the form $a + b\sqrt{3}$ where a and b are integers, find, without using a calculator, the value of a and the value of b

Show your working clearly.

(3)



1

$$(8 - \sqrt{6})w = 50$$

Without using a calculator, find the value of w

Give your answer in the form $\frac{a+b\sqrt{6}}{c}$ where a and b are integers and c is prime.

Show your working clearly.

(3)



4-MARK QUESTIONS

5 questions in this booklet



- 1 Given that $\frac{a + \sqrt{3}}{2 - \sqrt{3}} = 11 + b\sqrt{3}$ where a and b are integers,

find the value of a and the value of b .

(4)



- 1 Given that $\frac{a - \sqrt{48}}{\sqrt{3} + 1}$ can be written in the form $b\sqrt{3} - 9$ where a and b are integers,

find the value of a and the value of b
Show your working clearly.

(4)



1

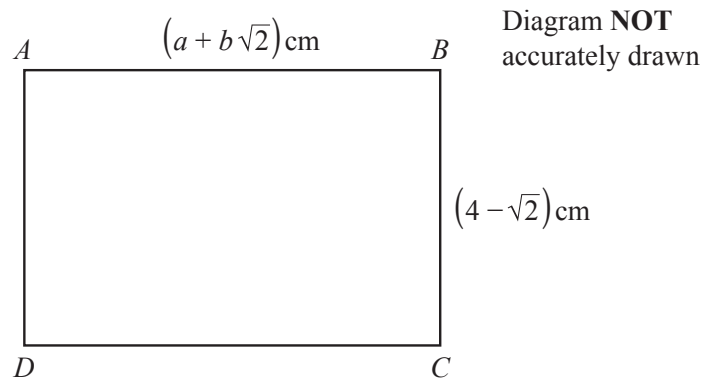
**Figure 1**

Figure 1 shows the rectangle $ABCD$.

$AD = BC = (4 - \sqrt{2})$ cm and $AB = DC = (a + b\sqrt{2})$ cm where a and b are integers.

The area of the rectangle is $(10 + \sqrt{2})$ cm²

Find the value of a and the value of b

Show your working clearly.

(4)



1

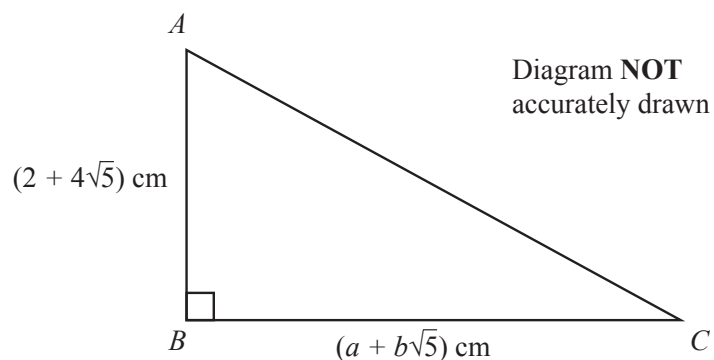
**Figure 1**

Figure 1 shows the triangle ABC

$\angle ABC = 90^\circ$ $AB = (2 + 4\sqrt{5})$ cm $BC = (a + b\sqrt{5})$ cm where a and b are integers.

The area of triangle $ABC = (34 + 11\sqrt{5})$ cm²

Without using a calculator, find the value of a and the value of b

(4)

- 1 Without using a calculator, solve the inequality $\sqrt{50}x - \sqrt{18} > 6x + 5$

Give your answer in an exact form with a rationalised denominator.
Show your working clearly.

(4)



5-MARK QUESTIONS

4 questions in this booklet



7 Solve the equation

$$\log_7(8x^2 - 6x + 3) - \log_{49}x^2 = 3\log_7 2$$

(5)



10 Solve the equation

$$\log_4 x + \log_{16} x + \log_2 x = 10.5$$

Show your working clearly.

(5)



- 1 Given that $\frac{a + 2\sqrt{5}}{3 - \sqrt{5}} = \frac{11 + b\sqrt{5}}{2}$ where a is an integer and b is prime,

find the value of a and the value of b
Show your working clearly.

(5)

- 3 Given that $\frac{a+b\sqrt{5}}{6-2\sqrt{5}} = \frac{9+4\sqrt{5}}{c}$ where a , b and c are prime numbers,

find the value of a , the value of b and the value of c

(5)



7-MARK QUESTIONS

6 questions in this booklet



3 (a) Write down the value of $\log_3 9$

(1)

(b) Solve the equation $\log_3 9t = \log_9 \left(\frac{12}{t} \right)^2 + 2$ where $t > 0$

Give your answer in the form $a\sqrt{b}$ where a and b are prime numbers.

(6)

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9 Showing your working clearly, use algebra to solve the equations

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$

(7)



4 (i) Solve the equation $16\log_r 4 = \log_4 r$

(2)

(ii) Solve the equation $\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = 1 + \log_5 x + \log_5 x^2$

(5)

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8 Solve the equation $\log_3 x - 2\log_x 3 = 1$

(7)



5 Solve the equation

$$\log_3 \sqrt{x-5} + \log_9 (x+3) - 1 = 0$$

Show clear algebraic working.

(7)



10 Solve the equation

$$\log_4 x^3 + 8\log_x 64 = 22$$

(7)



8-MARK QUESTIONS

7 questions in this booklet



- 8 Given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a (2n - 1)$$

- (a) find the value of n .

(3)

Given that $\log_p x = 3$ and $\log_p y - 3 \log_p 2 = 4$

- (b) (i) express x in terms of p ,

(1)

- (ii) express xy in terms of p .

(4)

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8 Use an algebraic method to solve the simultaneous equations

$$\log_4 a + 3 \log_8 b = \frac{5}{2}$$

$$2^a = \frac{16^4}{4^{b^2}}$$

(8)

7 (a) Write down the value of $\log_2 16$

(1)

Given that $4 + 2\log_4 x = \log_2 y$

(b) show that $y = 16x$

(4)

(c) Hence solve the equation $4 + 2\log_4 x = \log_2(4x + 5)$

(3)

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6 Given that $\frac{a + \sqrt{5}}{\sqrt{5} - 2} = 11 + 5\sqrt{5}$

- (a) without using a calculator, find the value of a
Show your working clearly.

(2)

Triangle PQR is such that

$$PR = (x + 3) \text{ cm} \quad QR = x \text{ cm} \quad \text{angle } QPR = 30^\circ \quad \text{angle } PQR = 45^\circ$$

- (b) Show that $x = 3 + 3\sqrt{2}$

(3)

Given that $\sin 105^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$ and that the area of triangle PQR is $A \text{ cm}^2$

- (c) find the exact value of A in the form $\frac{9}{8}(p\sqrt{6} + q\sqrt{2} + r\sqrt{3} + s)$

where p, q, r and s are integers.

(3)

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9 Giving each value in your solution to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln(x + 3) - 2y - 1 = 0$$

(8)



6 Solve the equation

$$\log_2 x^3 + \log_4 x^2 - 3 \log_x 2 = 0$$

giving your answers to 3 significant figures.

(8)



11 Solve the simultaneous equations

$$2\log_4 x = \log_3 3y^2$$

$$\log_2 x^3 + 8\log_9 y = 13$$

Show your working clearly.

(8)



9-MARK QUESTIONS

4 questions in this booklet



5 (a) Show that $\log_4 32 = \frac{5}{2}$

(2)

(b) Hence, or otherwise, find the exact solutions of the equation

$$\log_2 x - \log_4 32 + \frac{1}{4} \log_x 16 = 0$$

(7)

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9 (a) Find the value of a such that $\log_a 8 = \frac{3}{4}$

(2)

(b) Show that

$$3x \log_2 x - 4 \log_{16} 8 + 6x \log_4 8 - \log_2 x = \log_2 (8x)^{3x-1}$$

(4)

(c) Hence solve the equation $3x \log_2 x - 4 \log_{16} 8 + 6x \log_4 8 - \log_2 x = 0$

(3)

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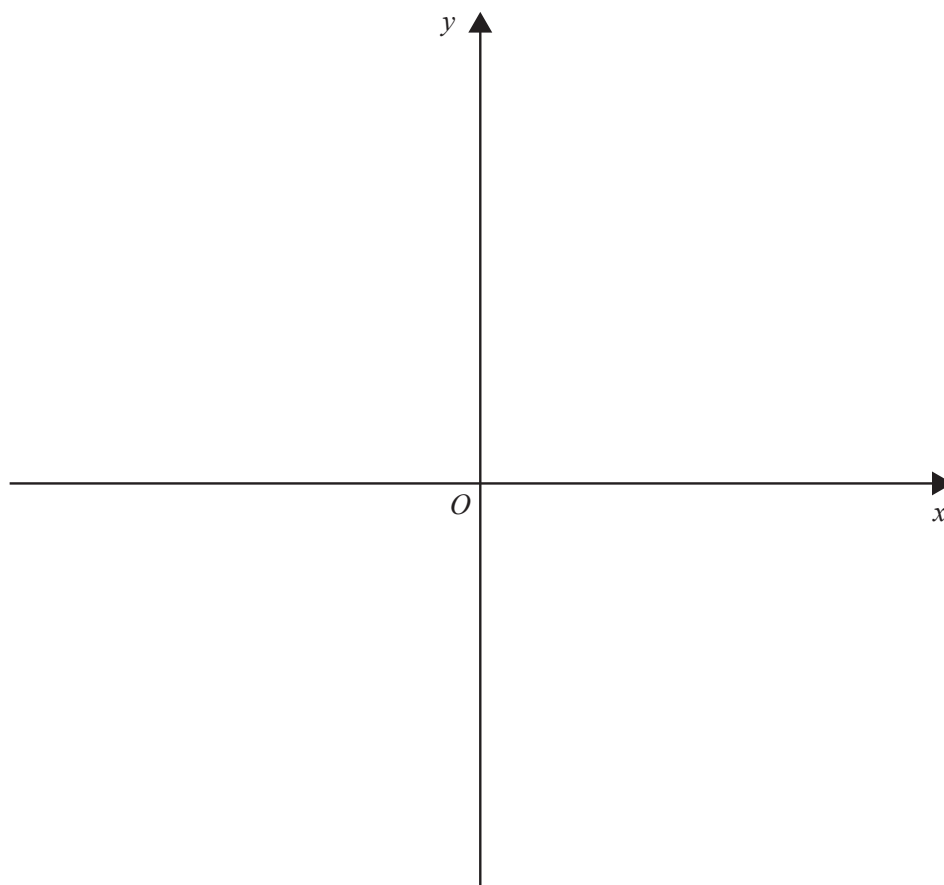
7 The curve C has equation $y = -\log_4(x + 4)$

- (a) Using the axes below, sketch the graph of C .
Label the coordinates of the points of intersection of C with the coordinate axes
and the equation of any asymptote to C .

(4)

- (b) Solve the equation $\log_{(x+4)} 256 - \log_4(x + 4) = 0$

(5)



- 9 (i) Solve the equation $3(\log_a 9 + \log_a 27) = 1$

Give your answer in the form $a = b^c$ where b is prime and c is an integer.

(3)

- (ii) Solve the equation $\log_4 p + \log_p 256 = -4$

(6)

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10-MARK QUESTIONS

1 question in this booklet



8 (a) Solve $5p^2 - 9p + 4 = 0$

(2)

(b) Hence solve $5^{2x+1} - 9(5^x) + 4 = 0$

Give your answers to 3 significant figures where appropriate.

(4)

The curve with equation $y = 5^{2x+1} + 5^x$ intersects the curve with equation $y = 2(5^{x+1}) - 4$ at two points.

(c) Find the coordinates of each of these two points.

Give your answers to 3 significant figures where appropriate.

(4)



11-MARK QUESTIONS

3 questions in this booklet



7 A curve C has equation $y = \log_{10}(x + 2)$

(a) Using the axes below, sketch the graph of C .

Label the coordinates of the points of intersection of C with the coordinate axes.

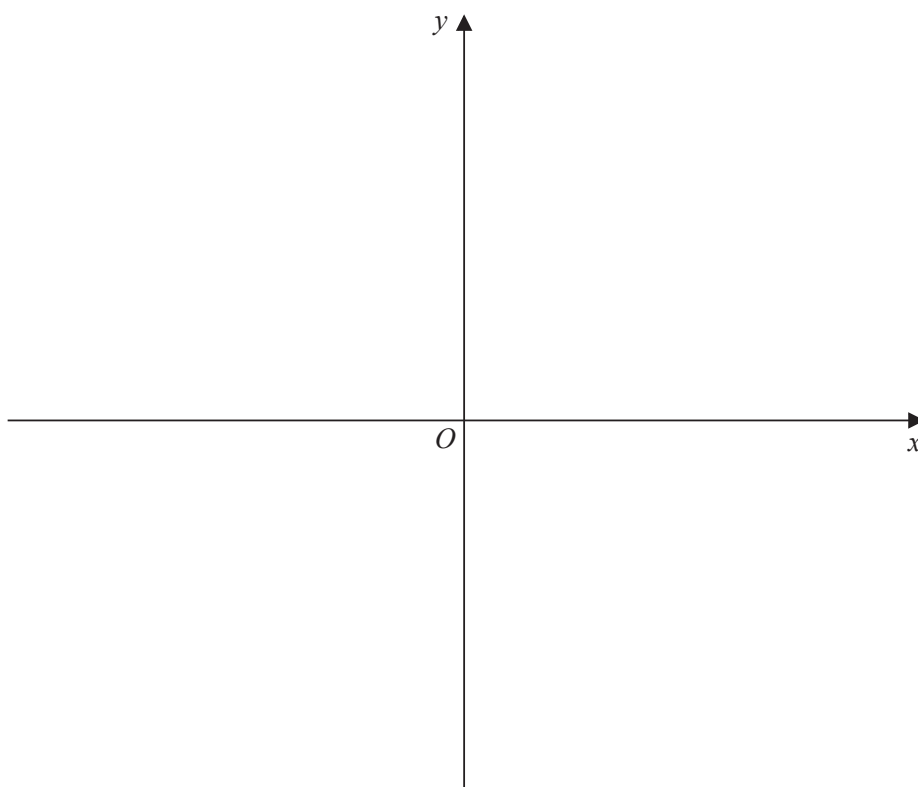
(2)

(b) Solve the equation $2(\log_a 4 + \log_a 16) = 1$

(3)

(c) Solve the equation $5\log_q 16 + 4\log_2 q = 24$

(6)



6 (i) Solve the equation $5(\log_b 9 + \log_b 3) = 3$

(4)

(ii) Solve the equation $3\log_3 x + 3\log_x 27 = 8\log_4 128$
Give your answers in exact form.

(7)

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10 Solve the equation

(i) $\log_4(6y-5) = 3$

(2)

(ii) $\log_4(4-3x)^2 - \log_2(x^2-5) - 3 = 0$

Show clear algebraic working.

Give your answer to 3 significant figures.

(9)

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12-MARK QUESTIONS

1 question in this booklet



9 (a) Solve the equation $2\log_p 9 + 3\log_3 p = 8$

(6)

Given that $\log_2 3 = \log_4 3^k$

(b) find the value of k

(2)

(c) Show that

$$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3 = \log_4 \left(\frac{x^{6x-5}}{3^{6x-20}} \right)$$

(4)

13-MARK QUESTIONS

1 question in this booklet



10 (a) Show that $\frac{9^{3y}}{243} = 3^{(6y-5)}$

(4)

(b) Solve the simultaneous equations

$$\frac{9^{3y}}{243} = 27^{(x-2)}$$

$$\log_{10} \sqrt{6xy} = \log_4 2$$

(9)

MARK SCHEMES

official Pearson Edexcel mark schemes



Question	Scheme	Marks
2	$\frac{4+2\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}}$ $= \frac{20+10\sqrt{3}+8\sqrt{3}+12}{25-4 \times 3} =$ (At least one intermediate step must be shown for M1 to be awarded)	M1 dM1A1 [3]
Total 3 marks		

Additional Guidance	
M1	For the correct use of $5+2\sqrt{3}$ seen as intention to multiply both the numerator and denominator. This need not be on one line. Do not accept $\frac{4+2\sqrt{3}}{5-2\sqrt{3}} \times (5+2\sqrt{3})$ for this mark unless multiplication of the numerator and denominator is later seen explicitly. This mark can be implied by later correct work, for example $\frac{20+10\sqrt{3}+8\sqrt{3}+12}{25-4 \times 3}$ Ignore poor/missing bracketing provided the work is correct.
M1	Multiplies out both denominator and numerator. The multiplication must be correct for the award of this mark.
A1	For the correct final answer $\frac{32+18\sqrt{3}}{13}$ There must be no errors seen for the award of this mark.

Question number	Scheme	Marks
1	$\frac{2\sqrt{3}-4}{3\sqrt{3}+5} \times \frac{3\sqrt{3}-5}{3\sqrt{3}-5}$ $= \frac{18-10\sqrt{3}-12\sqrt{3}+20}{27-25} \left(= \frac{38-22\sqrt{3}}{2} \right) \text{ oe}$ $= 19-11\sqrt{3} \text{ correct working throughout only}$	M1 dM1 A1 (3)
ALT	$2\sqrt{3}-4 = (3\sqrt{3}+5)(a+b\sqrt{3})$ $2\sqrt{3}-4 = 5a+9b+3\sqrt{3}a+5\sqrt{3}b \Rightarrow "5a+9b" = -4; "3a+5b" = 2$ $15a+27b = -12 \quad \text{or} \quad 25a+45b = -20$ $15a+25b = 10 \quad \quad \quad 27a+45b = 18$ $2b = -22 \Rightarrow b = -11 \quad \quad \quad 2a = 38 \Rightarrow a = 19$ $15a-297 = -12 \Rightarrow a = 19 \quad \quad \quad 57+5b = 2 \Rightarrow b = -11$ $= 19-11\sqrt{3} \text{ correct working throughout only}$	M1 dM1 A1 (3)
Total 3 marks		

Marks	Notes
M1	For multiplying by $\frac{3\sqrt{3}-5}{3\sqrt{3}-5}$ This may be seen as two separate calculations. Note, multiplying by $\frac{5-3\sqrt{3}}{5-3\sqrt{3}}$ is valid and will lead to all terms being the opposite and should be marked in the same way.
dM1	Dependent on M1 for attempting to multiply out the numerator and denominator. There may be up to 2 errors or omissions. $\frac{38-22\sqrt{3}}{2}$ is sufficient working
A1	For $19-11\sqrt{3}$ (Allow $a=19, b=-11$) As this result can be achieved with a calculator, there can be no incorrect working shown for this mark to be awarded.
ALT	
M1	For $(3\sqrt{3}+5)(a+b\sqrt{3})$
dM1	For attempting to multiply out $(3\sqrt{3}+5)(a+b\sqrt{3})$ to reach an expression that can be used to compare coefficients. There may be up to 2 errors or omissions.
A1	For forming 2 correct simultaneous equations and solving correctly to reach $19-11\sqrt{3}$ (Allow $a=19, b=-11$). Two common examples are shown in the scheme. As this result can be achieved with a calculator, there can be no incorrect working shown for this mark to be awarded.

Question number	Scheme	Marks
1	$w = \frac{50}{8 - \sqrt{6}} \times \frac{8 + \sqrt{6}}{8 + \sqrt{6}}$ $= \frac{400 + 50\sqrt{6}}{58}$ $= \frac{200 + 25\sqrt{6}}{29}$	M1 M1 A1 [3]
Total 3 marks		

Part	Mark	Additional Guidance
1	M1	For making w the subject, and applying a multiplication to rationalise the denominator $\frac{50}{8 - \sqrt{6}} \times \frac{8 + \sqrt{6}}{8 + \sqrt{6}}$
	M1	For carrying out the multiplication of the numerator and the denominator to reach single integer values in both, allow one processing error.
	A1	For correctly manipulating the fraction to reach the correct answer.

Question number	Scheme	Marks
1	$\frac{(a+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{2a+\sqrt{3}(a+2)+3}{1}$ $2a+\sqrt{3}(a+2)+3=11+b\sqrt{3} \Rightarrow 11=2a+3, \quad b=a+2$ Solves the equations in a and $b \Rightarrow a=4, \quad b=6$ ALT $\frac{a+\sqrt{3}}{2-\sqrt{3}} = 11+b\sqrt{3} \Rightarrow a+\sqrt{3} = (2-\sqrt{3})(11+b\sqrt{3})$ $\Rightarrow a+\sqrt{3} = (22-3b) + (2b-11)\sqrt{3}$ $\Rightarrow a=22-3b \text{ and } 1=2b-11$ Solves the equations in a and $b \Rightarrow a=4, \quad b=6$	M1 M1M1 A1 [4] {M1} {M1} {M1} {A1} [4]
Total 4 marks		
(a)	M1 Multiply by $\frac{(2+\sqrt{3})}{(2+\sqrt{3})}$ M1 For either $11=2a+3$ or $b=a+2$ M1 For $11=2a+3$ and $b=a+2$ A1 $a=4, \quad b=6$ ALT M1 Multiply by $2-\sqrt{3}$ M1 For either $a=22-3b$ or $1=2b-11$ M1 For $a=22-3b$ and $1=2b-11$ A1 $a=4, \quad b=6$	

Question	Scheme	Marks
1	$\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ Method A $\frac{a - \sqrt{48}}{\sqrt{3} + 1} = b\sqrt{3} - 9 \Rightarrow a - 4\sqrt{3} = 3b - 9 + \sqrt{3}(b - 9)$ $[a = 3b - 9 \text{ and } -4 = b - 9]$ $b = 5, a = 6$ Method B $\frac{(a - 4\sqrt{3})}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{a\sqrt{3} - a - 12 + 4\sqrt{3}}{2} = b\sqrt{3} - 9$ $b = 5, a = 6$	B1 M1 M1 A1 [4] [M1M1 A1]
Total 4 marks		

Question	Notes	Marks
1	$\frac{a - \sqrt{48}}{\sqrt{3} + 1} = b\sqrt{3} - 9$	
	Simplifies $\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ - Seen anywhere.	B1
	Method A – Both sides multiplied by $\sqrt{3} + 1$, collects terms and equates rational and irrational parts and obtains two equations at least one of which must be correct. $a - 4\sqrt{3} = (b\sqrt{3} - 9)(\sqrt{3} + 1) = (3b - 9) + \sqrt{3}(b - 9)$ $\Rightarrow a = 3b - 9 \text{ and } -4 = b - 9$	M1
	Solves their equations. The equation $-4 = b - 9$ must be solved correctly and the result substituted into the second equation to find a Allow one processing error here. This is an A mark in Epen	M1
	For $a = 6$ and $b = 5$	A1 [4]
	Simplifies $\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$	B1
	Method B – rationalises the denominator, collects terms and equates rational and irrational parts and obtains two equations at least one of which must be correct. $\frac{(a - 4\sqrt{3})(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{\sqrt{3}(a + 4) - (a + 12)}{2} = b\sqrt{3} - 9$ $\Rightarrow \frac{-(a + 12)}{2} = -9 \quad \frac{a + 4}{2} = b \quad \text{oe}$	M1
	Solves their equations: The equation $\frac{-(a + 12)}{2} = -9$ must be solved correctly and the result substituted into the second equation to find b Allow one processing error here. This is an A mark in Epen	M1
	For $a = 6$ and $b = 5$	A1 [4]
Total 4 marks		

Question	Scheme	Marks
1	$(4 - \sqrt{2})(a + b\sqrt{2}) = 10 + \sqrt{2} \Rightarrow a + b\sqrt{2} = \frac{10 + \sqrt{2}}{4 - \sqrt{2}}$	B1
	$\frac{(10 + \sqrt{2})}{(4 - \sqrt{2})} \times \frac{(4 + \sqrt{2})}{(4 + \sqrt{2})} = \frac{42 + 14\sqrt{2}}{14} = 3 + \sqrt{2} = a + b\sqrt{2}$	M1
	$\Rightarrow a = 3, \quad b = 1$	A1A1 [4]
	ALT	
	$(4 - \sqrt{2})(a + b\sqrt{2}) = 4a + 4b\sqrt{2} - a\sqrt{2} - 2b$ $\Rightarrow 4a - 2b + \sqrt{2}(4b - a) = 10 + \sqrt{2}$ $4a - 2b = 10$ $-a + 4b = 1$ $\Rightarrow a = 3, \quad b = 1$	B1 M1 A1A1 [4]
Total 4 marks		

Mark	Notes
B1	For forming the correct equation $[a + b\sqrt{2} =] \frac{10 + \sqrt{2}}{4 - \sqrt{2}}$ Allow as an expression with $a + b\sqrt{2}$ omitted.
M1	For multiplying the denominator and the numerator by $4 + \sqrt{2}$ The calculation must be shown. Minimum working is: $\frac{(10 + \sqrt{2})}{(4 - \sqrt{2})} \times \frac{(4 + \sqrt{2})}{(4 + \sqrt{2})} = \frac{42 + 14\sqrt{2}}{14}$
A1	Either $a = 3$ OR $b = 1$ Allow embedded.
A1	Both $a = 3$ AND $b = 1$ Allow embedded.
ALT	
B1	For forming a simplified expression in a and b for the area. Minimum required is expression of the form $4a + 4b\sqrt{2} - a\sqrt{2} - 2b$
M1	For solving the simultaneous equations in a and b Condone arithmetic errors, but process for solving must be correct.
A1	Either $a = 3$ OR $b = 1$ Allow embedded.
A1	Both $a = 3$ AND $b = 1$ Allow embedded.

Question	Scheme	Marks
1	$34 + 11\sqrt{5} = \frac{1}{2} \times (2 + 4\sqrt{5})(a + b\sqrt{5})$	M1
	$\Rightarrow a + b\sqrt{5} = \frac{2 \times (34 + 11\sqrt{5})}{2 + 4\sqrt{5}} \quad \text{or} \quad a + b\sqrt{5} = \frac{(34 + 11\sqrt{5})}{1 + 2\sqrt{5}} \quad \text{oe}$	M1
	$= \frac{(34 + 11\sqrt{5})}{(1 + 2\sqrt{5})} \times \frac{(1 - 2\sqrt{5})}{(1 - 2\sqrt{5})} = \frac{34 - 68\sqrt{5} + 11\sqrt{5} - 110}{-19} \quad \text{oe} \quad \text{or}$ $= \frac{(68 + 22\sqrt{5})}{(2 + 4\sqrt{5})} \times \frac{(2 - 4\sqrt{5})}{(2 - 4\sqrt{5})} = \frac{136 - 272\sqrt{5} + 44\sqrt{5} - 440}{-76} \quad \text{oe}$ $= 4 + 3\sqrt{5} \quad \text{or} \quad a = 4, \quad b = 3$	M1 A1 [4]
	ALT	
	$34 + 11\sqrt{5} = \frac{1}{2} \times (2 + 4\sqrt{5})(a + b\sqrt{5}) \quad \text{oe}$	M1
	$(34 + 11\sqrt{5}) = a + b\sqrt{5} + 2a\sqrt{5} + 10b \quad \text{oe eg} \quad 68 + 22\sqrt{5} = 2a + 2b\sqrt{5} + 4a\sqrt{5} + 20b$	M1
	$(34 + 11\sqrt{5}) = (a + 10b) + (2a + b)\sqrt{5} \quad \text{oe eg} \quad (68 + 22\sqrt{5}) = 2a + 20b + (4a + 2b)\sqrt{5}$	
	$\Rightarrow 34 = a + 10b \quad \text{and} \quad 11 = 2a + b \quad \text{oe eg} \quad \Rightarrow 68 = 2a + 20b \quad \text{and} \quad 11 = 4a + 2b$	M1
	Solves simultaneous equations by any method	A1
	$= 4 + 3\sqrt{5} \quad \text{or} \quad a = 4, \quad b = 3$	[4]
Total 4 marks		

Mark	Notes
M1	For the correct statement for the area, this may be explicit or implied by working, but must be a fully correct statement. Allow $a + b\sqrt{5}$ to be written as BC or denoted by a letter.
M1	For correctly rearranging to the form $a + b\sqrt{5} = \frac{2 \times (34 + 11\sqrt{5})}{2 + 4\sqrt{5}} \text{ oe or } a + b\sqrt{5} = \frac{(34 + 11\sqrt{5})}{1 + 2\sqrt{5}} \text{ oe}$ This mark may also be awarded for rearranging an initial formula, missing the $\frac{1}{2}$ ie for correctly rearranging $34 + 11\sqrt{5} = (2 + 4\sqrt{5})(a + b\sqrt{5})$ to $a + b\sqrt{5} = \frac{(34 + 11\sqrt{5})}{2 + 4\sqrt{5}} \text{ oe}$ Allow $a + b\sqrt{5}$ to be written as BC or denoted by a letter.
M1	For multiplying eg $\frac{(34 + 11\sqrt{5})}{(1 + 2\sqrt{5})} \times \frac{(1 - 2\sqrt{5})}{(1 - 2\sqrt{5})}$ or $\frac{(68 + 22\sqrt{5})}{(2 + 4\sqrt{5})} \times \frac{(2 - 4\sqrt{5})}{(2 - 4\sqrt{5})}$ - allow the 2 to remain factorised. or, if missing the $\frac{1}{2}$ for the first method mark $\frac{(34 + 11\sqrt{5})}{(2 + 4\sqrt{5})} \times \frac{(2 - 4\sqrt{5})}{(2 - 4\sqrt{5})} = \frac{68 - 136\sqrt{5} + 22\sqrt{5} - 220}{-76} \text{ oe}$ Allow one error in multiplying out. Look for equivalences. The question precludes the use of a calculator, so method must be shown.
A1	$4 + 3\sqrt{5}$ or $a = 4, b = 3$
ALT	
M1	For the correct statement for the area, this may be explicit or implied by working, but must be a fully correct statement. Allow $a + b\sqrt{5}$ to be written as BC or denoted by a letter.
M1	For multiplying out correctly as shown in the scheme. But if the initial formula is missing the $\frac{1}{2}$, the mark may also be awarded for $(34 + 11\sqrt{5}) = 2a + 2b\sqrt{5} + 4a\sqrt{5} + 20b$
M1	For correctly equating coefficients and any valid complete method to solve a pair of simultaneous equations, each equation in a and b, allow one error. This work may also be done with the $\frac{1}{2}$ missing from the formula and that doesn't count as 'one error'. The question precludes the use of a calculator, so method must be shown.
A1	$4 + 3\sqrt{5}$ or $a = 4, b = 3$

Question	Scheme	Marks
1	$\sqrt{50}x - \sqrt{18} > 6x + 5 \Rightarrow \sqrt{50}x - 6x > 5 + \sqrt{18} \Rightarrow x > \frac{5 + \sqrt{18}}{\sqrt{50} - 6}$ $\frac{5 + \sqrt{18}}{\sqrt{50} - 6} \times \frac{\sqrt{50} + 6}{\sqrt{50} + 6} = \frac{5\sqrt{50} + 30 + \sqrt{18}\sqrt{50} + 6\sqrt{18}}{14} = \frac{43\sqrt{2} + 60}{14}$ $x > \frac{43\sqrt{2} + 60}{14}$	M1 M1dM1 A1 [4]
Total 4 marks		

Mark	Notes
M1	Collects like terms and obtains a value for x in the minimally acceptable form: $x > \frac{a + \sqrt{18}}{\sqrt{50} - 6} \quad \text{o.e. for example } x > \frac{a + 3\sqrt{2}}{5\sqrt{2} - 6} \text{ where } a \text{ is an integer.}$ Please watch out for reversed signs For example, accept $\frac{-a - \sqrt{18}}{-\sqrt{50} + 6}$ Ignore the inequality and accept $>, <, =$ for this mark
M1	For showing the intent to multiply the numerator and denominator by the conjugate of their $(\sqrt{50} + 6 \text{ or } 5\sqrt{2} + 6)$ but minimally of the form $\sqrt{A} + B$ or $C\sqrt{2} + D$ e.g., $\frac{5 + \sqrt{18}}{\sqrt{50} - 6} \times \frac{\sqrt{50} + 6}{\sqrt{50} + 6}$ scores this mark. That is all that is required for this mark. Ignore $>, <, =$ for this mark
dM1	For explicitly multiplying out and simplifying their expression to obtain a value for x in the required form. Allow no more than one error in this simplification. However, it must be simplified as far as $\frac{A + \sqrt{B}}{C}$ or $\frac{P + Q\sqrt{R}}{T}$ where A, B, C, P, Q, R and T are integers Ignore $>, <, =$ for this mark
A1	For obtaining the value of x as given with the correct inequality. For candidates who give an estimated value [8.629....], please isw if you see the exact value first.

ALT 1 – Squares both sides [Ignore <, >, = for the first 3 marks]	
M1	Squares both sides without errors. $(\sqrt{50x} - \sqrt{18})^2 > (6x + 5)^2 \Rightarrow 50x^2 - 60x + 18 > 36x^2 + 60x + 25$
M1	Collects up their like terms and forms a 3TQ $\Rightarrow 14x^2 - 120x - 7 > 0$
dM1	Solves their 3TQ by either valid method using the formula or completing the square. $x = \frac{60 \pm 43\sqrt{2}}{14} \quad \text{oe eg., } x = \frac{60 \pm \sqrt{3698}}{14} \quad \text{Accept } \pm \text{ for this mark}$ We MUST see a method for the award of this mark. Do not award for roots appearing with no working. Ignore >, <, = for this mark
A1	For obtaining the value of x as given with the correct inequality. This has to be $x > \frac{43\sqrt{2} + 60}{14}$ For candidates who give an estimated value [8.629....] , please isw if you see the exact value first.
ALT 2 – Collects up like terms and squares [Ignore <, >, = for the first 3 marks]	
M1	Squares both sides without error. $(\sqrt{50x} - 6x)^2 > (5 + \sqrt{18})^2 \Rightarrow 50x^2 - 12\sqrt{50}x^2 + 36x^2 > 25 + 10\sqrt{18} + 18$
M1	Collects up like terms and forms a 2TQ $86x^2 - 12\sqrt{50}x^2 - (43 + 10\sqrt{18}) > 0 \Rightarrow [(86 - 60\sqrt{2})x^2 - (43 + 30\sqrt{18}) > 0]$
dM1	Solves their 2TQ by either valid method using the formula or completing the square. $x = \frac{43\sqrt{2} \pm 60}{14} \quad \text{oe eg., } x = \frac{\sqrt{3698} \pm 60}{14}$ We MUST see a method for the award of this mark. Do not award for roots appearing with no working.
A1	For obtaining the value of x as given with the correct inequality. This has to be $x > \frac{43\sqrt{2} + 60}{14}$ For candidates who give an estimated value [8.629....] , please isw if you see the exact value first.

Question number	Scheme	Marks
7	$\frac{\log_7 x^2}{\log_7 49}$ $\log_7 \left(\frac{8x^2 - 6x + 3}{x} \right), \log_7 2^3$ $\frac{8x^2 - 6x + 3}{x} = 2^3$ $8x^2 - 14x + 3 = 0$ $(4x - 1)(2x - 3) = 0$ $x = \frac{1}{4}, \frac{3}{2}$	B1 M1 A1 M1 A1 [5]

Mark	Additional Guidance	
B1	For changing the base of the log either to base 7 or base 49	
	$\log_{49} x^2 = \frac{\log_7 x^2}{\log_7 49} = \frac{\log_7 x^2}{2}$ OR $\log_{49} x^2 = \frac{2 \log_7 x}{\log_7 49} = \log_7 x$	$\log_7 (8x^2 - 6x + 3) = \frac{\log_{49} (8x^2 - 6x + 3)}{\log_{49} 7}$ $= 2 \log_{49} (8x^2 - 6x + 3)$ AND $\log_7 2 = \frac{\log_{49} 2}{\log_{49} 7} = 2 \log_{49} 2$
M1	For combining the LHS together into one log and dealing with the powers on both sides	
	$\left[\frac{1}{2} \log_7 x^2 = \log_7 x \right] \Rightarrow$ $\log_7 \left(\frac{8x^2 - 6x + 3}{x} \right), \log_7 2^3$	$\log_{49} \left(\frac{[8x^2 - 6x + 3]^2}{x^2} \right), \log_{49} 2^6$
dM1	For forming a 3TQ with their expressions which must have come from an acceptable attempt to deal with the logs This is an A mark in Epen	
	$8x^2 - 14x + 3 = 0$	$(8x^2 - 6x + 3)^2 = 64x^2 \Rightarrow$ $8x^2 - 6x + 3 = \pm 8x \Rightarrow 8x^2 - 14x + 3 = 0$ If this method is used they must reject the negative root of $64x^2$ (i.e. $-8x$) because it will form a quadratic equation with no real roots. $\{8x^2 + 2x + 3 = 0 \Rightarrow b^2 - 4ac = -92\}$
dM1	For attempting to solve their 3TQ $8x^2 - 14x + 3 = (4x - 1)(2x - 3) = 0 \Rightarrow x = \dots, \dots$	
A1	$x = \frac{3}{2}, \frac{1}{4}$	

Question number	Scheme	Marks
10	$\frac{\log_2 x}{\log_2 4} + \frac{\log_2 x}{\log_2 16} + \log_2 x = 10.5$	M1
	$\frac{\log_2 x}{2} + \frac{\log_2 x}{4} + \log_2 x = 10.5$	M1
	$\frac{7}{4} \log_2 x = 10.5$	M1
	$x = 2^{'6'}$	M1
	$x = 64$	A1 (5)
Total 5 marks		

Mark	Notes
The first two marks are common to both methods	
M1	Changes the base of any log correctly seen anywhere. Accept change to $\log_x$ or even $\log_{10}$ Base 2 $\frac{\log_2 x}{\log_2 4}, \frac{\log_2 x}{\log_2 16}$ Base 4 $\frac{\log_4 x}{\log_4 16}, \frac{\log_4 x}{\log_4 2}$ Base 16 $\frac{\log_{16} x}{\log_{16} 4}, \frac{\log_{16} x}{\log_{16} 2}$
M1	For forming an equation (in any form) in a single base in any base <u>For example;</u> Base 2 $\frac{\log_2 x}{2} + \frac{\log_2 x}{4} + \log_2 x = 10.5$ or e.g., $\frac{\log_2 x}{\log_2 4} + \frac{\log_2 x}{4} + \log_2 x = 10.5$ Base 4 e.g $\log_4 x + \frac{\log_4 x}{2} + \frac{\log_4 x}{\frac{1}{2}} = 10.5$ Base 16 $\frac{\log_{16} x}{\frac{1}{2}} + \log_{16} x + \frac{\log_{16} x}{\frac{1}{4}} = 10.5$ NB - This is an A mark in Epen

Method 1	
M1	For simplifying to the form $A \log_{(\text{any base})} = B$ where A and B are constants For example; Base 2 $1.75 \log_2 x = 10.5$ Base 41 $3.5 \log_4 x = 10.5$ Base 16 $7 \log_{16} x = 10.5$
M1	For undoing the log <u>For example:</u> Base 2 $x = 2^6$ Base 4 $x = 4^3$ Base 16 $x = 16^{1.5}$
Method 2	
M1	Uses the addition law to simplify to the form: $\log_n (x^a \times x^b \times x^c) = 10.5 \Rightarrow (a + b + c) \log_n x = 10.5$ For example: B Base 2 $\log_2 \left(x^{\frac{1}{2}} \times x^{\frac{1}{4}} \times x \right) = 10.5 \Rightarrow \frac{7}{4} \log_2 x = 10.5$ Base 4 $\log_4 \left(x \times x^{\frac{1}{2}} \times 2x \right) = 10.5 \Rightarrow \frac{7}{2} \log_4 x = 10.5$ Base 16 $\log_{16} (2x \times x \times 5x) = 10.5 \Rightarrow 7 \log_{16} x = 10.5$
M1	For undoing the log: Base 2 $2^{10.5} = x^{\frac{7}{4}}$ Base 4 $4^{10.5} = x^{\frac{7}{2}}$ Base 16 $16^{10.5} = x^7$
A1	For $x = 64$

Question	Scheme	Marks
1	$\frac{(a+2\sqrt{5})}{(3-\sqrt{5})} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{3a+a\sqrt{5}+6\sqrt{5}+10}{9-5} \left(= \frac{3a+10+(6+a)\sqrt{5}}{4} \right)$ $\left(\frac{3a+10+(6+a)\sqrt{5}}{4} = \frac{11+b\sqrt{5}}{2} \right)$ $\Rightarrow \frac{3a+10}{4} = \frac{11}{2} \quad \text{oe} \Rightarrow a=4 \Rightarrow \frac{6+a}{4} = \frac{b}{2} \quad \text{oe} \Rightarrow b=5$	M1 M1M1A1A1 [5]
	ALT $\left(\frac{(a+2\sqrt{5})}{(3-\sqrt{5})} = \frac{11+b\sqrt{5}}{2} \Rightarrow \right) 2(a+2\sqrt{5}) = (3-\sqrt{5})(11+b\sqrt{5})$ $2a+4\sqrt{5} = 33+3b\sqrt{5}-11\sqrt{5}-5b \quad (= (33-5b)+(3b-11)\sqrt{5})$ $\Rightarrow 4 = 3b-11 \Rightarrow b=5$ $\Rightarrow 2a = 33-5b \Rightarrow a=4$	M1 M1M1A1A1 [5]
Total 5 marks		

Mark	Notes
M1	For multiplying both numerator & denominator of $\frac{(a+2\sqrt{5})}{(3-\sqrt{5})}$ through by $\frac{(3+\sqrt{5})}{(3+\sqrt{5})}$ to give $\frac{3a+a\sqrt{5}+6\sqrt{5}+10}{9-5}$. Allow one error on the numerator. The denominator must be correct.
M1	For correctly equating their coefficients with $\frac{11+b\sqrt{5}}{2}$. Although this is not a dependent mark, there must be at least one equation in a and b
M1	For a complete and correct attempt to solve one of their equations to find a value for a or a value for b . Although this is not a dependent mark, there must be at least one equation in a and b.
A1	For either $a = 4$ or $b = 5$
A1	For both $a = 4$ and $b = 5$
Students may also multiply by $-3-\sqrt{5}$. This produces the correct answer and is the same as the main MS, but all the terms are negative. Mark to exactly the same principles.	
ALT	
M1	Correctly removes the denominators from the given equation and multiplies out as shown to give the equation $2a+4\sqrt{5}=33+3b\sqrt{5}-11\sqrt{5}-5b$ Allow one error.
M1	For correctly equating their coefficients on either side of the equation. Although this is not a dependent mark, there must be at least one equation in a and b
M1	For a correct and complete attempt to solve one of their equations to find a value for a or a value for b . Although this is not a dependent mark, there must be at least one equation in a and b
A1	For either $a = 4$ or $b = 5$
A1	For both $a = 4$ and $b = 5$

3	$\frac{a+b\sqrt{5}}{6-2\sqrt{5}} \times \frac{6+2\sqrt{5}}{6+2\sqrt{5}} = \frac{6a+10b+2a\sqrt{5}+6b\sqrt{5}}{36-20} = \left[\frac{(6a+10b)+\sqrt{5}(2a+6b)}{16} \right]$ $\frac{(6a+10b)+\sqrt{5}(2a+6b)}{16} = \left[\frac{(3a+5b)+\sqrt{5}(a+3b)}{8} \right] = \frac{9+4\sqrt{5}}{C}$ $\Rightarrow 9 = 6a+10b \quad \text{OR} \quad 9 = 3a+5b$ $\Rightarrow 4 = 2a+6b \quad \quad \quad 4 = a+3b$ $\Rightarrow a = \frac{7}{8}, \quad b = \frac{3}{8} \quad \text{OR} \quad \Rightarrow a = \frac{7}{4}, \quad b = \frac{3}{4}$ $\frac{\frac{7}{8} + \frac{3}{8}\sqrt{5}}{6-2\sqrt{5}} = \frac{9+4\sqrt{5}}{2 \times 8} \Rightarrow \left[\frac{\frac{7}{4} + \frac{3}{4}\sqrt{5}}{6-2\sqrt{5}} = \frac{9+4\sqrt{5}}{2 \times 4} \right] = \frac{7+3\sqrt{5}}{6-2\sqrt{5}} = \frac{9+4\sqrt{5}}{2}$ $\Rightarrow a = 7, b = 3 \text{ and } c = 2$ ALT $c(a+b\sqrt{5}) = (9+4\sqrt{5})(6-2\sqrt{5})$ $= 54 - 18\sqrt{5} + 24\sqrt{5} - 40$ $c(a+b\sqrt{5}) = 14 + 6\sqrt{5}, = 2(7+3\sqrt{5})$ $\Rightarrow c = 2 \quad a = 7 \quad b = 3$	M1 M1 M1 M1 A1 [5] [M1 M1 M1,M1 A1]
Total 5 marks		

Mark	Additional Guidance
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M1	Multiplies numerator and denominator by $6 + 2\sqrt{5}$ with no errors
M1	Equates numerator coefficients to find two simultaneous equations in a and b
M1	Solves their simultaneous equations by any method provided the equations of the form; $ma + nb = \dots$ and $pa + qb = \dots$ where $m \neq p$ and $n \neq q$ They must reach both a value for a and a value for b for this mark
M1	For multiplying through by 8 or 4 to give a prime number in the denominator of $\frac{9 + 4\sqrt{5}}{c}$
A1	For the correct values of a , b and c Allow embedded values.
ALT	
M1	‘Cross multiplies’ through by $(6 - 2\sqrt{5})$ AND by c
M1	Multiplies out both sides with no errors
M1	Collects up like terms to obtain $c(a + b\sqrt{5}) = A + B\sqrt{5}$
M1	Factorises their expression to obtain $P(Q + R\sqrt{5})$ where P , Q and R are integers or deduces by inspection that $a = 7$ and $c = 2$
A1	For the correct values – accept if embedded in the expression.

Question number	Scheme	Marks
3 (a)	$\log_3 9 = 2$	B1 [1]
(b)	$\log_3 9t = \log_9 \left(\frac{12}{t} \right)^2 + 2 \Rightarrow \log_3 9 + \log_3 t = 2(\log_9 12 - \log_9 t) + 2$ $\log_3 9 + \log_3 t = 2 \left(\frac{\log_3 12}{\log_3 9} - \frac{\log_3 t}{\log_3 9} \right) + 2$ $\Rightarrow \log_3 9 + \log_3 t = \log_3 12 - \log_3 t + 2$ $\Rightarrow 2 \log_3 t = \log_3 12 \Rightarrow \log_3 t^2 = \log_3 12$ $\Rightarrow t^2 = 12 \Rightarrow t = 2\sqrt{3}$	M1M1 M1 A1 M1A1 [6]
Total 7 marks		
(a) B1	$(\log_3 9 =) 2$	
(b) M1	The M marks can be seen anywhere in the solution Use of $\log AB = \log A + \log B$ or $\log \frac{A}{B} = \log A - \log B$	
M1	Use of $\log A^n = n \log A$	
M1	Use of $\log_a x = \frac{\log_b x}{\log_b a}$	
A1	Simplifying to $2 \log_3 t = \log_3 12$ oe or $\log_3 \left(\frac{9t^2}{12} \right) = 2$ oe	
M1	Simplify to $t^2 = \dots$	
A1	$t = 2\sqrt{3}$	

Question number	Scheme	Marks
9	$\frac{2^{4x}}{2^{3y}} = \frac{1}{2^2}$ $2^{4x-2y} = 2^{-2} \quad (\rightarrow 4x-3y = -2)$ $2^{2x}2^y = 2^4$ $2^{2x+y} = 2^4 \quad \rightarrow (2x+y = 4)$ A fully correct method using for solving simultaneously leading to either $10x = 10$ or $5y = 10$ $4x-3y = -2 \Rightarrow 10x = 10 \text{ or } 4x-3y = -2 \Rightarrow 5y = 10$ $6x+3y = 12 \quad \quad \quad 4x+2y = 8$ $y = 2$ $x = 1$	M1 dM1 M1 dM1 ddddM1 A1 A1 [7]
	Alternative Method $4^x = \frac{16}{2^y}$ $\frac{4^{2x}}{8^y} = \frac{1}{4}$ $\left(\frac{16}{2^y}\right)^2 \times \frac{1}{8^y} = \frac{1}{4}$ $8^y \times 2^{2y} = 4 \times 16^2$ $2^{3y} \times 2^{2y} = 2^2 \times 2^8$ $(2^{5y} = 2^{10}) \quad y = 2$ $(4^x \times 4 = 16) \quad x = 1$	M1 M1 ddM1 dddM1 ddddM1 A1 A1

Part	Mark	Additional Guidance
(a)	M1	For correctly changing any two indices into powers of 2 and simplifying. Accept any two of 2^2 or 2^{4x} or 2^{3y}
	dM1	Dependent on previous method mark. A fully correct method using index laws to simplify their expressions as powers of 2 and an attempt to write this as a linear equation.
	M1	For correctly changing both indices to powers of 2, as shown
	dM1	Dependent on previous method mark. A fully correct method using index laws to simplify their expressions as powers of 2 and an attempt to write this as a linear equation.
	ddddM1	Dependent on all previous method marks
	A1	$y = 2$
	A1	$x = 1$
ALT	M1	For a correct rearrangement of the 2 nd equation as shown
	M1	For converting the 16^x into 4^{2x} as shown.
	ddM1	Dependent on both previous method marks. Substitution of $\frac{16}{2^y}$ into the second equation, this need not be fully simplified.
	dddM1	Dependent on all previous method marks. An attempt to rearrange the equation, must have at least one side of the equation shown correct.
	ddddM1	Dependent on all previous method marks. An attempt to convert all into powers of 2, must see at least 2 of 2^{3y} , 2^{2y} , 2^2 , 2^8 correctly written.
	A1	$y = 2$
	A1	$x = 1$



Question Number	Scheme	Marks
4(i)	$\frac{16}{\log_4 r} = \log_4 r \Rightarrow 16 = (\log_4 r)^2 \Rightarrow \log_4 r = \pm 4$	M1
	$r = 4^4 = 256 \quad \text{or} \quad r = 4^{-4} = \frac{1}{256}$	A1 (2)
(ii)	$\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = \log_5 (9 \times 12 \times 15 \times 18) = \log_5 29160$	M1
	$1 + \log_5 x + \log_5 x^2 = \log_5 5 + \log_5 x + \log_5 x^2 = \log_5 5x^3$	M1A1
	$5x^3 = 29160$	dM1
	$x = 18$	A1 (5) [7]
ALT 1	$\text{LHS} = \log_5 29160$	M1
	$\text{RHS} = 1 + \log_5 x^3$	M1
	$\left(\frac{\log_{10} 29160}{\log_{10} 5} \right) = 6.387... (= \log_5 x^3 + 1)$	A1
	$5.387... = 3 \log_5 x$	dM1
	$\log_5 x = 1.795...$	
	$x = 18$	A1
ALT 2	$\text{LHS} = \log_5 29160$	M1
	$\text{RHS} = \log_5 5 + \log_5 x^3$	M1A1
	$\log_5 29160 = \log_5 5 + \log_5 5832$	
	$5832 = x^3$	dM1
	$x = 18$	A1
ALT 3	$\text{LHS} = \log_5 5832 + \log_5 5$	M1
	$\text{RHS} = 1 + \log_5 x^3$	M1
	$\text{LHS} = \log_5 5832 + 1$	A1
	$\log_5 5832 = \log_5 x^3$	
	$5832 = x^3$	dM1
	$x = 18$	A1
ALT 4	$\log_5 29160 - \log_5 x^3 = 1$	M1M1
	$\log_5 \frac{29160}{x^3} = 1$	A1
	$\frac{29160}{x^3} = 5 \Rightarrow x^3 = 5832$	dM1
	$x = 18$	A1

(i)	
M1	Change base (can have base 4 or base r provided the same for both logs), multiply to remove the fraction and solve to $\log_4 r = \dots$ (or $\log_r 4 = \dots$) (One answer only is sufficient)
A1	Complete to the correct answers, both needed
(ii)	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
M1	Change 1 to $\log_5 5$ and obtain a single log for the RHS
A1	Correct single log for RHS (Requires second M mark, not first)
dM1	Use LHS = RHS to obtain an equation without logs Depends on both previous M marks
A1	Correct answer
ALT 1	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
M1	Combine the two logs on RHS
A1	Correct numerical value for LHS . This will need a calculator so change of base need not be seen. Equation need not be formed yet. Correct final answer implies correct value here. Otherwise min 3 sf needed
	This mark requires the first M mark to have been given – the second M mark can be M0
dM1	Use LHS = RHS to obtain a value for $3\log_5 x$ or $\log_5 x$
	Depends on both previous M marks
A1	Correct answer. This will be exact if all numbers stored on the calculator so accept 18 only.
ALT 2	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
	Alternatively we may see $\text{LHS} = \log_5 5 + \log_5 5832$ without ever seeing $\text{LHS} = \log_5 29160$
M1	Combine the 2 logs on RHS and change 1 to $\log_5 5$
A1	Correct RHS (Requires second M mark, not first)
dM1	Use LHS = RHS to obtain a value for x^3 Depends on both previous M marks
A1	Correct answer
ALT 3	
M1	Split $\log_5 15$ and combine all logs apart from $\log_5 5$ to a single log
M1	Combine the two logs on RHS
A1	Change $\log_5 5$ to 1 and have the correct log on LHS
	This mark requires the first M mark to have been given – the second M mark can be M0
dM1	Use LHS = RHS to obtain a value for x^3 Depends on both previous M marks
A1	Correct answer
ALT 4	
M1	Combine the LHS logs to a single log. Numbers should be multiplied – if added award M0
M1	Combine the two logs from the RHS
A1	Obtain the equation shown
dM1	Obtain a value for x^3 Depends on both previous M marks
A1	Correct answer

Question number	Scheme	Marks
8	$\log_x 3 = \frac{1}{\log_3 x}$ Let $y = \log_3 x$ So $y - \frac{2}{y} = 1$ $y^2 - y - 2 = 0$ $(y - 2)(y + 1) = 0$ $\log_3 x = 2 \text{ or } \log_3 x = -1$ $x = 9 \text{ or } x = \frac{1}{3}$	B1 M1 A1 M1 M1 A1 A1 [7]
Notes		
B1	For use of $\log_a x = \frac{1}{\log_b a}$	
M1	For $y - \frac{2}{y} = 1$ oe	
A1	For rearranging to a 3 TQ	
M1	For solving the 3 TQ	
M1	For either $\log_3 x = 2$ or $\log_3 x = -1$	
A1	For $x = 9$	
A1	For $x = \frac{1}{3}$	

Question number	Scheme	Marks
5	$\log_3 \sqrt{x-5} + \log_9 (x+3) - 1 = 0$ $\frac{1}{2} \log_3 (x-5) + \frac{\log_3 (x+3)}{\log_3 9} = 1 \Rightarrow \left(\frac{1}{2} \log_3 (x-5) + \frac{\log_3 (x+3)}{2} = 1 \right)$ $\log_3 (x-5) + \log_3 (x+3) = 2 \Rightarrow \log_3 [(x-5)(x+3)] = 2$ $\Rightarrow (x-5)(x+3) = 3^2 \Rightarrow x^2 - 2x - 24 = 0$ $(x+4)(x-6) = 0 \Rightarrow x = 6$ (reject $x = -4$)	M1M1 M1 M1A1 dM1A1 [7]
Total 7 marks		

Mark	Notes
Method 1 – Works in base 3	
M1	Uses $n \log A = \log A^n$ correctly to write $\log_3 \sqrt{x-5} = \frac{1}{2} \log_3 (x-5)$
M1	For an attempt to change the base of $\log_9 (x+3)$ to base 3 using $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_9 (x+3) = \frac{\log_3 (x+3)}{\log_3 9} = \frac{\log_3 (x+3)}{2}$ [accept $\frac{\log_3 (x+3)}{p}$ where $p \neq 1$]
M1	Uses $\log A + \log B = \log AB$ to correctly combine the logs $\log_3 (x-5) + \log_3 (x+3) = \log_3 (x-5)(x+3)$
M1	For removing the logs in the equation to obtain $(x-5)(x+3) = 3^2$ and rearranging to a 3TQ
A1	For obtaining a correct 3TQ. $x^2 - 2x - 24 = 0$
Method 2 – Works in base 9	
M1	Uses $n \log A = \log A^n$ correctly to write $\log_3 \sqrt{x-5} = \frac{1}{2} \log_3 (x-5)$
M1	For an attempt to change the base of $\log_3 \sqrt{x-5}$ or $\frac{1}{2} \log_3 (x-5)$ to base 9 using $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_3 \sqrt{x-5} = \frac{\log_9 \sqrt{x-5}}{\log_9 3} = \frac{\log_9 \sqrt{x-5}}{1/2} = 2 \log_9 \sqrt{x-5}$ [accept $q \log_9 \sqrt{x-5}$ where $q \neq 1$] $\frac{1}{2} \log_3 (x-5) = \frac{1}{2} \times \frac{\log_9 (x-5)}{\log_9 3} = \frac{1}{2} \times \frac{\log_9 (x-5)}{1/2} = \log_9 (x-5)$
M1	Uses $\log A + \log B = \log AB$ to correctly combine the logs $\log_9 (x-5) + \log_9 (x+3) = \log_9 (x-5)(x+3)$
M1	For removing the logs in the equation to obtain $(x-5)(x+3) = 9$ and rearranging to a 3TQ
A1	For obtaining a correct 3TQ. $x^2 - 2x - 24 = 0$
Attempt to solve the quadratic equation	
dM1	For an attempt to solve their 3TQ. See General Guidance for the definition of an attempt. Dependent on at least one previous M mark scored.
A1	$x = 6$ Must reject $x = -4$ if this solution is also included.

Question	Scheme	Marks
10	$8\log_x 64 = \frac{8\log_4 64}{\log_4 x}$	M1
	$\log_4 x^3 = 3\log_4 x$	M1
	$\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \frac{8\log_4 64}{\log_4 x} = 22$	M1
	$\Rightarrow 3(\log_4 x)^2 + 8\log_4 64 = 22\log_4 x \Rightarrow 3(\log_4 x)^2 - 22\log_4 x + 24 = 0$	
	$3(\log_4 x)^2 - 22\log_4 x + 24 = 0 \Rightarrow (3\log_4 x - 4)(\log_4 x - 6) = 0$	M1
	$\Rightarrow \log_4 x = \frac{4}{3}, 6$	A1
	$x = 4^{\frac{4}{3}}$ or awrt 6.35 and $x = 4096$	M1A1 [7]
Total 7 marks		

NOTE WELL! This can be solved using a modern calculator. No working = no marks.
Award marks only for work explicitly seen.

Mark	Notes
Works in base 4	
M1	For changing the base of the log correctly to base 4 $3\log_4 x + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \frac{8\log_4 4^3}{\log_4 x} = 22$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_4 64 = 3\log_4 4$
M1	For multiplying through by $\log_4 x$ and forming a 3TQ in log base 4
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 They must obtain two values for their log.
A1	For both correct values of $\log x$ $\left[\frac{4}{3}, 6 \right]$
M1	For undoing either log correctly. Allow this mark for any erroneous log they find, but undo correctly.
A1	For both values of x Accept $4^{\frac{4}{3}}$ or awrt 6.35 and 4^6 or 4096

Works in base x	
M1	For changing the base of the log correctly to base x or vice versa. $\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow \frac{3\log_x x}{\log_x 4} + 8\log_x 4^3 = 22$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_x 64 = 3\log_x 4$
M1	For multiplying through by $\log_x 4$ and forming a 3TQ in log base x $\frac{3}{\log_x 4} + 24\log_x 4 = 22 \Rightarrow 24(\log_x 4)^2 - 22\log_x 4 + 3 = 0$
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 $24(\log_x 4)^2 - 22\log_x 4 + 3 = 0 \Rightarrow (4\log_x 4 - 3)(6\log_x 4 - 1) = 0 \Rightarrow \log_x 4 = \dots, \dots$ They must obtain two values for their log.
A1	For both correct values. $\log_x 4 = \frac{3}{4}, \frac{1}{6}$
M1	For undoing either log correctly, but they must obtain a value for $x = \dots$ $x = 4^{\frac{4}{3}}$ or $x = 4^6$ Allow this mark for any erroneous log they find, but undo correctly.
A1	Accept $4^{\frac{4}{3}}$ or awrt 6.35 and 4^6 or 4096
Works in base 2	
M1	Changes the base of at least one log to base 2. $\frac{3}{2}\log_2 x, \frac{8\log_x x^6}{\log_2 x}$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_2 64 = 6\log_2 2$
M1	For multiplying through by $\log_2 x$ and forming a 3TQ in base 2 $\frac{3}{2}(\log_2 x)^2 - 22\log_2 x + 48 = 0 \Rightarrow [3(\log_2 x)^2 - 44\log_2 x + 96 = 0]$
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 $(12\log_2 x - 1)(8\log_2 x - 3) = 0 \Rightarrow \log_2 x = \dots, \dots$
A1	For both correct values. $\log_2 x = \frac{8}{3}, 12$
M1	For undoing either log correctly. $x = 2^{\frac{8}{3}}$ or $x = 2^{12}$ Allow this mark for any erroneous log they find, but undo correctly.
A1	Accept $2^{\frac{8}{3}}$ or awrt 6.35 and 2^{12} or 4096

Question number	Scheme	Marks
8 (a)	$\log_a n = \log_a 3(2n-1)$	M1
	$n = 3(2n-1)$	M1
	$n = \frac{3}{5}$	A1 (3)
(b)(i)	$x = p^3$	B1 (1)
(b)(ii)	$\log_p y - \log_p 2^3 = 4 \Rightarrow \log_p \left(\frac{y}{2^3} \right) = 4$ or $\log_p \left(\frac{y}{8} \right) = 4$	M1
	$\frac{y}{2^3} = p^4 \Rightarrow (y = 2^3 p^4 \text{ or } 8p^4)$	M1
	$xy = 8p^7$	M1A1 (4)
	ALT (b)(ii)	
	$\log_p x + \log_p y - 3\log_p 2 = 4 + 3 \Rightarrow \log_p \left(\frac{xy}{2^3} \right) = 7$	{M1}
	$\frac{xy}{2^3} = p^7$	{M1}
	$xy = 8p^7$	{M1A1} (4)
Total 8 marks		

Part	Mark	Notes
(a)	M1	Uses the addition law of logs correctly $\log_a n = \log_a 3 + \log_a (2n-1) \Rightarrow \log_a n = \log_a 3(2n-1)$ Accept also $\log_a n = \log_a 3 + \log_a (2n-1) \Rightarrow 0 = \log_a \left(\frac{n}{3(2n-1)} \right) = (\log_a 1)$
	M1	For obtaining a linear equation from their log equation and attempting to find a value for n . $n = 3(2n-1)$ leading to a numerical value for n
	A1	For $n = \frac{3}{5}$
(b)(i)	B1	For $x = p^3$
(b)(ii)	M1	For stating that $3\log_p 2 = \log_p 8$ or $\log_p 2^3$ and for using the addition law correctly to combine the LHS: $\log_p y - \log_p 2^3 = 4 \Rightarrow \log_p \left(\frac{y}{2^3} \right) = 4$ or $\log_p \left(\frac{y}{8} \right) = 4$
	M1	Correctly removes logs on both sides to obtain: $\frac{y}{2^3} = p^4 \Rightarrow (y = 2^3 p^4 \text{ or } 8p^4)$
	M1	For correctly finding the product of their x and their y : $xy = 'p^3 \times 8p^4'$
	A1	For the correct answer of $xy = 8p^7$
	ALT	
	M1	For stating that $3\log_p 2 = \log_p 8$ or $\log_p 2^3$ and states $\log_p x + \log_p y - 3\log_p 2 = 3 + 4$ Uses the addition law correctly to combine the LHS $\log_p x + \log_p y - 3\log_p 2 = 4 + 3 \Rightarrow \log_p \left(\frac{xy}{2^3} \right) = 7$
	M1	Correctly remove logs on both sides to obtain: $\frac{xy}{2^3} = p^7$
	M1	Correctly rearrange their expression to make xy the subject
	A1	For the correct answer of $xy = 8p^7$

Question number	Scheme	Marks
8	$\log_4 a + 2\log_4 b = \frac{5}{2}$	M1
	$\log_4(ab^2) = \frac{5}{2}$	M1
	$32 = ab^2$	A1
	$2^a = \frac{2^{16}}{2^{2b^2}}$	M1
	$a = 16 - 2b^2$ or $b^2 = 8 - \frac{1}{2}a$	A1
	$32 = a(8 - \frac{1}{2}a)$ or $32 = (16 - 2b^2)b^2$	M1
	$a^2 - 16a + 64 = 0$ or $2b^4 - 16b^2 + 32 = 0$	A1
	$a = 8$ $b = 2$	A1
Total 8 marks		

Mark	Guidance
Log equation Method 1 – Works in base 4	
M1	For an attempt to change the base of $3\log_8 b$ to base 4 using $\log_a x = \frac{\log_b x}{\log_b a}$ $3\log_8 b = \frac{3\log_4 b}{\log_4 8} = \frac{3\log_4 b}{\frac{3}{2}} = 2\log_4 b$ [accept $p\log_4 b$ where $p \neq 3$]
M1	Uses $n\log A = \log A^n$ and $\log A + \log B = \log AB$ to combine the logs correctly $\log_4(ab^2) = \frac{5}{2}$ [ft their p provided $p \neq 1$]
A1	For removing the logs in the equation to obtain $32 = ab^2$ o.e. e.g. $a^2b^4 = 1024$
Method 2 – Works in base 8	
M1	For an attempt to change the base of $\log_4 a$ to base 8 using $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_4 a = \frac{\log_8 a}{\frac{2}{3}} = \frac{3\log_8 a}{2}$ [accept $q\log_8 a$ where $q \neq 1$]
M1	Uses $n\log A = \log A^n$ and $\log A + \log B = \log AB$ correctly to combine the logs $\log_8(a^{\frac{3}{2}}b^3) = \frac{5}{2}$ [ft their q]
A1	For removing the logs in the equation to obtain $a^{\frac{3}{2}}b^3 = 8^{\frac{5}{2}}$ and rearranges (raises both sides to the power of $\frac{2}{3}$) to obtain $32 = ab^2$
Second equation	
M1	For attempting to change the second equation to powers of 2 or 4: $2^a = \frac{2^{16}}{2^{2b^2}} \Rightarrow \left[2^a = 2^{(16-2b^2)} \right]$ or $4^{\frac{a}{2}} = \frac{4^8}{4^{b^2}} = \left(4^{\frac{a}{2}} = 4^{8-b^2} \right)$ At least one correct change of term e.g either 2^{16} or 2^{2b^2} OR either $4^{\frac{a}{2}}$ or 4^8
A1	Combines the powers to achieve $a = 16 - 2b^2$ or $\frac{a}{2} = 8 - b^2$ oe
Attempt to solve the simultaneous equations	
M1	For an attempt to solve their equations simultaneously, both of which must be in terms of a and b^2 , to obtain a 3TQ in either a or b^2 . $32 = a(8 - \frac{1}{2}a) \Rightarrow a^2 - 16a + 64 = 0$ or $32 = (16 - 2b^2)b^2 \Rightarrow 2b^4 - 16b^2 + 32 = 0$
M1	For an attempt to solve their 3TQ in either a or b^2 by any method. See General Guidance for the definition of an attempt For example: $a^2 - 16a + 64 = 0 \Rightarrow (a - 8)(a - 8) = 0 \Rightarrow a = \dots$ $2b^4 - 16b^2 + 32 = 0 \Rightarrow b^4 - 8b^2 + 16 = 0 \Rightarrow (b^2 - 4)(b^2 - 4) = 0 \Rightarrow b = \dots$
A1	For $a = 8$ and $b = 2$ [If $b = \pm 2$ is given as the final answer, withhold this mark].

Question	Scheme	Marks
7(a)	4	B1 [1]
(b)	Working in $\log_2$ $2 \log_4 x = \frac{2 \log_2 x}{\log_2 4} = \frac{2 \log_2 x}{2} = [\log_2 x]$ $\log_2 16 + \frac{2 \log_2 x}{2} = \log_2 y$ $\log_2 16x = \log_2 y$ OR $\log_2 \left(\frac{x}{y}\right) = -4 \Rightarrow \frac{x}{y} = 2^{-4}$ $y = 16x^*$	M1 M1 M1 A1 cso [4]
	ALT Working in $\log_4$ $\log_2 y = \frac{\log_4 y}{\log_4 2} = \frac{\log_4 y}{\frac{1}{2}} = 2 \log_4 y = [\log_4 y^2]$ $\log_4 256 + \log_4 x^2 = \log_4 y^2$ $\log_4 (256x^2) = \log_4 y^2$ OR $2 \log_4 \left(\frac{y}{x}\right) = 4 \Rightarrow \frac{y}{x} = 4^2$ $256x^2 = y^2 \Rightarrow y = 16x^*$	[M1 M1 M1 A1]
(c)	$16x = 4x + 5$ $16x = 4x + 5 \Rightarrow 12x = 5 \Rightarrow x = \dots$ $x = \frac{5}{12}$	B1 M1 A1 [3]
Total 8 marks		

Part	Marks	Scheme
(a)	B1	States 4 only
(b)	M1	For an attempt to change the base of $\log_4 x$ to base 2 using $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_4 x = \frac{\log_2 x}{\log_2 4} \left[= \frac{\log_2 x}{2} \right]$
	M1	An attempt to rewrite the equation in terms of $\log_2$ $\log_2 16 + \frac{2 \log_2 x}{2} = \log_2 y$ F.t. their '2' from attempted change of base.
	M1	Uses $\log A + \log B = \log AB$ to correctly combine the logs $\log_2 16x = \log_2 y$ OR Uses $\log A - \log B = \log \frac{A}{B}$ to correctly combine the logs and removes logs $\log_2 \left(\frac{x}{y}\right) = -4$ and $\frac{x}{y} = 2^{-4}$ (this approach will score the second and third M marks at this stage)
	A1	For correctly obtaining $y = 16x^*$



Alt – working in $\log_4$		
	M1	For an attempt to change the base of $\log_2 y$ to base 4 using $\log_a y = \frac{\log_b y}{\log_b a}$ $\log_2 y = \frac{\log_4 y}{\log_4 2} \left[= \frac{\log_4 y}{\frac{1}{2}} = 2 \log_4 y \right]$
	M1	For dealing with the indices and writing $4 = \log_4 256$ $\log_4 256 + \log_4 x^2 = \log_4 y^2$
	M1	Uses $\log A + \log B = \log AB$ to correctly combine the logs $\log_4 (256x^2) = \log_4 y^2$ OR Uses $\log A - \log B = \log \frac{A}{B}$ to correctly combine the logs and removes logs $2 \log_4 \left(\frac{y}{x} \right) = 4$ and $\frac{y}{x} = 4^2$ (this approach will score the second and third M marks at this stage)
	A1	For correctly obtaining $y = 16x^*$
(c)	B1	For writing down $16x = 4x + 5$
	M1	For an attempt to solve the equation $16x = 4x + 5 \Rightarrow 12x = 5 \Rightarrow x = \dots$
	A1	For $x = \frac{5}{12}$
	Note: This is a ‘hence’ question. Condone candidates working without using given results, but the first mark is not awarded until the candidate reaches $16x = 4x + 5$	

Question number	Scheme		Marks
6 (a)	Method 1 $\left(\frac{a+\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}\right) = \frac{\sqrt{5}a+2a+5+2\sqrt{5}}{(1)}$ $= \left[2a+5+(2+a)\sqrt{5}\right]$ $a = 3$	Method 2 $((11+5\sqrt{5})(\sqrt{5}-2))$ $11\sqrt{5}-22+25-10\sqrt{5}$ $a = 3$	M1 A1 [2]
(b)	(angle $PQR = 45^\circ$ $\frac{x+3}{\sin 45^\circ} = \frac{x}{\sin 30^\circ} \Rightarrow \frac{x+3}{\frac{\sqrt{2}}{2}} = \frac{x}{\frac{1}{2}} \Rightarrow x+3 = \sqrt{2}x$ $\Rightarrow 3 = x(\sqrt{2}-1) \Rightarrow x = \frac{3}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \Rightarrow x = \dots$ $x = 3+3\sqrt{2}$ OR $\frac{x+3}{\sin 45^\circ} = \frac{x}{\sin 30^\circ} \Rightarrow \frac{x+3}{\frac{\sqrt{2}}{2}} = \frac{x}{\frac{1}{2}} \Rightarrow \sqrt{2}x+3\sqrt{2} = 2x$ $x(2-\sqrt{2}) = 3\sqrt{2} \Rightarrow x = \frac{3\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} \Rightarrow x =$ $x = 3+3\sqrt{2}$		M1 M1 A1* cso [3] [M1 M1 A1]
(c)	$\frac{1}{2} \times (3+3\sqrt{2}) \times (6+3\sqrt{2}) \times \sin(180-30-45)^\circ$ $\frac{1}{2} \times (3+3\sqrt{2}) \times (6+3\sqrt{2}) \times \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$ $\frac{9}{8}(4\sqrt{6}+4\sqrt{2}+3\sqrt{12}+6) \text{ or } \frac{1}{8}(36\sqrt{6}+36\sqrt{2}+54\sqrt{12}+54)$ oe with two out of four terms correct. $\frac{9}{8}(4\sqrt{6}+4\sqrt{2}+6\sqrt{3}+6)$		M1 dM1 A1 cao [3]
Total 8 marks			

Part	Mark	Additional Guidance
Throughout this question allow missing degree signs.		
(a)	M1	Any three out of four terms correct in the numerator of method 1 or in the expansion of method 2. If they use method 1 we must see the expansion come from multiplying numerator and denominator by $\sqrt{5} + 2$
	A1	Cao If the answer $a = 3$ is stated without sufficient working seen award M0A0.
(b)	M1	For using a correct sine rule, either way around, using the exact values of $\sin 45$ and $\sin 30$, and attempting to simplify to $kx + 3 = \sqrt{2}x$ where k is a constant. Allow $\frac{kx}{2} + \frac{3}{2} = \frac{\sqrt{2}x}{2}$ for this mark
	M1	For factorising their $x + 3 = \sqrt{2}x$ [which must have come from using sine rule] to obtain $3 = x(\sqrt{2} - k)$, rearranging and multiplying numerator and denominator by $\sqrt{2} + k$ This is a given answer – this step must be seen explicitly
	A1*	cso, no errors or emissions.
(c)	M1	Follow general guidance, if area formula is quoted, allow one slip in substitution. If formula not quoted, must be a fully correct substitution. For this mark only allow the angle to be $(180 - 30 - 45)^\circ$ If they work in terms of x and obtain $\frac{1}{2} \times x \times (x + 3) \times \sin 105^\circ$ then do not award the first M mark until they have substituted the given value of $3 + 3\sqrt{2}$ for x .
	dM1	Replaces $\sin 105^\circ$ with the given value and attempts to multiply out. The mark can be awarded for any two terms correct, four terms must be present.
	A1cao	Must be in the given form with the terms in any order.



Mark	Additional Guidance
M1	For correctly rearranging and taking logs to obtain $2y = \ln(x-2)$ or $y = \frac{1}{2} \ln(x-2)$
M1	For correct substitution of their expression for y or $2y$ into $\ln(x+3) - 2y - 1 = 0$
M1	For correct use of $\log_a \frac{x}{y} = \log_a x - \log_a y$ with their equation.
M1	For correctly converting their log equation to exponential form.
ddddM1	For correctly multiplying throughout to eliminate any denominators and collecting x terms on one side of the equation and making x subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $x = 4.91$ or better e.g. exact form (Calc gives: 4.90988...) accept any answer which rounds to 4.91
M1	For correct substitution of their value for x into a correct equation leading to a value for y
A1	For $y = 0.53$ Accept any answer which rounds 0.53 (calculator gives: 0.5340565...)
ALT 1	
M1	Correctly rearranging to make x the subject and correctly substituting into $\ln(x+3) - 2y - 1 = 0$
M1	A correct rearrangement of their equation in y to give an equation which then allows them to correctly convert their log equation to exponential form.
M1	Correctly applying the multiplication law for powers
M1	Correctly rearranging their equation and correctly factorising e^{2y}
ddddM1	For dividing by $e - 1$ and converting to log form to make y the subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $y = 0.53$ or better e.g. exact form (Calc gives: 0.5340565...), accept any answer which rounds to 0.53
M1	For correct substitution of y into a correct equation leading to a value for x
A1	For $x = 4.91$ Accept any answer which rounds to 4.91 (Calc gives: 4.90988...)
ALT 2	
M1	Correctly rearranges and converts to exponential form to obtain $x = e^{2y+1} - 3$
M1	A correct substitution of their x into $e^{2y} - x + 2 = 0$
M1	Correct use of the multiplication law for powers
M1	Correct rearrangement of their equation and factorisation of e^{2y}
ddddM1	For dividing by $e - 1$ and converting to log form to make y the subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $y = 0.53$ or better e.g. exact form (Calc gives: 0.5340565...), accept any answer which rounds to 0.53
Final 2 marks	As ALT1
ALT 3	
M1	Correctly rearranges to make $2y$ the subject $2y = \ln(x+3) - 1$
M1	Correctly raises each side to be a power of e $e^{2y} = e^{\ln(x+3)-1}$
M1	Correct use of the multiplication rule for powers and simplification to give $e^{2y} = \frac{x+3}{e}$
M1	Correct substitution of their expression for e^{2y} (usually equating the two equations)
ddddM1	For correctly multiplying throughout to eliminate any denominators and collecting x terms on one side of the equation and making x subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $x = 4.91$ or better e.g. exact form (Calc gives: 4.90988...) accept any answer which rounds to 4.91
Final 2 marks	As main scheme

Question	Scheme	Marks
6	$\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \log_2 x^3 + \frac{\log_2 x^2}{\log_2 4} - \frac{3\log_2 2}{\log_2 x} = 0$	M1
	$= 3\log_2 x + \frac{2\log_2 x}{\log_2 4} - \frac{\log_2 2^3}{\log_2 x} = 0$	M1
	$3\log_2 x + \frac{2\log_2 x}{2} - \frac{3}{\log_2 x} = 0$	B1
	$\Rightarrow 3(\log_2 x)^2 + (\log_2 x)^2 - 3 = 0$	M1
	$\Rightarrow 4(\log_2 x)^2 = 3 \Rightarrow (\log_2 x)^2 = \frac{3}{4} \Rightarrow \log_2 x = \pm\sqrt{\frac{3}{4}}$	M1
	$\Rightarrow x = 2^{\sqrt{\frac{3}{4}}} \approx 1.82 \text{ or } x = 2^{-\sqrt{\frac{3}{4}}} \approx 0.549$	M1A1A1
		[8]
Total 8 marks		

Mark	Notes NB: Candidates will frequently use a substitution for their chosen log.
Working in log base 2	
Correct answer/s with no working scores no marks.	
M1	For changing the base of the log correctly in at least one term $\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \log_2 x^3 + \frac{\log_2 x^2}{\log_2 4} - \frac{3\log_2 2}{\log_2 x} = 0$
M1	For using the power law in at least one term. $\log_2 x^3 = 3\log_2 x \text{ or } \log_2 x^2 = 2\log_2 x \text{ or } 3\log_2 2 = \log_2 2^3$ $\Rightarrow 3\log_2 x + \log_2 x - \frac{\log_2 2^3}{\log_2 x} = 0$
B1	For either $\log_2 4 = 2$ or $\log_2 8 = 3$
M1	For multiplying through by $\log_2 x$ $3(\log_2 x)^2 + (\log_2 x)^2 - 3 = 0$
M1	For simplifying and obtaining two values for $\log_2 x$ using a valid method. NB: If they discard one value at any stage do not award this mark. $4(\log_2 x)^2 - 3 = 0 \Rightarrow \log_2 x = \pm\sqrt{\frac{3}{4}}$
M1	For removing the log to find at least one value for x $x = 2^{\sqrt{\frac{3}{4}}} \text{ or } x = 2^{-\sqrt{\frac{3}{4}}}$
A1	For either awrt $x = 1.82$ or 0.549
A1	For awrt both $x = 1.82$ and 0.549

Working in log base 4	
M1	For changing the base of the log correctly in at least one term $\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \frac{\log_4 x^3}{\log_4 2} + \log_4 x^2 - \frac{3\log_4 2}{\log_4 2} = 0$
M1	For using the power law in at least one term $\frac{3\log_4 x}{\log_4 2} + 2\log_4 x - \frac{3\log_4 2}{\log_4 2} = 0$
B1	For either $\log_4 2 = \frac{1}{2}$ or $\log_4 8 = \frac{3}{2}$
M1	For multiplying through by $\log_4 x$ $6(\log_4 x)^2 + 2(\log_4 x^2)^2 - \frac{3}{2} = 0$
M1	For simplifying and obtaining two values for $\log_4 x$ $8(\log_4 x)^2 = \frac{3}{2} \Rightarrow (\log_4 x)^2 = \frac{3}{16} \Rightarrow \log_4 x = \pm \sqrt{\frac{3}{16}}$ NB: If they discard one value at any stage do not award this mark.
M1	For removing the log to find at least one value for x using a valid method. $x = 4^{\sqrt{\frac{3}{16}}} \approx \dots \text{ or } x = 4^{-\sqrt{\frac{3}{16}}} \approx \dots$
A1	For either awrt $x = 1.82$ or 0.549
A1	For awrt both $x = 1.82$ and 0.549
Working in log base x	
M1	For changing the base of the log correctly in at least one term $\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \frac{\log_x x^3}{\log_x 2} + \frac{\log_x x^2}{\log_x 4} - 3\log_x 2 = 0$
M1	For using the power law in at least one term $\frac{3\log_x x}{\log_x 2} + \frac{2\log_x x}{2\log_x 2} - 3\log_x 2 = 0$
B1	For $\log_x x = 1$ $\frac{3}{\log_x 2} + \frac{1}{\log_x 2} - 3\log_x 2 = 0 \Rightarrow \frac{4}{\log_x 2} - 3\log_x 2 = 0$
M1	For multiplying through by $\log_x 2$ $4 - 3(\log_x 2)^2 = 0$
M1	For obtaining two values of $\log_x 2$ using a valid method $\left(\log_x 2 = \pm \sqrt{\frac{4}{3}} \right)$ NB: If they discard one value at any stage do not award this mark.
M1	For removing the log to find at least one value for x . $2 = x^{\sqrt{\frac{4}{3}}} \Rightarrow x = 2^{\sqrt{\frac{3}{4}}} \approx \dots \text{ or } 2 = x^{-\sqrt{\frac{4}{3}}} \Rightarrow x = 2^{-\sqrt{\frac{3}{4}}} \approx \dots$
A1	For either awrt $x = 1.82$ or 0.549
A1	For awrt both $x = 1.82$ and 0.549

Eg3	$2\log_4 x = \log_3 3y^2$ $2\log_4 x = \frac{\log_9 3y^2}{\log_9 3} \Rightarrow \log_4 x = \log_9 3y^2 \Rightarrow \log_4 x = \log_9 3 + \log_9 y^2$ $\Rightarrow \log_4 x = 0.5 + 2\log_9 y \text{ oe}$ $\log_2 x^3 + 8\log_9 y = 13$ $\frac{\log_4 x^3}{\log_4 2} + 8\log_9 y = 13 \Rightarrow \log_4 x^3 + 4\log_9 y = \frac{13}{2}$ $\Rightarrow 3\log_4 x + 4\log_9 y = \frac{13}{2} \text{ oe}$ Solves the simultaneous equations by any method to find a value for $\log_4 x$ or $\log_9 y$ $3\log_4 x + 4\log_9 y = \frac{13}{2} \text{ oe}$ $\log_4 x = 0.5 + 2\log_9 y \text{ oe}$ $\Rightarrow \log_4 x = \frac{15}{10} \text{ or } \log_9 y = \frac{5}{10} \text{ oe}$ $x = 4^{\frac{15}{10}} = 8 \quad y = 9^{\frac{5}{10}} = 3$	M1B1 M1M1 dddM1 A1 M1 A1 [8]
An aid to marking (please also see more detailed notes).		
Correct application of addition/subtraction law, anywhere in working. Once seen, ignore any other incorrect applications.		M1
Any correct evaluation of a $\log_a a^b = b$ seen. Look for implicit evaluation during change of base of log. a, b rational.		B1
Correct application of the power law seen once anywhere. Once seen, ignore any other incorrect applications.		M1
Any correct change of base seen once anywhere. Once seen, ignore any other incorrect applications.		M1
Solve a pair of simultaneous equations – one error permitted, see detailed notes for required form.		dddM1
One correct log equation following from this solution.		A1
Undoing a log of the required form – see detailed notes.		M1
Both correct values of x and y .		A1
Note also that: $2\log_4 x = \log_2 x$ or $8\log_9 y = 4\log_3 y$ or $\log_2 x^3 = 2\log_4 x^3$ or $\log_3 3y^2 = 2\log_9 3y^2$ will normally imply B1 and 3 rd M1		

ALT1	$2\log_4 x = \log_3 3y^2$ $\frac{2\log_2 x}{\log_2 4} = \log_3 3y^2 \Rightarrow \log_2 x = \log_3 3y^2$ $\log_2 x^3 + 8\log_9 y = 13 \Rightarrow 3\log_2 x + 8\log_9 y = 13$ $3\log_3 3y^2 + 8\log_9 y = 13$ $\log_3 27y^6 + 8\frac{\log_3 y}{\log_3 9} = 13$ $\log_3 27y^6 + 4\log_3 y = 13$ $\log_3 27y^6 + \log_3 y^4 = 13$ $\log_3 27y^{10} = 13$ $27y^{10} = 3^{13}$ $y = \sqrt[10]{\frac{3^{13}}{27}} = 3$ $\log_2 x = \log_3 3 \times 3^2 \Rightarrow \log_2 x = 3$ $x = 2^3 = 8$	M1 B1 M1 M1 dddM1 A1 M1 A1 [8]
ALT2	$2\log_4 x = \log_3 3y^2$ $3\log_4 x = \frac{3}{2}\log_3 3y^2 \Rightarrow \log_4 x^3 = \frac{3}{2}\log_3 3y^2$ $\frac{\log_2 x^3}{\log_2 4} = \frac{3}{2}\log_3 3y^2 \Rightarrow \log_4 x^3 = 3\log_3 3y^2$ $3\log_3 3y^2 + 8\log_9 y = 13$ $\log_3 27y^6 + 8\frac{\log_3 y}{\log_3 9} = 13$ $\log_3 27y^6 + 4\log_3 y = 13$ $\log_3 27y^6 + \log_3 y^4 = 13$ $\log_3 27y^{10} = 13$ $27y^{10} = 3^{13}$ $y = \sqrt[10]{\frac{3^{13}}{27}} = 3$ $\log_2 x = \log_3 3 \times 3^2 \Rightarrow \log_2 x = 3$ $x = 2^3 = 8$	M1 B1 M1 M1 dddM1 A1 M1 A1 [8]
Total 8 marks		

Mark	Notes
M1	For any correct application of the addition (or subtraction) law eg $\log_3 3y^2 = \log_3 3 + \log_3 y^2$ Seen anywhere in their working once, ignore any other incorrect applications.
B1	For $\log_3 3 = 1$ or if students do a change of base first, this mark can also be awarded for example for $\log_2 4 = 2$ or $\log_3 9 = 2$ or $\log_4 2 = 0.5$ or $\log_9 3 = 0.5$ - note that this evaluation may not be explicitly seen, but seen in the simplification of an equation, particularly following a change of base of logs. This mark may be awarded for any correct use of $\log_a a^b = b$ seen. a, b rational.
M1	For applying the power law correctly with either $\log_p x^3$ or $\log_q y^2$ $p = 2$ or 4 $q = 3$ or 9 Seen anywhere in their working once, ignore any other incorrect applications.
M1	For changing the base of any log in either equation. eg $2\log_4 x = \frac{2\log_2 x}{\log_2 4}$ or $8\log_9 y = \frac{8\log_3 y}{\log_3 9}$ or $\log_2 x^3 = \frac{\log_4 x^3}{\log_4 2}$ or $\log_3 3y^2 = \frac{\log_9 3y^2}{\log_9 3}$ This mark may be awarded for any correct change of base of logs seen. Seen anywhere in their working once, ignore any other incorrect applications.
dddM1	For solving their simultaneous equations by any valid complete method. Allow one error. Candidates must reach a value for $\log_p x^n$ or $\log_q y^m$ $p = 2$ or 4 $q = 3$ or 9 m, n constants Dependent on all 3 previous method marks. The equations must be in a form with $\log_p ax^n$ and $\log_q by^m$ in $p = 2$ or 4 $q = 3$ or 9 All methods of solving (substitution and elimination being the most common) are valid.
A1	For the correct values of $\log_p x^n$ or $\log_q y^m$ $p = 2$ or 4 $q = 3$ or 9 m, n constants Markers will need to check the solution of candidates' simultaneous equations, but more than one error seen will result in dddM0 and A0. The question asks students to show their working clearly. This accuracy mark would generally only be gained after clearly seeing the work for the previous 4 method marks. If there is a solution where it is clear the student is working more concisely and fully correctly, perhaps taking shortcuts or missing the odd step – please send this to Review.
M1	For a correct method to 'undo' the logs on any equation of the form $\log_p ax^n = E$ or $\log_q by^m = F$ $p = 2$ or 4 $q = 3$ or 9 , E, m and n are non-zero constants.
A1	For the correct values of x and y

ALTS	For any similar methods mark using the same principles, but send to Review, if not clear how to implement these.
ALT1 M1	For change of base of log eg $2\log_4 x = 2\frac{\log_2 x}{\log_2 4}$ or $8\log_9 y = 8\frac{\log_3 y}{\log_3 9}$ This mark may be awarded for any correct change of base of logs seen.
B1	For $\log_2 4 = 2$ or $\log_3 9 = 2$. This mark may be awarded for any correct use of $\log_a a^b = b$ seen. a, b rational.
M1	For applying the power law correctly with $\log_2 x^3$ or in reverse $3\log_3 3y^2 \Rightarrow \log_3 27y^6$
M1	For correctly using $\log_2 x = \log_3 3y^2$ in $3\log_2 x + 8\log_9 y = 13$
dddM1	For using the addition (or subtraction) law in reverse $\log_3 27y^6 + \log_3 y^4 \Rightarrow \log_3 27y^{10}$
A1	For $\log_3 27y^{10} = 13$
M1	For a correct method to undo logs.
A1	For the correct values of x and y
ALT2 M1	For applying the power law correctly in reverse $3\log_2 x = \log_2 x^3$
B1	For $\log_2 4 = 2$ or $\log_3 9 = 2$. This mark may be awarded for any correct use of $\log_a a^b = b$ seen. a, b rational.
M1	For change of base of log eg $2\log_4 x = 2\frac{\log_2 x}{\log_2 4}$ or $8\log_9 y = 8\frac{\log_3 y}{\log_3 9}$ This mark may be awarded for any correct change of base of logs seen.
M1	For correctly using $\log_4 x^3 = 3\log_3 3y^2$ in $3\log_3 3y^2 + 8\log_9 y = 13$
dddM1	For using the addition (or subtraction) law in reverse $\log_3 27y^6 + \log_3 y^4 \Rightarrow \log_3 27y^{10}$
A1	For $\log_3 27y^{10} = 13$
M1	For a correct method to undo logs.
A1	For the correct values of x and y



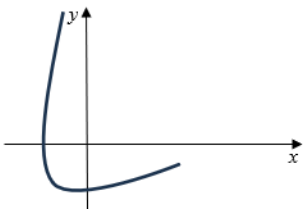
Question number	Scheme	Marks
5 (a)	$\log_4 32 = \frac{\log_2 32}{\log_2 4} = \frac{5}{2}^*$ or $\log_4 32 = \log_4 4^{\frac{5}{2}} = \frac{5}{2}^*$ or $\log_4 32 = \log_{2^2} 2^5 = \frac{5}{2}^*$ ALT $\log_4 32 = a \Rightarrow 4^a = 32 \Rightarrow a = \frac{5}{2}^*$	M1A1cso [2] {M1}{A1} cso [2]
(b)	$\log_2 x - \log_4 32 + \frac{1}{4} \log_x 16 = 0$ Let $\log_2 x = y$ $y - \frac{5}{2} + \frac{1}{4} \left(\frac{\log_2 16}{\log_2 x} \right) = 0 \quad \text{or} \quad y - \frac{5}{2} + \frac{1}{\log_2 x} = 0$ $\Rightarrow y - \frac{5}{2} + \frac{1}{y} = 0$ $\Rightarrow 2y^2 - 5y + 2 = 0$ $\Rightarrow (2y - 1)(y - 2) = 0$ $\Rightarrow y = \log_2 x = \frac{1}{2} \text{ or } 2$ $\Rightarrow x = 2^{\frac{1}{2}} = \sqrt{2} \quad \text{and} \quad x = 2^2 = 4$	M1 M1A1 M1 M1 M1A1 [7]
Total 9 marks		
(a) M1 ALT M1 A1 cso (b) M1 M1 A1 M1 M1 A1	For $\log_4 32 = \frac{\log_2 32}{\log_2 4}$ or $\log_4 32 = \log_4 4^{\frac{5}{2}}$ or $\log_4 32 = \log_{2^2} 2^5$ For $4^a = 32$ Obtains the given answer with no errors in the working Use of $\log_a x = \frac{\log_b x}{\log_b a}$ or $\log_a b = \frac{1}{\log_b a}$ Forming a 3TQ $2y^2 - 5y + 2 = 0$ Solving the 3TQ For $y = \log_2 x = \frac{1}{2}$ or 2 Either $x = 2^{\frac{1}{2}} = \sqrt{2}$ or $x = 2^2 = 4$ Both $x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$	

Question	Scheme	Marks
9 (a)	$\left(\log_a 8 = \frac{3}{4} \Rightarrow\right) a^{\frac{3}{4}} = 8 \left(\Rightarrow a = \left(\sqrt[3]{8}\right)^4\right) = 16$	M1A1 [2]
(b)	$(3x \log_2 x - 4 \log_{16} 8 + 6x \log_4 8 - \log_2 x) 3x \log_2 x - \frac{4 \log_2 8}{\log_2 16} + \frac{6x \log_2 8}{\log_2 4} - \log_2 x$ $\Rightarrow 3x \log_2 x - \log_2 8 + 3x \log_2 8 - \log_2 x$ $= (3x-1) \log_2 x + (3x-1) \log_2 8 \quad \text{or} \quad 3x \log_2 8x - (1) \log_2 8x$ $\Rightarrow (3x-1) \log_2 8x \quad \text{or} \quad \log_2 (8x)^{3x} + \log_2 (8x)^{-1} = \log_2 (8x)^{3x-1} *$	M1 M1 M1A1 cso [4]
ALT	$(3x \log_2 x - 4 \log_{16} 8 + 6x \log_4 8 - \log_2 x) 3x \log_2 x - \frac{4 \log_2 8}{\log_2 16} + \frac{6x \log_2 x}{\log_2 4} - \log_2 x$ $\Rightarrow 3x \log_2 x - \log_2 8 + 3x \log_2 8 - \log_2 x = \log_2 x^{3x} - \log_2 8 + \log_2 8^{3x} - \log_2 x$ $(\Rightarrow \log_2 (8x)^{3x} - \log_2 8x \Rightarrow) \log_2 \left(\frac{(8x)^{3x}}{8x}\right) \quad \text{or} \quad \log_2 ((8x)^{3x} \times (8x)^{-1})$ $\text{or} \log_2 x^{3x-1} 8^{3x-1} = \log_2 (8x)^{3x-1} *$	M1 M1 M1A1 cso [4]
“Box 3” of part b We will see unanticipated methods once live marking begins. If the answer is correct and there is no incorrect working, check the work carefully, to ascertain if they’ve shown enough steps to demonstrate use of the three main log laws this question tests and award full marks. If in any doubt at all – the response MUST be sent to review. Other than this exception, please mark to the following rules. Also use these rules if students don’t gain the 2nd or 3rd M under the main or ALT schemes. • M1 for any correct change of base to base 2 • M1 for any two correct applications of the power law $ax \log_2 8x + b \log_2 8x \Rightarrow \log_2 (8x)^{ax+b}$ or for $(ax+b) \log_2 8x \Rightarrow \log_2 (8x)^{ax+b}$ $a, b \neq 0$ • M1 for any two correct applications of the addition or subtraction law In each case – ignore any incorrect working. Poor or incorrect bracketing may not be recovered in this question. (general principle of marking is usually that it can).		
(c)	$\left[\log_2 (8x)^{3x-1} = 0 \Rightarrow \log_2 (8x)^{3x-1} = \log_2 8^0 \quad \text{or} \quad (3x-1) \log_2 (8x) = 0\right]$ $\Rightarrow 3x-1=0 \Rightarrow x = \frac{1}{3}$ $\Rightarrow 8x=1 \Rightarrow x = \frac{1}{8}$	M1A1 A1 [3]
Total 9 marks		

Part	Mark	Notes
(a)	M1	For undoing the log to obtain $a^{\frac{3}{4}} = 8$ or $(a =) 8^{\frac{4}{3}}$
	A1	For $a = 16$ M1 A1 may be awarded for just seeing $a = 16$ unless from a string of incorrect work. Poor notation may be seen.
(b)	M1	For correctly changing the base to base 2 on at least one log, anywhere in their work. This may be implied and the mark awarded for eg $4\log_{16} 8 \Rightarrow \frac{4\log_2 8}{\log_2 16}$ or $\frac{4\log_2 8}{4}$ or $\log_2 8$ or $\frac{12}{4}$ or 3 or $6x\log_4 8 \Rightarrow \frac{6x\log_2 8}{\log_2 4}$ or $\frac{6x\log_2 8}{2}$ or $3x\log_2 8$ or $9x$
	M1	For $(3x-1)\log_2 x + (3x-1)\log_2 8$
	M1	For correctly applying the power log law to their expression of the form $(ax+b)\log_2 8x$ $a, b \neq 0$ In general, poor bracketing may be recovered, but as this question is a show question, it generally cannot be recovered eg do not accept $8x^{3x}$
	A1 CSO	For the correct expression, minimum steps as shown with no errors or omissions.
ALT	M1	For correctly changing the base to base 2 on at least one log. This may be implied and the mark awarded for $4\log_{16} 8 \Rightarrow \frac{4\log_2 8}{\log_2 16}$ or $\frac{4\log_2 8}{4}$ or $\log_2 8$ or $\frac{12}{4}$ or 3 or $6x\log_4 8 \Rightarrow \frac{6x\log_2 8}{\log_2 4}$ or $\frac{6x\log_2 8}{2}$ or $3x\log_2 8$ or $9x$
	M1	For $\log_2 x^{(3x)} - \log_2 8 + \log_2 8^{(3x)} - \log_2 x$
	M1	For correctly applying the subtraction law to an expression of the form (see the MS for minimum steps) $\log_2 x^{(ax)} - \log_2 8^b + \log_2 8^{(ax)} - \log_2 x^b$ The step in brackets doesn't need to be shown. Do not permit only eg $\log_2 (x^{3x} \div 8 \times 8^{3x} \div x)$ as a sufficient minimum step. In general, poor bracketing may be recovered, but as this question is a show question, it generally cannot be recovered eg do not accept $8x^{3x} \times 8x^{-1}$
	A1	For the correct expression, minimum steps as shown with no errors or omissions. .
	SC4	For working on both sides with no errors and achieving lhs = rhs Note, this is an exception to what we normally allow in a show that question (ie working on both sides until agreement)
(c)	M1	For setting $3x-1=0$ or $8x=1$ This mark may be implied by a correct answer.
	A1	For either $x = \frac{1}{3}$ or $\frac{1}{8}$
	A1	For both $x = \frac{1}{3}$ and $\frac{1}{8}$



Question	Scheme	Marks
7(a)		B1 Shape and position B1 Intersection with x-axis B1 Intersection with y-axis B1 Asymptote [4]
(b)	$\log_{(x+4)} 256 - \log_4 (x+4) = 0 \Rightarrow \frac{\log_4 256}{\log_4 (x+4)} - \log_4 (x+4) = 0$ $\Rightarrow 4 - (\log_4 (x+4))^2 = 0 \Rightarrow (\log_4 (x+4))^2 = 4$ $\Rightarrow \log_4 (x+4) = \pm 2$ $x+4 = 4^2 \Rightarrow x = 12$ $x+4 = 4^{-2} \Rightarrow x = -\frac{63}{16} \text{ or } -3.9375$	M1 M1 M1 A1 A1 [5]
Total 9 marks		

Part	Mark	Notes
(a)	B1	For the correct shape in the correct position. Ignore intersections and the asymptote for this mark. Award as long as the curve is the correct way around in quadrants 2, 3 and 4. The ends must not turn back in themselves. Do Not accept for example:  If there are two or more lines, and candidates have not made clear which ONE they want marked– award B0
	B1	For a line/curve passing through $(-3, 0)$ or accept $x = -3$ Do not accept stopping at the axis.
	B1	For a line/curve passing through $(0, -1)$ or accept $y = -1$ Do not accept stopping at the axis.
	B1	For the correct equation only of the asymptote. If they also give a horizontal asymptote then do not isw – it is B0
(b)	Method 1 - Works in log base 4	
	M1	For changing the base of the log to base 4
	M1	For forming a quadratic in terms of $(x + 4)$ Accept a substitution for the log
	M1	For taking the square root and undoing the log. Accept just the positive root for this mark.
	A1	For either $x = 12$ or $-\frac{63}{16}$
	A1	For both $x = 12$ and $-\frac{63}{16}$
	Method 2 – Works in log base $(x + 4)$	
	M1	For changing the base of the log to base $(x + 4)$ $\log_{(x+4)} 256 - \frac{\log_{(x+4)} (x + 4)}{\log_{(x+4)} 4} = 0 \Rightarrow$
	M1	Changes $\log_{(x+4)} (x + 4) = 1$ and $\log_{(x+4)} 256 = 4\log_{(x+4)} 4$ And forming a quadratic equation with log base $(x + 4)$ $(\log_{(x+4)} 4)^2 = \frac{1}{4} \quad \text{Accept a substitution for the log}$
	M1	Takes the square root (accept just the positive root) and undoes the log $\log_{(x+4)} 4 = \pm \frac{1}{2} \Rightarrow 4 = (x + 4)^{\frac{1}{2}} \quad \text{and} \quad 4 = (x + 4)^{-\frac{1}{2}}$
	A1	For either $x = 12$ or $-\frac{63}{16}$
	A1	For both $x = 12$ and $-\frac{63}{16}$

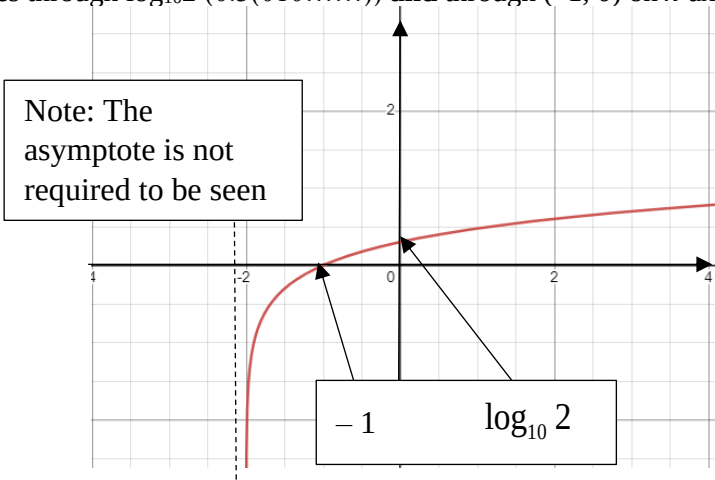
Method 3 works in an unspecified base. If it is just log, assume base 10	
M1	For changing the base $\frac{\log 256}{\log(x+4)} - \frac{\log(x+4)}{\log 4} = 0$
M1	Forms a QE in terms of $(x+4)$ $4(\log 4)^2 = (\log(x+4))^2$ oe
M1	Takes square root (accept just the positive root) and undoes the log $\pm 2 \log 4 = \log(x+4) \Rightarrow \log 4 = \pm \frac{1}{2} \log(x+4)$ $\Rightarrow \log 4 = \log(x+4)^{\pm \frac{1}{2}} \Rightarrow 4 = (x+4)^{\pm \frac{1}{2}}$ $\Rightarrow 16 = x+4, \quad \frac{1}{16} = x+4$
A1	For either $x = 12$ or $-\frac{63}{16}$
,A1	For both correct values $x = 12$ and $-\frac{63}{16}$



Part	Mark	Additional Guidance
When marking (i) stick to 1 ALTERNATIVE only – eg it's not possible to gain M2 for an application of a power rule and an addition rule with logs. Look out for equivalent working, particularly the use of $\log_{a^b} c = \frac{1}{b} \log_a c$ Send any work that you may think uses a different method worth partial marks to Review		
(i)	M1	For correct use of the rule $\log_a A^b = b \log_a A$ This mark may be awarded anywhere the power rule is correctly used once in working, in either direction, even if there's other contradictory work.
	M1	For a rearrangement of $3(b \log_a 3 + c \log_a 3) = 1$ to $a^{\frac{1}{3R}} = 3$ or $\log_3 a = 3R$ $R > 1$ seen anywhere in the working, even if there's other contradictory work. Allow errors in rearrangement. May be implied by any correct value for a not in the required form eg $14348907, (3^5)^3, (3^3)^5, 9^{7.5}$
	A1	$(a =) 3^{15}$
ALT	M1	For correct use the rule $\log_a A + \log_a B = \log_a AB$ This mark may be awarded anywhere the addition rule is correctly used once in working, in either direction, even if there's other contradictory work.
	M1	For a rearrangement of $3(\log_a (C \times D)) = 1$ to $a^{\frac{1}{3}} = CD$ seen anywhere in the working, even if there's other contradictory work. Allow errors in rearrangement. May be implied by any correct value for x not in the required form eg $14348907, (3^5)^3, (3^3)^5, 9^{7.5}$
	A1	$(a =) 3^{15}$
(ii)	M1	Correct use of change of base of logs. This mark may be awarded anywhere a change of base of logs is used correctly once in working, even if there's other contradictory work.
	dM1	Rearranges their equation to a 3 term quadratic ($= 0$). Allow errors in rearrangement so long as reach an expression of the form $p(\log_4 p)^2 + q \log_4 p + r = 0$ or $s(\log_p 4)^2 + t \log_p 4 + u = 0$ $p, q, r, s, t, u \neq 0$
	A1	Correct 3 term quadratic ($= 0$) as shown.
	M1	A minimally acceptable attempt to solve their 3 term quadratic, see general guidance. Students may have used an alternative variable here for $\log_4 p$ or $\log_p 4$ The mark is for a minimally acceptable attempt to solve a 3TQ – see general guidance. The correct value from the correct quadratic implies this mark, otherwise method must be shown.
	M1	Correctly converts their log equation into a value for p . ie $\log_4 p = F \rightarrow p =$ or $\log_p 4 = G \rightarrow p =$
	A1	For $(p =) \frac{1}{16}$ oe

Question	Scheme	Marks
8 (a)	$(5p-4)(p-1)=0$ $p=\frac{4}{5}, p=1$	M1A1 [2]
(b)	$5^{2x+1}-9(5^x)+4=0 \Rightarrow 5 \times 5^{2x}-9(5^x)+4=0$ $5^x=1 \quad x=0$ $5^x=\frac{4}{5}, \quad x \ln 5 = \ln\left(\frac{4}{5}\right) \quad x=-0.1386...=-0.139$	M1 A1 M1A1 [4]
(c)	$5^{2x+1}+5^x=2(5^{x+1})-4$ $5^{2x+1}-9(5^x)+4=0$ $x=0 \quad y=5+1=6 \quad (\text{or } y=2 \times 5-4=6) \quad (0,6)$ $x=-0.1386... \quad 5^x=\frac{4}{5} \quad y=5 \times \left(\frac{4}{5}\right)^2 + \frac{4}{5}=4 \quad (\text{or } y=10 \times \frac{4}{5}-4=4)$ $(-0.139,4)$	M1 A1 M1 A1 [4]
Total 10 marks		

Additional Guidance		
(a)	M1	For an attempt to solve the given quadratic. Note: they must arrive at two values of x that must follow from their factorisation. Minimally acceptable attempt are as follows; $(5p \pm 4)(p \pm 1) \Rightarrow p = \pm \frac{4}{5}, \pm 1$ $(5p \pm 1)(p \pm 4) \Rightarrow p = \pm \frac{1}{5}, \pm 4$ $(5p \pm 2)(p \pm 2) \Rightarrow p = \pm \frac{2}{5}, \pm 2$
	A1	For $p = \frac{4}{5}, p=1$ or $p=0.8, p=1$
(b)	M1	For changing $5^{2x+1} = 5 \cdot 5^{2x}$ forming the 3TQ shown $5 \times 5^{2x} - 9(5^x) + 4 = 0$ and using the factorisation in (a) to find one value for x .
	A1	For $x=0$
	M1	For finding the second value of x arising from their factorisation.
	A1	For $x=-0.139$
(c)	M1	For setting the equation of the curve = to the equation of the line, forming a 3TQ and finding one set of coordinates. Follow through their factorisation from part (a).
	A1	For the coordinates of $(0, 6)$
	M1	For finding the second set of coordinates.
	A1	For $(-0.139, 4)$

Question number	Scheme	Marks
7 (a)	Shape of the curve, crossing negative x-axis and positive y-axis with the asymptotic nature of the curve shown. passes through $\log_{10} 2$ (0.3010.....) and through $(-1, 0)$ on x-axis  Note: The asymptote is not required to be seen -1 $\log_{10} 2$	B1 B1 [2]
(b)	$(2\log_a 4 + 2\log_a 4^2)$ $2\log_a 4 + 4\log_a 4$ $\log_a 4 = \frac{1}{6}$ $a^{\frac{1}{6}} = 4$ $a = 4096$ ALT $(2)\log_a 64 = 1 \Rightarrow \log_a 64 = \frac{1}{2}$ $\Rightarrow a^{\frac{1}{2}} = 64$ $\Rightarrow a = 4096$ or $(2\log_a 2^2 + 2\log_a 2^4)$ $4\log_a 2 + 8\log_a 2$ $\log_a 2 = \frac{1}{12}$ oe $a^{\frac{1}{12}} = 2$	M1 dM1 A1 [M1 dM1 A1] [3]
(c)	$\log_q 16 \rightarrow \frac{\log_2 16}{\log_2 q}$ $\left(5 \frac{\log_2 16}{\log_2 q} + 4\log_2 q = 24\right)$ $20 + 4(\log_2 q)^2 = 24\log_2 q$ $(\log_2 q - 5)(\log_2 q - 1) = 0$ $(\log_2 q = 5) \rightarrow q = 32$ $(\log_2 q = 1) \rightarrow q = 2$ $\log_2 q \rightarrow \frac{\log_q q}{\log_q 2}$ or $\frac{1}{\log_q 2}$ $\left(5\log_q 16 + 4 \frac{\log_q q}{\log_q 2} = 24\right)$ $20(\log_q 2)^2 + 4 = 24\log_q 2$ $(5\log_q 2 - 1)(\log_q 2 - 1) = 0$ $\left(\log_q 2 = \frac{1}{5}\right) \rightarrow q = 32$ $(\log_q 2 = 1) \rightarrow q = 2$	M1 M1 A1 dM1 M1 A1 [6]
Total 11 marks		

Part	Mark	Additional Guidance
(a)	B1	The curve must not bend back on itself and should cross both axes once on negative x and once on positive y . It needs to demonstrate an asymptotic nature. Accept an asymptote drawn correctly as well as their curve.
	B1	It is enough for -1 to be indicated on the x -axis with the curve passing through this point and for $\log_{10} 2$ to be marked on the y -axis (decimal 0.3..... allowed) There must be a curve drawn (even incorrectly) to achieve this mark. Do not accept intersections unless they are marked on the graph in the correct place.
(b)	M1	For the use of the power rule with logs to obtain $2\log_a 4$ from $\log_a 16$ OR $4\log_a 2$ from $\log_a 16$
	dM1	A correct rearrangement of their equation to obtain for example $a^{\frac{1}{6}} = 4$ or $a^{\frac{1}{12}} = 2$ or even of the type $a^1 = 64^2$ There are many ways of completing this. Look for a correct value for the corresponding correct power of a . This mark is dependent on the first M mark. Look for correct log work for their values.
	A1	For $a = 4096$
ALT	M1	For use of the correct log rule to obtain $\log_a 64$
	dM1	A correct rearrangement of their equation to obtain $a^{\frac{1}{2}} = 64$ This mark is dependent on the first M mark. Look for correct log work for their values.
	A1	For $a = 4096$
(c)	M1	For a fully correct change of base of log to either base 2 or base q
	M1	For an attempt to rearrange and form a 3TQ quadratic. Allow one arithmetical slip only.
	A1	For the correct 3TQ.
	dM1	Dependent on the second M mark – an acceptable attempt to solve the quadratic equation – see general guidance.
	M1	For a correct application using either of their solutions moving from log form to exponential form. Note this is an independent mark.
	A1	Both correct solutions.

Question number	Scheme	Marks
6 (i)	$5(2\log_b 3 + \log_b 3) = 3$ $\left(3\log_b 3 = \frac{3}{5}\right) \Rightarrow \log_b 3 = \frac{1}{5}$ $b^{\frac{1}{5}} = 3$ $b = 243 \text{ or } 3^5$ ALT $5\log_b 27 = 3$ $\log_b 27 = \frac{3}{5}$ $b^{\frac{3}{5}} = 27$ $b = 243 \text{ or } b = 27^{\frac{5}{3}}$	M1 M1 M1 A1 M1 M1 M1 A1 [4]
(ii)	$3\log_3 x + 3\frac{\log_3 27}{\log_3 x} = 28$ $3(\log_3 x)^2 + 9 = 28\log_3 x$ $3(\log_3 x)^2 - 28\log_3 x + 9 (= 0) \Rightarrow (3\log_3 x - 1)(\log_3 x - 9) (= 0)$ $(\log_3 x =) \frac{1}{3} \text{ and } (\log_3 x =) 9$ $x = 3^{\frac{1}{3}}, 3^9 \text{ oe exact form}$	M1, B1 dM1 ddM1 A1 M1 A1 [7]
Total 11 marks		

Part	Mark	Additional Guidance
(i)	M1	Rewrites 9 as 3^2 and correctly uses the power log rule as shown.
	M1	Collects terms as rearranges to give $\log_b p = r$, allow 1 error in rearrangement.
	M1	Correctly converts their log equation to an exponential.
	A1	Cao
ALT	M1	Uses the log addition law correctly to combine into a single log.
	M1	Collects terms as rearranges to give $\log_b p = r$, allow 1 error in rearrangement.
	M1	Rearranges to the form $b^{\frac{p}{q}} = r$, allow 1 error in rearrangement.
	A1	Cao
(ii)	B1	Replacement of $8\log_4 128 = 28$, wherever we see
	M1	For a correct change of the same base of logs on the lhs of the equation e.g. $3\log_3 x + 3\frac{\log_3 27}{\log_3 x} = 28$ or $\frac{3}{\log_x 3} + 3\log_x 27 = 28$ Allow slips on the coefficients
	dM1	For forming a 3-term quadratic in any form e.g. $3(\log_3 x)^2 + 9 = 28\log_3 x$ or $3 + 9(\log_x 3)^2 = 28(\log_x 3)$
	ddM1	For solving their 3-term quadratic using a valid method – see general guidance. (Correct answers from a correct 3TQ scores this mark)
	A1	For $(\log_3 x) = \frac{1}{3}$ and $(\log_3 x) = 9$ or $(\log_x 3) = 3$ and $(\log_x 3) = \frac{1}{9}$
	M1	For correctly rearranging either of their equations, removes logs to find $x =$
	A1	For both correct values or exact form. E.g. $x = 19683, x = \sqrt[3]{3}$ isw

Question number	Scheme	Marks
10(i)	$\log_4(6y-5) = 3 \Rightarrow 6y-5 = 64 \Rightarrow y = \frac{23}{2}$ oe	M1A1 [2]
(ii)	$\log_4(4-3x)^2 - \frac{\log_4(x^2-5)}{\log_4 2} = 3$ or $\frac{\log_2(4-3x)^2}{\log_2 4} - \log_2(x^2-5) = 3$ $\frac{1}{2}\log_4(4-3x)^2 - \log_4(x^2-5) = \frac{3}{2}$ or $\frac{1}{2}\log_2(4-3x)^2 - \log_2(x^2-5) = 3$ $\log_4(4-3x) - \log_4(x^2-5) = \frac{3}{2}$ or $\log_2(4-3x) - \log_2(x^2-5) = 3$ $\log_4\left(\frac{4-3x}{x^2-5}\right) = \frac{3}{2}$ or $\log_2\left(\frac{4-3x}{x^2-5}\right) = 3$ $\frac{4-3x}{x^2-5} = 4^{\frac{3}{2}} = 8$ or $\frac{4-3x}{x^2-5} = 2^3 = 8$ $8x^2 + 3x - 44 = 0$ $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 8 \times -44}}{2 \times 8} = \frac{-3 \pm \sqrt{1417}}{16} \Rightarrow \{x = 2.1651... \text{ or } -2.540...\}$ $\therefore x = -2.54$ (awrt)	M1 B1 M1 M1 M1 A1 M1A1 A1 [9]
Total 11 marks		

Part	Mark	Additional Guidance
(i)	M1	For a complete correct method to find the value of y
	A1	For the correct value
For part (ii) award the marks when you see them		
(ii)	M1	For change of base correctly to either 4 or 2
	B1	For replacing $\log_2 4$ with 2 or replacing $\log_4 2$ with $\frac{1}{2}$
	M1	For applying the power law correctly e.g.: $\frac{1}{2}\log_4(4-3x)^2 \Rightarrow \log_4(4-3x)$ or $\frac{1}{2}\log_2(4-3x)^2 \Rightarrow \log_2(4-3x)$
	M1	For applying the subtraction/addition law correctly
	M1	For correctly removed logs
	A1	For the correct 3TQ, “=0 “could be implied by later work
	M1	For a correct clear method to solve their 3TQ, allow one/two solutions from a calculator correct to 1dp (awrt) from a correct 3TQ
	A1	For two correct solutions, exact (allow embedded in a quadratic formula) or $x = 2.17$ (awrt), $x = -2.54$ (awrt) or JUST $x = -2.54$ (awrt) seen only, from a correct 3TQ, scores M1A1A1(rejected 2.17 is implied)
	A1	For the correct solution. Accept awrt -2.54 The positive root must be rejected as it is not feasible.

Question	Scheme	Marks
9 (a)	$2 \log_p 9 + 3 \log_3 p = 8$ $2 \frac{\log_3 9}{\log_3 p} + 3 \log_3 p = 8$ $2 \log_3 9 + 3 (\log_3 p)^2 = 8 \log_3 p$ $3 (\log_3 p)^2 - 8 \log_3 p + 4 = 0$ $(3 \log_3 p - 2)(\log_3 p - 2) = 0$ $\log_3 p = \frac{2}{3} \quad p = 3^{\frac{2}{3}} = \sqrt[3]{9} \quad (= 2.08)$ $\log_3 p = 2 \quad p = 3^2 = 9$	 M1 M1 A1 M1 A1 A1 [6]
(b)	$\log_2 3 = \frac{\log_4 3}{\log_4 2} = \frac{\log_4 3}{\frac{1}{2}} = 2 \log_4 3 = \log_4 3^2 \Rightarrow k = 2$	 M1A1 [2]
(c)	$6x \log_4 x - 3x \log_2 3 - 5 \log_4 x + 10 \log_2 3$ $= 6x \log_4 x - 5 \log_4 x - 3x \log_4 3^2 + 10 \log_4 3^2$ $= \log_4 x^{6x} - \log_4 x^5 - \log_4 3^{6x} + \log_4 3^{20}$ $= \log_4 \frac{x^{6x} \times 3^{20}}{x^5 \times 3^{6x}}$ $= \log_4 \frac{x^{6x-5}}{3^{6x-20}} *$	 M1 M1 M1 A1 [4]
Total 12 marks		

Additional Guidance		
(a)	Method 1 – Changing to $\log_3$	
	M1	For changing the base of $\log_p 9$ to $\log_3 9$
	M1	For forming a 3TQ
	A1	For the correct 3TQ $3(\log_3 p)^2 - 8\log_3 p + 4 = 0$ in any form. Accept for example, $2\log_3 9 + 3(\log_3 p)^2 = 8\log_3 p$
	M1	For an attempt to solve their 3TQ. They must obtain two values for $\log_3 p$ Minimally acceptable attempts to solve the 3TQ are defined in General Guidance
	A1	For either $p = 9$ or $p = \sqrt[3]{9}$ [accept 2.08]
	A1	For both $p = 9$ and $p = \sqrt[3]{9}$ [accept 2.08]
	Method 2 - Changing to $\log_p$	
	M1	For changing the base of $\log_3 p$ to $\log_p 3 \Rightarrow 3 \frac{\log_p p}{\log_p 3}$
	M1	For forming a 3TQ $4(\log_p 3)^2 + 3 = 8\log_p 3$
	A1	For the correct 3TQ in any form. $4(\log_p 3)^2 - 8\log_p 3 + 3 = 0$
	M1	For an attempt to solve their 3TQ. They must obtain two values for $\log_p 3$ Minimally acceptable attempts to solve the 3TQ are defined in General Guidance
	A1	For either $p = 9$ or $p = \sqrt[3]{9}$ [accept 2.08]
	A1	For both $p = 9$ and $p = \sqrt[3]{9}$ [accept 2.08]
(b)	Method 1 – Changing the base to $\log_4 3$	
	M1	For changing $\log_2 3 \Rightarrow \log_4 3$ and finding the value of k
	A1	For $k = 2$
	Method 2 – Changing the base to $\log_2 3$	
	M1	For changing the base of the log and obtaining a value for k $\log_2 3 = \log_4 3^k \Rightarrow \log_2 3 = k \log_4 3 \Rightarrow k = \frac{\log_2 3}{\log_4 3}$ $\Rightarrow k = \frac{\log_2 3}{\frac{\log_2 3}{\log_2 4}} \Rightarrow k = \frac{\log_2 3}{\frac{\log_2 3}{2}} = 2$
	A1	For $k = 2$
(c)	Mark this part in any order	
	M1	For changing the base of both terms with $\log_2$ to $\log_4$
	M1	For dealing with all the powers
	M1	For combining the logs together into one log term.
	A1	For the final correct answer. This is a show question, there must be no errors for the award of the final A mark

Question	Scheme	Marks
10(a)	$\frac{1}{243} = 3^{-5}$ $9^{3y} = 3^{6y}$ $\frac{9^{3y}}{243} = 3^{-5} \times 3^{6y} \Rightarrow \frac{9^{3y}}{243} = 3^{(6y-5)} *$	B1 B1 M1A1 cso [4]
(b)	$27^{(x-2)} = 3^{3(x-2)} = 3^{(3x-6)}$ $6y - 5 = 3x - 6 \Rightarrow (6y - 3x - 1 = 0)$ $\log_4 2 = \frac{1}{2}$ $\log_{10} \sqrt{6xy} = \frac{1}{2} \log_{10} (6xy) \Rightarrow \log_{10} (6xy) = 1$ $1 = \log_{10} 10 \Rightarrow \log_{10} (6xy) = \log_{10} 10 \Rightarrow 6xy = 10$	M1 M1 B1 M1 M1
$3x - 6y - 1 = 0$ $6xy = 10$		
	Method A	
	$6y = \frac{10}{x} \Rightarrow 3x - \frac{10}{x} - 1 = 0 \Rightarrow 3x^2 - x - 10 = 0$ $3x^2 - x - 10 = (3x + 5)(x - 2) = 0 \Rightarrow x = 2, -\frac{5}{3}$ $3 \times 2 - 6y - 1 = 0 \Rightarrow 6y = 5 \Rightarrow y = \frac{5}{6}$ $3 \times \left(-\frac{5}{3}\right) - 6y - 1 = 0 \Rightarrow -6y = 6 \Rightarrow y = -1$ $x = 2 \quad y = \frac{5}{6} \text{ or } x = -\frac{5}{3} \quad y = -1$	M1 M1 A1 A1 [9]
	Method B	
	$3x = \frac{5}{y} \Rightarrow \frac{5}{y} - 6y - 1 = 0 \Rightarrow 6y^2 + y - 5 = 0$ $6y^2 + y - 5 = (6y - 5)(y + 1) = 0 \Rightarrow y = \frac{5}{6}, -1$ $3x - 6 \times \frac{5}{6} - 1 = 0 \Rightarrow 3x = 6 \Rightarrow x = 2$ $3x - 6 \times (-1) - 1 = 0 \Rightarrow 3x = -5 \Rightarrow x = -\frac{5}{3}$ $x = 2 \quad y = \frac{5}{6} \text{ or } x = -\frac{5}{3} \quad y = -1$	M1 M1 A1 A1 [9]
Total 13 marks		

Question	Scheme	Marks
10	$\frac{9^{3y}}{243} = 27^{(x-2)}$ $\log_{10} \sqrt{6xy} = \log_4 2$	
(a)	Writes down $\frac{1}{243} = 3^{-5}$ May be seen as $\frac{1}{243} = \frac{1}{3^5}$ later correctly used as 3^{-5} when combining terms.	B1
	Writes down $9^{3y} = 3^{6y}$	B1
	Combines the terms $\frac{9^{3y}}{243} = 3^{-5} \times 3^{6y} = 3^{(6y-5)}$	M1
	For the correct expression with no errors. $\frac{9^{3y}}{243} = 3^{(6y-5)} *$	A1 cso [4]
(b)	For dealing with the power of 3 to give $27^{(x-2)} = 3^{3(x-2)} = 3^{(3x-6)}$	M1
	For equating the powers of 3 to give the equation $6y - 5 = 3x - 6 \Rightarrow (6y - 3x - 1 = 0)$	M1
	For stating $\log_4 2 = \frac{1}{2}$	B1
	For dealing with the square root $\log_{10} \sqrt{6xy} = \frac{1}{2} \log_{10} (6xy) = \left(\frac{1}{2}\right) \Rightarrow \log_{10} (6xy) = 1$	M1
	For correctly removing all logarithms from the second equation $1 = \log_{10} 10 \Rightarrow \log_{10} (6xy) = \log_{10} 10 \Rightarrow 6xy = 10$	M1
$3x - 6y - 1 = 0$ $6xy = 10$		
	Method A	
	For substituting $6y = \frac{10}{x}$ into the linear equation to give $3x - \frac{10}{x} - 1 = 0$ and attempting to form a 3TQ $3x^2 - x - 10 = 0$ OR for substituting $y = \frac{1}{6}(3x - 1)$ into $6xy = 10$ to give $x(3x - 1) = 10$ and attempting to form a 3TQ $3x^2 - x - 10 = 0$	M1
	For attempting to solve their 3TQ $3x^2 - x - 10 = (3x + 5)(x - 2) = 0 \Rightarrow x = 2, -\frac{5}{3}$	M1

	For finding the values of y $3 \times 2 - 6y - 1 = 0 \Rightarrow 6y = 5 \Rightarrow y = \frac{5}{6}$ $3 \times \left(-\frac{5}{3}\right) - 6y - 1 = 0 \Rightarrow -6y = 6 \Rightarrow y = -1$ $x = 2 \quad y = \frac{5}{6} \text{ or } x = -\frac{5}{3} \quad y = -1$ Answers must be given in pairs, pairing may be implied from working.	A1 A1 [9]
	Method B	
	For substituting $3x = \frac{10}{2y} \Rightarrow 3x = \frac{5}{y} \text{ into the linear equation to give } \frac{5}{y} - 6y - 1 = 0$ and attempting to form a 3TQ $6y^2 + y - 5 = 0$ OR for substituting $x = \frac{1}{3}(6y + 1)$ into $6xy = 10$ to give $2(6y + 1)y = 10 \text{ and attempting to form a 3TQ } 6y^2 + y - 5 = 0$	M1
	For attempting to solve their 3TQ $6y^2 + y - 5 = (6y - 5)(y + 1) = 0 \Rightarrow y = \frac{5}{6}, -1$	M1
	For finding the values of x $3x - 6 \times \frac{5}{6} - 1 = 0 \Rightarrow 3x = 6 \Rightarrow x = 2$ $3x - 6 \times (-1) - 1 = 0 \Rightarrow 3x = -5 \Rightarrow x = -\frac{5}{3}$ $x = 2 \quad y = \frac{5}{6} \text{ or } x = -\frac{5}{3} \quad y = -1$	A1 A1 [9]
Total 13 marks		