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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

The Quadratic Function (2)

International GCSE Further Pure Mathematics Topic 2

41 questions · 381 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

41 questions · 381 marks · lowest tariff first



4-MARK QUESTIONS

3 questions in this booklet



- 1 Find the set of values of k for which the equation

$$2kx^2 + 5kx + 5k - 3 = 0 \quad \text{where } k \neq 0$$

has real roots.

(4)



1 $f(x) = 2x^2 + (k + 8)x + k$

Show that for all values of k , the equation $f(x) = 0$ has distinct real roots.

(4)



- 1 The quadratic equation

$$kx^2 - (2k + 6)x + 16 = 0$$

has equal roots.

Find the two possible values of k

(4)



5-MARK QUESTIONS

5 questions in this booklet



5

$$f(x) = 3x^2 - 9x + 5$$

Given that $f(x)$ can be written in the form $a(x - b)^2 + c$, where a , b and c are constants, find

(a) the value of a , the value of b and the value of c .

(3)

(b) Hence write down

(i) the minimum value of $f(x)$,

(ii) the value of x at which this minimum occurs.

(2)



2

$$f(x) = 7 + 4x - x^2$$

(a) Write $f(x)$ in the form $a - b(x + c)^2$ where a , b and c are integers to be found.

(3)

(b) Hence, or otherwise, find

(i) the maximum value of $f(x)$

(ii) the value of x for which this maximum occurs.

(2)

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2

$$f(x) = 2x^2 + (p - 1)x - 2p \quad \text{where } p \text{ is a constant.}$$

Find the set of values of p for which the equation $f(x) = 0$ has two distinct real roots.

(5)



For these marks you must use the spaces in your working.

- 1 The equation $kx^2 + 8x + 3k = 0$ where k is a constant, has real unequal roots.

Find the set of values of k giving your answer in an exact simplified form.

(5)



2

$$f(x) = 2x^2 + 4x + 9$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$, where A , B and C are integers,

- (a) find the value of A , the value of B and the value of C

(3)

- (b) Hence, or otherwise, find

- (i) the value of x for which $\frac{1}{f(x)}$ is a maximum

- (ii) the maximum value of $\frac{1}{f(x)}$

(2)

6-MARK QUESTIONS

2 questions in this booklet



You must write down all the stages in your working.

- 1 The quadratic equation

$$3(k+2)x^2 + (k+5)x + k = 0$$

has real roots.

Find the set of possible values of k .

(6)



2

$$f(x) = 2 + \frac{4}{5}x - \frac{1}{25}x^2$$

Given that $f(x)$ can be expressed in the form $A - B(x + C)^2$ where A , B and C are constants,

(a) find the value of A , the value of B and the value of C .

(4)

(b) Hence write down

(i) the maximum value of $f(x)$,

(ii) the value of x for which this maximum occurs.

(2)



7-MARK QUESTIONS

5 questions in this booklet



1

$$f(x) = (x - 3)[x^2 + (p - 2)x + q]$$

Given that $f(0) = -12$

(a) find the value of q .

(2)

(b) Find the range of values of p for which the cubic equation $f(x) = 0$ has only one real root.

(5)

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8

$$f(x) = 3x^2 - x + 4$$

$$g(x) = x^2 - px + q$$

The roots of the quadratic equation $f(x) = 0$ are α and β

The roots of the quadratic equation $g(x) = 0$ are $\left(\alpha + \frac{1}{\alpha}\right)$ and $\left(\beta + \frac{1}{\beta}\right)$

Without solving the equation $f(x) = 0$

(a) show that $p = \frac{7}{12}$

(3)

(b) Find the value of q

(4)

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- 1 The roots of the equation $4x^2 - 3x - 8 = 0$ are α and β

Without solving this equation, form a quadratic equation, with integer coefficients, which has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

(7)



2

$$f(x) = 2x^2 - 12x + 5$$

Given that $f(x)$ can be written in the form $f(x) = a(x + b)^2 + c$ where a , b and c are integers,

(a) find the value of a , the value of b and the value of c

(3)

(b) Hence find the set of values of x for which $f(x) - 37 > 0$

(4)

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2

$$g(x) = 2x^2 + \frac{1}{2}x - 3$$

- (a) Express $g(x)$ in the form $p(x+q)^2 + r$ where p , q and r are rational numbers to be found.

(3)

- (b) Find

- (i) the minimum value of $g(x)$
(ii) the value of x at which this minimum occurs.

(2)

$$h(x) = 2x^6 + \frac{1}{2}x^3 - 3$$

- (c) Hence, or otherwise, write down

- (i) the minimum value of $h(x)$
(ii) the value of x at which this minimum occurs.

(2)

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8-MARK QUESTIONS

4 questions in this booklet



5 Given that α and β are such that $\alpha + \beta = \frac{7}{2}$ and $\alpha\beta = 2$

(a) form a quadratic equation with integer coefficients that has roots α and β , (2)

(b) form a quadratic equation with integer coefficients that has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. (6)

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5 Given that k is a non-zero constant

curve C has equation $kx^2 - xy + (k + 1)x = 1$

straight line l has equation $y = \frac{k}{2}x + 1$

The point A is the only point that lies on both C and l .

(a) Find the value of k

(6)

(b) Hence, find the coordinates of A .

(2)

- 2 The quadratic equation $3x^2 - 5x + 1 = 0$ has roots α and β

Without solving the equation,

form a quadratic equation with integer coefficients, that has roots $\frac{\alpha}{2\beta}$ and $\frac{\beta}{2\alpha}$ (8)



- 5 (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$ (2)

The quadratic equation $2x^2 - 6x - 7 = 0$ has roots α and β

Without solving the equation

- (b) form a quadratic equation, with integer coefficients, which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ (6)

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9-MARK QUESTIONS

1 question in this booklet



5 The roots of the quadratic equation $2x^2 + (6 + 2p)x + 2p = 0$ are α and β

(a) Write down an expression in terms of p for

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(2)

(b) Show that $(\alpha - \beta)^2 = 9 + 2p + p^2$

(4)

Given that $(\alpha - \beta) = 3$

(c) find the possible values of p

(3)

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10-MARK QUESTIONS

5 questions in this booklet



- 6 (a) Show that $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ (3)

The quadratic equation $x^2 - 7kx + k^2 = 0$, where k is a positive constant, has roots α and β where $\alpha > \beta$

- (b) Show that $\alpha - \beta = 3k\sqrt{5}$ (3)

- (c) Hence form a quadratic equation with roots $\alpha + 1$ and $\beta - 1$

Give your equation in the form $x^2 + px + q = 0$ where p and q should be given in terms of k .

(4)

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4 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β

(a) Without solving the equation, show that $\alpha^2 + \beta^2 = 1$

(4)

(b) Without solving the equation, find the value of $\alpha^4 + \beta^4$

(3)

(c) Hence form a quadratic equation with integer coefficients that has roots α^4 and β^4

(3)

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- 8 (a) Using the binomial expansion, or otherwise, find the complete expansion of

$$(x + y)^3 \quad (1)$$

The quadratic equation

$$2x^2 + 3x + 4 = 0$$

has roots α and β

- (b) Without solving the equation, find the value of

$$\alpha^3 + \beta^3 \quad (4)$$

- (c) Hence, form a quadratic equation with integer coefficients that has roots

$$\frac{\alpha}{\beta^2} \text{ and } \frac{\beta}{\alpha^2} \quad (5)$$

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6 $f(x) = 2x^2 + 5x - p$

The equation $f(x) = 0$ has roots α and β

Given that $\alpha^3 + \beta^3 = -\frac{215}{8}$

(a) find the value of p

(5)

Without solving the equation $f(x) = 0$

(b) form a quadratic equation, with integer coefficients, that has roots

$$\frac{\alpha + \beta}{\alpha^2} \text{ and } \frac{\alpha + \beta}{\beta^2}$$

(5)

10 The roots of a quadratic equation are α and β where

$$\alpha + \beta = -\frac{5}{2} \text{ and } \alpha^3 + \beta^3 = \frac{115}{8}$$

(a) Show that $\alpha\beta = 4$

(3)

(b) Form a quadratic equation with integer coefficients, that has roots

$$\frac{\alpha^2 + 1}{\beta} \text{ and } \frac{\beta^2 + 1}{\alpha}$$

(7)



11-MARK QUESTIONS

3 questions in this booklet



- 2 (a) Express $x^2 + 4x - 8$ in the form $(x + a)^2 + b$ where a and b are constants whose values are to be found.

(2)

- (b) Use algebra to solve the simultaneous equations

$$y = x^2 + 4x - 8$$

$$y = 2x + 7$$

(5)

The curve C has equation $y = x^2 + 4x - 8$

The straight line L has equation $y = 2x + 7$

Using the same axes and the results of part (a) and part (b),

- (c) sketch C and L , showing clearly the coordinates of the turning point of C and the coordinates of the points of intersection of C and L .

(4)

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6

$$f(x) = 4x^2 - 3x - 5$$

The equation $f(x) = 0$ has roots α and β

Without solving the equation $f(x) = 0$

- (a) form an equation, with integer coefficients, that has roots $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$ (6)

$$g(x) = 4x^2 + px + q \quad \text{where } p \text{ and } q \text{ are constants}$$

Given that the equation $g(x) = 0$ has roots $3\alpha + \beta$ and $\alpha + 3\beta$

- (b) find the value of p and the value of q (5)



8 The quadratic equation

$$x^2 - 4k\sqrt{2}x + 2k^4 - 1 = 0$$

where k is a positive constant, has roots α and β

Given that $\alpha^2 + \beta^2 = 66$ and that $\alpha^3 + \beta^3 = p\sqrt{2}$ where p is an integer,

find the value of p

(11)

12-MARK QUESTIONS

5 questions in this booklet



11 The quadratic equation $x^2 - px + q = 0$ where $p > 0$, has roots α and β .

Given that $2\alpha\beta = 3$ and that $4(\alpha^2 + \beta^2) = k^2 - 6k - 3$ where $k > 3$

(a) (i) write down the value of q ,

(ii) find an expression, in terms of k , for p .

(5)

Given also that $7\alpha\beta = 3(\alpha + \beta)$

(b) find the value of k .

(2)

(c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha}{\alpha + \beta} \text{ and } \frac{\beta}{\alpha + \beta}$$

(5)

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- 5 (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$ (2)

The roots of the equation $2x^2 + 3x + 6 = 0$ are α and β

Without solving the equation,

- (b) find the value of $\alpha^3 + \beta^3$ (2)

- (c) Show that $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$ (2)

- (d) Form a quadratic equation with integer coefficients that has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$ (6)

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- 7 (a) Find the set of values of k for which the equation $kx^2 - 4x + 2k = 7$ has real roots

(4)

Given that the roots of the equation $kx^2 - 4x + 2k = 7$ are α and β ,

- (b) form a quadratic equation with roots $\frac{\alpha + 1}{\alpha}$ and $\frac{\beta + 1}{\beta}$

Give each coefficient in terms of k .

(8)

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11 The roots of a quadratic equation E are α and β where $\alpha > \beta > 0$

Given that $\alpha - \beta = 2\sqrt{6}$ and $\alpha^2 + \beta^2 = 30$

(a) show that

(i) $\alpha\beta = 3$ (4)

(ii) $\alpha + \beta = 6$ (2)

(b) Without solving E

(i) find the value of $\alpha^4 + \beta^4$ (2)

(ii) find the exact value of $\alpha^4 - \beta^4$ (2)

Given that $\alpha^4 = P + Q\sqrt{6}$ where P and Q are positive integers,

(c) find the value of P and the value of Q (2)

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10 The quadratic equation $2x^2 + kx + 4 = 0$ has roots α and β such that

$$k < 0 \text{ and } \alpha > \beta$$

Given that $\alpha^2 - \beta^2 = \frac{7\sqrt{17}}{4}$

(a) show that $k = -7$

(8)

(b) Hence form a quadratic equation that has roots

$$(\alpha - \beta) \text{ and } (\alpha + \beta)$$

(4)

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13-MARK QUESTIONS

4 questions in this booklet



10

$$f(x) = 6x - x^2 \quad x \in \mathbb{R}$$

Given that $f(x)$ can be written in the form $D(x + E)^2 + F$ where D , E and F are integers,

(a) find the value of D , the value of E and the value of F .

(3)

(b) Find

(i) the maximum value of $f(x)$,

(ii) the value of x for which the maximum occurs.

(2)

The curve C has equation $y = f(x)$

The curve S has equation $y = x^2 - 4x + 8$

The curve S intersects the curve C at two points.

(c) Find the coordinates of each of these two points.

(4)

The finite region R is bounded by the curve C and the curve S .

(d) Use algebraic integration to find the area of R .

(4)

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7

$$f(x) = x^2 - 9x + 14$$

Given that $f(x)$ can be written in the form $(x + a)^2 + b$, where a and b are constants,

(a) find the value of a and the value of b .

(2)

(b) Hence, or otherwise, find

(i) the minimum value of $f(x)$

(ii) the value of x for which this minimum occurs.

(2)

The curve C has equation $y = f(x)$

The line l has equation $y = x + 5$

(c) Use algebra to find the coordinates of the points of intersection of C and l .

(4)

(d) Use algebraic integration to find the exact area of the finite region bounded by C and l .

(5)

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7

$$\begin{aligned} f(x) &= 2x^2 + px + q && \text{where } p \text{ and } q \text{ are integers} \\ g(x) &= 14x^2 + 37x + 14 \end{aligned}$$

The equation $f(x) = 0$ has roots α and β

The equation $g(x) = 0$ has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

Given that $p + q = -4$ where $p > 0$ and without solving the equation $g(x) = 0$

(a) find

(i) the value of p

(ii) the value of q

(9)

Given also that $\alpha > \beta$

(b) show that $\alpha^2 - \beta^2 = -\frac{3\sqrt{65}}{4}$

(4)

3

$$f(x) = 8x^2 + 10x - 3$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$ where A , B and C are constants,

- (a) find the value of A , the value of B and the value of C .

(3)

- (b) Hence, or otherwise, find,

- (i) the value of x for which $f(x)$ has a minimum,
(ii) the minimum value of $f(x)$.

(2)

The curve C has equation $y = f(x)$.

- (c) Find the x coordinate of each of the points where C crosses the x -axis.

(2)

The straight line l has equation $y = 2x + 13$

- (d) Use algebra to find the coordinates of the two points of intersection of C and l .

(4)

Using the same axes and the results of parts (b), (c) and (d),

- (e) sketch the curve C and the straight line l .

(2)

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14-MARK QUESTIONS

2 questions in this booklet



- 8 The quadratic equation $3x^2 - kx - 1 = 0$, where k is a positive integer, has roots α and β

(a) Show that $\alpha^2 + \beta^2 = \frac{k^2 + 6}{9}$

(3)

Given that $\alpha^4 + \beta^4 = \frac{466}{81}$

- (b) find the value of k

(5)

- (c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha^3 + \beta}{\beta} \text{ and } \frac{\beta^3 + \alpha}{\alpha}$$

(6)

11

$$f(x) = 10 + 6x - x^2$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$ where A , B and C are constants,

- (a) find the value of A , the value of B and the value of C

(4)

- (b) Hence, or otherwise, find

- (i) the value of x for which $f(x)$ has its greatest value

- (ii) the greatest value of $f(x)$

(2)

The curve C has equation $y = f(x)$

The curve S with equation $y = x^2 - x + 13$ intersects curve C at two points.

- (c) Find the x coordinate of each of these two points.

(3)

- (d) Use algebraic integration to find the exact area of the finite region bounded by the curve C and the curve S .

(5)

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15-MARK QUESTIONS

1 question in this booklet



10 The roots of the equation $x^2 + 3x - 5 = 0$ are α and β .

(a) Without solving the equation, find

(i) the value of $\alpha^2 + \beta^2$

(ii) the value of $\alpha^4 + \beta^4$

(5)

Given that $\alpha > \beta$ and without solving the equation

(b) show that $\alpha - \beta = \sqrt{29}$

(2)

(c) Factorise $\alpha^4 - \beta^4$ completely.

(3)

(d) Hence find the exact value of $\alpha^4 - \beta^4$

(2)

Given that $\beta^4 = p + q\sqrt{29}$ where p and q are positive constants

(e) find the value of p and the value of q .

(3)

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18-MARK QUESTIONS

1 question in this booklet



10

$$f(x) = 3 - 4x - 9x^2$$

Given that $f(x)$ can be expressed in the form $A - B(x + C)^2$ where A , B and C are positive constants

- (a) find the value of A , the value of B and the value of C

(4)

- (b) Hence write down the maximum value of $f(x)$

(1)

The equation $f(x) = 0$ has roots α and β

Without solving the equation $f(x) = 0$

- (c) form a quadratic equation, with integer coefficients, that has roots $\frac{3\alpha}{\beta}$ and $\frac{3\beta}{\alpha}$

(6)

- (d) Show that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

(1)

$$g(x) = 3x^2 + qx + r \quad \text{where } q \text{ and } r \text{ are constants}$$

The equation $g(x) = 0$ has roots $\alpha^2 - \beta$ and $\beta^2 - \alpha$ where α and β are the roots of the equation $f(x) = 0$

- (e) Using your answer to part (d), find in simplified exact form, the value of q and the value of r

(6)

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MARK SCHEMES

official Pearson Edexcel mark schemes



Question number	Scheme	Marks
1	$b^2 - 4ac \geq 0 \Rightarrow (5k)^2 - 4(2k)(5k - 3) [\geq 0]$	M1
	$[-15k^2 + 24k] \Rightarrow 3k(-5k + 8)$	M1
	$'0' < k \leq \frac{8}{5}$	M1
	$0 < k \leq \frac{8}{5}$	A1 (4)
Total 4 marks		

Mark	Notes
M1	For correct substitution of a , b and c into $b^2 - 4ac$ Allow with any inequality, equals or even no sign at all.
M1	For solving their quadratic equation using any valid method, (provided the QE is either a 2TQ or a 3TQ). They must reach TWO critical values for the award of this mark. See General Guidance for the definition of a valid attempt to solve a QE If they use a calculator, then the Quadratic equation and the two critical values must be correct for the award of this mark.
M1	For forming an 'inside' region with their critical values. Allow use of either $<$ or $\leq$ here $'0' < k \leq \frac{8}{5}$ Allow for example $'0' \leq k \leq \frac{8}{5}$
A1	For $0 < k \leq \frac{8}{5}$ allow $0 < k < \frac{8}{5}$

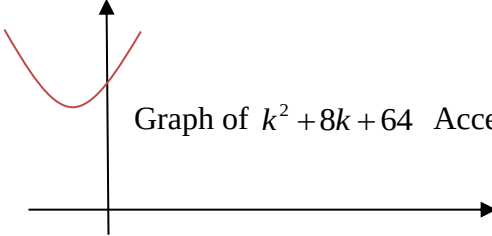
Question	Scheme	Marks
1	$b^2 - 4ac = (k+8)^2 - 4 \times 2 \times k = [k^2 + 8k + 64]$	M1
	$k^2 + 8k + 64 = (k+4)^2 - 16 + 64 = (k+4)^2 + 48$	dM1A1
	Conclusion: Irrespective of the value of k , $(k+4)^2 \geq 0$ and so $b^2 - 4ac > 0$ therefore the equation will always have distinct (real) roots.	A1 [4]
Total 4 marks		

Mark	Notes
M1	Applies the discriminant on the given QE Ignore $= 0$, ≥ 0 , > 0 This must be correct for this mark $[(k+8)^2 - 4 \times 2 \times k]$. Ignore subsequent simplification errors.
dM1	Completes the square ONLY on their expression for the discriminant provided it is a 3TQ The expression must be of the form $(k+4)^2 \pm X$ where $X \neq 0, 64$ NB: This mark is dependent on the first M mark. Use of calculus to find $k = -4$ is M0
A1	For the correct expression.
A1	For a suitable conclusion. Only accept for this final mark a conclusion as follows [or an equivalent]. $(k+4)^2 \geq 0$ and so $(k+4)^2 + 48 > 0$ hence it will always have (real), distinct roots. Note: $(k+4)^2 + 48 \geq 0$ is A0



Alternative methods for last 3 marks.	
ALT 1 finds discriminant of $k^2 + 8k + 64$	
M1	Finds discriminant of their $k^2 + 8k + 64 \Rightarrow D = 64 - 4 \times 1 \times 64 = -192$
A1	Obtains the correct expression in terms of k with the correct value of D
A1	Conclusion: $D < 0$ so $k^2 + 8k + 64$ does not have real roots. k^2 is positive so we know it has a minimum so $k^2 + 8k + 64 > 0$ This means that $k^2 + 8k + 64 > 0$ and therefore the discriminant will be > 0 meaning that the equation has distinct real roots for all values of k .

ALT 2 – Uses calculus	
M1	$y = k^2 + 8k + 64 \Rightarrow \frac{dy}{dk} = 2k + 8 = 0$ at max/min
A1	So coordinates of max/min are $(-4, 48)$ and finds second derivative $\frac{d^2y}{dk^2} = 2$
A1	Conclusion: $\frac{d^2y}{dk^2} = 2 > 0$ hence it is a minimum. $(-4, 48)$ is the minimum point and so $k^2 + 8k + 64 > 0$ This means that $k^2 + 8k + 64 > 0$ and therefore the discriminant will be > 0 meaning that the equation has distinct real roots for all values of k .

ALT 3 – Uses a graph	
M1	Draws a sketch.  Graph of $k^2 + 8k + 64$ Accept anywhere in positive y
A1	The graph is placed correctly with the min point in the 4 th quadrant.
A1	Conclusion: the graph is always positive with its min point as shown in the diagram. This means that $k^2 + 8k + 64 > 0$ and therefore the discriminant will be > 0 meaning that the equation has distinct real roots for all values of k .

Question	Scheme	Marks
1	$[\pm(2k+6)]^2 - 4 \times k \times 16$ $4k^2 - 40k + 36 = 0$ $\rightarrow \{4\}(k-1)(k-9)\{=0\} \rightarrow k = \dots$ $k = 1, 9$	M1 dM1 M1 A1 [4]
Total 4 marks		

Mark	Notes
M1	Allow working on expression, equation or any inequality for all M marks Applies $b^2 - 4ac = [\pm(2k+6)]^2 - 4 \times k \times 16$ and might be seen embedded in an attempt at the quadratic formula.
dM1	Forms a 3TQ in k by attempting to expand and collect the like terms.
M1	Attempts to solve their 3TQ in k to find at least one real value of k by any correct method (See General Principles). Condone labelling their k as x for this method mark. If there is no method shown, (use of a calculator) then the 3TQ must be correct and both k values must be fully correct ($k = 1, k = 9$) for evidence for this mark.
A1	For $k = 1, 9$ selected as their final answers. (Extra inequality answers for k must be rejected)

Question	Scheme	Marks
1 ALT I (Via completing Square)	$k\left(x - \frac{k+3}{k}\right)^2 - \frac{(k+3)^2}{k} + 16$ $16 - \frac{(k+3)^2}{k} = 0 \rightarrow k^2 - 10k + 9 = 0$ $(k-1)(k-9)\{=0\} \rightarrow k = \dots$ $k = 1, 9$	M1 dM1 M1 A1 [4]
Total 4 marks		

Mark	Notes
M1	Attempts to complete the square for $kx^2 - (2k+6)x + 16$ (See General Principles) Similarly, divides both sides of the given quadratic equation by k first, then attempts to complete the square: $\left(x - \frac{k+3}{k}\right)^2 - \frac{(k+3)^2}{k^2} + \frac{16}{k}$
dM1	Sets the y coordinate of their turning point to be zero ($= 0$ can be implied) attempts to expand and collect like terms.
M1	Solves their 3TQ or polynomial to find at least one real value of k by any correct method (See General Principles) If there is no method shown, (use of a calculator) then their 3TQ must be correct and both k values must be fully correct ($k = 1, k = 9$) for evidence for this mark.
A1	For $k = 1, 9$ only

Question	Scheme	Marks
1 ALT II (Via sum and product of roots)	$\alpha + \beta = \pm \frac{2k+6}{k}, \alpha \times \beta = \frac{16}{k} \rightarrow \left\{ \alpha + \alpha = \pm \frac{2k+6}{k}, \alpha \times \alpha = \frac{16}{k} \right\}$	M1
	$\left(\frac{k+3}{k}\right)^2 = \frac{16}{k} \rightarrow k^2 - 10k + 9 = 0$	dM1
	$(k-1)(k-9) = 0 \rightarrow k = \dots$	M1
	$k = 1, 9$	A1 [4]
Total 4 marks		

Mark	Notes
M1	For: $\alpha + \beta = \pm \frac{2k+6}{k}, \alpha \times \beta = \frac{16}{k} \rightarrow \left\{ \alpha + \alpha = \pm \frac{2k+6}{k}, \alpha \times \alpha = \frac{16}{k} \right\}$
dM1	Uses the fact that the quadratic equation has equal roots $\alpha = \beta$, sets up a correct equation in k , expands, collects like terms to form a 3TQ (or a Cubic equation) e.g. $\left(\frac{2k+6}{2k}\right)^2 = \frac{16}{k} \rightarrow 4k^3 - 40k^2 + 36k = 0$ or $4k^2 - 40k + 36 = 0$
M1	Solves their 3TQ or Cubic equation to find at least one real value of k by any correct method (See General Principles) If there is no method shown, (use of a calculator) then their 3TQ or Cubic must be correct and both k values must be fully correct ($k = 1, k = 9$) for evidence for this mark.
A1	For $k = 1, 9$ only

ALT III: (Via differentiation)

M1: Differentiates $kx^2 - (2k+6)x + 16$ with respect to x to achieve a linear expression

dM1: Sets their derivative to 0, solves for x and substitutes x back to $kx^2 - (2k+6)x + 16 = 0$ to form a 3TQ.

M1A1 same as ALT II



Question	Scheme	Marks
5 (a)	$f(x) = 3(x^2 - 3x) + 5 = 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 5 = 3\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$ $a = 3, b = \frac{3}{2}, c = -\frac{7}{4}$ ALT Expand and equate coefficients	M1M1A1 [3]
(b)	(i) $f(x)_{\min} = -\frac{7}{4}$ (ii) $x = \frac{3}{2}$ (Allow both if obtained by differentiation)	B1ft B1ft [2]
Total 5 marks		

Additional Guidance		
(a)	M1	For an attempt to take out 3 as a common factor to achieve; $3\left(x^2 - 3x + \frac{5}{3}\right)$ or $3(x^2 - 3x) + 5$ This must be correct for this mark
	M1	For an acceptable attempt to complete the square. Minimally acceptable attempts are; $3\left(x \pm \frac{3}{2}\right)^2 \pm l$ or $3\left[\left(x \pm \frac{3}{2}\right)^2 \pm m\right]$ where l and m are constants
	A1	For either $3\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$ or $a = 3, b = \frac{3}{2}, c = -\frac{7}{4}$ The values of a, b and c can be embedded and need not be explicitly stated.
	ALT	
	M1	For a correct expansion of $a(x - b)^2 + c$ and set = to $3x^2 - 9x + 5$ $a(x - b)^2 + c = ax^2 - 2abx + ab^2 + c \Rightarrow 3x^2 - 9x + 5 = ax^2 - 2abx + ab^2 + c$
	dM1	For an attempt to equate coefficients. Minimally acceptable attempt; $a = 3, 9 = 2 \times 3 \times b \Rightarrow b = \dots, 5 = 3 \times b^2 + c \Rightarrow c = \dots$
(b)	A1	For either $3\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$ or $a = 3, b = \frac{3}{2}, c = -\frac{7}{4}$ The values of a, b and c can be embedded and need not be explicitly stated.
	(i) B1ft	For $f(x)_{\min} = -\frac{7}{4}, f(x)_{\min} = \text{'their } c\text{'}$ Note this a follow through mark.
	(ii) B1ft	For $x = \frac{3}{2}, x = -\text{'their } b\text{'}$ Note this a follow through mark.
Allow B1B1 for correct values obtained using differentiation. Their answers must be clearly labelled and there must be no ambiguity for the minimum value of $f(x)$ and the value of x at which the minimum occurs.		

Question number	Scheme	Marks
2 (a)	$7 + 4x - x^2 = 11 - (x - 2)^2$ $[a = 11, b = 1, c = -2]$ ALT $7 + 4x - x^2 = a - b(x^2 + 2cx + c^2)$ $a - bc^2 = 7 \quad b = 1 \quad bc = 4 \quad \text{So } a = 11, b = 1, c = -2$ $7 + 4x - x^2 = 11 - (x - 2)^2$	M1A1A1 [3] {M1} {A1} {A1} [3]
(b)	(i) 11 (ii) 2	B1ft B1ft [2]
Total 5 marks		
(a) M1 A1 A1 ALT M1 A1 A1	An attempt to factorise to make x^2 positive e.g. $-(x \pm p)^2 \pm q$ Complete the square to obtain an expression in the form $-(x \pm 2)^2 \pm q$ NB Any expression in this form will score M1A1 $11 - (x - 2)^2$ or $a = 11, b = 1, c = -2$ Expands $a - b(x + c)$ $a - bc^2 = 7 \quad b = 1 \quad bc = 4$ $11 - (x - 2)^2$ or $a = 11, b = 1, c = -2$	
(b) (i) B1ft	Mark parts b(i) and b(ii) together 11 follow through their a	
(b) (ii) B1ft	2 follow through their c NB Answer of Max = (2, 11) score B1B1	

Question number	Scheme	Marks
2	$(p - 1)^2 - 4 \times 2 \times -2p > 0 \Rightarrow p^2 + 14p + 1 > 0$ critical values $p = \frac{-14 \pm \sqrt{14^2 - 4 \times 1 \times 1}}{2} = -7 \pm 4\sqrt{3}$ $p < -7 - 4\sqrt{3}$ OR $p > -7 + 4\sqrt{3}$	M1A1 M1 M1A1 [5]
Total 5 marks		

Mark	Notes
M1	Uses $b^2 - 4ac$ on the given quadratic equation with correct a, b, c ; $a = 2$ $b = p - 1$ $c = -2p$ and a correct substitution to obtain $(p - 1)^2 - 4 \times 2 \times -2p$ Note: Accept for this mark any inequality, equals sign and even $b^2 - 4ac$ used on its own.
A1	For the correct 3TQ with the correct inequality. Note: Allow $\geq$ or $\leq$ in place of $>$ and $<$ for this mark. $p^2 + 14p + 1 > 0$ or $-p^2 - 14p - 1 < 0$
M1	For an attempt to solve their 3TQ (provided it is a 3TQ) in terms of p by any acceptable method to obtain 2 values for p . See General Guidance for the definition of an attempt by factorisation, formula or completing the square. Use of calculators: if their 3TQ is incorrect, do not award this mark if working is not seen. $p = \frac{-14 \pm \sqrt{14^2 - 4 \times 1 \times 1}}{2} = -7 \pm 4\sqrt{3}$ oe Accept awrt -13.9 , awrt -0.072
M1	For forming the correct inequalities with their critical values, provided they have been obtained from a 3TQ. $p < -7 - 4\sqrt{3}$ OR $p > -7 + 4\sqrt{3}$ oe ft their values from their $p^2 + 14p + 1 > 0$ or $-p^2 - 14p - 1 < 0$ Note: Condone use of AND for this mark. Condone $\geq$ or $\leq$ in place of $>$ and $<$ for this mark. Accept awrt -13.9 , awrt -0.072
A1	For the correct inequality. $p < -7 - 4\sqrt{3}$ oe OR $p > -7 + 4\sqrt{3}$ oe Note: Must not indicate AND for this mark. Accept awrt -13.9 , awrt -0.072

Question	Scheme	Marks
1	$b^2 - 4ac > 0 \Rightarrow 8^2 - 4 \times k \times 3k > 0$	M1
	$64 - 12k^2 > 0$	
	cvs: $k = \pm \sqrt{\frac{64}{12}} = \left[\pm \frac{4\sqrt{3}}{3} \right]$ or $\left[\pm \frac{4}{\sqrt{3}} \right]$	M1A1
	$\Rightarrow -\frac{4\sqrt{3}}{3} < k < \frac{4\sqrt{3}}{3}$	M1A1 [5]
Total 5 marks		

Mark	Notes
M1	Applies the correct values, with the correct inequality to $b^2 - 4ac > 0$
M1	Attempts to find two critical values by solving the quadratic equation, which must be of the form $k^2 = \text{constant}$ [oe] using a correct method. Accept as a minimum solution $k = \pm \sqrt{\text{constant}}$ Allow simplified or unsimplified. Ignore any inequalities, equal signs etc
A1	For the correct critical values simplified or unsimplified Award this mark for correct critical values. Ignore $=, <, >$ or even $\leq, \geq$ M0M1A1 is a possible marking pattern.
M1	Simplifies the critical values to the required form and writes down the inside region for their TWO critical values. If they solve a linear equation for k this mark is not available. Allow use of x for this mark and also allow $\leq$ in place of $<$.
A1	For the correct region specified correctly in either of the two forms specified in terms of k . That is, a continuous inside region. Accept $-\frac{4}{\sqrt{3}} < k < \frac{4}{\sqrt{3}}$ oe Accept also for example $k > -\frac{4}{\sqrt{3}}$ AND $k < \frac{4}{\sqrt{3}}$

$$(2x)^2 = (3x)^2 + (4x)^2 - 2 \times 3x \times 4x \times \cos A \text{ or}$$

e.g. $(\cos A =) \frac{(3x)^2 + (4x)^2 - (2x)^2}{2 \times 3x \times 4x} \Rightarrow \angle A = 28.95502...^\circ$

All the values here are correct to give 28.95502, but the labelling of angle A is incorrect.

Allow angles in degrees to be rounded to the nearest whole number here and angles in radians to be rounded to 1 dp.

Where candidates do not actually work the angle out, if and only if the fully correct expression for $\cos(\text{angle})$ or $\text{angle} = \cos^{-1}$ is seen in the 1st M1, look for any of the following used in the 2nd M1, then this A mark can be awarded.

$$\sin(\cos^{-1}(\text{ANS})) \text{ or } \sin(\cos^{-1}(((\text{angle}) A))) \text{ or } \sin(\cos^{-1}(((\text{angle}) B)))$$

$$\text{or } \sin(\cos^{-1}(((\text{angle}) C)))$$

$$\text{or for } \sin\left(\cos^{-1}\left(-\frac{1}{4}\right)\right) \text{ or } \sin\left(\cos^{-1}\left(\frac{11}{16}\right)\right) \text{ or } \sin\left(\cos^{-1}\left(\frac{7}{8}\right)\right) \text{ seen}$$

dM1	For using the correct formula for the area of a triangle using 50 and their angle (or use of $\sin \cos^{-1}$ correctly and the correct 2 sides for their angle and a fully correct rearrangement to find x^2 or x If 1 st M1 has been gained and a poor rearrangement occurs to give an incorrect angle ft their angle. This is a general principle of marking. Sight of 17.2(1.....) following M1 A1 and the correct formula written will imply this mark. The same condition applies, if the angle is not actually worked out as for the A mark. Dependent on previous method mark.
A1	For the correct value of x . Accept awrt 4.15 [cm]
ALT1	
M1	For fully correct substitution, as shown into Heron's formula.
A1	For the correct expression for s
x must be present, but may be recovered (indicated by A1)	
dM1	Applies Heron's formula correctly with their value of s and 50 and a fully correct rearrangement to find x^2 or x
A1	For the correct value of x . Allow all awrt 4.15.

ALT2	
M1	For a fully correct substitution into the cosine formula as shown. ie including an equals sign. OR for a fully correct expression for $\cos ABC$ or $\cos BAC$ or $\cos ACB$ Allow students to use just B , A and C for angles or any labelling.
A1	For one of the correct values for $\sin(\text{relevant angle})$
Note for M1 A1 The x can be consistently omitted or the x can be recovered (also indicated by a correct value for $\sin$) or values in the correct proportions can be used as an alternative (also indicated by a correct value for $\sin$). M1 may also be awarded if the values in the expression or equation lead to the correct $\sin$ value for those values, regardless of the labelling of the angle. This may also lead to A1 $(2x)^2 = (3x)^2 + (4x)^2 - 2 \times 3x \times 4x \times \cos A \quad \text{or}$ e.g. $(\cos A =) \frac{(3x)^2 + (4x)^2 - (2x)^2}{2 \times 3x \times 4x} \Rightarrow \sin A = \sqrt{\frac{15}{64}}$ All the values here are correct to give $\Rightarrow (\sin ABC =) \sqrt{\frac{15}{64}}$, but the labelling of angle A is incorrect. Allow values to be rounded to 1 dp.	
dM1	For using the correct formula for the area of a triangle using 50 and their $\sin$ value correctly and the correct 2 sides for their angle and a fully correct rearrangement to find x^2 or x If M1 has been gained but their $\sin$ value is incorrect, students must be seen to have substituted into $\cos^2(\text{angle}) + \sin^2(\text{angle}) = 1$. Though a poor rearrangement to give an incorrect $\sin$ value, will potentially allow this mark to be awarded. Sight of 17.2(1.....) following M1 A1 will imply this mark. Dependent on previous method mark.
A1	For the correct value of x . Accept awrt 4.15 [cm]

Question	Scheme	Marks
For part (a) of this question, mark using the scheme which gives the most marks.		
2 (a)	$A = 2 \quad B = 1 \quad C = 7$	B1B1B1
ALT	$2x^2 + 4x + 9 = 2(x^2 + 2x) + 9 = 2[(x+1)^2 - 1] + 9$ $\Rightarrow f(x) = 2(x+1)^2 + 7 \quad A = 2 \quad B = 1 \quad C = 7$	M1M1 A1 [3]
(b)(i)	-1	B1ft
(ii)	$\frac{1}{7}$	B1ft [2]
Total 5 marks		

Part	Mark	Notes
• Mark using the B scheme first. • If not full marks – use the MMA scheme also, if appropriate. • Same score – apply the B marks. • Higher score – apply the marks from the MMA scheme. Allow the candidate to state A, B and C or the values to be embedded within an expression.		
(a) Different marks on ePen	B1	For one of A , B , or C correct.
	B1	For two of A , B , or C correct.
	B1	For all of A , B , and C correct.
ALT	M1	For correctly factorising the given expression to achieve either: $2(x^2 + 2x) + 9$ or $2\left(x^2 + 2x + \frac{9}{2}\right)$
	M1	Completes the square correctly, regardless of any factor on the outside – follow through their factorisation. $(x^2 + ax + b)$ or $(x^2 + ax) + c \Rightarrow$ ie $\left[\left(x + \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b\right]$ or $\left[\left(x + \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2\right] + c \quad a, b, c \neq 0$
	A1	For all of A , B and C correct.
Note, the question does not ask students to show working nor preclude the use of a calculator, so if values are simply listed, these can be given marks from the B scheme throughout.		
(b)(i)	B1ft	For the correct value or follow through their $-B$
(ii)	B1ft	For the correct value or follow through their $\frac{1}{C}$
Only if no labelling of (i) and (ii)	SC2	If candidate clearly writes max value of $\frac{1}{7}$ when $x = -1$. Allow ft
	SC1	If candidate clearly writes $\left(-1, \frac{1}{7}\right)$ Allow ft Marked as 1 st B1
If no labelling of (i) and (ii) is present for parts (b) – marks may be awarded for the values presented in the correct order. B0 B0 if not labelled and work doesn't meet this condition.		



Question number	Scheme	Marks
1	$b^2 - 4ac \geq 0$ $(k+5)^2 - 4k(3k+6) \geq 0$ $k^2 + 10k + 25 - 12k^2 - 24k \geq 0$ $11k^2 + 14k - 25 \leq 0$ $(11k+25)(k-1) \leq 0$ [Critical values are $-\frac{25}{11}$ and 1] $-\frac{25}{11} \leq k \leq 1$ oe	M1 M1 A1 M1 M1A1
Total 6 marks		

Mark	Notes
M1	Uses $b^2 - 4ac$ on the given quadratic equation with correct a , b and c ; $a = 3(k+2)$ $b = k+5$ $c = k$ and a correct substitution to obtain $(k+5)^2 - 4 \times 3 \times (k+2)(k)$ Note: Accept for this mark any inequality, equals sign and even $b^2 - 4ac$ used on its own.
M1	For attempting to expand the brackets and form a 3TQ in terms of k . Allow as a minimum at least one term correct. $k^2 + 10k + 25 - 12k^2 - 24k \Rightarrow (-11k^2 - 14k + 25)$ M0M1 is possible here.
A1	For the correct 3TQ with the correct inequality. Note: Allow $>$ or $<$ in place of $\geq$ and $\leq$ for this mark $-11k^2 - 14k + 25 \geq 0$ or $11k^2 + 14k - 25 \leq 0$
M1	For an attempt to solve their 3TQ, (provided it is a 3TQ) in terms of k by any acceptable method. See General Guidance for the definition of an attempt by factorisation, formula or completing the square. Use of calculators: if their 3TQ is incorrect, do not award this mark if working is not seen. $(11k+25)(k-1) = 0 \Rightarrow k = 1, -\frac{25}{11}$
M1	For forming the correct inequality with their critical values, provided they have been obtained from a 3TQ, must be a closed region. $\left(-\frac{25}{11} \leq k \leq 1\right)$ ft their values from their $-11k^2 + 14k - 25 \geq 0$ or $11k^2 - 14k + 25 \leq 0$
A1	For the correct inequality. $-\frac{25}{11} \leq k \leq 1$

Question number	Scheme	Marks
2 (a)	$2 - \frac{1}{25}(x^2 - 20x)$ $2 \mp \frac{1}{25}[(x \pm 10)^2 - 100]$ $6 - \frac{1}{25}(x - 10)^2 \quad \text{So } A = 6 \quad B = \frac{1}{25} \quad C = -10$	M1 A1 A1 A1 (4)
ALT	$A - Bx^2 - 2BCx - BC^2 = 2 + \frac{4}{5}x - \frac{1}{25}x^2$ $B = \frac{1}{25}$ $-\frac{2}{25}C = \frac{4}{5} \quad C = -10$ $-\frac{1}{25}(100) + A = 2 \quad A = 6$	{M1} {A1} {A1} {A1} (4)
(b)(i)	6	B1 ft
(b)(ii)	10	B1 ft
		(2)
Total 6 marks		

Part	Mark	Guidance
(a)	M1	For a complete method to complete the square to achieve as a minimum $2 \mp \frac{1}{25}(x \pm 10)^2 - p$ or $\mp \frac{1}{25}[(x \pm 10)^2 - q]$ where p and q are constants
	A1	For one correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated explicitly or embedded
	A1	For two correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated explicitly or embedded
	A1	Fully correct $A = 6$ $B = \frac{1}{25}$ and $C = -10$ OR $6 - \frac{1}{25}(x - 10)^2$ oe
	ALT – equates coefficients	
	M1	For an attempt to expand $A - B(x + C)^2$ AND equate coefficients to the given $f(x) \Rightarrow A - Bx^2 - 2BCx - BC^2 = 2 + \frac{4}{5}x - \frac{1}{25}x^2$ Allow $A \pm Bx^2 \pm 2BCx \pm BC^2$ for the expansion of $A - B(x + C)^2$ There must be an attempt to equate at least one coefficient. $-B = -\frac{1}{25} \Rightarrow B = \dots$ $-2BC = \frac{4}{5} \Rightarrow C = \dots$ $A - BC^2 = 2 \Rightarrow A = \dots$
	A1	For one correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated explicitly or embedded
	A1	For two correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated explicitly or embedded
	A1	Fully correct $A = 6$ $B = \frac{1}{25}$ and $C = -10$ OR $6 - \frac{1}{25}(x - 10)^2$ oe
(b)(i)	B1ft	For the value of 6 or ft their A
(ii)	B1ft	For the value of 10 or ft their C

Question	Working	Marks
1 (a)	$-3 \times q = -12 \Rightarrow q = 4$	M1A1 [2]
(b)	$b^2 - 4ac < 0, (p-2)^2 - 4 \times 1 \times 4 < 0$ To find critical values; $p^2 - 4p - 12 = 0 \Rightarrow (p-6)(p+2) = 0 \Rightarrow p = 6, -2$ (Chooses inside region from their critical values) $-2 < p < 6$	M1 dM1A1 ddM1 A1 [5]
Alternative		
(b)	$b^2 - 4ac < 0, (p-2)^2 - 4 \times 1 \times 4 < 0$ $(p-2)^2 < 16 \Rightarrow -4 < (p-2) < 4$ $-2 < p < 6$	M1 dM1A1 ddM1A1
Total 7 marks		

(a)	M1	Substitute $x = 0$ and $f(x) = -12$ and proceed to eliminate the term in p to obtain a linear equation in q .
	A1	$q = 4$ (score M1 A1 if no working shown)
(b)	For the first 3 marks allow $b^2 - 4ac < 0$ or $b^2 - 4ac = 0$ or $b^2 - 4ac > 0$	
	M1	Attempt to use $b^2 - 4ac (< 0)$ with $b = p - 2, a = 1$ and $c = 4$ (ft from (a)). Correct formula shown and used or correct substitution.
	dM1	Attempt to solve their quadratic inequality or equation. Depends on previous M mark. See general guidance for solving quadratic equations.
	A1	Correct critical values, $(p =) -2, (p =) 6$
	ddM1	Correct use of their critical values, showing that the inside region has been selected. Both M marks needed. Allow with $\leq$ or $<$ or set language statement.
	A1	$-2 < p < 6$ must be in terms of p . Allow $p \in (-2, 6)$ or $-2 < p$ AND $p < 6$ or $\{-2 < p \cap p < 6\}$
Alternative		
(b)	For the first 3 marks allow $b^2 - 4ac < 0$ or $b^2 - 4ac = 0$ or $b^2 - 4ac > 0$	
	M1	Attempt to use $b^2 - 4ac < 0$ with $b = p - 2, a = 1$ and $c = 4$ (ft from (a)). Correct formula shown and used or correct substitution.
	dM1	Write in the form $(p-2)^2 < 16$ and attempt to take square root of both sides (both positive and negative options are needed), or as $(p-2)^2 - 16 < 0$ and attempt to factorise $[(p-2)-4][(p-2)+4] < 0$ Depends on first M mark.
	A1	Correct critical values, $p = -2, p = 6$ (Accept if wrong variable is used.)
	ddM1	Correct use of their critical values, showing that the inside region has been selected. Both M marks needed. Allow with $\leq$ or $<$ or set language statement.
	A1	$-2 < p < 6$ (in terms of p) with options as above.

Question number	Scheme	Marks
8 (a)	$\alpha + \beta = \frac{1}{3} \quad \alpha\beta = \frac{4}{3}$ $p = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$ $= \frac{1}{3} + \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{7}{12} *$	B1 M1 A1 cso* (3)
(b)	$q = \left(\alpha + \frac{1}{\alpha} \right) \left(\beta + \frac{1}{\beta} \right) = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta^2 + \alpha^2}{\alpha\beta}$ $= \alpha\beta + \frac{1}{\alpha\beta} + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ $= \frac{4}{3} + \frac{3}{4} + \frac{\frac{1}{9} - \frac{8}{3}}{\frac{4}{3}} = \frac{1}{6} \text{ oe}$	M1 A1 (M1 on ePen) dM1 A1 (4) [7]

Part	Mark	Additional Guidance
(a)	B1	$\alpha + \beta = \frac{1}{3}$, $\alpha\beta = \frac{4}{3}$ can be explicit or implicitly used later
	M1	For an initial expression correctly adding the roots of $g(x)$ and an attempt to simplify. Minimum attempt must involve terms $\alpha + \beta$ and an attempt to simplify, bringing the fraction part to a common denominator $\alpha\beta$ written as a single fraction.
	A1*	cso*
(b)	M1	For multiplying the roots of $g(x)$. Minimum attempt must involve a correct multiplication of the brackets with terms $\alpha\beta + \frac{1}{\alpha\beta}$ and an attempt to bring the fraction part to a common denominator $\alpha\beta$
	A1 (M1 ePen)	Correct expression as shown.
	dM1	Substitutes their values for $\alpha + \beta$ and $\alpha\beta$ correctly into their expression (this expression must be ready for substitution of both $\alpha + \beta$ and $\alpha\beta$)
	A1	$\frac{1}{6}$ oe

Question number	Scheme	Marks
1	$\alpha + \beta = \frac{3}{4}, \alpha\beta = -2$ Sum: $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{3}{4}}{-2} = -\frac{3}{8} = \left(-\frac{b}{a}\right)$ Product: $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = -\frac{1}{2} = \left(\frac{c}{a}\right)$ Equation: $x^2 + \frac{3}{8}x + \left(-\frac{1}{2}\right) = 0 \Rightarrow 8x^2 + 3x - 4 = 0$	B1B1 M1A1 B1ft M1A1ft [7]
Total 7 marks		

Mark	Notes
B1	For correct value for $\alpha + \beta$
B1	For correct value for $\alpha\beta$
M1	For the sum $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\text{their } \frac{3}{4}}{\text{their } -2}$ Allow use of their stated $\alpha + \beta$ and $\alpha\beta$
A1	For the correct sum $-\frac{3}{8}$
B1ft	For the correct value of the product $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = -\frac{1}{2}$ FT their stated $\alpha + \beta$ and $\alpha\beta$
M1	For correctly forming an equation with their sum and product $x^2 - \frac{'-3'}{8}x + -\frac{'1'}{2} (= 0)$ Condone the absence of $=0$ for this mark.
A1ft	For the correct equation $8x^2 + 3x - 4 = 0$ oe Follow through from their sum and product. Must be integer coefficients and constant.

Question number	Scheme	Marks
2 (a)	$f(x) = 2(x^2 - 6x) + 5$ OR $f(x) = 2\left(x^2 - 6x + \frac{5}{2}\right)$ $f(x) = 2([x-3]^2 - k) + 5$ OR $f(x) = 2\left([x-3]^2 - k + \frac{5}{2}\right)$ $f(x) = 2[x-3]^2 - 13$ $a = 2, b = -3, c = -13$ Allow values embedded in $f(x)$	M1 M1 A1 [3]
ALT – Equating coefficients		
	$ax^2 + 2abx + (ab^2 + c) \equiv 2x^2 - 12x + 5 \Rightarrow a = 2$ $2ab = -12 \Rightarrow 2 \times 2 \times b = -12 \Rightarrow b = -3$ $ab^2 + c = 5 \Rightarrow 2 \times (-3)^2 + c = 5 \Rightarrow c = 5 - 18 = -13$	[M1 M1 A1]
(b)	$2[x-3]^2 - 13 > 37 \Rightarrow 2[x-3]^2 = 50 \Rightarrow [x-3]^2 = 25 \Rightarrow x = 3 \pm \dots$ Critical values $x = -2, 8$ $\{x < -2\} \cup \{x > 8\}$	M1 A1 M1A1 [4]
ALT - solving inequality without use of completed square form		
	$2x^2 - 12x - 32 > 0 \Rightarrow x^2 - 6x - 16 > 0 \Rightarrow (x+2)(x-8) > 0$ Critical values $x = -2, 8$ $\{x < -2\} \cup \{x > 8\}$	[M1 A1 M1A1]
Total 7 marks		

Part	Mark	Notes
(a)	M1	Starts process to complete the square by taking out 2 as a common factor $f(x) = 2(x^2 - 6x) + 5$ OR $f(x) = 2\left(x^2 - 6x + \frac{5}{2}\right)$
	M1	Attempts to complete the square: $f(x) = 2([x-3]^2 - k) + 5$ OR $f(x) = 2\left([x-3]^2 - k + \frac{5}{2}\right)$
	A1	Correctly completes the square to obtain: $a = 2, b = -3, c = -13$ Allow embedded answers i.e. $f(x) = 2[x-3]^2 - 13$
ALT – Equating coefficients		
	M1	Expands the given form and equates to $f(x)$. Begins process of comparing coefficients, establishing that $a = 2$. $ax^2 + 2abx + (ab^2 + c) \equiv 2x^2 - 12x + 5 \Rightarrow a = 2$
	M1	Equates coefficient of x and solves for b . $2ab = -12 \Rightarrow 2 \times 2 \times b = -12 \Rightarrow b = -3$ Equates constant terms and attempts to solve for c .

		$ab^2 + c = 5 \Rightarrow 2 \times (-3)^2 + c = 5 \Rightarrow c = 5 - 18 = -13$
	A1	For correct values of a, b and c . $a = 2, b = -3, c = -13$ Allow embedded answers i.e. $f(x) = 2[x - 3]^2 - 13$
(b)	M1	Sets $f(x) > 37$ and uses their result from part (a) [provided it is in the form $f(x) = 2(x \pm P)^2 \pm Q$] and attempts to find two critical values $2[x - '3']^2 - '13' > 37 \Rightarrow 2[x - '3']^2 = 50 \Rightarrow [x - 3]^2 = 25 \Rightarrow x = '3' \pm \dots$ Condone use of $=$ rather than $>$
	A1	For both critical values of $x = -2, 8$
	M1	For choosing the outside region for their cv's [provided there are two values] $\{x < '-2'\} \cup \{x > '8'\}$ or any other correct notation. Condone 'AND' for this mark
	A1	For the correct region with correct values $\{x < -2\} \cup \{x > 8\}$ or any other correct notation. Must not use 'AND' for this mark.
Alt – solving inequality without use of completed square form		
	M1	Sets $f(x) > 37$ and attempts to find two critical values. See general guidance on what constitutes an attempt to solve a quadratic. Condone use of $=$ rather than $>$
	A1	For both critical values of $x = -2, 8$
	M1	For choosing the outside region for their cv's [provided there are two values] $\{x < '-2'\} \cup \{x > '8'\}$ or any other correct notation. Condone 'AND' for this mark
	A1	For the correct region with correct values $\{x < -2\} \cup \{x > 8\}$ or any other correct notation. Must not use 'AND' for this mark.

Question	Scheme	Marks
For part (a) of this question, mark using the scheme which gives the most marks.		
2a	$p = 2, \quad q = \frac{1}{8}, \quad r = -\frac{97}{32}$	B1B1B1
ALT	$\left(g(x) = 2x^2 + \frac{1}{2}x - 3 = \right) 2\left(x^2 + \frac{1}{4}x - \frac{3}{2}\right)$ or $2\left(x^2 + \frac{1}{4}x\right) - 3$	M1
	$(\text{"2"})\left(\left[x + \frac{1}{8}\right]^2 - \frac{1}{64} - \frac{3}{2}\right)$ or $(\text{"2"})\left(\left[x + \frac{1}{8}\right]^2 - \frac{1}{64}\right) - 3$	M1
	$2\left[x + \frac{1}{8}\right]^2 - \frac{97}{32}$ or $p = 2, \quad q = \frac{1}{8}, \quad \text{oe } r = -\frac{97}{32}$ oe	A1 [3]
(b)	(i) $(g(x)_{\min}) = -\frac{97}{32}$	B1ft
	(ii) $(x) = -\frac{1}{8}$	B1ft [2]
(c)	(i) $h(x)_{\min} = -\frac{97}{32}$	B1ft
	(ii) $x = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$	B1ft [2]
Total 7 marks		

Part	Mark	Notes
• Mark using the B scheme first. • If not full marks – use the MMA scheme also, if appropriate. • Same score – apply the B marks. • Higher score – apply the marks from the MMA scheme.		
(a) Different to Epen marks	B1	For one of p, q , or r correct.
	B1	For two of p, q , or r correct.
	B1	For all of p, q , or r correct – note q and r can be written as equivalent fractions.
ALT	M1	Correctly takes out 2 as a common factor to give either of the 2 expressions shown.
	M1	Completes the square correctly, regardless of any factor on the outside – follow through their factorisation. $(x^2 + ax + b)$ or $(x^2 + ax) + c \Rightarrow$ ie $\left[\left(x + \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b\right]$ or $\left[\left(x + \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2\right] + c \quad a, b, c \neq 0$
	A1	Fully correct answer – note q and r can be written as equivalent fractions.
Note part (a) does not ask students to show working nor preclude the use of a calculator, so if values are simply listed, these can be given marks from the B scheme		
(b) (i)	B1ft	For the correct value or ft their r
(ii)	B1ft	For the correct value or ft their $-q$
If no labelling of (i) and (ii) is present for parts (b) and (c) – marks may be awarded for the values presented in the correct order.		
B0 B0 if not labelled and work doesn't meet this condition.		
(c) (i)	B1ft	For the correct value or ft their $-\frac{97}{32}$
(ii)	B1ft	For the correct value or ft the cube root of their $-\frac{1}{8}$ (evaluated).



Question Number	Scheme	Marks
5		
(a)	$x^2 - \frac{7}{2}x + 2 (=0)$	M1
	$2x^2 - 7x + 4 = 0$	A1 (2)
(b)	$\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$	B1
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1
	$\frac{\frac{49}{4} - 4}{2}, = \frac{33}{8}$	dM1A1
	$x^2 - \frac{33}{8}x + 1 (=0)$	M1
	$8x^2 - 33x + 8 = 0$	A1 (6)
		[8]
(a)		
M1	Use $x^2 - (\text{sum of roots})x + \text{product of roots} (=0 \text{ may be missing})$	
A1	Correct equation as shown or any integer multiple of this. Must have $= 0$	
	NB: A correct equation with no working scores 2	
ALT		
M1	Eliminate α (or β) between the 2 equations and multiply through by α (or β)	
A1	A correct quadratic equation with integer coefficients. Unknown can be α (or β)	
NB;	isw any attempt to solve their equation.	
(b)		
B1	Correct product of roots, seen explicitly or used.	
M1	Attempt a single fraction for the sum of the roots with the numerator ready for substitution of known quantities. Denominator must be $\alpha\beta$.	
dM1	Substitute numbers in their single fraction.	
A1	Correct value for sum (as shown or equivalent fraction)	
M1	Use $x^2 - (\text{sum of roots})x + \text{product of roots} (=0 \text{ may be missing})$	
A1	Correct equation as shown or any integer multiple of this. Must have $= 0$	



Question number	Scheme	Marks
5 a	$\left(y = 1 + \frac{k}{2}x \Rightarrow\right)$ eg $kx^2 - x\left(1 + \frac{k}{2}x\right) + (k+1)x - 1 = 0$ or eg $\frac{kx^2 + (k+1)x - 1}{x} = \frac{k}{2}x + 1$ $kx^2 - x - \frac{k}{2}x^2 + kx + x - 1 = 0$ or $2kx^2 + 2(k+1)x - 2 = kx^2 + 2x$ oe And an attempt to simplify must be made. $kx^2 + 2kx - 2 = 0$ oe $(b^2 - 4ac = 0 \Rightarrow) ("2k")^2 - 4("k")(" - 2") = 0$ or $(x+1)^2 = 1 + \frac{2}{k}$ $4k(k+2) = 0$ or $k+2 = 0$ or $1 + \frac{2}{k} = 0$ $k = -2$	M1 M1 A1 M1 dM1 A1 (6)
ALT	Realises if the line intersects the curve only once, it is a tangent at point A – final 3 marks	
Final 3 marks	$y = \frac{kx^2 + (k+1)x - 1}{x} = \left(kx + k + 1 - \frac{1}{x}\right)$ and $\left(\frac{dy}{dx} = \right) \frac{k}{2}$ $\left(\frac{dy}{dx} = \right) k(+0) + \frac{1}{x^2} = \frac{k}{2} \Rightarrow x^2 = -\frac{2}{k}$ $k\left(-\frac{2}{k}\right) + 2k\sqrt{-\frac{2}{k}} - 2 = 0 \Rightarrow 2k\sqrt{-\frac{2}{k}} = 4 \Rightarrow 4k^2\left(-\frac{2}{k}\right) = 16 \Rightarrow -8k = 16$ $k = -2$	M1 dM1 A1
b	When $k = -2$ $-2x^2 - 4x - 2 = 0$ or $(x+1)^2 = 1 + \frac{2}{-2}$ $(x+1)^2 = 0$ $x = -1 \Rightarrow y = 2$ So $(-1, 2)$	M1 A1 (2)
Total 8 marks		



Part (a)	Mark	Additional Guidance
	M1	For correctly substituting $y = 1 + \frac{k}{2}x$ into $kx^2 - xy + (k+1)x = 1$
	M1	For a complete expansion of $x\left(1 + \frac{k}{2}x\right)$ and $(k+1)x$. Allow one error in the expansion. For multiplying the equation throughout by $2x$ or $-2x$, x or $-x$ to eliminate any algebraic denominators, allow one processing error. There must be an attempt to simplify follow either of these steps. If any other manipulation is seen, full marks can be awarded if the steps lead to a correct equation or a three term quadratic with one processing error made. If there is relevant work, not leading to either of these, please send to review.
	A1	For a correct three term quadratic. oe
	M1	For clear and correct use of $b^2 - 4ac = 0$ or a full and correct completing the square process, using their expressions for a , b and c . $a, b, c \neq 0$
	dM1	For factorising or any attempt to solve their quadratic in k , or setting their $1 + \frac{2}{k} = 0$ See general guidance for acceptable attempt to solve a quadratic. For the case of a 2TQ with terms in k and k^2 (which a fully correct solution delivers), we need to see a correct factorisation in the case of this mark or, as the candidate is told k is no zero, they may also correctly divide throughout by k . Dependent on the previous method mark. Allow this mark to be implied if k is correct. Otherwise, the method must be shown to gain this mark.
	A1	$k = -2$, must dismiss $k = 0$, if found.
ALT	States and use the gradient of C is $\frac{k}{2}$.	
	M1	As main scheme.
	M1	As main scheme.
	A1	As main scheme.
	M1	Differentiates C and puts the gradient $= \frac{k}{2}$ to get an equation of the form $mk + \frac{n}{x^2} = \frac{k}{2} \quad m, n \neq 0$ (doesn't need to be simplified) but must lead to an expression of the form $x^2 = -\frac{l}{k} \quad l \neq 0$
	dM1	Substitutes their expression for x^2 into their quadratic equation and rearranges to give an expression of the form $-pk = q$ or $pk = -q \quad p > 0, k > 0$
	A1	$k = -2$ (must be the only solution given).
(b)	M1	For correct substitution of their value of k into their 3 term quadratic from part a and a minimally acceptable attempt to solve for x . See general guidance.
	A1	For $(-1, 2)$ or $x = -1$ and $y = 2$



Question	Scheme	Marks
2	$\alpha + \beta = -\left(-\frac{5}{3}\right) = \frac{5}{3}, \quad \alpha\beta = \frac{1}{3}$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow \left(\frac{5}{3}\right)^2 - 2 \times \frac{1}{3} = \frac{19}{9}$ PRODUCT: $\frac{\alpha}{2\beta} \times \frac{\beta}{2\alpha} = \frac{1}{4}$ SUM: $\frac{\alpha}{2\beta} + \frac{\beta}{2\alpha} = \frac{2\alpha^2 + 2\beta^2}{4\alpha\beta} = \left[\frac{\alpha^2 + \beta^2}{2\alpha\beta}\right] \Rightarrow \frac{\alpha}{2\beta} + \frac{\beta}{2\alpha} = \frac{\frac{19}{9}}{\frac{1}{4}} = \frac{19}{6}$ EQUATION: $x^2 - \frac{19}{6}x + \frac{1}{4} = 0 \Rightarrow 12x^2 - 38x + 3 = 0$ o.e.	B1,B1 B1 B1 M1A1 M1A1 [8]
Total 8 marks		

Mark	Notes
B1	For the correct sum of $\frac{5}{3}$ This may not be explicitly seen – look for it in their working
B1	For the correct product of $\frac{1}{3}$ This may not be explicitly seen – look for it in their working
B1	For the correct value of $\alpha^2 + \beta^2$ $\left[\frac{19}{9}\right]$ but does not need to be evaluated so accept $\left[\left(\frac{5}{3}\right)^2 - 2 \times \frac{1}{3}\right]$ seen anywhere in their working. This can be embedded anywhere in their work This can also be implied from a correct value of the SUM $\frac{\alpha}{2\beta} + \frac{\beta}{2\alpha} = \frac{19}{6}$
B1	For the correct PRODUCT of $\frac{1}{4}$
M1	For the correct algebra $\left[\frac{2\alpha^2 + 2\beta^2}{4\alpha\beta}\right]$ and substitution of their values of $\alpha^2 + \beta^2$ [even if the algebra on $\alpha^2 + \beta^2$ is incorrect] to find the SUM. Ft their $\frac{19}{9}$ even if it comes from incorrect algebra on $\alpha^2 + \beta^2$ earlier.
A1	For the value of $\frac{19}{6}$
M1	For a correct equation in any form. Need not = 0 for this mark
A1	For a correct equation with integer coefficients = 0

Question number	Scheme	Marks
5(a)	$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) =$ $\beta^3\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 - 3\alpha^2\beta - 3\alpha\beta^2 = \alpha^3 + \beta^3$	M1 A1cso [2]
ALT (a)	$(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $= (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$ $= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ $= \alpha^3 + \alpha\beta^2 - \alpha^2\beta + \beta\alpha^2 + \beta^3 - \beta^2\alpha$ $= \alpha^3 + \beta^3$	M1 A1 cso [2]
(b)	$\alpha\beta = -\frac{7}{2} \quad \text{and} \quad \alpha + \beta = 3$ Product: $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = -\frac{7}{2}$ Sum: $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \left\{ \frac{\alpha^3 + \beta^3}{\alpha\beta} \right\} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} = \frac{3^3 - 3 \times -\frac{7}{2} \times 3}{-\frac{7}{2}} = -\frac{117}{7}$ Equation: $x^2 + \frac{117}{7}x - \frac{7}{2} = 0 \Rightarrow 14x^2 + 234x - 49 = 0$	B1 B1ft M1A1 M1A1 [6]
Total 8 marks		



Part	Mark	Additional Guidance
(a)	M1	For correctly expanded $(\alpha + \beta)^3$ $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$
	A1cso	For obtaining the given result without any errors
ALT(a)	M1	Factorises $(\alpha + \beta)$ first then attempts to expand all brackets
	A1cso	For obtaining the given result without any errors
(b)	B1	For the correct sum and product of the given equation Allow calling α, β as x_1, x_2 or other letters
	B1ft	For the product $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = '-\frac{7}{2}'$, follows through their value of $\alpha\beta$
	M1	For the sum $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} = \frac{('3')^3 - (3) \times \left('-\frac{7}{2}' \right) \times ('3')}{\left('-\frac{7}{2}' \right)} = \left(-\frac{117}{7} \right)$ Correct algebra and correct substitution of values of their $\alpha\beta$ and $\alpha + \beta$
	A1	For the correct sum $-\frac{117}{7}$
	M1	For correctly forming an equation with their sum and product $x^2 - \text{sum } x + \text{product} = 0$ $x^2 + \frac{117}{7}x - \frac{7}{2} = 0$ Condone the absence of = 0 for this mark.
	A1	For the correct equation $14x^2 + 234x - 49 = 0$ oe Must be integers coefficients



Question number	Scheme	Marks
5 a	$\alpha + \beta = -\frac{b}{a} = \frac{-(6+2p)}{2} = -3 - p$	B1
	$\alpha\beta = \frac{c}{a} = \frac{2p}{2} = p$	B1 (2)
b	$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$	M1
	$= (\alpha + \beta)^2 - 4\alpha\beta$	M1
	$= (-3 - p)^2 - 4p$	M1
	$= 9 + 2p + p^2$ *	A1 cso (4)
c	$(\alpha - \beta) = 3$ implies $(\alpha - \beta)^2 = 9$	B1
	So $9 = 9 + 2p + p^2 \Rightarrow 0 = 2p + p^2 \Rightarrow 0 = p(2 + p)$	M1
	$p = 0$ or $p = -2$	A1 (3)
Total 9 marks		

Part	Mark	Notes
(a)	B1	For the sum $\alpha + \beta = -3 - p$ Need not be simplified. E.g., accept $\frac{-(6+2p)}{2}$
	B1	For the product $\alpha\beta = p$ (Accept $\frac{2p}{2}$)
(b)	M1	For the correct algebra on $(\alpha - \beta)^2$ $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$ Accept un-simplified and terms in any order
	M1	For the correct algebra on $(\alpha - \beta)^2$ using $(\alpha + \beta)^2$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ Note: $(\alpha + \beta)^2 = 9 + 6p + p^2$ which you may see substituted in without seeing the algebra
	M1	For substituting in their sum and product from part (a) $(\alpha - \beta)^2 = (-3 - p)^2 - 4p$
	A1 cso	For obtaining the given result $(\alpha - \beta)^2 = 9 + 2p + p^2$ Note: condone absence of seeing the LHS. Note: This is a given result. There must be no errors or omissions (including in algebra) in their work for this mark
(c)	B1	For $(\alpha - \beta)^2 = 9$
	M1	For setting the given answer = 9 $9 + 2p + p^2 = 9 \Rightarrow 2p + p^2 = 0 \Rightarrow p = \dots$ Two values.
	A1	For $p = 0$ or $p = -2$

Question number	Scheme	Marks
6 (a)	$(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ *	M1 M1 A1 cso (3)
(b)	$\alpha + \beta = 7k$ and $\alpha\beta = k^2$ $\alpha - \beta = \sqrt{49k^2 - 4k^2}$ $= \sqrt{45}k = 3k\sqrt{5}$ *	B1 M1 A1 cso (3)
(c)	Sum = $\alpha + \beta$ ($= 7k$) Product $(\alpha + 1)(\beta - 1) = \alpha\beta - (\alpha - \beta) - 1 \Rightarrow (\alpha + 1)(\beta - 1) = k^2 - 3k\sqrt{5} - 1$ So $x^2 - 7kx + k^2 - 3k\sqrt{5} - 1 = 0$	B1ft M1 M1A1 (4)
Total 10 marks		

Note: You may see a method based on the difference of two squares for part (a)

i.e. $(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$

Solution

$$\begin{aligned}
 (\alpha - \beta)^2 &= (\alpha + \beta)^2 - 4\alpha\beta \Rightarrow (\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta \\
 (\alpha + \beta)^2 - (\alpha - \beta)^2 &= ([\alpha + \beta] + [\alpha - \beta])([\alpha + \beta] - [\alpha - \beta]) \\
 &= (2\alpha)(2\beta) \\
 &= 4\alpha\beta \\
 \text{LHS} &= \text{RHS (hence shown)}
 \end{aligned}$$

If this is a full and correct solution as shown (no errors) – award full marks – otherwise, please send to Review.



Part	Mark	Notes
(a)	M1	For expanding $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 \Rightarrow (\alpha^2 + \beta^2 - 2\alpha\beta)$ or expanding $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ This must be correct for this mark.
	M1	Replaces $\alpha^2 + \beta^2$ with $(\alpha + \beta)^2 - 2\alpha\beta$ in their expansion of $(\alpha - \beta)^2$. And attempts to collect terms $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$
	A1 cso	For the correct given expression with no errors seen. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ *
	ALT	
	M1	For expanding $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 \Rightarrow (\alpha^2 + \beta^2 - 2\alpha\beta)$ or expanding $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ This must be correct for this mark.
	M1	For expanding the RHS and equates to their expansion of $(\alpha - \beta)^2$ and attempting to simplify. $\alpha^2 + \beta^2 - 2\alpha\beta = (\alpha^2 + \beta^2 + 2\alpha\beta) - 4\alpha\beta$
	A1 cso	Both sides of the equivalence are shown to be equal with no errors seen. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ *
(b)	B1	For both $\alpha + \beta = 7k$ and $\alpha\beta = k^2$ This may be implied in later work e.g. by use of $49k^2$ and $4k^2$
	M1	For substituting their values for the sum and product into the given expression for $(\alpha - \beta)^2$, simplifying and square rooting both sides. $(\alpha - \beta)^2 = (7k)^2 - 4k^2 \Rightarrow \alpha - \beta = \sqrt{(7k)^2 - 4k^2} = \sqrt{45k^2}$ Condone $\pm\sqrt{45k^2}$ for this mark.
	A1 cso	For the correct value of $\alpha - \beta = 3k\sqrt{5}$ *
(c)	B1ft	For the sum $(\alpha + 1 + \beta - 1) = \alpha + \beta = 7k$ Ft their $\alpha + \beta$
	M1	For the product in terms of k . Correctly multiplying out $(\alpha + 1)(\beta - 1)$ and substituting in their value $\alpha\beta$ and the correct value $\alpha - \beta = 3k\sqrt{5}$ $(\alpha + 1)(\beta - 1) = \alpha\beta - (\alpha - \beta) - 1 \Rightarrow (\alpha + 1)(\beta - 1) = k^2 - 3k\sqrt{5} - 1$
	M1	For correctly forming an equation with their sum and product $x^2 - '7k'x + 'k^2 - 3k\sqrt{5} - 1' (= 0)$ Condone the absence of $= 0$ for this mark
	A1	For the correct equation $x^2 - 7kx + k^2 - 3k\sqrt{5} - 1 = 0$ Allow $p = -7k$ $q = k^2 - 3k\sqrt{5} - 1$



Question number	Scheme	Marks
4 (a)	$\alpha + \beta = -2 \quad \alpha\beta = \frac{3}{2}$ $(\alpha + \beta)^2 - 2\alpha\beta = \alpha^2 + \beta^2$ $(-2)^2 - 2\left(\frac{3}{2}\right) = 4 - 3 = 1 *$	B1 M1 M1 A1cso (4)
ALT	$2\alpha^2 + 4\alpha + 3 = 0 \quad \text{and} \quad 2\beta^2 + 4\beta + 3 = 0$ $2\alpha^2 + \beta^2 + 4\alpha + \beta + 6 = 0$ $\alpha^2 + \beta^2 = -2 - 2 - 3 = 1 *$	{B1} {M1} {M1} {A1cso} (4)
(b)	$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)(\alpha^2 + \beta^2) - 2\alpha^2\beta^2$ $1 \times 1 - 2\left(\frac{3}{2}\right)^2 = -\frac{7}{2}$	M1 M1 A1 (3)
ALT	$\alpha^2 + \beta^2 = 1 = \alpha^4 + 2\alpha^2\beta^2 + \beta^4 = \alpha^4 + \beta^4 + 2 \times \frac{9}{4}$ $\alpha^4 + \beta^4 = 1 - \frac{9}{2} = -\frac{7}{2}$	{M1} {M1} {A1} (3)
(c)	Product of the roots: $\alpha^4\beta^4 = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$ (Sum of the roots: $\alpha^4 + \beta^4 = -\frac{7}{2}$) $16x^2 + 56x + 81 = 0$	M1 M1 A1ft (3)
Total 10 marks		

Part	Mark	Notes
(a)	B1	For $\alpha + \beta = -2$ and $\alpha\beta = \frac{3}{2}$
	M1	For $(\alpha + \beta)^2 - 2\alpha\beta = \alpha^2 + \beta^2$ This must be correct
	M1	For substituting their sum and product into their expansion for $\alpha^2 + \beta^2$ $(-2)^2 - 2\left(\frac{3}{2}\right)$
	A1cso	For obtaining the given expression $\alpha^2 + \beta^2 = 1$
	ALT	
	B1	For $2\alpha^2 + 4\alpha + 3 = 0$ and $2\beta^2 + 4\beta + 3 = 0$
	M1	For $2\alpha^2 + \beta^2 + 4\alpha + \beta + 6 = 0$
	M1	For $-2 -2 -3$
	A1cso	For obtaining the given expression $\alpha^2 + \beta^2 = 1$
(b)	M1	For $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)(\alpha^2 + \beta^2) - 2\alpha^2\beta^2$ This must be correct
	M1	For substituting $\alpha^2 + \beta^2 = 1$ and their product from (a) into their expansion for $\alpha^4 + \beta^4$ $\alpha^4 + \beta^4 = 1 \times 1 - 2\left(\frac{3}{2}\right)^2$
	A1	For the correct value $\alpha^4 + \beta^4 = -\frac{7}{2}$
	ALT	
	M1	For $1 = \alpha^4 + 2\alpha^2\beta^2 + \beta^4$
	M1	For $1 = \alpha^4 + \beta^4 + 2 \times \frac{9}{4}$
	A1	For $\alpha^4 + \beta^4 = -\frac{7}{2}$
(d)	M1	For product of the roots: $\alpha^4\beta^4 = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$
	M1	For use of $x^2 - (\text{their sum})x + (\text{their product}) = [0]$
	A1	For $16x^2 + 56x + 81 = 0$ Must include $= 0$

Question number	Scheme	Marks
8(a)	$x^3 + 3x^2y + 3xy^2 + y^3$	B1 [1]
(b)	$\alpha + \beta = -\frac{3}{2}$ and $\alpha\beta = \frac{4}{2}$ oe $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\left(-\frac{3}{2}\right)^3 - 3\left(2\right)\left(-\frac{3}{2}\right)$ oe $\frac{45}{8}$ oe	B1 M1 dM1 A1 [4]
(c)	$\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} =$ $\frac{\alpha^3 + \beta^3}{\alpha^2\beta^2} = \frac{\left(\frac{45}{8}\right)}{2^2}$ $\frac{45}{32}$ oe $\frac{\alpha}{\beta^2} \times \frac{\beta}{\alpha^2} = \frac{1}{\alpha\beta} = \frac{1}{2}$ $\frac{1}{2}$ oe $x^2 - \frac{45}{32}x + \frac{1}{2} (= 0)$ $32x^2 - 45x + 16 = 0$ oe	M1 A1ft B1ft M1 A1 [5]
Total 10 marks		



Part	Mark	Additional Guidance
(a)	B1	For the correct expansion simplified or un-simplified. For example, allow $x^3 + 2x^2y + x^2y + xy^2 + 2xy^2 + y^3$
(b)	B1	Product and sum both correct, written explicitly or used later in working.
	M1	An attempt to rearrange their expression from part (a) to achieve as a minimum $\alpha^3 + \beta^3 = (\alpha + \beta)^3 \pm 3\alpha\beta(\alpha + \beta)$ Note: Accept alternative algebraic arrangements of $\alpha^3 + \beta^3$ but please check carefully that the algebra is correct.
	dM1	Substitution of their sum and product into an expression which must be of the form $(\alpha + \beta)^3 \pm 3\alpha\beta(\alpha + \beta)$ Note: If they substitute correctly into alternative arrangements check that they are correct.
	A1	For $\frac{45}{8}$ oe
(c)	M1	Correct algebra to obtain $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}$ and substitution of their values for the sum and product.
	A1ft	For $\frac{45}{32}$ oe Note: only follow through their values for product and sum into correct algebra .
	B1ft	For the correct algebra and substitution of their $\alpha\beta = 2$ to obtain $\frac{\alpha}{\beta^2} \times \frac{\beta}{\alpha^2} = \frac{1}{\alpha\beta} = \frac{1}{2}$ ft their $\alpha\beta$
	M1	Use of their sum and product to correctly form an expression as shown, $x^2 - \frac{45}{32}x + \frac{1}{2} (= 0)$ Allow missing = 0 for this mark or even = y for this mark.
	A1	Correct equation or any multiple of it, for example, $64x^2 - 90x + 32 = 0$ The coefficients, must be integers.
	ALT Starts with $\left(x - \frac{\alpha}{\beta^2}\right)\left(x - \frac{\beta}{\alpha^2}\right) = 0 \Rightarrow x^2 - x\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) + \frac{\beta}{\alpha^2} \times \frac{\alpha}{\beta^2} = 0$	
	M1	Correct algebra to obtain $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}$ and substitution of their values for the sum and product.
	A1ft	For $\frac{45}{32}$ oe Note: only follow through their values for product and sum into correct algebra .
	B1ft	For the correct algebra and substitution of their $\alpha\beta = 2$ to obtain $\frac{\alpha}{\beta^2} \times \frac{\beta}{\alpha^2} = \frac{1}{\alpha\beta} = \frac{1}{2}$ ft their $\alpha\beta$
	M1	For $x^2 - x\left(\frac{45}{32}\right) + \frac{1}{2} = 0$ Accept this expression on it's own or even = y for this mark
	A1	Correct equation or any multiple of it, for example, $64x^2 - 90x + 32 = 0$ The coefficients, must be integers.

Question	Scheme	Marks
6(a)	$\alpha + \beta = -\frac{5}{2} \quad \alpha\beta = -\frac{p}{2}$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\alpha^3 + \beta^3 = \left(-\frac{5}{2}\right)^3 - 3\left(-\frac{p}{2}\right)\left(-\frac{5}{2}\right) = -\frac{125}{8} - \frac{15p}{4} = -\frac{215}{8}$ $\Rightarrow p = 3$ ALT $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$	B1 M1 M1M1 A1 [5] [M1]
(b)	<u>Sum:</u> $\frac{\alpha + \beta}{\alpha^2} + \frac{\alpha + \beta}{\beta^2} = \frac{\alpha\beta^2 + \beta^3 + \alpha^3 + \alpha^2\beta}{\alpha^2\beta^2} = \frac{\alpha^3 + \beta^3 + \alpha\beta(\alpha + \beta)}{\alpha^2\beta^2}$ ALT $\frac{\alpha + \beta}{\alpha^2} + \frac{\alpha + \beta}{\beta^2} = \frac{\alpha^2(\alpha + \beta) + \beta^2(\alpha + \beta)}{\alpha^2\beta^2} = \frac{(\alpha^2 + \beta^2)(\alpha + \beta)}{\alpha^2\beta^2}$ $= \frac{((\alpha + \beta)^2 - 2\alpha\beta)(\alpha + \beta)}{\alpha^2\beta^2}$ $\frac{-\frac{215}{8} + \left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{\left(-\frac{3}{2}\right)^2} = -\frac{185}{18} \quad \text{allow } \frac{185}{18} \text{ if they start with a negative.}$ <u>Product:</u> $\left(\frac{\alpha + \beta}{\alpha^2}\right) \times \left(\frac{\alpha + \beta}{\beta^2}\right) = \frac{(\alpha + \beta)^2}{\alpha^2\beta^2} = \frac{\left(-\frac{5}{2}\right)^2}{\left(-\frac{3}{2}\right)^2} = \frac{25}{9}$ <u>Equation:</u> $x^2 + \frac{185}{18}x + \frac{25}{9} = 0 \Rightarrow 18x^2 + 185x + 50 = 0 \text{ oe}$	M1 M1 B1 M1A1 [5]
Total 10 marks		

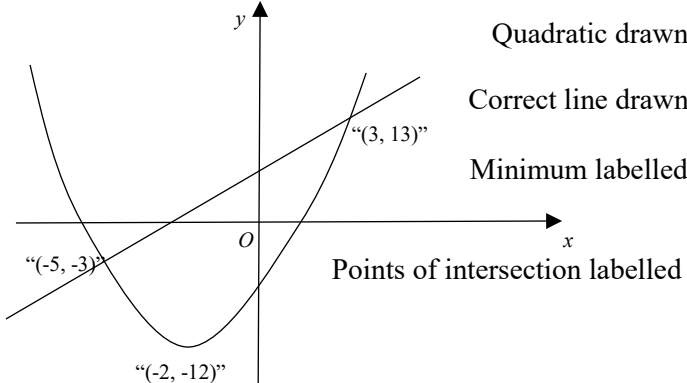
Part	Mark	Notes
(a)	B1	For the correct expression/values for BOTH the sum and product. This must be identified, or implied from their working.
	M1	For the correct algebra to find $\alpha^3 + \beta^3$ in terms of $\alpha + \beta$ and $\alpha\beta$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\alpha^3 + \beta^3 = (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)$ - Or any other algebra, but do not award this mark until the values of $\alpha + \beta$ and $\alpha\beta$ can be substituted in directly.
	M1	For substituting their values of the sum and product into their expression for $\alpha^3 + \beta^3$
	M1	For solving the linear equation in p Allow one slip in their working.
	A1	For the correct value of p with no errors.
(b)	M1	For the correct algebra for the sum of roots. This must be such that the given value of $\alpha^3 + \beta^3$, with their values of $\alpha\beta$ and $(\alpha + \beta)$ can be substituted in. If they use the ALT they will not need $\alpha^3 + \beta^3$ Some candidates will reverse the sign at this stage in anticipation of the reversal required in the equation.
	M1	For substituting in the given value for $\alpha^3 + \beta^3$, and their values for $\alpha + \beta$ and $\alpha\beta$ into their expansion for the sum. If they use the ALT they will not need $\alpha^3 + \beta^3$
	B1ft	For the correct value of the product of roots. You must follow through their p . You do not need to check the calculation once you see the correct algebra with a correct substitution.
	M1	For forming an equation with their sum and product. Ft the sign of their sum. Some candidates reverse the sign when finding the sum. Watch out for that! Accept this without = 0
	A1	For a correct equation including = 0 There is no follow through on this mark. NB accept a correct equivalent equation provided it has integer coefficients. e.g. $36x^2 + 370x + 100 = 0$
	SC – they solve the equation and find values for α and β Award marks for correct work seen above in part (b) - If they do not obtain an expansion for the sum into which $\alpha^3 + \beta^3$, $\alpha\beta$ and $(\alpha + \beta)$ cannot be directly substituted – first M0 - If they do not substitute $\alpha^3 + \beta^3$, $\alpha\beta$ and $(\alpha + \beta)$ but other values based on α and β – second M0 - If they cannot substitute $\alpha\beta$ and $(\alpha + \beta)$ into the product – B0 Then allow marks for forming the equation as above.	



Question	Scheme	Marks
10(a)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\frac{115}{8} = \left(-\frac{5}{2}\right)^3 - 3\alpha\beta\left(-\frac{5}{2}\right)$ $\Rightarrow \alpha\beta = \frac{-\frac{115}{8} + \left(-\frac{5}{2}\right)^3}{3 \times \left(-\frac{5}{2}\right)}$ $\Rightarrow \alpha\beta = 4$	M1 dM1 A1 cso [3]
(b)	Sum: $\frac{\alpha^2 + 1}{\beta} + \frac{\beta^2 + 1}{\alpha} = \frac{\alpha^3 + \beta^3 + \alpha + \beta}{\alpha\beta}$ $\Rightarrow \frac{\alpha^2 + 1}{\beta} + \frac{\beta^2 + 1}{\alpha} = \frac{\frac{115}{8} + \left(-\frac{5}{2}\right)}{4} = \frac{95}{32}$ Product: $\frac{\alpha^2 + 1}{\beta} \times \frac{\beta^2 + 1}{\alpha} = \frac{\alpha^2\beta^2 + \alpha^2 + \beta^2 + 1}{\alpha\beta}$ $\Rightarrow \frac{\alpha^2 + 1}{\beta} \times \frac{\beta^2 + 1}{\alpha} = \frac{\alpha^2\beta^2 + \left[(\alpha + \beta)^2 - 2\alpha\beta\right] + 1}{\alpha\beta}$ $\Rightarrow \frac{\alpha^2 + 1}{\beta} \times \frac{\beta^2 + 1}{\alpha} = \frac{4^2 + \left(-\frac{5}{2}\right)^2 - 2 \times 4 + 1}{4} = \frac{61}{16}$ Equation: $x^2 - \left(\frac{95}{32}\right)x + \left(\frac{61}{16}\right) = 0 \Rightarrow 32x^2 - 95x + 122 = 0 \quad \text{oe}$	M1A1 M1 M1 A1 M1A1 [7]
Total 10 marks		

Part	Mark	Notes
(a)		Part (a) is a ‘Show that’ question. You must see sufficient work for the award of both M marks for the award of the A mark
	M1	For the correct algebra and substitution of the given values into a correct expression for $\alpha^3 + \beta^3$ There is more than one acceptable form of this expansion. They must be able to substitute the given values of $\alpha + \beta$ and $\alpha^3 + \beta^3$ with $\alpha\beta$ as the value to find in any version they use. For example: $\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$
	dM1	For an attempt to solve the linear equation to find a value for $\alpha\beta$ Allow one processing error for this mark. Note, this is a dependent M mark.
	A1 CS0	For $\alpha\beta = 4$ This is a show question, you must check their algebra carefully.
(b)	M1	For the correct algebra and substitution of the given values to find the sum.
	A1	For the correct sum $= \frac{95}{32}$
	M1	For the correct algebra and substitution to find $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left[-\frac{7}{4}\right]$
	M1	For the correct algebra and substitution of the given values to find the product
	A1	For the correct product $= \frac{61}{16}$
	M1	For forming an equation using their sum and product correctly. $x^2 - (\text{their sum})x + (\text{their product}) = (0)$ Allow missing = 0 for this mark
	A1	For a correct equation with integer coefficients. For example: $64x^2 - 190x + 244 = 0$



Question number	Scheme	Marks
2 (a)	$(x+2)^2 - 4 - 8$ $(x+2)^2 - 12 \quad a = 2 \quad b = -12$	M1 A1 (2)
(b)	$x^2 + 4x - 8 = 2x + 7$ $x^2 + 2x - 15 = 0$ $(x-3)(x+5) = 0$ or any valid method $x = 3, y = 13 \quad x = -5, y = -3$	M1 dM1 M1 A1 A1 (5)
(c)		B1 B1 B1 ft B1 ft (4) [11]

Part	Mark	Additional Guidance
(a)	M1	Use general guidance, allow an expression of the form $(x \pm \frac{a}{2})^2 \pm q \pm 8 \quad q \neq 0$
	A1	Correct expression as shown, a and b need not be explicitly stated
(b)	M1	Correctly equates the 2 expressions
	dM1	Rearranges to a $3TQ = 0$ (allow any $3TQ$ if intention of rearrangement is clear)
	M1	Uses any valid method to solve – see general guidance
	A1	For either pair of values stated
	A1	For all four values, correctly paired or written as coordinates.
		For the final A1 A1, do not allow recovery of y values from part c.
(c)	B1	Correctly shaped quadratic curve, with a clear minimum point, drawn anywhere on their axis, mark intention.
	B1	Correct line – must have a positive y intercept, a positive gradient and a negative x intercept
	B1ft	Correctly labelled coordinates for their minimum, ft their answer from b, must correctly ft their answer from a, ie minimum point labelled $(-a, b)$
	B1ft	Correctly labelled coordinates for their intersections.
		The coordinates must be clearly indicated and not inferred from a scale on the graph. Ignore any labelling of intersections with axes.

Question Number	Scheme	Marks
6(a)	$\alpha + \beta = \frac{3}{4} \quad \alpha\beta = -\frac{5}{4}$ $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2(\alpha^2 + \beta^2)}{\alpha\beta} = \frac{2((\alpha + \beta)^2 - 2\alpha\beta)}{\alpha\beta}$ $= \frac{2\left(\frac{9}{16} + \frac{5}{2}\right)}{-\frac{5}{4}} = -\frac{49}{10}$ $\frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4$ $x^2 + \frac{49}{10}x + 4 (= 0)$ $10x^2 + 49x + 40 = 0$	B1 M1 A1 B1 M1 A1 (6)
(b)	$4(\alpha + \beta) = 3 = -\frac{p}{4} \quad p = -12$ $(3\alpha + \beta) \times (\alpha + 3\beta) = 3(\alpha^2 + \beta^2) + 10\alpha\beta$ $= 3\left(\frac{9}{16} + \frac{5}{2}\right) - 10 \times \left(\frac{5}{4}\right) = -\frac{53}{16} \quad \text{oe}$ $\frac{q}{4} = -\frac{53}{16} \quad q = -\frac{53}{4} \quad \text{oe}$	M1A1 M1 A1 A1 (5)
(a)B1	Correct values for $\alpha + \beta$ and $\alpha\beta$. Award if values not seen explicitly but embedded in the sum/product calculations for the new equation.	
M1	Attempt the sum of the roots of the new equation. Must reach a correct expression ready for substitution of values of $\alpha + \beta$ and $\alpha\beta$	
A1	Correct value for the sum. Allow if $\alpha + \beta = -\frac{3}{4}$ has been used.	
B1	Correct value for the product of roots of the new equation.	
M1	Use $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$ may be missing	
A1	Correct final equation, as shown or an integer multiple of this. Must have = 0 now.	
(b)	Allow if $\alpha + \beta = -\frac{3}{4}$ has been used.	
M1	Use their value of $\alpha + \beta$ to obtain a value for the sum of the roots of $g(x) = 0$ and equate to $\pm p/4$	
A1	Obtain the correct value of p	
M1	Attempt the product of the roots of $g(x) = 0$ and obtain an expression ready for substitution of values. May use work from (a) for value of $\alpha^2 + \beta^2$ so the expression shown is sufficient.	
A1	Obtain the correct value for the product	
A1	Correct value of q	

Question number	Scheme	Marks
8	$\alpha + \beta = 4k\sqrt{2}$ and $\alpha\beta = 2k^4 - 1$ $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow (4k\sqrt{2})^2 = 66 + 2(2k^4 - 1)$ $k^4 - 8k^2 + 16 = 0$ $(k^2 - 4)^2 = 0 \Rightarrow k = 2$ $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(66 - \alpha\beta)$ $\alpha^3 + \beta^3 = (8\sqrt{2})^3 - 3 \times 31 \times 8\sqrt{2} = 280\sqrt{2}$ or $8\sqrt{2}(66 - 31) = 280\sqrt{2}$ $p = 280$	B1 B1 M1 A1 M1 M1 A1 M1 A1 M1 A1 (11)
Total 11 marks		

Mark	Notes
B1	For either $\alpha + \beta = 4k\sqrt{2}$ or $\alpha\beta = 2k^4 - 1$
B1	For both $\alpha + \beta = 4k\sqrt{2}$ and $\alpha\beta = 2k^4 - 1$
M1	For the correct algebra on $\alpha^2 + \beta^2$ (in any order) and substitution of their values of $\alpha\beta$ and $\alpha + \beta$ providing both sum and product are in terms of k . $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow \alpha^2 + \beta^2 = ('4k\sqrt{2}')^2 - 2('2k^4 - 1')$
A1	For obtaining $(4k\sqrt{2})^2 = 66 + 2(2k^4 - 1)$ in any order.
M1	For simplifying to form a 3TQ in k^4 i.e., $4k^4 - 32k^2 + 64 = 0$ oe Accept as a minimum $4k^4 - 32k^2 \pm Q = (0)$ $Q \neq 0$
M1	For factorising or solving the 3TQ using any valid method. See General Guidance.
A1	For $k = 2$ If they also give $k = -2$ withhold this mark.

Method 1	
M1	For expanding $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$
A1	For obtaining $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(66 - \alpha\beta)$ This must be such that $\alpha\beta$ and $\alpha + \beta$ can be substituted in directly.
M1	For substitution of $\alpha + \beta$ and $\alpha\beta$ for their positive value of k into a correct expansion of $\alpha^3 + \beta^3$ NOTE: If they do not obtain $k = 2$, then full substitution of their numerical value for k into $\alpha + \beta$ and $\alpha\beta$ must be seen for the award of this mark. For example: $\alpha^3 + \beta^3 = (4 \times \text{'their } k \text{' } \sqrt{2})^3 - 3 \times (2 \times \text{'their } k^4 \text{' } - 1) \times (4 \times \text{'their } k \text{' } \sqrt{2})$
A1	For $p = 280$
Method 2	
M1	Finds the exact value of α and β Solves the equations $(\alpha\beta = 2k^4 - 1 \text{ and } \alpha + \beta = 4k\sqrt{2})$ simultaneously to give a value for α and β $\alpha = \frac{31}{\beta} \Rightarrow \alpha + \beta = \frac{31}{\beta} + \beta = 8\sqrt{2} \Rightarrow \beta^2 - 8\sqrt{2}\beta + 31 = 0$ $\Rightarrow \beta = \dots \quad \alpha = \dots$
A1	For $\alpha = 1 + 4\sqrt{2} \quad \beta = -1 + 4\sqrt{2}$ OR $\beta = 1 + 4\sqrt{2} \quad \alpha = -1 + 4\sqrt{2}$
M1	Substitutes these values into $\alpha^3 + \beta^3 = p\sqrt{2}$ to find a value for p $(1 + 4\sqrt{2})^3 + (-1 + 4\sqrt{2})^3 = 24\sqrt{2} + 256\sqrt{2} = 280\sqrt{2} \Rightarrow p = \dots$
A1	$p = 280$

Question number	Scheme	Marks
11 (a)	(i) $2\alpha\beta = 3 \Rightarrow q = \alpha\beta \Rightarrow q = \frac{3}{2}$ (ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$, $\left[\alpha^2 + \beta^2 = \frac{k^2 - 6k - 3}{4} \right]$ $(\alpha + \beta)^2 = \frac{k^2 - 6k - 3}{4} + 3 = \frac{k^2 - 6k + 9}{4} = \frac{(k - 3)^2}{4}$ $\Rightarrow \alpha + \beta = \frac{k - 3}{2} \Rightarrow p = \frac{k - 3}{2}$	B1 M1A1 M1A1 [5]
(b)	$7\left(\frac{3}{2}\right) = 3\left(\frac{k - 3}{2}\right) \Rightarrow 7 = k - 3 \Rightarrow k = 10$	M1A1 [2]
(c)	$\alpha + \beta = \frac{10 - 3}{2} = \frac{7}{2}$ Sum: $\frac{\alpha}{\alpha + \beta} + \frac{\beta}{\alpha + \beta} = \frac{\alpha + \beta}{\alpha + \beta} = 1$ Product: $\frac{\alpha}{\alpha + \beta} \times \frac{\beta}{\alpha + \beta} = \frac{\alpha\beta}{(\alpha + \beta)^2} = \frac{\frac{3}{2}}{\left(\frac{7}{2}\right)^2} = \frac{6}{49}$ Equation: $x^2 - x + \frac{6}{49} (= 0)$ $49x^2 - 49x + 6 = 0$	M1 B1 M1 M1 A1 [5]

(a)	B1	$q = \frac{3}{2}$
(i)		
(ii)	M1	State or use any correct arrangement of $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ and substitute $\alpha\beta = \frac{3}{2}$ and $\alpha^2 + \beta^2 = \frac{k^2 - 6k - 3}{4}$ oe p may be seen instead of $\alpha + \beta$ at any stage of the working. Condone $-p = \alpha + \beta$ for this mark only.
	A1	Any correct expression for $(\alpha + \beta)^2$ or $4(\alpha + \beta)^2$ in terms of k
	M1	Correct method to find an expression for p in terms of k from their expression for $(\alpha + \beta)^2$ or $4(\alpha + \beta)^2$. Ignore negative square root. Must identify $p = \alpha + \beta$.
	A1	A correct expression for p in terms of k . The question does not demand simplification, but if it is left as a square root there must be an indication that p must be the positive root (ie no $\pm$ present).

(b)	M1	Substitute $\alpha\beta = \frac{3}{2}$ and their expression for $\alpha + \beta$ from (a)(ii) into $7\alpha\beta = 3(\alpha + \beta)$ and obtain an equation in k
	A1	$k = 10$ Do not accept if there is an additional root, such as $k = -4$.
(c)	M1	Find a value for $\alpha + \beta$. Need not be simplified. Follow through their k and their expression for p .
	B1	Show or state sum of roots = 1 (stated or explicitly used).
	M1	Write product of roots as $\frac{\frac{3}{2}}{\left(\frac{7}{2}\right)^2}$, ft their $\alpha + \beta = \frac{7}{2}$.
	M1	Use equation is $x^2 - (\text{sum of roots}) + (\text{product of roots}) (= 0)$ ($= 0$ may be missing). There must be some attempt to substitute values, not just expressions in terms of α and β . May be implied by an equation with fractions correctly cleared.
	A1	$49x^2 - 49x + 6 = 0$ Correct equation with integer coefficients. Must have $= 0$.

Question number	Scheme	Marks
5 (a)	$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \Rightarrow \alpha^3 + \beta^3$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)^*$	M1A1 cso [2]
(b)	$\alpha + \beta = -\frac{3}{2} \quad \alpha\beta = \frac{6}{2} = 3$ $\left[\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \right]$ $\alpha^3 + \beta^3 = \left(-\frac{3}{2}\right)^3 - 3 \times 3 \times \left(-\frac{3}{2}\right) = \frac{81}{8}$	B1 B1 [2]
(c)	$(\alpha^2 + \beta^2)^2 = \alpha^4 + 2\alpha^2\beta^2 + \beta^4 \Rightarrow \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2^*$	M1A1 cso [2]
(d)	$\alpha^2 + \beta^2 = \left(-\frac{3}{2}\right)^2 - 2 \times 3 = -\frac{15}{4}$ Sum $(\alpha^3 - \beta) + (\beta^3 - \alpha) = \alpha^3 + \beta^3 - (\alpha + \beta) = \frac{81}{8} - \left(-\frac{3}{2}\right) = \frac{93}{8}$ Product $(\alpha^3 - \beta) \times (\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta$ $= 27 - \left[\left(-\frac{15}{4}\right)^2 - 2 \times 3^2 \right] + 3 = \frac{543}{16}$ Equation $x^2 - \frac{93}{8}x + \frac{543}{16} = 0 \Rightarrow 16x^2 - 186x + 543 = 0$	B1 B1 M1 A1 M1A1 [6]
Total 12 marks		
(a) M1 A1 cso (b) B1 B1 (c) M1 A1 cso	$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ Obtains the given answer with no errors in the working $\alpha + \beta = -\frac{3}{2} \quad \text{and} \quad \alpha\beta = \frac{6}{2} = 3$ $\alpha^3 + \beta^3 = \frac{81}{8}$ $(\alpha^2 + \beta^2)^2 = \alpha^4 + 2\alpha^2\beta^2 + \beta^4$ Obtains the given answer with no errors in the working	

(d)	
B1	$\alpha^2 + \beta^2 = -\frac{15}{4}$ (May be implied by $\alpha^4 + \beta^4 = -\frac{63}{16}$)
B1	$(\alpha^3 - \beta) + (\beta^3 - \alpha) = \frac{93}{8}$
M1	$(\alpha^3 - \beta) \times (\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta$
A1	$(\alpha^3 - \beta) \times (\beta^3 - \alpha) = \frac{543}{16}$
M1	$x^2 - \text{'sum' } x + \text{'product' } (= 0)$
A1	$16x^2 - 186x + 543 = 0$

Question number	Scheme	Marks
7 (a)	$16 - 4k(2k - 7) \geq 0$ $2k^2 - 7k - 4 \leq 0$ $(2k + 1)(k - 4) \leq 0$ $-\frac{1}{2} \leq k \leq 4$ Accept $-\frac{1}{2} < k < 4$	M1 M1 M1 A1 (4)
(b)	$\alpha + \beta = \frac{4}{k}$ $\alpha\beta = \frac{2k - 7}{k}$ $\frac{\alpha + 1}{\alpha} + \frac{\beta + 1}{\beta} = \frac{2\alpha\beta + \alpha + \beta}{\alpha\beta} = \frac{2\left(\frac{2k - 7}{k}\right) + \frac{4}{k}}{\frac{2k - 7}{k}}$ $\frac{4k - 10}{k} \times \frac{k}{2k - 7} = \frac{2(2k - 5)}{2k - 7}$ $\frac{\alpha + 1}{\alpha} \times \frac{\beta + 1}{\beta} = \frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta} = \frac{\frac{2k - 7}{k} + \frac{4}{k} + 1}{\frac{2k - 7}{k}}$ $\frac{3k - 3}{k} \times \frac{k}{2k - 7} = \frac{3(k - 1)}{2k - 7}$ $(2k - 7)x^2 - 2(2k - 5)x + 3(k - 1) = 0$ ALTERNATIVE METHOD Let $w = \frac{x + 1}{x}$ $x = \frac{1}{w - 1}$ Hence $k\left(\frac{1}{w - 1}\right)^2 - 4\left(\frac{1}{w - 1}\right) + 2k - 7 = 0$ $\frac{k}{(w - 1)^2} - \frac{4}{w - 1} + 2k - 7 = 0$ $k - 4(w - 1) + (2k - 7)(w - 1)^2 = 0$ $k - 4w + 4 + (2k - 7)(w^2 - 2w + 1) = 0$ $k - 4w + 4 + 2kw^2 - 4kw + 2k - 7w^2 + 14w - 7 = 0$ $(2k - 7)x^2 - 2(2k - 5)x + 3(k - 1) = 0$	B1 M1 M1 A1 M1 M1 A1 A1 (8) B1 M1 M1 A1 M1 M1 A1 A1 (8) [12]

	Notes
(a)	
M1	For use of $b^2 - 4ac$ (Ignore inequality for this mark)
M1	For a 3 TQ ≤ 0 oe
	For solving their 3TQ (Ignore inequality for this mark) May be implied by
M1	$-\frac{1}{2}$ and 4 seen as critical values
A1	For $-\frac{1}{2} \leq k \leq 4$ Allow $<$ instead of $\leq$ Accept $-\frac{1}{2} < k < 4$
(b)	
B1	For $\alpha + \beta = \frac{4}{k}$ and $\alpha\beta = \frac{2k-7}{k}$
M1	For $\frac{\alpha+1}{\alpha} + \frac{\beta+1}{\beta} = \frac{2\alpha\beta + \alpha + \beta}{\alpha\beta}$
M1	For substitution into $\frac{2\alpha\beta + \alpha + \beta}{\alpha\beta}$ with some attempt to simplify
A1	For $\frac{2(2k-5)}{2k-7}$
M1	For $\frac{\alpha+1}{\alpha} \times \frac{\beta+1}{\beta} = \frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta}$
M1	For substitution into $\frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta}$ with some attempt to simplify
A1	For $\frac{3(k-1)}{2k-7}$
A1	For $(2k-7)x^2 - 2(2k-5)x + 3(k-1) = 0$
	Alternative
B1	For $w = \frac{x+1}{x}$
M1	For rearranging to make x the subject
M1	For substitution of $x = \frac{1}{w-1}$ into the quadratic
A1	For $\frac{k}{(w-1)^2} - \frac{4}{w-1} + 2k-7 = 0$
M1	For multiplying by $(w-1)^2$
M1	For expanding brackets
A1	For $k - 4w + 4 + 2kw^2 - 4kw + 2k - 7w^2 + 14w - 7 = 0$
A1	For $(2k-7)x^2 - 2(2k-5)x + 3(k-1) = 0$

Question	Scheme	Marks
11 (a)(i)	$(\alpha - \beta = 2\sqrt{6} \Rightarrow (\alpha - \beta)^2 = 24) \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 24 \quad \text{oe} \quad (1)$ $\alpha^2 + \beta^2 = 30 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 30 \quad \text{oe} \quad (2)$ $(2) - (1) \quad 6 = 2\alpha\beta \Rightarrow \alpha\beta = 3 \text{ cso}$	M1 M1 dM1A1* [4]
ALT1	$((\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta \Rightarrow) 2\alpha\beta = \alpha^2 + \beta^2 - (\alpha - \beta)^2 \quad \text{oe}$ $30 - (2\sqrt{6})^2 = 6$ $2\alpha\beta = 6 \Rightarrow \alpha\beta = 3$	M1 M1 dM1A1 [4]
(ii)	$30 = (\alpha + \beta)^2 - 2 \times 3 \Rightarrow (\alpha + \beta)^2 = 36 \Rightarrow \alpha + \beta = 6 \quad [\alpha > \beta > 0] \text{ cso}$	M1A1* [2]
ALT2 (i)	$\alpha - \beta = 2\sqrt{6} \rightarrow \alpha = 2\sqrt{6} + \beta$ $\alpha^2 + \beta^2 = 30 \rightarrow (2\sqrt{6} + \beta)^2 + \beta^2 = 30$ $2\beta^2 + 4\sqrt{6}\beta - 6 = 0 \quad \text{oe} \quad \text{eg} \quad \beta^2 + 2\sqrt{6}\beta - 3 = 0$ $(\beta =) \frac{-2\sqrt{6} \pm \sqrt{(2\sqrt{6})^2 - 4(1)(-3)}}{2} \quad \text{oe} \rightarrow \beta (= 3 - \sqrt{6})$ $\alpha = 3 + \sqrt{6}$ $\alpha\beta = (3 + \sqrt{6})(3 - \sqrt{6}) = 9 + 6\sqrt{3} - 6\sqrt{3} - 6 = 3$	M1 M1 dM1 A1* [4]
(ii)	$\alpha + \beta = 3 + \sqrt{6} + 3 - \sqrt{6} = 6$	M1A1 [2]
(b)(i)	$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = 30^2 - 2 \times 3^2 = 882$	M1A1 [2]
(b) ii)	$\alpha^4 - \beta^4 = (\alpha^2 + \beta^2)(\alpha^2 - \beta^2) = (\alpha^2 + \beta^2)(\alpha - \beta)(\alpha + \beta)$ $\alpha^4 - \beta^4 = 30 \times 6 \times 2\sqrt{6} = 360\sqrt{6}$	M1 A1 [2]
(c)	$(\alpha^4 + \beta^4) + (\alpha^4 - \beta^4) = 2\alpha^4 = 882 + 360\sqrt{6} \Rightarrow \alpha^4 = 441 + 180\sqrt{6}$	M1A1 [2]
ALT1	$\alpha - \beta = 2\sqrt{6}, \alpha + \beta = 6 \Rightarrow \alpha = 3 + \sqrt{6}$ $\alpha^4 = (3 + \sqrt{6})^4$ $\alpha^4 = 441 + 180\sqrt{6}$	M1 A1 [2]
ALT2	$\alpha - \beta = 2\sqrt{6}, \alpha + \beta = 6 \Rightarrow \beta = 3 - \sqrt{6}$ $\alpha^4 = \beta^4 + 360\sqrt{6} = (3 - \sqrt{6})^4 + 360\sqrt{6}$ $\alpha^4 = 441 + 180\sqrt{6}$	M1 A1 [2]
Total 12 marks		

Part	Mark	Notes
(a) (i)	M1	For forming an equation of the form $(\alpha + \beta)^2 \pm p\alpha\beta = 24$ oe or $(\alpha + \beta)^2 \pm t\alpha\beta = 30$ oe
	M1	For this equation being fully correct. Both marks may be implied by early substitution of values.
	dM1	For correctly solving their simultaneous equations in $\alpha + \beta$ and $\alpha\beta$ to find a value for $\alpha\beta$, dependent on 1 st method mark.
	A1*cs0	For the correct value of $\alpha\beta$
ALT1	M1	For forming an equation of the form $q\alpha\beta = \alpha^2 + \beta^2 - (\alpha - \beta)^2$ oe
	M1	For this equation being fully correct. Both marks may be implied by early substitution of values.
	dM1	For correct substitution into their equation, dependent on 1 st method mark.
	A1*cs0	For the correct value of $\alpha\beta$
(a) (ii)	M1	Correctly uses either of their equations (must be of the required form) to substitute the given value of $\alpha\beta$ and obtains a value for $\alpha + \beta$
	A1*cs0	For the correct value of $\alpha + \beta$
ALT2 (i)	M1	For an attempt to eliminate α or β and arrive at an unsimplified quadratic equation in one variable. Allow one error
	M1	For the correct quadratic equation
	dM1	For a fully correct method to solve their quadratic equation in α or β , dependent on the 1 st method mark
	A1*cs0	For correctly finding α and β and showing the minimum steps shown to find $\alpha\beta$
	M1	For finding $\alpha + \beta$ with their values
	A1*cs0	For finding $\alpha + \beta$ with the correct values, minimum steps as shown.
(b) (i)	M1	For the correct algebra $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$, if they use more complex algebra, it must be correct and fully ready for substitution of given values. Do not allow $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$ unless recovered
	A1	For substituting the given values to find the correct value for $\alpha^4 + \beta^4$
(b) (ii)	M1	For the correct algebra to write $\alpha^4 - \beta^4$ in terms of $(\alpha^2 + \beta^2)$, $(\alpha - \beta)$ and $(\alpha + \beta)$
	A1	For the correct value of $\alpha^4 - \beta^4$
(c)	M1	For adding together $\alpha^4 + \beta^4$ and $\alpha^4 - \beta^4$ to eliminate β^4 and reach α^4 either as an expression or implied by adding their values together and dividing by 2. If students subtract $\alpha^4 + \beta^4$ and $\alpha^4 - \beta^4$ to eliminate α^4 and reach β^4 , they must then reach α^4 as an expression or implied by subtracting their values, using one of the expressions from part b and dividing by 2.
	A1	For the correct value of $\alpha^4 = 441 + 180\sqrt{6}$
ALT1	M1	For $\alpha^4 = (3 + \sqrt{6})^4$
	A1	For the correct value of $\alpha^4 = 441 + 180\sqrt{6}$
ALT2	M1	For $\alpha^4 = \beta^4 + 360\sqrt{6} = (3 - \sqrt{6})^4 + 360\sqrt{6}$
	A1	For the correct value of $\alpha^4 = 441 + 180\sqrt{6}$

Question	Scheme	Marks
10(a)	$\alpha + \beta = -\frac{k}{2} \quad \alpha\beta = 2$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta, \quad \alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$ $\Rightarrow \alpha^2 - \beta^2 = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$ $\Rightarrow \alpha^2 - \beta^2 = \left(-\frac{k}{2}\right)\sqrt{\left(-\frac{k}{2}\right)^2 - 4 \times 2} = \frac{7\sqrt{17}}{4}$ $\Rightarrow \frac{k^4}{4} - 8k^2 - \frac{833}{4} = 0 \Rightarrow k^4 - 32k^2 - 833 = 0 \text{ or any equivalent}$ $\Rightarrow (k^2 - 49)(k^2 + 17) = 0 \Rightarrow k = -7 *$	B1 B1B1 M1 M1A1 M1A1 cso [8]
(b)	Product: $\left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 = \frac{7\sqrt{17}}{4} \right]$ from (a) Sum: $(\alpha - \beta) + (\alpha + \beta) = \frac{\sqrt{17}}{2} + \frac{7}{2} = \left(\frac{\sqrt{17} + 7}{2} \right)$ $x^2 - \left(\frac{\sqrt{17} + 7}{2} \right)x + \frac{7\sqrt{17}}{4} = 0 \Rightarrow 4x^2 - 2(\sqrt{17} + 7)x + 7\sqrt{17} = 0$	B1 M1 M1A1 [4]
Total 12 marks		

There is an alternative solution using the roots of the equation.

$$\begin{aligned}
 \text{Roots are } & \frac{-k \pm \sqrt{k^2 - 4 \times 2 \times 4}}{2 \times 2} \\
 \Rightarrow & \left(\frac{-k + \sqrt{k^2 - 32}}{4} \right)^2 - \left(\frac{-k - \sqrt{k^2 - 32}}{4} \right)^2 = \frac{7\sqrt{17}}{4} \\
 \Rightarrow & \left(-k + \sqrt{k^2 - 32} \right)^2 - \left(-k - \sqrt{k^2 - 32} \right)^2 = 28\sqrt{17} \\
 \Rightarrow & k^2 - 2k\sqrt{k^2 - 32} + k^2 - 32 - k^2 - 2k\sqrt{k^2 - 32} - k^2 + 32 = 28\sqrt{17} \\
 \Rightarrow & -4k\sqrt{k^2 - 32} = 28\sqrt{17} \Rightarrow k\sqrt{k^2 - 32} = 7\sqrt{17} \\
 \Rightarrow & k^2(k^2 - 32) = 833 \Rightarrow k^4 - 32k^2 - 833 = 0 \\
 \Rightarrow & (k^2 - 49)(k^2 + 17) = 0 \Rightarrow k = -7 *
 \end{aligned}$$



Part	Mark	Notes
(a)	B1	For the correct sum in terms of k and the correct product. Look out for these embedded in their work.
	B1	For the correct algebra on $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ Substitution not required for this mark, although it can be implied from correct substitution without the actual algebra seen.
	B1	For the correct algebra on $\alpha^2 - \beta^2$ Award also for $(\alpha + \beta)(\alpha - \beta) = \frac{7\sqrt{17}}{4}$ Substitution not required for this mark, although it can be implied from correct substitution without the actual algebra seen.
	M1	For substituting in the values of their sum and product into their $\alpha^2 - \beta^2$ This can be implied from sight of $\left(-\frac{k}{2}\right)\sqrt{\left(-\frac{k}{2}\right)^2 - 4 \times 2} = \frac{7\sqrt{17}}{4}$
	M1	For forming a 3TQ in terms of k^2 in any form
	A1	For the correct 3TQ Accept it in any form as long as it is 3 terms. For example, accept $\frac{k^4}{16} - 2k^2 - \frac{833}{16} = 0$ etc
	M1	For any acceptable attempt seen to solve their 3TQ [see General Guidance] If there is no working, just $k = -7$ following a 3TQ then award M0 However, accept evidence of $k^2 = 49$ [with $k^2 = -17$]
	A1 cso	For the correct value of k * Note: This is a given value
(b)	B1	For a value for the product using their results from (a) or just writes it down.
	M1	For a correct method to find the value of the sum using their values for $(\alpha - \beta)$ and $(\alpha + \beta)$
	M1	For forming a 3TQ with their sum and product. $= 0$ is not required for the award of this mark
	A1	For a correct equation simplified or unsimplified. For example, accept, $x^2 - \left(\frac{\sqrt{17} + 7}{2}\right)x + \frac{7\sqrt{17}}{4} = 0$ as well as $4x^2 - 2(\sqrt{17} + 7)x + 7\sqrt{17} = 0$ o.e.

Question number	Scheme	Marks
10 (a)	$6x - x^2 = -(x^2 - 6x)$ $-(x^2 - 6x) = -(x-3)^2 - 9 \Rightarrow f(x) = -(x-3)^2 + 9$ $D = -1, E = -3 \text{ and } F = 9$	M1A1A1 [3]
(b)	(i) $f(x)_{\max} = 9$ (ii) $x = 3$	B1ft B1ft [2]
(c)	$6x - x^2 = x^2 - 4x + 8 \Rightarrow 2x^2 - 10x + 8 = 0$ $2x^2 - 10x + 8 = (2x-2)(x-4) \Rightarrow x = 1, x = 4$ $y = 5, y = 8$ Coordinates are (1, 5) and (4, 8)	M1 M1A1 A1 [4]
(d)	$\text{Area} = \int_1^4 (6x - x^2) dx - \int_1^4 (x^2 - 4x + 8) dx = \int_1^4 [-2x^2 + 10x - 8] dx$ $= \left[\frac{-2x^3}{3} + \frac{10x^2}{2} - 8x \right]_1^4$ $= \left(\frac{-2 \times 4^3}{3} + \frac{10 \times 4^2}{2} - 8 \times 4 \right) - \left(\frac{-2 \times 1^3}{3} + \frac{10 \times 1^2}{2} - 8 \times 1 \right) = 9 \text{ (units}^2\text{)}$	M1 M1 M1A1 [4]
Total 13 marks		
(a) M1 A1 A1 (b) B1 ft B1 ft (c) M1 M1 A1 A1 (d) M1 M1 M1 A1	An attempt to factorise to make x^2 positive e.g. $-(x \pm a)^2 \pm b$ Complete the square to obtain an expression in the form $-(x \pm 3)^2 \pm q$ NB Any expression in this form will score M1A1 $D = -1, E = -3 \text{ and } F = 9$ $(f(x)_{\max} =) 9$ or follow through their value for F . $(x =) 3$ or follow through their value for E . Equating the two curves and simplifying to a 3 term quadratic Solve their 3 term quadratic $x = 1, x = 4$ (1, 5) and (4, 8) Use of $\int_a^b (f(x) - g(x)) dx$ or $\int_a^b f(x) dx - \int_a^b g(x) dx$ Ignore limits ($f(x)$ and $g(x)$ can be either way round) Attempt the integration. Limits not needed. Substitute the correct limits. 9 (units ²) NB A correct answer with no working will score 4 out of 4	

Question Number	Scheme	Marks
7(a)	$x^2 - 9x + 14 = \left(x - \frac{9}{2}\right)^2 + 14 - \frac{81}{4} = \left(x - \frac{9}{2}\right)^2 - \frac{25}{4}$	M1
	$a = -\frac{9}{2}, b = -\frac{25}{4} \quad \text{oe}$	A1 (2)
(b)	(i) least value of $f(x) = -\frac{25}{4}$	B1ft
	(ii) least value when $x = \frac{9}{2}$	B1ft (2)
(c)	$x + 5 = x^2 - 9x + 14$	M1
	$x^2 - 10x + 9 = 0 \Rightarrow (x - 9)(x - 1) = 0$	M1
	Points are (9,14) (1,6)	A1A1 (4)
(d)	$\text{Area } \int_1^9 ((x+5) - (x^2 - 9x + 14)) dx = \int_1^9 (-x^2 + 10x - 9) dx$	M1
	$= \left[-\frac{x^3}{3} + 5x^2 - 9x \right]_1^9$	M1A1
	$= (-243 + 405 - 81) - \left(-\frac{1}{3} + 5 - 9\right) = 85\frac{1}{3}$	M1A1 (5)
[13]		

Part	Mark	Notes
(a)	M1	For attempting to complete the square to achieve as a minimum $x^2 - 9x + 14 = \left(x \pm \frac{9}{2}\right)^2 + 14 - k \quad \text{where } k \text{ is a constant}$
	A1	For the correct expression $x^2 - 9x + 14 = \left(x - \frac{9}{2}\right)^2 - \frac{25}{4}$ or $a = -\frac{9}{2}, b = -\frac{25}{4}$ oe stated
(b)(i)	B1ft	For the correct value $f(x) = -\frac{25}{4}$ follow through their value of $-\frac{25}{4}$,
(ii)	B1ft	For the correct value of $x = \frac{9}{2}$ provided they have $\left(x - \frac{9}{2}\right)^2$...in part (a). Follow through their value of $\frac{9}{2}$,

(c)	M1	For equating the equation of the line with the equation of C $x+5 = x^2 - 9x + 14 \Rightarrow x^2 - 10x + 9 = 0$ and attempting to form a 3TQ
	M1	Attempts to solve their 3TQ by any method, provided it is the result of equating the line with C $x^2 - 10x + 9 = 0 \Rightarrow (x-9)(x-1) = 0$
	A1	For the correct coordinates of either (9,14) or (1,6)
	A1	For both correct pairs of coordinates (9,14) and (1,6)
(d)	M1	For a correct expression for the required area with both limits correct. (ft their limits from (c)) Award this mark if they have 'curve – line' but otherwise correct. $\int_1^9 ((x+5) - (x^2 - 9x + 14)) dx = \left[\int_1^9 (-x^2 + 10x - 9) dx \right]$, accept $\int_1^9 (x^2 - 10x + 9) dx$ OR Area under the trapezium – curve $\frac{1}{2} \times 8 \times (6+14) - \int_1^9 (x^2 - 9x + 14) dx$
	M1	For attempting to integrate the equation for the combined expression or the curve only.
	A1	For the correct integrated expression for required area. Ignore limits for this mark – even if they are absent altogether. $\text{Area} = \left[-\frac{x^3}{3} + 5x^2 - 9x \right]_1^9 \quad \text{accept} \quad \left[\frac{x^3}{3} - 5x^2 + 9x \right]_1^9$ OR $\frac{1}{2} \times 8 \times (6+14) - \left(\frac{x^3}{3} - \frac{9x^2}{2} + 14x \right)_1^9$ OR $\left(\frac{x^2}{2} + 5x \right)_1^9 - \left(\frac{x^3}{3} - \frac{9x^2}{2} + 14x \right)_1^9 \quad \text{or} \quad \left(\frac{x^3}{3} - \frac{9x^2}{2} + 14x \right)_1^9 - \left(\frac{x^2}{2} + 5x \right)_1^9$
	M1	For substituting their limits (x – coordinates from part(c)) into their integrated expression. $= (-243 + 405 - 81) - \left(-\frac{1}{3} + 5 - 9 \right) = \dots$ OR $80 - \left[\left(\frac{9^3}{3} - \frac{9 \times 9^2}{2} + 14 \times 9 \right) - \left(\frac{1^3}{3} - \frac{9 \times 1^2}{2} + 14 \times 1 \right) \right] = \dots$ OR $\left[\left(\frac{9^2}{2} + 5 \times 9 \right) - \left(\frac{1^2}{2} + 5 \times 1 \right) \right] - \left[\left(\frac{9^3}{3} - \frac{9 \times 9^2}{2} + 14 \times 9 \right) - \left(\frac{1^3}{3} - \frac{9 \times 1^2}{2} + 14 \times 1 \right) \right] = \dots$
	A1	For the correct area of $85\frac{1}{3}$ or $\frac{256}{3}$ If they get a value of $-85\frac{1}{3}$ they must give a final value of $85\frac{1}{3}$ for this mark.



Question	Scheme	Mark
7 (a)	$\alpha + \beta = -\frac{p}{2}$ and $\alpha\beta = \frac{q}{2}$	B1
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{37}{14}$	B1
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(-\frac{p}{2}\right)^2 - 2\left(\frac{q}{2}\right)}{\frac{q}{2}} = -\frac{37}{14}$	A1
	$\frac{\left(-\frac{p}{2}\right)^2 - 2\left(\frac{q}{2}\right)}{\frac{q}{2}} = \frac{p^2 - 4q}{2q} = \frac{p^2 + 4p + 12}{2(-p - 4)} = -\frac{37}{14} \Rightarrow 7p^2 - 9p - 36 = 0$ [$7p^2 - 9p - 36 = 0$ OR $7q^2 + 65q + 112 = 0$] $p = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 7 \times (-36)}}{2 \times 7} \Rightarrow p = 3, q = -7$	dM1A1 M1A1A1ft [9]
(b)	$\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$	M1
	$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$	M1
	$(\alpha - \beta)(\alpha + \beta) = \sqrt{\left(-\frac{3}{2}\right)^2 - 4 \times \left(-\frac{7}{2}\right)} \times \left(-\frac{3}{2}\right) = -\frac{3\sqrt{65}}{4}$	M1A1 [4]
Total		13 marks

Part	Mark	Notes
(a)	B1	For the sum and product of roots of the equation $f(x) = 0$ in terms of p and q $\alpha + \beta = -\frac{p}{2}$ and $\alpha\beta = \frac{q}{2}$
	B1	For the sum of roots of the equation $g(x) = 0$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{37}{14}$
	M1	For the correct algebra to find the sum of roots of $g(x) = 0$ ready for substitution of values. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

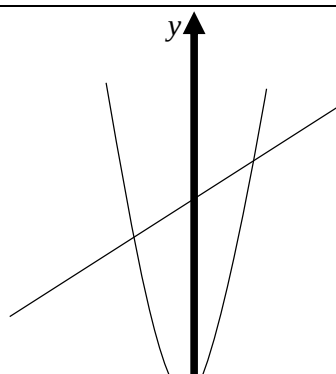
		This can be implied by correct substitution of their values for the sum and product.
	A1	For the correct value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ in terms of p and q [Simplification not required] $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(-\frac{p}{2}\right)^2 - 2\left(\frac{q}{2}\right)}{\frac{q}{2}} = -\frac{37}{14}$
	dM1	For substituting in for either p or for q into the above equation correctly to form a 3TQ in just p or q [ft their $-\frac{37}{14}$ and their $-\frac{p}{2}$ and $\frac{q}{2}$]. The algebra must be correct here. $\frac{\left(-\frac{p}{2}\right)^2 - 2\left(\frac{q}{2}\right)}{\frac{q}{2}} = \frac{p^2 - 4q}{2q} = \frac{p^2 + 4p + 12}{2(-p - 4)} = -\frac{37}{14} \Rightarrow [7p^2 - 9p - 36 = 0]$ This mark is dependent on the previous M mark.
	A1	For the correct 3TQ $7p^2 - 9p - 36 = 0$ OR $7q^2 + 65q + 112 = 0$
	M1	For attempting to solve their 3TQ by any valid method $(7p + 12)(p - 3) = 0 \Rightarrow p = \dots$ ($p = 3$) OR $p = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 7 \times (-36)}}{2 \times 7} \Rightarrow p = \dots, \dots$ Negative value of p not required
	A1	For the correct value of $p = 3$ or value of $q = -7$
	A1ft	For the correct value of $p = 3$ and value of $q = -7$ Ft on their value of p or q Ignore mislabelling of (i) and (ii) or even no labelling of parts at all.
(b)	M1	For factorising $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$ correctly
	M1	For the correct algebra on $(\alpha - \beta)^2$ i.e. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ Look for this expansion which may be embedded in their working.
	M1	For the correct substitution of their sum and product into $(\alpha - \beta)(\alpha + \beta)$ $(\alpha - \beta)(\alpha + \beta) = \sqrt{\left(-\frac{3}{2}\right)^2 - 4 \times \left(-\frac{7}{2}\right)} \times \left(-\frac{3}{2}\right) = \left[\sqrt{\frac{65}{4}} \times \left(-\frac{3}{2}\right)\right]$
	A1	For the correct answer only $\alpha^2 - \beta^2 = -\frac{3\sqrt{65}}{4}$

Question number	Scheme	Marks
3 a	$8\left(x^2 + \frac{10}{8}x\right) - 3$	M1
	$8\left(x + \frac{5}{8}\right)^2 - \frac{200}{64} - 3 = 8\left(x + \frac{5}{8}\right)^2 - \frac{49}{8}$	M1
	So $A = 8$ $B = \frac{5}{8}$ $C = -\frac{49}{8}$	A1 (3)
b (i)	$x = -\frac{5}{8}$	B1 ft
(ii)	$f(x)_{\min} = -\frac{49}{8}$	B1 ft (2)
c	$8\left(x + \frac{5}{8}\right)^2 - \frac{49}{8} = 0 \Rightarrow x = -\frac{3}{2}$ $x = \frac{1}{4}$	M1 A1 (2)
d	$8x^2 + 10x - 3 = 2x + 13 \Rightarrow 8x^2 + 8x - 16 = 0$ oe	M1
	$x^2 + x - 2 = 0 = (x + 2)(x - 1) \Rightarrow x = -2, x = 1$	M1 A1
	$y = 9$ $y = 15$ Coordinates are $(-2, 9)$ and $(1, 15)$	A1 (4)
e	Correct curve or line	B1
	Correct curve and line	B1 (2)
Total 13 marks		

Part	Mark	Notes
(a)	M1	For factorising in the form $8(x^2 + bx) + c$ or $8(x^2 + bx + d)$
	M1	For completing the square in the form $8\left(x + \frac{1}{2}b\right)^2 + e$ [see GG]
	A1	For $A = 8$ $B = \frac{5}{8}$ $C = -\frac{49}{8}$ Accept these listed or embedded.
	ALT	
	M1	For expanding $[f(x)] = A(x + B)^2 + C = Ax^2 + 2ABx + (AB^2 + C)$ This must be correct for this mark
	M1	Equates coefficients: $Ax^2 + 2ABx + (AB^2 + C) = 8x^2 + 10x - 3$ $\Rightarrow A = 8, 2AB = 10, AB^2 + C = -3$ At least two out of three must be correct.
	A1	For $A = 8$ $B = \frac{5}{8}$ $C = -\frac{49}{8}$

(b)		For $x = -\frac{5}{8}$ ft their $-B$ Allow differentiation:
(i)	B1ft	$\frac{dy}{dx} = 16x + 10 = 0 \Rightarrow x = -\frac{10}{16}$ which must be correct.
(ii)	B1ft	For $f(x)_{\min} = -\frac{49}{8}$ ft their C If they differentiate, allow a ft from their differentiation.
		Incorrect or no labelling of parts. If the responses are labelled incorrectly, mark as labelled. If there is no labelling, treat the first as (i) and the second as (ii) and mark accordingly.
(c)	M1	For setting the given $f(x) = 0$ and solving using any method. See General Guidance.
	A1	For $x = -\frac{3}{2}$ and $x = \frac{1}{4}$
(d)	M1	For setting $8x^2 + 10x - 3 = 2x + 13$ and forming a 3TQ which as a minimum must be $8x^2 + 8x \pm C$ where C is a constant, or any simplification, e.g., $x^2 + x \pm K$ where K is a constant.
	M1	For attempting to solve their 3TQ. See General Guidance.
	A1	For both $x = -2$, $x = 1$
	A1	For $(-2, 9)$ and $(1, 15)$ Accept $x = -2$, $y = 9$ and $x = 1$, $y = 15$ paired correctly.
		ALT – solves the equation in y
	M1	Substitutes $x = \frac{y-13}{2}$ into $y = 8x^2 + 10x - 3$ and forms a 3TQ which as a minimum must be of the form $2y^2 - 36y \pm X$ $X \neq 0$ $\Rightarrow y = 8\left(\frac{y-13}{2}\right)^2 + 10\left(\frac{y-13}{2}\right) - 3 \Rightarrow 2y^2 - 36y + 75 = 0$
	M1	For attempting to solve their 3TQ. See General Guidance.
	A1	For both $y = 9$ and $y = 15$
	A1	For $(-2, 9)$ and $(1, 15)$ Accept $x = -2$, $y = 9$ and $x = 1$, $y = 15$ paired correctly.
(e)	B1	For correct curve (intersections with x -axis are at $(-1.5, 0)$ and $(0.25, 0)$) or line (Intersections are $(-6.5, 0)$ and $(0, 13)$) drawn. We do not need to see any of these points marked. These are for guidance only. The question asks for a sketch, accept a reasonable attempt. - Accept a positive quadratic curve with the minimum point below the x-axis, and one branch either side of the y-axis. - Accept a straight line with a positive gradient where the intersection with the y-axis is positive.
	B1	For correct curve and line drawn

SKETCH OF CURVE AND LINE



Question	Scheme	Marks
8(a)	$\alpha + \beta = \frac{k}{3}$ and $\alpha\beta = -\frac{1}{3}$	B1
	$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	$\alpha^2 + \beta^2 = \left(\frac{k}{3}\right)^2 - 2\left(-\frac{1}{3}\right) = \frac{k^2}{9} + \frac{2}{3} = \frac{k^2 + 6}{9}^*$	A1 cso [3]
(b)	$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$	M1
	$\frac{466}{81} = \left(\frac{k^2 + 6}{9}\right)^2 - 2\left(-\frac{1}{3}\right)^2$	M1
	$\frac{466}{81} = \left(\frac{k^2 + 6}{9}\right)^2 - 2\left(-\frac{1}{3}\right)^2 \Rightarrow k^4 + 12k^2 - 448 = 0$	M1
	$k^4 + 12k^2 - 448 = (k^2 - 16)(k^2 + 28) = 0 \Rightarrow k^2 = 16 \Rightarrow k = 4$	M1A1 [5]
(c)	Sum: $\frac{\alpha^3 + \beta}{\beta} + \frac{\beta^3 + \alpha}{\alpha} = \frac{\alpha^4 + \beta^4 + 2\alpha\beta}{\alpha\beta}$	M1
	$\frac{\alpha^3 + \beta}{\beta} + \frac{\beta^3 + \alpha}{\alpha} = \frac{\frac{466}{81} + 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}} = -\frac{412}{27}$	A1
	Product: $\left(\frac{\alpha^3 + \beta}{\beta}\right)\left(\frac{\beta^3 + \alpha}{\alpha}\right) = \frac{\alpha^4 + \beta^4 + (\alpha\beta)^3 + \alpha\beta}{\alpha\beta}$	M1
	$\left(\frac{\alpha^3 + \beta}{\beta}\right)\left(\frac{\beta^3 + \alpha}{\alpha}\right) = \frac{\frac{466}{81} + \left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)} = -\frac{436}{27}$	A1
	3TQ: $x^2 + \frac{412}{27}x - \frac{436}{27} = 0 \Rightarrow 27x^2 + 412x - 436 = 0$	M1A1 [6]
Total 14 marks		

Question	Scheme	Marks
8(a)	$\alpha + \beta = \frac{k}{3}$ and $\alpha\beta = -\frac{1}{3}$	B1
	For the correct algebra on $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	For substituting <i>their</i> sum and <i>their</i> product to give a simplified expression for $\alpha^2 + \beta^2 = \left(\frac{k}{3}\right)^2 - 2\left(-\frac{1}{3}\right) = \frac{k^2}{9} + \frac{2}{3} = \frac{k^2 + 6}{9}^*$	A1 cso [3]

(b)	For the correct algebra on $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$ Or $\alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2$	M1
	For substituting the values for $\alpha^2 + \beta^2$ (in terms of k) and $\alpha\beta$ into their $\alpha^4 + \beta^4$ and equating to $\frac{466}{81}$ $\frac{466}{81} = \left(\frac{k^2 + 6}{9}\right)^2 - 2\left(-\frac{1}{3}\right)^2$ Or for substituting the values for $\alpha^2 + \beta^2$ (in terms of k), $\alpha + \beta$ and $\alpha\beta$ into their $\alpha^4 + \beta^4$ and equating to $\frac{466}{81}$	M1
	For forming a simplified quadratic. $\frac{466}{81} = \left(\frac{k^2 + 6}{9}\right)^2 - 2\left(-\frac{1}{3}\right)^2 \Rightarrow k^4 + 12k^2 - 448 = 0$ Or for $\left(\frac{k^2 + 6}{9}\right)^2 = \frac{448}{81}$ Their quadratic must come from an attempt at $\alpha^4 + \beta^4$	M1
	For solving their quadratic. $k^4 + 12k^2 - 448 = (k^2 - 16)(k^2 + 28) = 0 \Rightarrow k^2 = 16 \Rightarrow k = 4$ Their quadratic must come from an attempt at $\alpha^4 + \beta^4$	M1
	For the correct value of $k = 4$ If they give $k = \pm 4$ this is A0	A1 [5]
(c)	For the correct algebra on the sum. $\frac{\alpha^3 + \beta}{\beta} + \frac{\beta^3 + \alpha}{\alpha} = \frac{\alpha^4 + \beta^4 + 2\alpha\beta}{\alpha\beta}$	M1
	For the correct value of the sum. $\frac{\alpha^3 + \beta}{\beta} + \frac{\beta^3 + \alpha}{\alpha} = \frac{\frac{466}{81} + 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}} = -\frac{412}{27}$	A1
	For the correct algebra on the product. $\left(\frac{\alpha^3 + \beta}{\beta}\right)\left(\frac{\beta^3 + \alpha}{\alpha}\right) = \frac{\alpha^4 + \beta^4 + (\alpha\beta)^3 + \alpha\beta}{\alpha\beta}$	M1
	For the correct value of the product. $\left(\frac{\alpha^3 + \beta}{\beta}\right)\left(\frac{\beta^3 + \alpha}{\alpha}\right) = \frac{\frac{466}{81} + \left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)} = -\frac{436}{27}$	A1
	For the 3TQ from their values of the sum and product (used correctly) [= 0 not required for this mark].	M1

	$x^2 + \frac{412}{27}x - \frac{436}{27} = 0$	
	For the correct 3TQ. Must have = 0. $27x^2 + 412x - 436 = 0$ Must have integer coefficients. Accept a multiple of this.	A1 [6]
Total 14 marks		

Question	Scheme	Marks
11(a)	$f(x) = 10 + 6x - x^2 = -(x^2 - 6x) + 10 \Rightarrow A = -1$ $f(x) = -[(x-3)^2 - 9] + 10 = -(x-3)^2 + 19$ $B = -3, C = 19$	B1 M1 A1A1 [4]
(b)	(i) $x = 3$ (ii) $f(x)_{\text{greatest}} = 19$	B1ft B1ft [2]
(c)	$x^2 - x + 13 = 10 + 6x - x^2 \Rightarrow 2x^2 - 7x + 3 = 0$ $\Rightarrow (2x-1)(x-3) = 0 \Rightarrow x = \frac{1}{2}, 3$	M1 M1A1 [3]
(d)	$A = \int_{\frac{1}{2}}^3 (10 + 6x - x^2) dx - \int_{\frac{1}{2}}^3 (x^2 - x + 13) dx = \left[\int_{\frac{1}{2}}^3 -2x^2 + 7x - 3 dx \right]$ $A = \left[-\frac{2x^3}{3} + \frac{7x^2}{2} - 3x \right]_{\frac{1}{2}}^3$ $A = \left[\left(-\frac{2 \times 3^3}{3} + \frac{7 \times 3^2}{2} - 3 \times 3 \right) - \left(-\frac{2 \times 0.5^3}{3} + \frac{7 \times 0.5^2}{2} - 3 \times 0.5 \right) \right]$ $A = \frac{125}{24} \text{ oe}$	M1 M1A1 M1 A1 [5]
Total 14 marks		

Part	Mark	Notes
(a)	B1	For factorising – 1 and finding $A = -1$
	M1	For an attempt to complete the square – (See general guidance)
	A1	For either $B = -3$ or $C = 19$
	A1	For both $B = -3$ and $C = 19$ Accept all values embedded.
	NB: Correct values following no working – Award full marks in this part.	

(b)(i)	B1ft	$x = 3$ ft their B You may see differentiation to find a maximum x . Allow a correct value of x here even if it does not follow from their working.
(b)(ii)	B1ft	$f(x) = 19$ ft their C You may see differentiation to find a maximum x . Allow a correct value of $f(x)$ here even if it does not follow from their working. Allow $y = 19$.
(c)	M1	Sets the equation of curve C equal to the equation of curve S and forms a 3TQ
	M1	Attempts to solve their 3TQ using any correct method to find two values of x If their 3TQ is incorrect and no working is seen to solve it, withhold this mark.
	A1	For both $x = \frac{1}{2}$, 3 correct Accept coordinates – ignore y values.
	NB: Both correct values following no working – Award full marks in this part.	
(d)	M1	For a correct statement with their values of x in the correct position of the intent to integrate the two expressions and subtract the result, or to integrate $2x^2 - 7x + 3$ Ft their limits for this mark, the correct way around, but allow recovery at a later stage. If they deal with the two expressions separately, look for subtraction at the end of their solution.
	M1	For an attempt to integrate their combined expression or two separate expressions. They must achieve as a minimum $\pm Px^3 \pm Qx^2 \pm Rx$ for their integration whether they integrate two expressions separately (each) or one combined expression. Ignore limits for this mark.
	A1	For the correct integrated expression(s)– ignore limits for this mark Allow $\frac{2x^3}{3} - \frac{7x^2}{2} + 3x$
	M1	For substituting their limits into their integrated expression and subtracting the results of lower limit from upper limit. Note carefully: If they have the correct limits and correct integration allow this mark if the correct area is obtained without seeing explicit substitution. $\left(\pm \frac{125}{24}, \pm 5.208\ldots\right)$ If their limits are incorrect or the integration is incorrect or the final area is incorrect then explicit substitution must be seen for the award of this mark.
	A1	For the correct area $A = \frac{125}{24}$ oe

Question Number	Scheme	Marks
10(a)	$\alpha + \beta = -3 \quad \alpha\beta = -5$	B1
(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta, = 19$	M1,A1
(ii)	$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2, = 19^2 - 50 = 311$	M1,A1(5)
	OR: $\alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2, = 19^2 - 50 = 311$	
(b)	$(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 = 19 + 10$ OR $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 9 - (-20)$	M1
	$\alpha - \beta = \sqrt{29} *$	A1 cso (2)
(c)	$\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = (\alpha - \beta)(\alpha + \beta)(\alpha^2 + \beta^2)$	M1A1A1 (3)
(d)	$\alpha^4 - \beta^4 = \sqrt{29} \times (-3) \times 19 = -57\sqrt{29} \quad (-\sqrt{94221})$	M1A1 (2)
(e)	$2\beta^4 = \alpha^4 + \beta^4 - (\alpha^4 - \beta^4)$	M1
	$\beta^4 = \frac{1}{2}(311 + 57\sqrt{29}), = \frac{311}{2} + \frac{57}{2}\sqrt{29}$	A1,A1 (3)
	$p = \frac{311}{2} \quad q = \frac{57}{2}$	[15]
ALT	$\beta^4 = \left(\frac{-3 - \sqrt{29}}{2}\right)^4$ and use a correct binomial expansion	M1A1
	Correct final answer	A1
(a)B1	Correct sum and product of roots, seen explicitly or used (in (a)). Must be clear that sum is negative	
(i)M1	Correct algebra, ready for substitution of sum and product	
A1	Correct answer, condone use of $\alpha + \beta = 3$.	
(ii)M1	Correct algebra, ready for substitution	
A1	Correct answer, condone use of $\alpha + \beta = 3$.	
(b)M1	Correct algebra and substitution of their values	
A1cso	Correct answer from correct working. Must have seen sum = -3 here if not shown in (a)	
(c)	Factorise to 2 quadratic brackets or 2 linear and one quadratic bracket	
M1	Obtain 2 linear and 1 quadratic brackets with 2 of the 3 brackets correct	
A1	Third correct bracket Accept $(\alpha^2 + \beta^2)$ or $((\alpha + \beta)^2 - 2\alpha\beta)$	
(d)M1	Substitute their values for each of the 3 brackets obtained in (c)	
A1	Correct answer as shown or equivalent exact value	
(e)	Correct expression for $2\beta^4$ or β^4	
M1	Substitute their numbers to obtain a numerical expression for β^4 The expression must be exact but need not be simplified	
A1ft	NB A correct numerical expression for their values implies M1	
A1	Correct answer in the required form. p and q need not be shown explicitly.	

Question number	Scheme	Marks
10 (a)	$\left(3 - 9\left(\frac{4}{9}x + x^2\right) \text{ or } -9\left(-\frac{1}{3} + \frac{4}{9}x + x^2\right)\right)$ $3 - 9\left[\left(x + \frac{2}{9}\right)^2 - \frac{4}{81}\right] \text{ or } -9\left[\left(x + \frac{2}{9}\right)^2 - \frac{4}{81} - \frac{1}{3}\right]$ $\text{or } -1\left[9\left(x + \frac{2}{9}\right)^2 - \frac{36}{81} - 3\right] \text{ or } 3 + (1)\left[-9\left(x + \frac{2}{9}\right)^2 + \frac{36}{81}\right]$ $\frac{31}{9} - 9\left(x + \frac{2}{9}\right)^2 \quad A = \frac{31}{9} \quad B = 9 \quad C = \frac{2}{9} \text{ oe}$	M1 A1 A1 A1 [4]
(b)	$\frac{31}{9}$	B1ft [1]
(c)	$\alpha + \beta = -\frac{4}{9} \quad \alpha\beta = -\frac{3}{9} \text{ oe}$ $\frac{3\alpha}{\beta} + \frac{3\beta}{\alpha} \left(= \frac{3(\alpha^2 + \beta^2)}{\alpha\beta} \right) = \frac{3((\alpha + \beta)^2 - 2\alpha\beta)}{\alpha\beta}$ $\frac{3\left[\left(-\frac{4}{9}\right)^2 - 2\left(-\frac{3}{9}\right)\right]}{-\frac{3}{9}} \quad \left(= -\frac{70}{9} \right)$ $\left(\frac{3\alpha}{\beta} \times \frac{3\beta}{\alpha} = \right) 9$ $x^2 + \frac{70}{9}x + 9 (= 0)$ $9x^2 + 70x + 81 = 0$	B1 M1 A1ft B1 M1 A1cso [6]
(d)	$((x + y)^3 =) x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y)$	B1*cso [1]
(e)	$(\alpha^2 - \beta + \beta^2 - \alpha =)$ $(\alpha + \beta)^2 - 2\alpha\beta - (\alpha + \beta) \quad \left((\alpha + \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{70}{81} \right)$ $\left(-\frac{4}{9}\right)^2 - 2\left(-\frac{3}{9}\right) - \left(-\frac{4}{9}\right) \quad \left(= \frac{106}{81} \right)$ candidates may also transfer a value for $\alpha^2 + \beta^2$ from working in part (c) $[(\alpha^2 - \beta)(\beta^2 - \alpha) = \alpha^2\beta^2 - \beta^3 - \alpha^3 + \alpha\beta]$ $= (\alpha\beta)^2 + \alpha\beta - (\alpha^3 + \beta^3) \text{ or } (\alpha\beta)^2 + \alpha\beta - \alpha^3 - \beta^3$ $[(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)]$ $= (\alpha\beta)^2 + \alpha\beta - (\alpha + \beta)^3 + 3\alpha\beta(\alpha + \beta) \text{ or } (\alpha\beta)^2 + \alpha\beta - ((\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta))$ $\left[= \left(-\frac{3}{9}\right)^2 + \left(-\frac{3}{9}\right) - \left(-\frac{4}{9}\right)^3 + 3\left(-\frac{3}{9}\right)\left(-\frac{4}{9}\right) \left(= \frac{226}{729} \right) \right]$ $\frac{106}{81} = -\frac{q}{3} \quad \text{or} \quad \frac{226}{729} = \frac{r}{3}$ $q = -\frac{106}{27} \quad \text{and} \quad r = \frac{226}{243}$	M1 A1ft M1 M1 dddM1 A1 [6]
Total 18 marks		



Part	Mark	Additional Guidance
(a)	M1	For a method to complete the square to achieve as a minimum $3 \pm 9 \left[\left(x \pm \frac{2}{9} \right)^2 \pm p \right]$ or $\pm 9 \left[\left(x \pm \frac{2}{9} \right)^2 \pm q \right]$ or $\pm 1 \left[9 \left(x \pm \frac{2}{9} \right)^2 \pm r \right]$ or $3 \pm (1) \left[-9 \left(x \pm \frac{2}{9} \right)^2 + s \right]$ where p, q, r or s are constants > 0 .
	A1	For one of A, B or C correct.
	A1	For two of A, B or C correct.
	A1	For all of A, B or C correct For all accuracy marks, A, B and C may be either explicitly stated or embedded and if embedded correctly then stated incorrectly, we may isw. Correct A, B and C will imply 4 marks. 1 or 2 values correct, will need M1 to be gained first.
ALT	M1	For an attempt to expand $A - B(x + C)^2$ AND equate coefficients to the given $f(x) \Rightarrow A - Bx^2 - 2BCx - BC^2 = 3 - 4x - 9x^2$ Allow $A \pm Bx^2 \pm 2BCx \pm BC^2$ for the expansion of $A - B(x + C)^2$ Must be an attempt to correctly equate at least one coefficient using their expansion. Eg $-B = -9 \Rightarrow B = \dots$ $-2BC = -4 \Rightarrow C = \dots$ $A - BC^2 = 3 \Rightarrow A = \dots$
	A1A1A1	As main scheme.
(b)	B1ft	For their $\frac{31}{9}$ Note, strict 'hence' question, if $\frac{31}{9}$ is written with incorrect work in (a), B0. It is the candidate's final answer in part (a) which will be used to mark part (b) ft must be from $A - B(x + C)^2$ If $\frac{31}{9}$ is written with no work in (a), award the mark.
(c)	B1	Correct values for $\alpha + \beta$ and $\alpha\beta$. Award if and look for values not seen explicitly but embedded in the sum/product calculations for the new equation.
	M1	Reaches a correct expression ready for substitution of their values of $\alpha + \beta$ and $\alpha\beta$
	A1ft	Correctly substitutes their values for $\alpha + \beta$ and $\alpha\beta$ into a correct expression for the sum of roots. Simplification is not necessary.
	B1	For 9
	M1	Use $x^2 - (\text{their sum of their roots})x + \text{their product of roots}$ " $= 0$ " may be missing. This mark is not dependent and may be awarded for a clear substitution of anything we can see is their product and sum of roots.
	A1cso	For the given equation or with integer coefficients. Must have $= 0$.
Note. It is possible to get the correct equation from using $\alpha + \beta = +\frac{4}{9}$ This will always achieve final A0 as it's a correct equation from incorrect working.		

(d)	B1*cs0	Complete and full algebra to show the given identity. Minimum steps as shown in MS, no errors or omissions. Extra steps checked. For this question, where students have chosen to do full expansion of brackets, we will allow them to recover brackets if following work is correct.
(e)	Look carefully for students transferring their work in (c) on $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ as this can get the first 2 marks in (e), implied M1, A1ft. This work must be correct for their original sum and product of roots and must be shown. Examiners are not expected to check work if their substitution isn't shown.	
	M1	Reaches a correct expression ready for substitution of their values of $\alpha + \beta$ and $\alpha\beta$
	A1ft	Correctly substitutes their values for $\alpha + \beta$ and $\alpha\beta$ into a correct expression for sum of roots.
	M1	For the correct expression shown. Any equivalent, ready for substitution of $\alpha\beta$
	M1	For the correct expression shown. Any equiv ready for sub of $\alpha\beta$ and $\alpha + \beta$ For this mark and the previous M mark, we're looking for expressions preparing for substitution of candidates' values. You may see completion of this work in stages, eg when the values are subbed in later. Eg, you may see $\alpha^2\beta^2 + \alpha\beta - (\alpha^3 + \beta^3) \rightarrow \alpha^2\beta^2 + \alpha\beta - (\alpha + \beta)^3 + 3\alpha\beta(\alpha + \beta)$ then $\left(-\frac{3}{9}\right)^2 + \left(-\frac{3}{9}\right) - \left(-\frac{4}{9}\right)^3 + 3\left(-\frac{3}{9}\right)\left(-\frac{4}{9}\right)$ which is M2, $(\alpha\beta)^2$ is implied.
	dddM1	For equating their sum of roots to $-\frac{q}{3}$ or their product of roots to $\frac{r}{3}$. Dep on all method marks. This mark may sometimes be implied by a correct p or q
	A1	For both correct values for q and r

In general, A marks (or M marks) cannot come from incorrect working (though a correct method followed by incorrect simplification can usually go on to gain M marks).

In this question, you are looking for any of the forms given in the notes, at any point (and disregarding incorrect work around it) to award M1.

Only when this is awarded, you may award following A marks. The exception being a fully correct answer may be awarded 4 marks without M1 being present.

