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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Identities and Inequalities (3)

International GCSE Further Pure Mathematics Topic 3

39 questions · 307 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
4-MARK QUESTIONS		5 questions · 20 marks	4
1. Q1	Paper 1 June 2019	4	5
2. Q2	Paper 1 January 2020	4	6
3. Q2	Paper 1 November 2020	4	8
4. Q1	Paper 1R January 2022	4	9
5. Q1	Paper 1 November 2024	4	11
5-MARK QUESTIONS		2 questions · 10 marks	12
6. Q1	Paper 2R January 2022	5	13
7. Q2	Paper 2 June 2025	5	14
6-MARK QUESTIONS		8 questions · 48 marks	15
8. Q5	Paper 2 June 2019	6	16
9. Q3	Paper 1 January 2020	6	17
10. Q2	Paper 2 January 2020	6	18
11. Q1	Paper 2 June 2021	6	19
12. Q2	Paper 2R January 2023	6	20
13. Q2	Paper 2R June 2023	6	21
14. Q1	Paper 2 June 2024	6	22
15. Q2	Paper 2 November 2024	6	23
7-MARK QUESTIONS		5 questions · 35 marks	24
16. Q5	Paper 2R June 2019	7	25
17. Q4	Paper 1R June 2022	7	28
18. Q1	Paper 2 January 2023	7	29
19. Q5	Paper 2 June 2023	7	30
20. Q5	Paper 1R June 2025	7	33
8-MARK QUESTIONS		7 questions · 56 marks	34
21. Q4	Paper 1 January 2022	8	35
22. Q4	Paper 2R June 2022	8	38
23. Q2	Paper 1R June 2023	8	39
24. Q3	Paper 1 November 2023	8	40
25. Q4	Paper 2 November 2023	8	41
26. Q4	Paper 1 June 2025	8	44
27. Q5	Paper 2R June 2025	8	47



9-MARK QUESTIONS			4 questions · 36 marks	48
28.	Q5	Paper 1 January 2022	9	49
29.	Q2	Paper 1 June 2022	9	50
30.	Q4	Paper 1 January 2023	9	51
31.	Q8 <i>also Topic 8</i>	Paper 1 June 2023	9	52
11-MARK QUESTIONS			3 questions · 33 marks	53
32.	Q4	Paper 1R January 2020	11	54
33.	Q2	Paper 2R November 2020	11	55
34.	Q4	Paper 2R June 2024	11	56
12-MARK QUESTIONS			2 questions · 24 marks	57
35.	Q11	Paper 1 November 2021	12	58
36.	Q5 <i>also Topic 1</i>	Paper 1R January 2023	12	59
13-MARK QUESTIONS			1 questions · 13 marks	60
37.	Q6	Paper 2 June 2021	13	61
16-MARK QUESTIONS			2 questions · 32 marks	62
38.	Q5 <i>also Topic 8</i>	Paper 1R June 2023	16	63
39.	Q7 <i>also Topic 5</i>	Paper 2 November 2024	16	64



QUESTIONS

39 questions · 307 marks · lowest tariff first



4-MARK QUESTIONS

5 questions in this booklet



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

$$f(x) = x^3 + 2x^2 - 5x - 6$$

(a) Factorise $x^2 - x - 2$

(1)

(b) Hence, or otherwise, show that $(x^2 - x - 2)$ is a factor of $f(x)$.

(3)

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- 2 (a) On the grid below, draw the line with equation

(i) $5x + 2y = 10$ (ii) $y = x$

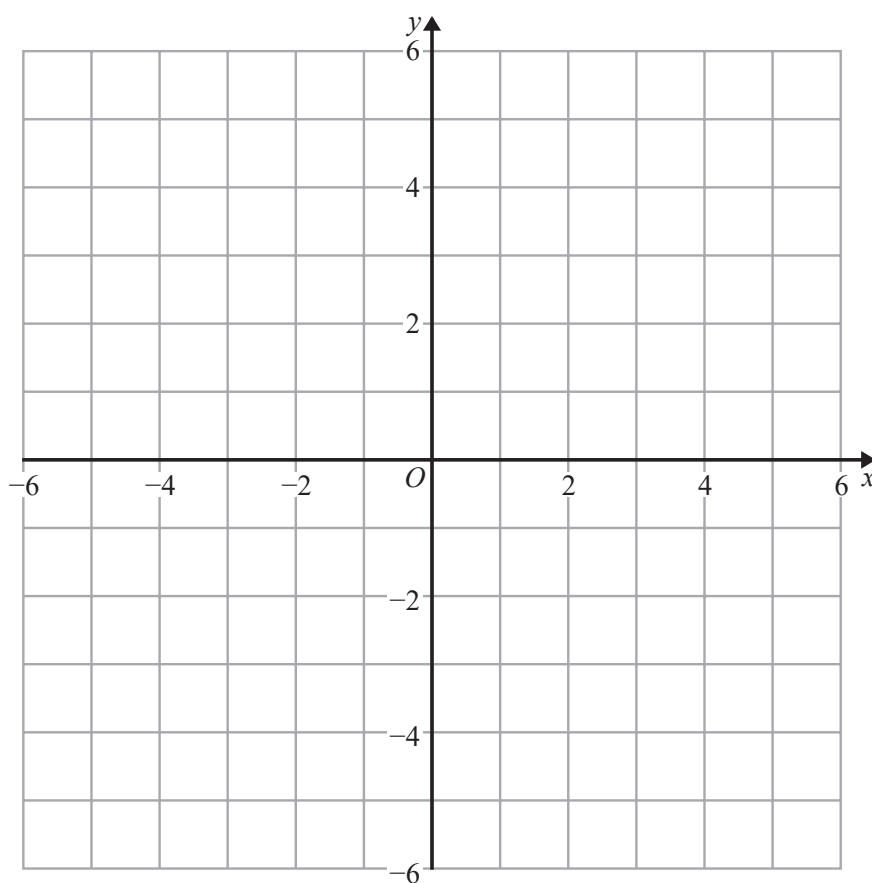
(2)

- (b) Show, by shading on the grid, the region R defined by the inequalities

$y \leq x$ $5x + 2y \leq 10$ $y \geq -2$ $x \geq 1$

Label the region R .

(2)



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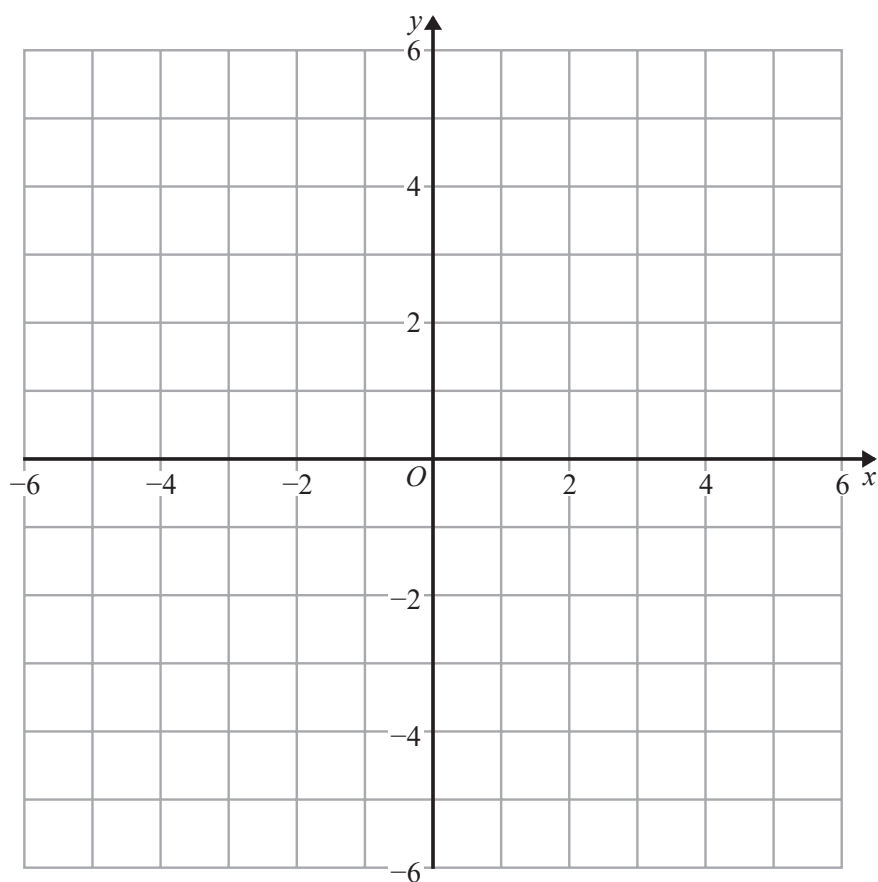
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Question 2 continued**Only use this grid if you need to redraw your graph.**

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- 2 (a) Using the axes below sketch the line with equation

(i) $y = 6$ (ii) $y + x = 10$ (iii) $y = 2x - 5$

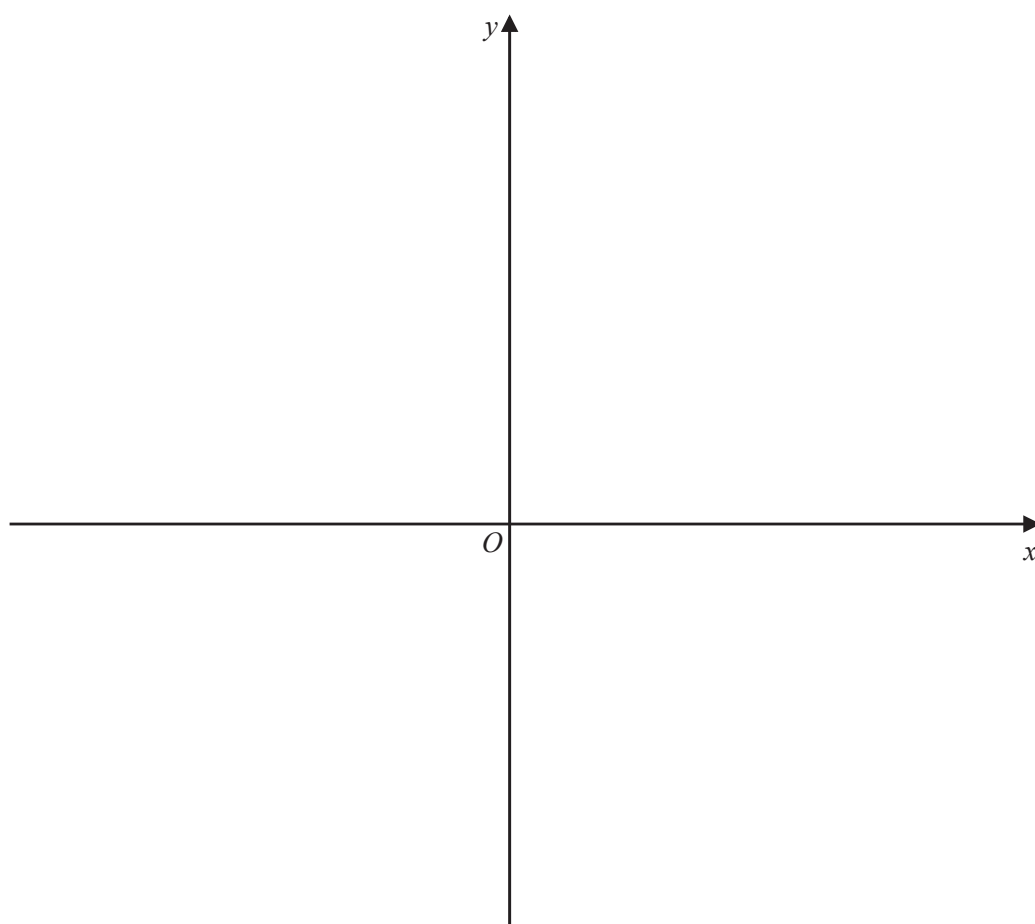
Show the coordinates of any point where each line crosses the coordinate axes.

(3)

- (b) Show, by shading on your sketch, the region R defined by the inequalities

$y \leq 6$ $y + x \leq 10$ $y \geq 2x - 5$ $x \geq 0$

(1)



- 1 (a) On the grid below, draw the graph of the line with equation

(i) $4x + 5y = 20$

(ii) $3y - 4x = -12$

(2)

- (b) Show, by shading on the grid, the region R defined by the inequalities

$4x + 5y \leq 20$

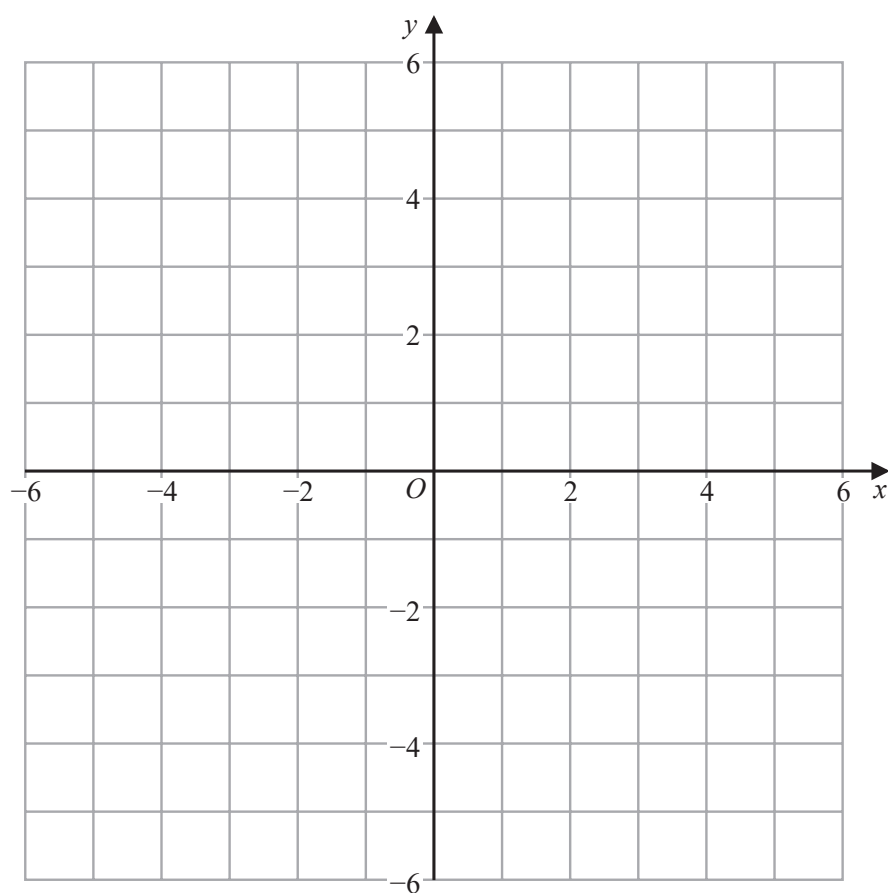
$3y - 4x \geq -12$

$y \leq 3$

$x \geq 1$

Label the region R .

(2)



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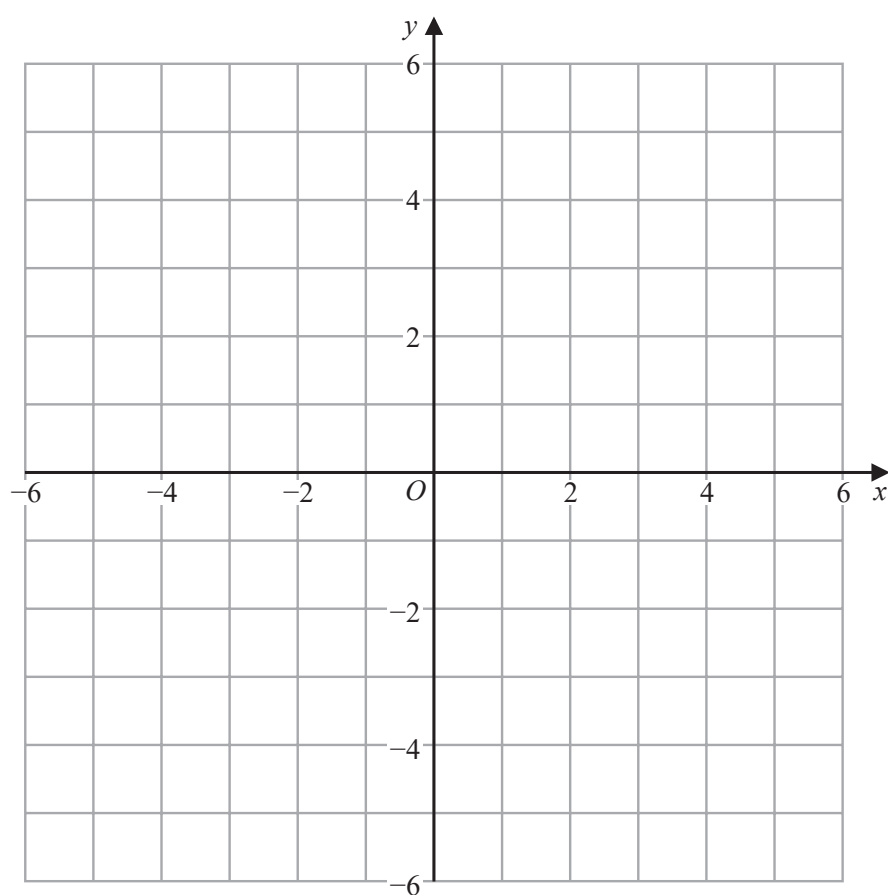
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Turn over for a spare grid if you need to redraw your graph.



Question 1 continued

Only use this grid if you need to redraw your graph.



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1 (a) On the grid below, draw the line with equation

(i) $3x + 4y = 24$ (ii) $2x - 5y + 10 = 0$

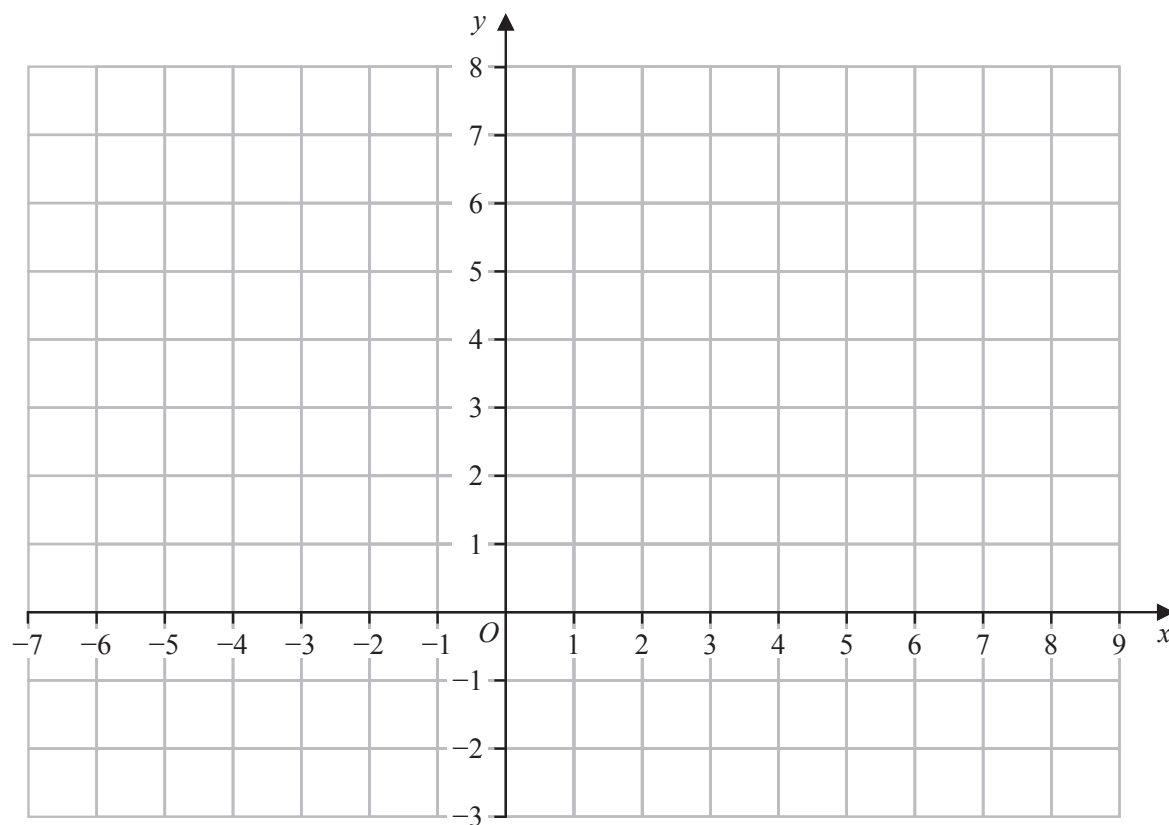
(2)

(b) Show, by shading on the grid, the region R defined by the inequalities

$$3x + 4y \leq 24 \quad 2x - 5y + 10 \geq 0 \quad y \leq 5 \quad x \geq -1$$

Label the region R

(2)



5-MARK QUESTIONS

2 questions in this booklet



1 Find the set of values for x for which

(a) $5x - 10 > 4x - 7$

(1)

(b) $2x^2 - 11x + 5 < 0$

(3)

(c) **both** $5x - 10 > 4x - 7$ **and** $2x^2 - 11x + 5 < 0$

(1)

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2 Find the set of values for x for which

(a) $9 - 3x > 11x + 2$

(1)

(b) $10x^2 + 7x < 12$

(3)

(c) **both** $9 - 3x > 11x + 2$ **and** $10x^2 + 7x < 12$

(1)

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6-MARK QUESTIONS

8 questions in this booklet



5 Use algebra to solve the equations

$$xy = 36$$

$$xy + x + 2y = 53$$

(6)



3 Given that $(x - 4)$ is a factor of $px^3 - 31x^2 + 25x + 12$ where p is a constant,

(a) show that $p = 6$

(2)

(b) Solve the equation $6x^3 - 31x^2 + 25x + 12 = 0$

Show clear algebraic working.

(4)

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2 Find the set of values of x for which

(a) $3 + 2x \leq x + 2$

(1)

(b) $8x^2 + 10x < 3$

(4)

(c) **both** $3 + 2x \leq x + 2$ **and** $8x^2 + 10x < 3$

(1)

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You must write down all the stages in your working.

1 Find the set of values for x for which

(a) $8x - 7 < 5x + 5$

(2)

(b) $2x^2 - 5x - 3 > 0$

(3)

(c) **both** $8x - 7 < 5x + 5$ **and** $2x^2 - 5x - 3 > 0$

(1)

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2 Find the set of values of x for which

(a) $3x - 8 < 5x + 3$

(1)

(b) $4x^2 - 7x + 1 > 6 - 2x^2$

(4)

(c) **both** $3x - 8 < 5x + 3$ **and** $4x^2 - 7x + 1 > 6 - 2x^2$

(1)

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2 Find the set of values of x for which

(a) $2(x + 1) < 5x - 2$

(2)

(b) $3x^2 - x \leq 10$

(3)

(c) **both** $2(x + 1) < 5x - 2$ **and** $3x^2 - x \leq 10$

(1)

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You must write down all the stages in your working.

1 $f(x) = 6x^3 - 13x^2 + ax - 10$ where a is a constant

Given that $(3x - 2)$ is a factor of $f(x)$

(a) show that $a = 21$

(2)

(b) Hence show algebraically that the curve $y = f(x)$ has only one intersection with the x -axis.

(4)

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- 2 The length of rectangle R is 2 cm greater than its width.

The area of R is greater than 8 cm^2 and the perimeter of R is less than 30 cm.

Given that the width of R is w cm,

find the set of possible values of w

Give your answer in the form $a < w < b$ where a and b are rational numbers.

(6)

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7-MARK QUESTIONS

5 questions in this booklet



- 5 (a) On the grid opposite, draw the graphs of the lines with equations

$$2x + 3y = 24 \quad y = 2x \quad 3y = 2x - 12 \quad (3)$$

- (b) Show, by shading on the grid, the region R defined by the inequalities

$$2x + 3y \leq 24 \quad y \leq 2x \quad 3y \geq 2x - 12 \quad y \geq 0 \quad (1)$$

For all points in R , with coordinates (x, y)

$$F = 2x + 5y$$

- (c) Find the greatest value of F . (3)

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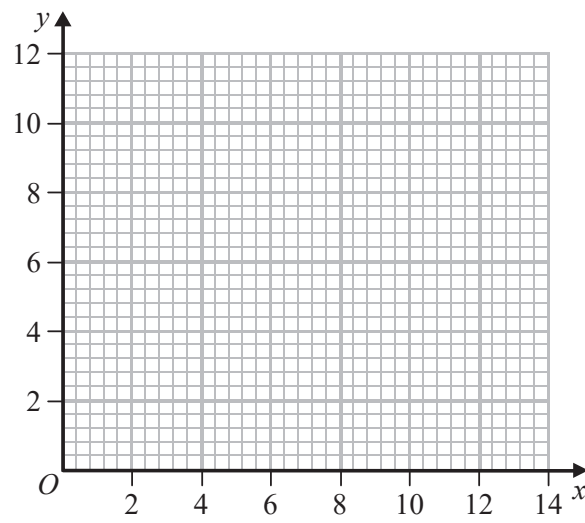
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Question 5 continued

Only use this grid if you need to redraw your graphs.



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4

Diagram **NOT**
accurately drawn $(x + 4)$ metres**Figure 2**

Figure 2 shows a rectangle with width x metres and length $(x + 4)$ metres.

The perimeter of the rectangle is P metres and the area of the rectangle is $A \text{ m}^2$

(a) Find, in terms of x , an expression for

(i) P (ii) A

(2)

The perimeter of the rectangle has to be less than 30 metres.

The area of the rectangle has to be greater than 12 m^2

(b) Find the set of possible values for x

Give your answer in the form $a < x < b$

(5)

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1 Find the set of values of x for which

(a) $2(3x - 1) < 4 - 3x$

(2)

(b) $3x^2 - 8x - 3 < 0$

(4)

(c) **both** $2(3x - 1) < 4 - 3x$ **and** $3x^2 - 8x - 3 < 0$

(1)

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5 (a) On the grid opposite draw the line with equation

(i) $y = 2x + 5$

(ii) $4y = x - 8$

(iii) $5y + 3x = 30$

(3)

(b) Show, by shading, the region R defined by the inequalities

$$y \leq 2x + 5$$

$$4y \geq x - 8$$

$$5y + 3x \leq 30$$

(1)

For all points in R with coordinates (x, y)

$$P = 2x - 5y$$

(c) Using your graph, find the least value of P

(3)

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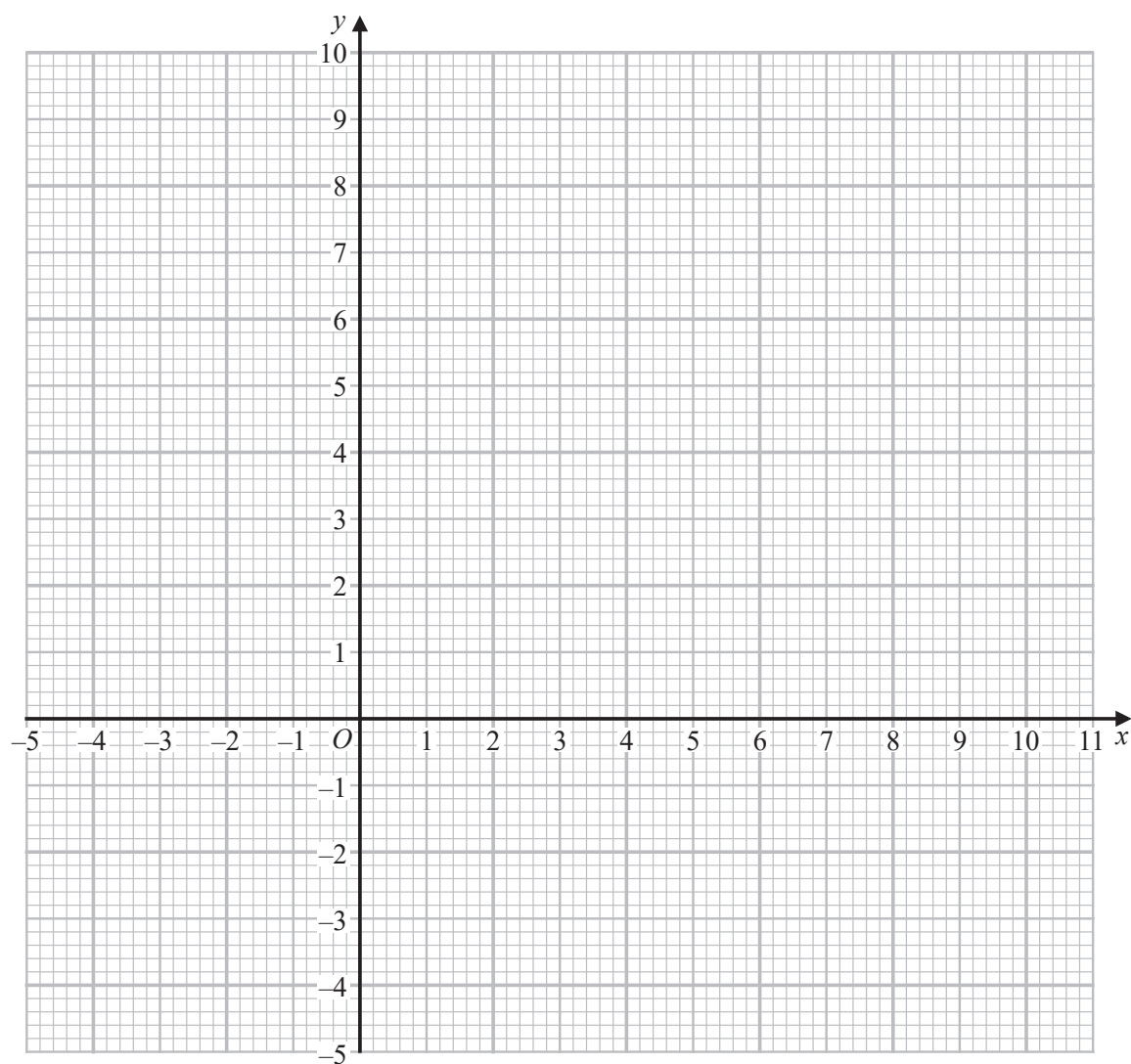
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Question 5 continued



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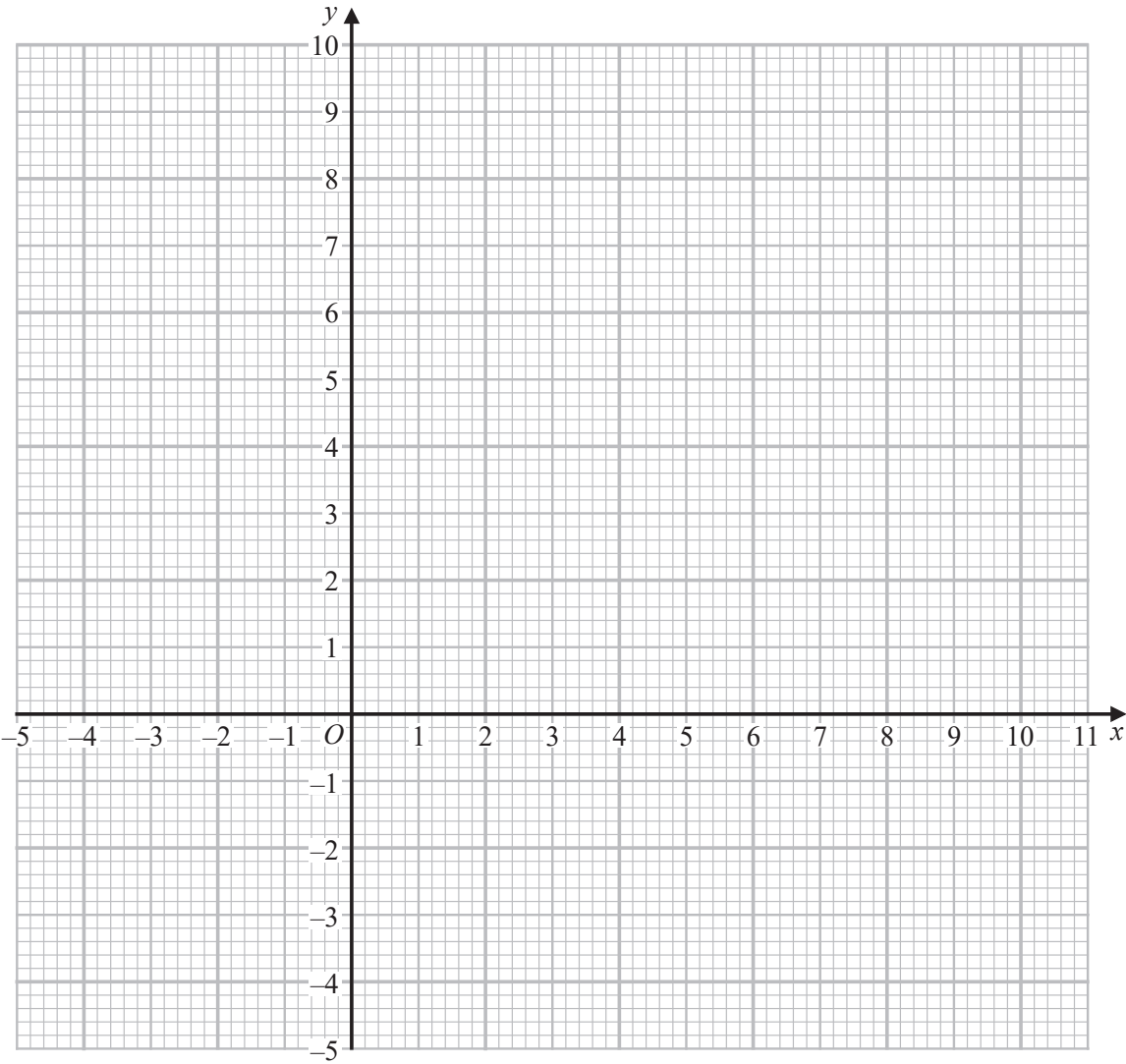
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Turn over for a spare grid if you need to redraw your graph.



Question 5 continued

Only use this grid if you need to redraw your graph.



5



Diagram **NOT**
accurately drawn

x cm

$(2x + 3)$ cm

Figure 1

Figure 1 shows a rectangle with width x cm and length $(2x + 3)$ cm.

The perimeter of the rectangle is P cm and the area of the rectangle is A cm²

$$P > 10 \quad \text{and} \quad A < 35$$

Find the set of possible values for x

Give your answer in the form $a < x < b$ where a and b are rational numbers.

Show clear algebraic working.

(7)

8-MARK QUESTIONS

7 questions in this booklet



4 (a) On the grid opposite, draw the line with equation

(i) $y = 2x - 4$

(ii) $2x + 3y = 12$

(iii) $y + 2x + 2 = 0$

(3)

(b) Show, by shading on the grid, the region R defined by the inequalities

$y \geq 2x - 4$

$2x + 3y \leq 12$

$y + 2x + 2 \geq 0$

(1)

For all points in R , with coordinates (x, y)

$$P = x - 2y$$

(c) find the least value of P

(4)

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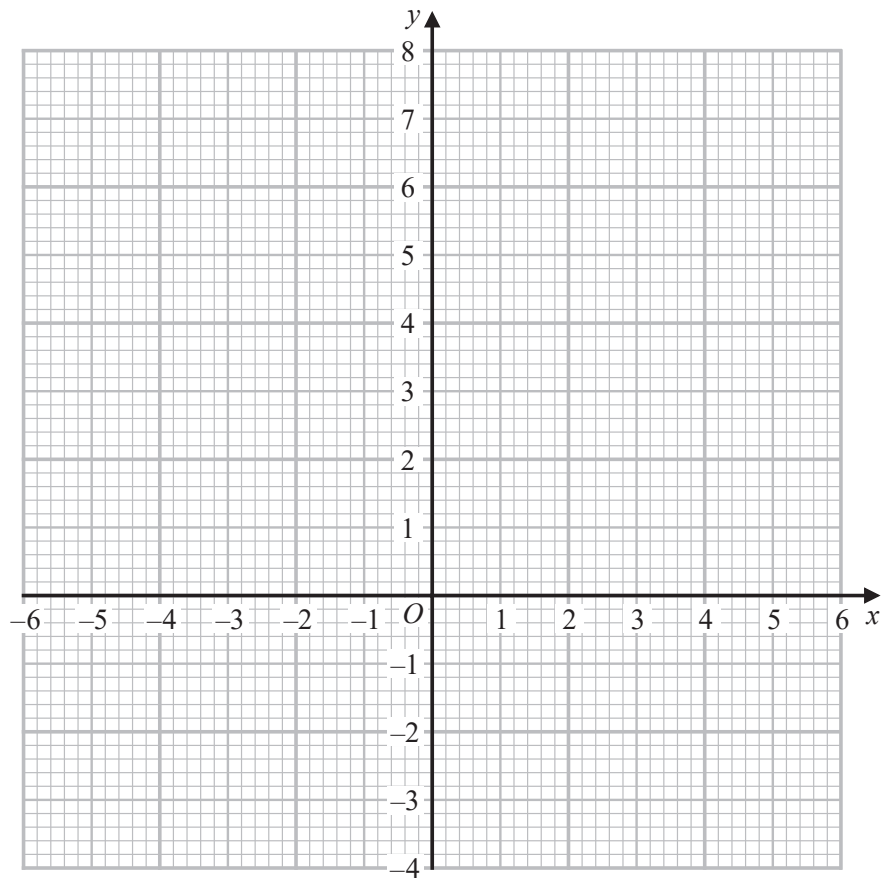
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Question 4 continued



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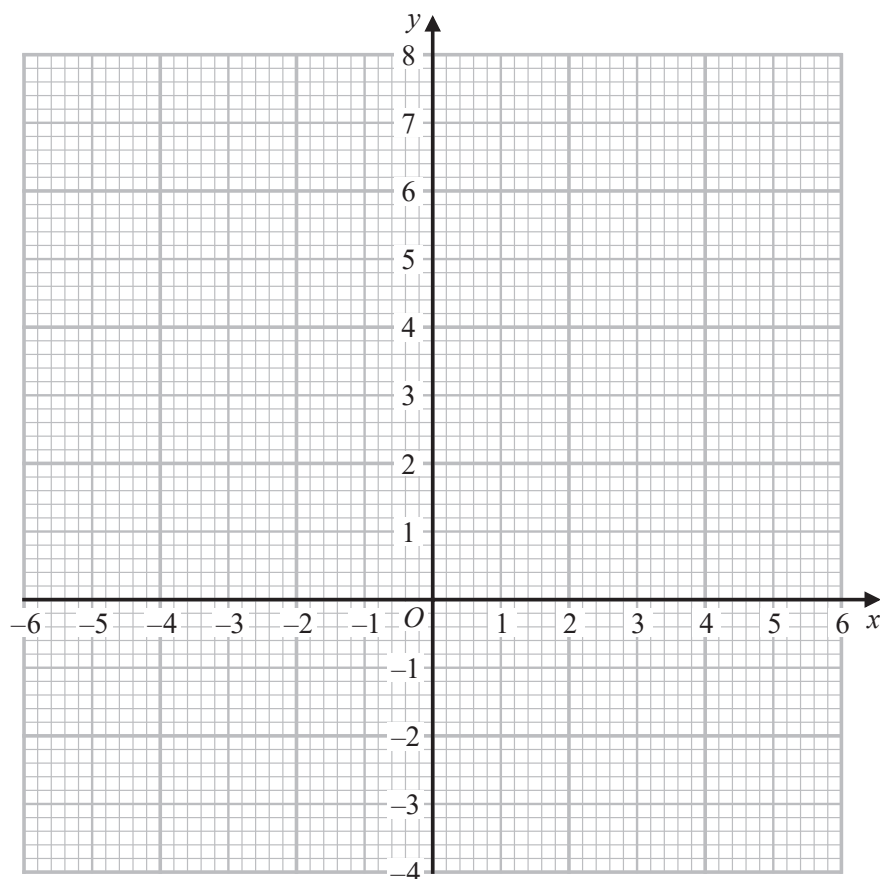
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Turn over for a spare grid if you need to redraw your graph.



Question 4 continued

Only use this grid if you need to redraw your graph.



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4 $f(x) = x^3 + px^2 + qx + 7$ where p and q are integers.

$(x + 1)$ is a factor of $f(x)$

The remainder when $f(x)$ is divided by $(x + 2)$ is -5

(a) Find the value of p and the value of q

(5)

(b) Hence, show that $f(x) = 0$ has only one real root.

(3)

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- 2 One solution to the following simultaneous equations

$$\begin{aligned}y &= px + 9 \\ 6x^2 - xy &= 5\end{aligned}$$

is $\left(-\frac{1}{2}, q\right)$, where p is an integer and q is a prime number.

- (a) Find the value of p and the value of q

(4)

- (b) Hence find the other solution to the equations.

(4)

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3 $g'(x) = mx^2 - 10x - 37$ where m is an integer

The curve $y = g(x)$ passes through the point with coordinates $(1, 20)$

Given that $(x-5)$ is a factor of $g(x)$

(a) show that $g(x) = 2x^3 - 5x^2 - 37x + 60$ (5)

(b) Hence, or otherwise, use algebra to solve the equation $g(x) = 0$ (3)

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4 (a) On the axes opposite, draw the line with equation

$$(i) \ y = -x - 1 \quad (ii) \ y - 3x + 8 = 0 \quad (iii) \ 2y = x + 8 \quad (3)$$

(b) Show, by shading on your graph, the region R defined by the inequalities

$$y \geq -x - 1 \quad \text{and} \quad y \geq 3x - 8 \quad \text{and} \quad 2y \leq x + 8 \quad (1)$$

For all points in R , with coordinates (x, y)

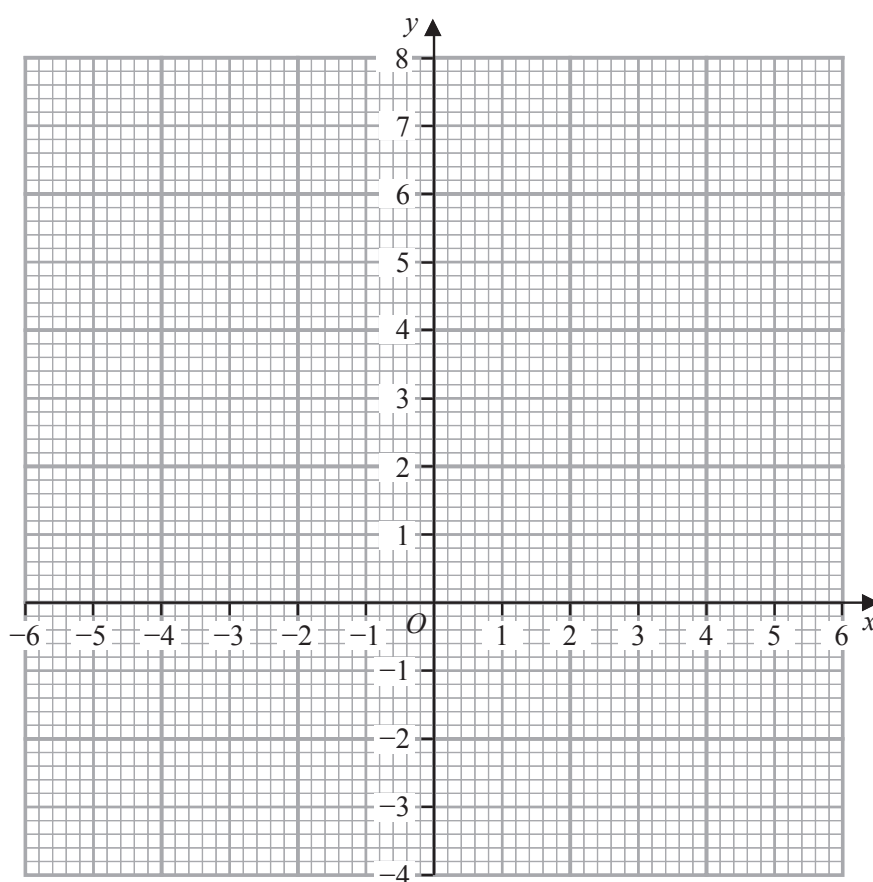
$$P = 2y - 3x$$

(c) Find

(i) the greatest value of P

(ii) the least value of $P \quad (4)$

Question 4 continued

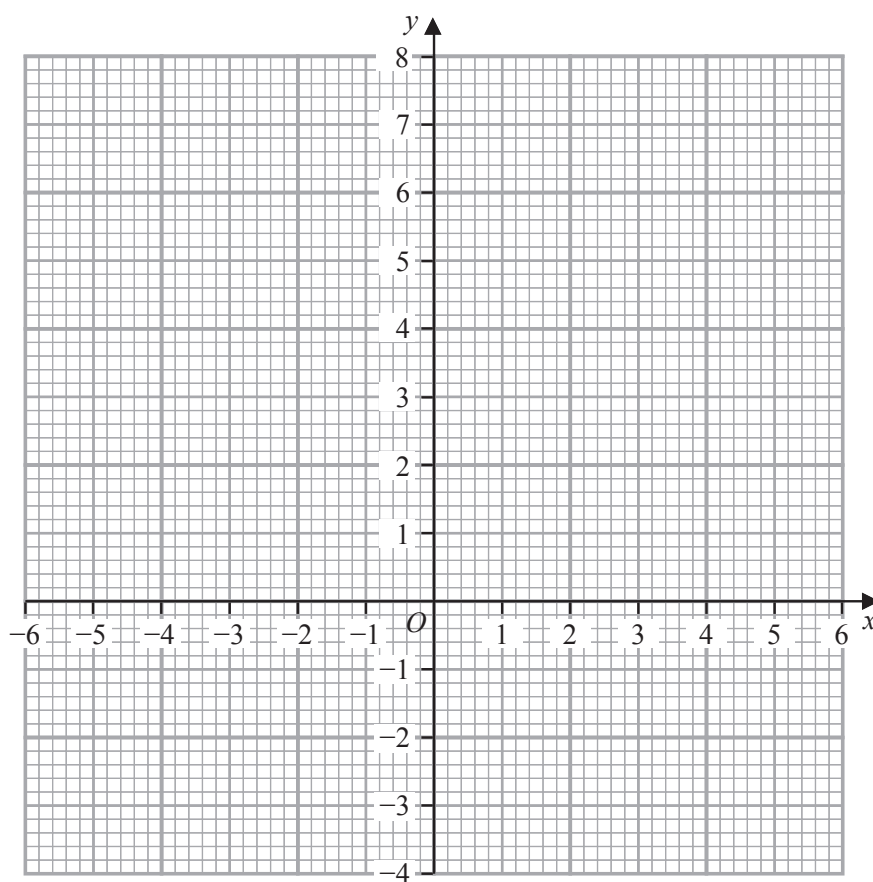


Turn over for a spare grid if you need to redraw your graph.



Question 4 continued

Only use this grid if you need to redraw your graph.



4 (a) On the grid opposite, draw the graph of the line with equation

(i) $3x + 2y = 18$

(ii) $x + 3y - 6 = 0$

(iii) $y = 3x$

(3)

(b) Show, by shading on the grid, the region R defined by the inequalities

$3x + 2y \leq 18$

$x + 3y - 6 \geq 0$

$y \leq 3x$

(1)

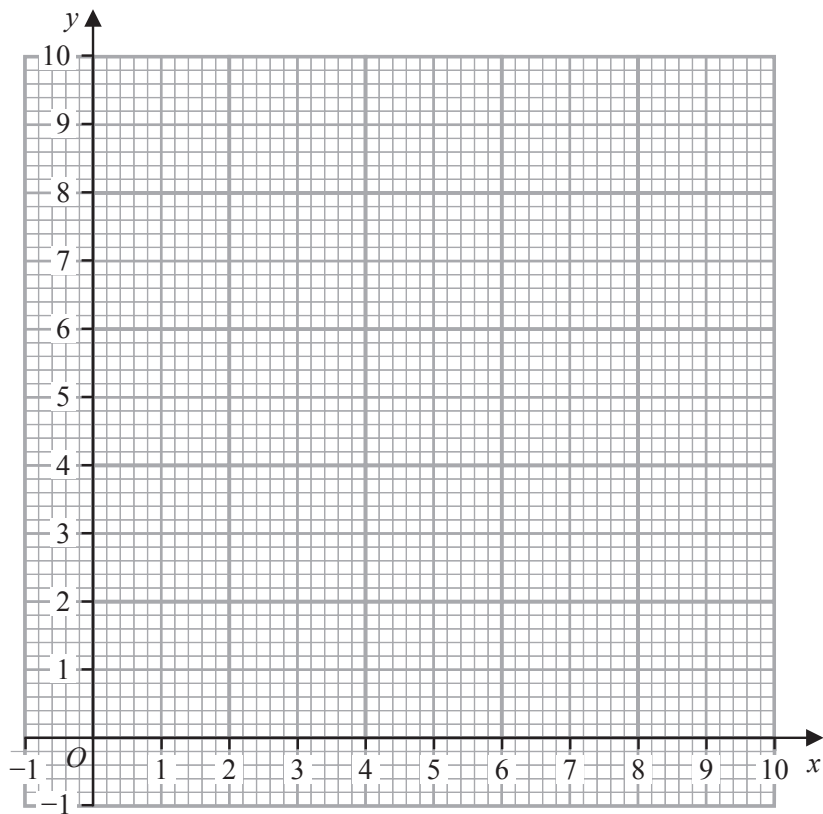
For all points in R , with coordinates (x, y)

$$P = 2x - y$$

(c) find the least value of P

(4)

Question 4 continued



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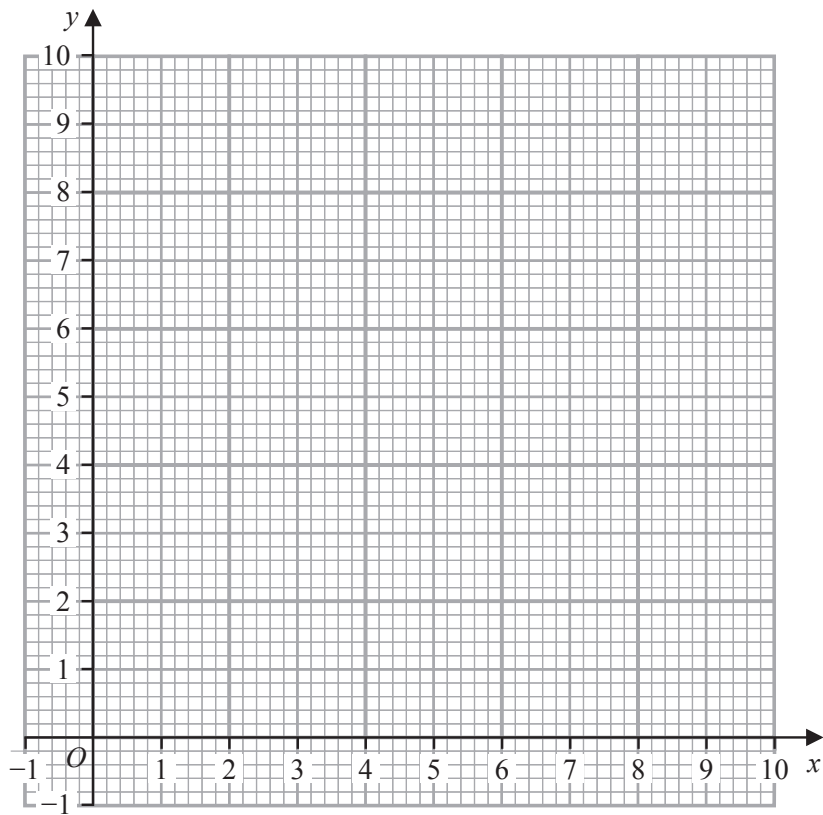
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Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Only use this grid if you need to redraw your graph.



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5

$$f(x) = x^3 + x^2 + x + c \quad \text{where } c \text{ is a constant}$$

The remainder when $f(x)$ is divided by $(x-2)$ is 4 times the remainder when $f(x)$ is divided by $(x+1)$

(a) Show that $c = 6$

(4)

Given that $(x+2)$ is a factor of $f(x)$

(b) show that the equation $f(x) = 0$ has only one real root.

(4)

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9-MARK QUESTIONS

4 questions in this booklet



5

$$f(x) = ax^3 + 5bx^2 + 8ax - 4b \quad \text{where } a \text{ and } b \text{ are integers}$$

Given that

$$(x + 2) \text{ is a factor of } f(x)$$

and that

$$\text{when } f(x) \text{ is divided by } (x + 3) \text{ the remainder is } 21$$

(a) show that $a = 2$ and find the value of b

(5)

(b) Use algebra to solve the equation $f(x) = 0$

(4)

2

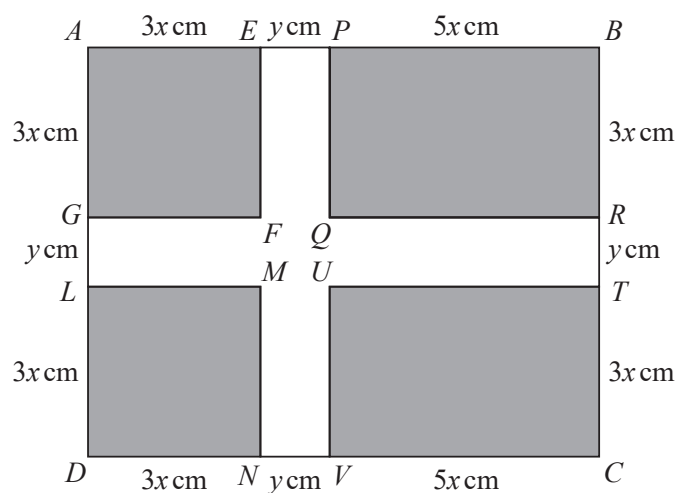


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows the design for a flag consisting of a white cross on a grey background.

$AEFG$ and $DLMN$ are squares with sides of length $3x$ cm.

$BPQR$ and $CTUV$ are rectangles with sides of length $5x$ cm and $3x$ cm.

The width of the cross is y cm.

The total area of the flag is H cm²

(a) Write down an expression, in terms of x and y , for H .

(1)

Given that the area of the cross is K cm²

(b) show that $K = 14xy + y^2$

(3)

The total area of the flag is to be 3432 cm² and the area of the cross is to be 1080 cm²

(c) Find the value of x and the value of y

(5)

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4

$$f(x) = x^3 + px^2 + qx + 6 \quad \text{where } p \text{ and } q \text{ are constants.}$$

Given that $(x - 1)$ is a factor of $f(x)$ and that when $f(x)$ is divided by $(x + 1)$ the remainder is 8

(a) (i) show that $p = -2$

(ii) find the value of q

(6)

(b) Hence, solve the equation $f(x) = 0$

(3)

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8

$$f'(x) = 18x^2 - 2x + 13$$

Given that $(2x - 1)$ is a factor of $f(x)$

show that the curve with equation $y = f(x)$ has only one intersection with the x -axis.

(9)



11-MARK QUESTIONS

3 questions in this booklet



4 $f(x) = 2x^3 + ax^2 + bx + 18$ where a and b are constants

When $f'(x)$ is divided by $(x - 2)$ the remainder is 5

Given that $(x - 2)$ is a factor of $f(x)$

(a) find the value of a and the value of b .

(6)

(b) Express $f(x)$ as a product of linear factors.

(3)

(c) Hence use algebra to solve the equation $f(x) = 0$

(2)

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2 $f(x) = x^3 + px + q$ where p and q are constants.

The remainder when $f(x)$ is divided by $(x - 1)$ is -12

The remainder when $f(x)$ is divided by $(x - 4)$ is 30

(a) Find the value of p and the value of q .

(6)

Using your values of p and q

(b) show that $f(3) = 0$

(1)

(c) Express $f(x)$ as a product of linear factors.

(3)

(d) Hence solve the equation $f(x) = 0$

(1)

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4 $f(x) = px^3 + qx^2 - 37x - 12q$ where p and q are constants.

When $f'(x)$ is divided by $(x + 2)$ the remainder is -33

Given that $(x + 5)$ is a factor of $f(x)$

(a) (i) show that $p = 2$

(ii) find the value of q

(6)

(b) Hence, use algebra to factorise $f(x)$ completely.

(3)

(c) Hence solve the equation $f(x) = 0$

(2)

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12-MARK QUESTIONS

2 questions in this booklet



11

$$f(x) = ax^2 - 14x - 10 \quad \text{where} \quad a \in \mathbb{Z}$$

Given that $(x - 4)$ is a factor of $f(x)$ and that when $f(x)$ is divided by $(x + 1)$ the remainder is 25

(a) show that $a = 6$

(6)

(b) Hence use algebra to solve the equation $f(x) = 0$

(6)

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5 $f(x) = 2ax^3 + x^2 - bx + 3a$ where a and b are integers.

Given that $(x + 2)$ and $(x - 1)$ are both factors of $f(x)$

(a) show that $a = 2$ and find the value of b (5)

(b) Hence factorise $f(x)$ completely. (2)

Hence, given that $h(y) = 2^{(3y+2)} + 2^{2y} - 11(2^y) + 6$

(c) solve the equation $h(y) = 0$

Where appropriate give your answers to 3 decimal places. (5)

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13-MARK QUESTIONS

1 question in this booklet



6

$$f(x) = x^3 + (p + 1)x^2 - 10x + q$$

where p and q are integers.

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that $9p + q + 6 = 0$

(3)

Given that $(x + p)$, where $p > 0$, is also a factor of $f(x)$

(b) show that $p^2 + 10p + q = 0$

(3)

(c) Hence find the value of p and the value of q .

(5)

(d) Using your values of p and q , factorise $f(x)$ completely.

(2)

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16-MARK QUESTIONS

2 questions in this booklet



5

$$f(x) = 2x^3 + ax^2 - 14x + b \text{ where } a \text{ and } b \text{ are constants.}$$

When $f(x)$ is divided by $(x - 4)$ the remainder is 39

Given that $(x - 1)$ is a factor of $f(x)$

(a) show that $a = -3$ and find the value of b

(5)

(b) Hence factorise $f(x)$ completely.

(4)

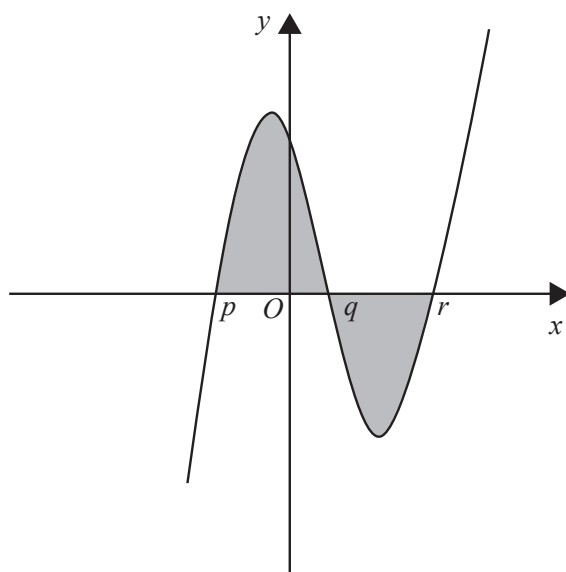


Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows part of the curve C with equation $y = f(x)$

Given that C crosses the x -axis at the points with coordinates $(p, 0)$, $(q, 0)$ and $(r, 0)$

(c) write down the value of p , the value of q and the value of r

(3)

The region shown shaded in Figure 3 is bounded by the curve and the x -axis.

(d) Use algebraic integration to find the exact area of the shaded region.

(4)

- 7 (a) Use the factor theorem to show that $(4x-1)$ is a factor of

$$f(x) = 64x^3 - 64x^2 + 3 \quad (2)$$

- (b) Hence, or otherwise, find the exact roots of the equation

$$f(x) = 0 \quad (4)$$

A geometric series G has first term a and common ratio r
The third term of G is 9 and the sum to infinity of G is 192

- (c) Show that $64r^3 - 64r^2 + 3 = 0$ (3)

Given that r is a rational number

- (d) write down the value of r (1)

- (e) show that $a = 144$ (2)

The sum to n terms of G is S_n

- (f) Using logarithms, find the least value of n such that $S_n > 191.9$ (4)

MARK SCHEMES

official Pearson Edexcel mark schemes



Question	Scheme	Marks
1 (a)	$x^2 - x - 2 = (x - 2)(x + 1)$	B1 [1]
(b)	$f(2) = 2^3 + 2^3 - 10 - 6 = 0$ $f(-1) = -1^3 + 2 \times 1^2 + 5 - 6 = 0$ M1 for either of these, A1 for both correct As $(x - 2)$ and $(x + 1)$ are factors of $f(x)$ then so is $(x^2 - x - 2)$	M1A1 A1 cso
	ALT 1 Divide $f(x)$ by $x^2 - x - 2$ Rem = 0	M1A1
	ALT 2 By inspection: $(x^2 - x - 2)(x + 3) = x^3 + 3x^2 - x^2 + 3x - 2x - 6$	M1A1
	Both Correct work and conclusion stated	A1
	Total 4 marks	

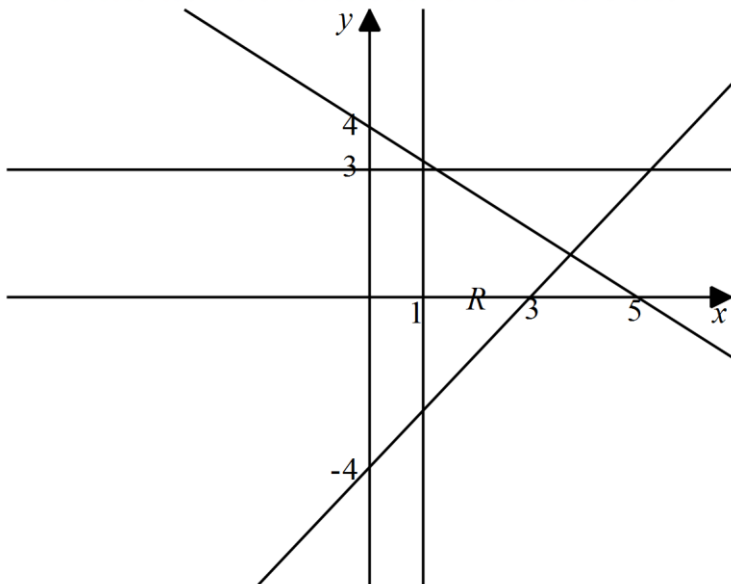
Additional Guidance		
(a)	B1	For the correct factorisation only $(x - 2)(x + 1)$ or $(x + 1)(x - 2)$
(b)	M1	For substituting either $x = 2$ or $x = -1$ into $f(x)$ AND equating = 0
	A1	For substituting both with no errors
	A1	For the correct conclusion. Note this is a show question so there must be no errors in substitution and there must be a conclusion.
	ALT 1	
	M1	An attempt to divide $f(x)$ by $x^2 - x - 2$ A minimally acceptable attempt in the division is to reach a quotient of $(x \pm 3)$
	A1	For the correct division and NO remainder
	A1	For the correct conclusion – there is no remainder hence $x^2 - x - 2$ is a factor of $f(x)$
	ALT 2	
	M1	Uses inspection and must reach $(x^2 - x - 2)(x \pm 3)$
	A1	For the correct factorisation with no errors.
	A1	For the correct conclusion – there is no remainder hence $x^2 - x - 2$ is a factor of $f(x)$

Question number	Scheme	Marks
2 (a)		B1 B1 (2)
(b)		B1 B1 (2) [4]

Part	Mark	Additional Guidance
(a)	B1 (i)	For either correct line drawn The correct intersection on the axis for $5x + 2y = 10$ are $(2, 0)$ and $(0, 5)$ Coordinates for the $y = x$ are $(0, 0)$ $(1, 1)$ $(2, 2)$ $(3, 3)$ etc
	B1 (ii)	For both lines drawn correctly
(b)	B1	For both lines $y = -2$ and $x = 1$ drawn correctly
	B1	The correct region shaded in or out You do not need to see the label R

Question Number	Scheme	Marks
2(a)		B1 B1 B1 (3)
(b)	Correct shading (in or out)	B1 (1) [4]

Part	Mark	Notes												
(a)	B1	For any one correct line from $y = 6$, $y + x = 10$, $y = 2x - 5$ <table border="1"> <thead> <tr> <th>Line</th><th>y intercept</th><th>x intercept</th></tr> </thead> <tbody> <tr> <td>$y = 6$</td><td>6</td><td>No intercept</td></tr> <tr> <td>$y + x = 10$</td><td>10</td><td>10</td></tr> <tr> <td>$y = 2x - 5$</td><td>-5</td><td>2.5</td></tr> </tbody> </table> Accept unambiguous indication on labelled axes. Note: - The line must cross both axes for the award of a mark - Accept an unruled line provided the intention is clear. Look for the intersections on the axes.	Line	y intercept	x intercept	$y = 6$	6	No intercept	$y + x = 10$	10	10	$y = 2x - 5$	-5	2.5
Line	y intercept	x intercept												
$y = 6$	6	No intercept												
$y + x = 10$	10	10												
$y = 2x - 5$	-5	2.5												
	B1	For any two correct lines from $y = 6$, $y + x = 10$, $y = 2x - 5$												
	B1	All three correct lines $y = 6$, $y + x = 10$, $y = 2x - 5$												
	B1	For the correct region shaded in or out. R does not need to be written onto the sketch.												

Question number	Scheme	Marks
1 (a)(i)	$4x + 5y = 20$ drawn	B1 (1)
(a)(ii)	$3y - 4x = -12$ drawn	B1 (1)
(b)	$y = 3$ and $x = 1$ drawn	B1
	Correct region defined (see below) 	B1ft (2)
Total 4 marks		

Part	Mark	Notes
(a) (i)	B1	Correct line
(ii)	B1	Correct line
(b)	B1	Both lines correct
	B1ft	Correctly shaded region ft correct $y = 3$ and $x = 1$, any line drawn with a positive gradient and negative y -intercept and any line drawn with a negative gradient and positive y -intercept where the region shaded is correct for the lines drawn.

Question number	Scheme	Marks
1 (a)(i)	$3x + 4y = 24$ drawn	B1 [1]
(a)(ii)	$2x - 5y + 10 = 0$ drawn	B1 [1]
(b)	$x = -1$ drawn	M1 (B1 on ePen)
	Correct region defined (see below) 	A1ft (B1 on ePen) [2]
Total 4 marks		

Part	Mark	Additional Guidance
(a) (i)	B1	Correct line – check the intersections with the axes, mark intention. The minimum length acceptable is reaching 2 sets of integer coordinates, for this mark: (0, 6) (4, 3) (8, 0)
(ii)	B1	Correct line – check the intersections with the axes, mark intention. The minimum length acceptable is reaching 2 sets of integer coordinates, for this mark: (-5, 0) (0, 2) (5, 4)
(b)	M1 (B1 on ePen)	$x = -1$ drawn ($y = 5$ doesn't need to be drawn) This line can be short, but must extend at least slightly above/below the x -axis. Note: this can be implied by further fully correct work
	A1ft (B1 on ePen)	Correctly shaded region – shaded in or out. Mark intention. The shading must reach below the x -axis, but doesn't need to extend to the bottom or right of grid, where it's unbounded. Accept dotted or solid lines. any ft must have all points such that $x > -1$ and $y < 5$ always. ft any line drawn with a positive gradient and positive y -intercept and any line drawn with a negative gradient and positive y -intercept where the region shaded is correct for their lines drawn. It is not sufficient to label R , with no shading.
Mark intention here means – if the candidate is clearly intending a line going through the correct points of intersection, the mark may be given. A line with lots of 'fuzziness' or which is too wavy or misses the integer coordinates by obviously more than around a quarter of a square (examiners are not expected to measure) cannot be awarded the marks.		

Question number	Scheme	Marks
1 (a)	$x > 3$	B1 (1)
(b)	$(2x-1)(x-5)$ Critical values are $x = \frac{1}{2}$ and $x = 5$	M1 A1
	$\frac{1}{2} < x < 5$	B1ft (3)
(c)	$3 < x < 5$	B1ft (1)
Total 5 marks		

Part	Mark	Notes
(a)	B1	For $x > 3$
(b)	M1	For solving the given 3TQ by any method. See General Guidance.
	A1	For the correct values; $x = \frac{1}{2}$ and $x = 5$
	B1ft	For $\frac{1}{2} < x < 5$ ft their solutions to the 3TQ.
(c)	B1ft	For $3 < x < 5$ ft from parts (a) and (b)

Question number	Scheme	Marks
2(a)	$x < \frac{1}{2}$ o.e	B1 (1)
(b)	$10x^2 + 7x - 12 (< 0)$ $(5x - 4)(2x + 3) (< 0)$ $x = \frac{4}{5} \quad x = -\frac{3}{2}$ $-\frac{3}{2} < x < \frac{4}{5}$ o.e	M1 M1A1 (3)
(c)	$-\frac{3}{2} < x < \frac{1}{2}$	B1FT (1)
Total 5 marks		

Part	Mark	Additional Guidance
(a)	B1	$x < \frac{1}{2}$ o.e
(b)	M1	Rearranges to a 3TQ and attempts to solve. See general guidance for what constitutes an attempt to solve. If they use a calculator to solve the 3TQ, the 3TQ must be correct and the values of x also correct. They must obtain two values of x for this mark
	M1	For forming an inside region with their critical values.
	A1	For the correct inequality. May be seen described in set notation.
(c)	B1FT	For $-\frac{3}{2} < x < \frac{1}{2}$ FT their (a) and (b) provided they overlap and it is an inside region. NB The final answer given must be consistent with their parts (a) and (b)

Question	Scheme	Marks
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Question Number	Scheme	Marks
5	$(x + 2y = 17) \quad x = \frac{36}{y} \quad \left(\text{or } y = \frac{36}{x} \right)$ $\frac{36}{y} + 2y = 17, \quad 36 + 2y^2 = 17y \quad \left(\text{or } 72 + x^2 = 17x \right)$ $2y^2 - 17y + 36 = 0 \quad \left(\text{or } x^2 - 17x + 72 = 0 \right)$ $(y - 4)(2y - 9) = 0 \quad \left(\text{or } (x - 8)(x - 9) \right)$ $y = 4 \quad x = 9$ $y = 4\frac{1}{2} \quad x = 8$	M1 M1 A1 dM1A1 A1 (6)
M1 M1 A1 M1 A1 A1	Rearrange $xy = 36$ to $x = \dots$ or $y = \dots$ Eliminate x or y from the linear equation and obtain a 3TQ, $= 0$ not needed Correct 3TQ, terms in any order. $= 0$ not needed Solve their 3TQ by any valid method. Obtain at least one value for y or x Either 2 correct values for x or y or a correct (x, y) pair Both pairs correct and pairing clear.	
ALT:	The following method may possibly be seen: $xy + x + 2y = 53 \quad \text{or} \quad 36 + x + 2y = 53 \quad \text{or} \quad x + 2y = 17 \text{ and } xy = 36 \text{ or } x \times 2y = 72$ Hence x and $2y$ are the roots of the equation $z^2 - 17z + 72 = 0$ $(z - 9)(z - 8) = 0 \quad \text{or} \quad z = 9 \text{ or } 8$ So $x = 8 \quad y = 4.5$ or $x = 9 \quad y = 4$	M1 M1A1 M1 A1A1 [6]
M1 M1 A1 M1 A1 A1	Substitute $xy = 36$ in the linear equation to obtain $x + 2y = 17$ and $xy = 36$ oe Obtain a 3TQ with roots x and $2y$ Correct 3TQ Solve their 3TQ by any valid method. Obtain at least one value for <i>for the roots</i> Either 2 correct values for x or y or a correct (x, y) pair Both pairs correct and pairing clear.	
	Special Case $x + 2y = 17 \quad xy = 36$ Use $xy = 36$ in the other equation to obtain $x + 2y = 17$ $\Rightarrow x = 9 \quad y = 4$ By inspection: Score M1M0A0M1A1A0 (Must see $x + 2y = 17$; otherwise no marks) If the second answer is also obtained correctly by inspection, award all marks	

Question number	Scheme	Marks
3 (a)	$x = 4$ $64p - 496 + 100 + 12 = 0$ $64p = 384$ $p = 6 *$	M1 A1 (2)
(b)	$(x - 4)(6x^2 - 7x - 3)$ $(x - 4)(2x - 3)(3x + 1)$ $x = 4, \frac{3}{2}, -\frac{1}{3}$	M1 A1 M1 A1 (4) [6]

Part	Mark	Additional Guidance	
(a)	M1	For substituting $x = 4$ into the given expression, equating the expression = 0 and attempting to solve for p	
	A1	For $p = 6 *$ This is a show question so every step must be seen	
(b)	M1	For attempting to divide $6x^3 - 31x^2 + 25x + 12$ by $(x - 4)$ $\begin{array}{r} 6x^2 - 7x + k \\ \Rightarrow x - 4 \overline{) 6x^3 - 31x^2 + 25x + 12} \end{array}$ (k is an integer)	
	A1	For finding the correct 3TQ $6x^2 - 7x - 3$	
	dM1	For an attempt to factorise their 3TQ to give $6x^2 - 7x - 3 = (2x - 3)(3x + 1)$	Condone $\left(x - \frac{3}{2}\right)\left(x + \frac{1}{3}\right)$
	A1	For the correct solution seen: $x = 4, \frac{3}{2}, -\frac{1}{2}$	
	ALT – equates coefficients		
	M1	For stating $6x^3 - 31x^2 + 25x + 12 = (x - 4)(Ax^2 + Bx + C) \Rightarrow$ $6x^3 - 31x^2 + 25x + 12 = Ax^3 + x^2(B - 4A) + x(C - 4B) - 4C$ Minimum required is $A = 6, B = -7$ and $C = k$	
	A1	For $A = 6, B = -7$ and $C = -3$	
	dM1	For an attempt to factorise their 3TQ to give $6x^2 - 7x - 3 = (2x - 3)(3x + 1)$	
	A1	For the correct solution seen: $x = 4, \frac{3}{2}, -\frac{1}{2}$	
	ALT – by inspection		
	M1	For finding the quadratic factor minimum required is $[(x - 4)](6x^2 - 7x + k)$	
	A1	For finding the correct 3TQ $6x^2 - 7x - 3$	
	dM1	For an attempt to factorise their 3TQ to give $6x^2 - 7x - 3 = (2x - 3)(3x + 1)$	
	A1	For the correct solution seen: $x = 4, \frac{3}{2}, -\frac{1}{2}$	
	Evidence of the 3TQ seen is required in part (b) $(x - 4)(2x - 3)(3x + 1) = 0 \Rightarrow x = 4, \frac{3}{2}, -\frac{1}{3}$ is M0		

Question Number	Scheme	Marks
2(a)	$x \leq -1$	B1 (1)
(b)	$8x^2 + 10x - 3 (< 0)$ $(4x - 1)(2x + 3) (< 0)$ $x = \frac{1}{4} \quad x = -\frac{3}{2}$ $-\frac{3}{2} < x < \frac{1}{4}$	M1 A1A1 A1ft (4)
(c)	$-\frac{3}{2} < x \leq -1$	B1 (1)
		[6]
(a) B1	For $x \leq -1$	
(b) NB M1 A1 A1 A1ft	Accept decimals in (b) and (c) The first 3 marks are for finding the critical values. Allow with $<$ or $=$ used. Attempt to obtain the critical values by solving their 3TQ by any valid method. Either CV correct Second CV correct. Award these 2 marks if correct CVs seen in an inequality. Inequality formed to indicate the values between their CVs. Must use $<$ (Can be written in set language).	
NB	If CVs incorrect and only shown in the inequality, award 0/4 if no working shown for solving their 3TQ: if working shown M1A0A0A1 is available.	
(c) B1	For $-\frac{3}{2} < x \leq -1$ (no ft)	

Question number	Scheme	Marks
1 (a)	$3x < 12$ $x < 4$	M1 A1 [2]
(b)	$(2x+1)(x-3) > 0$ Critical values are $x = -\frac{1}{2}$ and $x = 3$	M1 M1
	$x < -\frac{1}{2}$ $x > 3$	A1 [3]
(c)	$x < -\frac{1}{2}$ $3 < x < 4$	B1ft [1]
Total 6 marks		

Part	Mark	Guidance
(a)	M1	Attempts to solve the inequality to achieve $3x < 12$ Allow $3x < a$ where a is an integer
	A1	For $x < 4$
(b)	M1	Attempts to solve the inequality by any method to find critical values See General Guidance for acceptable methods. If a calculator is used, the critical values must be fully correct for this mark. Allow $=$ or $>$ for this mark or even no sign at all provided it is clear they are solving a quadratic. $(2x+1)(x-3) > 0 \Rightarrow x = \dots, \dots \left(x = -\frac{1}{2}, 3 \right)$
	M1	For forming a correct inequality, which must be an open interval, following through their two critical values which must have come from the solution of a 3TQ. $x < -\frac{1}{2} \quad x > 3$ Accept any correct notation. E.g., $x < -\frac{1}{2}$ or $x > 3$ Or $\left\{ x : x < -\frac{1}{2} \right\} \cup \{ x : x > 3 \}$ Condone $x < -\frac{1}{2}$ and $x > 3$ for this mark only
	A1	For the correct inequality with the correct critical values using any acceptable notation. Eg, $x < -\frac{1}{2} \quad x > 3$
(c)	B1ft	For $x < -\frac{1}{2} \quad 3 < x < 4$ ft their answers from parts (a) and (b) provided (b) is of the form $x < p$ and/or $x > q$ Note: If you have already penalised them for writing $x < -\frac{1}{2}$ and $x > 3$ in part (b) then allow $x < -\frac{1}{2}$ and $3 < x < 4$ for this mark.

Question	Scheme	Marks
2(a)	$3x - 8 < 5x + 3 \Rightarrow x > -\frac{11}{2}$	B1 [1]
(b)	$4x^2 - 7x + 1 > 6 - 2x^2 \Rightarrow$ $6x^2 - 7x - 5 > 0 \Rightarrow (3x - 5)(2x + 1) > 0$ cv's are $x = \frac{5}{3}, -\frac{1}{2} \Rightarrow x < -\frac{1}{2}, x > \frac{5}{3}$	M1M1 M1A1 [4]
(c)	$-\frac{11}{2} < x < -\frac{1}{2}, x > \frac{5}{3}$	B1 ft [1]
Total		6 marks

Question	Notes	Marks
2(a)	$3x - 8 < 5x + 3$ $\Rightarrow x > -\frac{11}{2}$	B1 [1]
(b)	For rearranging to form a 3TQ $4x^2 - 7x + 1 > 6 - 2x^2 \Rightarrow 6x^2 - 7x - 5 > 0$	M1
	For method to solve their 3TQ. See general guidance for what constitutes an attempt to solve. $6x^2 - 7x - 5 > 0 \Rightarrow (3x - 5)(2x + 1) > 0$	M1
	For writing down their critical values from their factorisation and the correct inequality for their cv's [outside region]. cv's are $x = \frac{5}{3}, -\frac{1}{2}$ $x < -\frac{1}{2}, x > \frac{5}{3}$	M1
	For the correct inequalities only $x < -\frac{1}{2}, x > \frac{5}{3}$ A0 if rejects one region or if inequalities are incorrectly combined.	A1 [4]
(c)	For the correct inequality only $-\frac{11}{2} < x < -\frac{1}{2}, x > \frac{5}{3}$ For follow through (a) must be a linear inequality and (b) must be of the form $x < a, x > b$ with $b > a$.	B1 ft [1]
Total		6 marks

Question	Scheme	Marks
2(a)	$2(x+1) < 5x-2 \Rightarrow 2x+2 < 5x-2 \Rightarrow 3x > 4 \Rightarrow x > \frac{4}{3}$	M1A1 [2]
(b)	$3x^2 - x - 10 = 0 \Rightarrow (3x+5)(x-2) = 0 \Rightarrow x = -\frac{5}{3}, 2$ $-\frac{5}{3} < x < 2$ allow $-\frac{5}{3} \leq x \leq 2$ $-\frac{5}{3} \leq x \leq 2$	M1 dM1 A1 [3]
(c)	$-\frac{4}{3} < x \leq 2$	B1ft [1]
Total 6 marks		

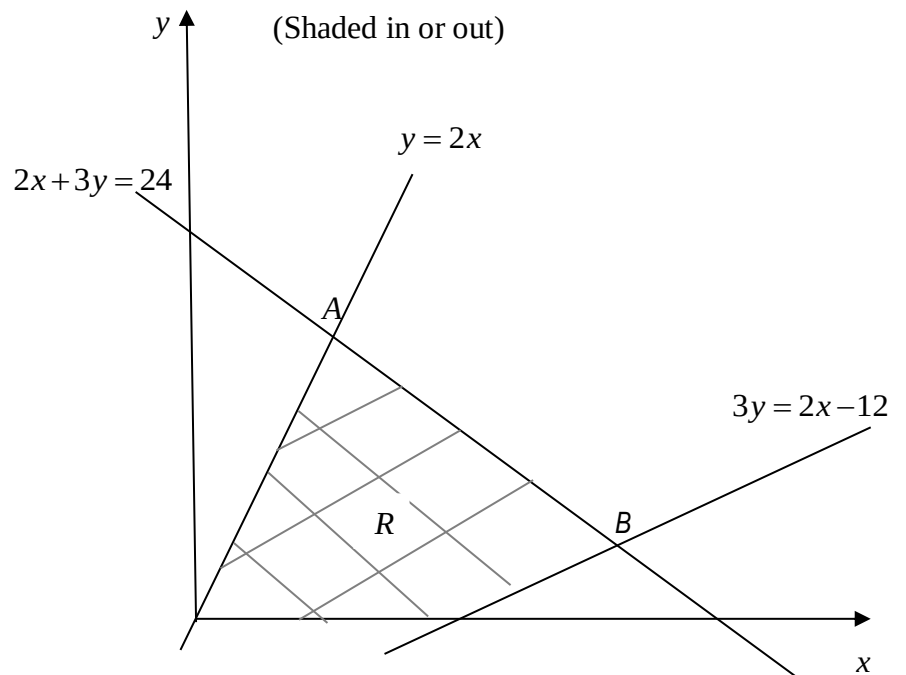
Part	Mark	Notes
(a)	M1	Attempts to solve the inequality with no more than one arithmetical error.
	A1	For the correct inequality.
(b)	M1	For attempting to solve the QE to find two critical values.
		For the dM and A marks allow any acceptable notation. For example; $-\frac{5}{3} \leq x \cap x \leq 2$ or, $-\frac{5}{3} \leq x$ and $x \leq 2$ The region must however indicate an INSIDE region
	dM1	Forms an inside region with their cv's This mark is dependent on the first M mark in (b)
	A1	For the correct region with the correct inequalities.
(c)	B1ft	For the correct combined inside region. Penalise the incorrect inequality from part (b) $<$ in place of $\leq$ only once. Even if the correct inequality does not follow from their work, award this mark.

Part	Mark	Notes
(a)	M1	Substitutes $\pm \frac{2}{3}$ into $f(x)$ and sets $= 0$
	A1	For using $x = \frac{2}{3}$ with no errors seen.
	ALT 1 – accepting a substitution	
	M1	Substitutes $\pm \frac{2}{3}$ into $f(x)$, $a = 21$
	A1	Shows that $f(x) = 0$
	ALT 2 – Polynomial division	
	M1	For obtaining as a minimum $a - k = 15$
	A1	For obtaining $a = 21$ with no errors seen
(b)	Polynomial division	
	M1	Uses polynomial division to achieve as a minimum $2x^2 \pm 3x \pm k$ where k is a constant NB Award this seen in part (a) if they use polynomial division
	A1	For the correct quadratic factor $2x^2 - 3x + 5$
	OR Equates coefficients	
	M1	Achieves the correct values for a and b
	A1	For the correct quadratic factor $2x^2 - 3x + 5$
	Uses the discriminant	
	M1	Finds the value of the discriminant using their QE Allow the discriminant to be embedded as part of the Quadratic formula. If they do not achieve the correct 3TQ – please check their discriminant carefully. NB: It must be evaluated,
	A1	For a value of -31 for the discriminant with a conclusion. Minimally acceptable conclusion: - Discriminant < 0 hence no real roots. - The only intersection [root/solution] is at $x = \frac{2}{3}$
	Completes square	
	M1	Completes the square on their 3TQ as far as: $\left(x - \frac{3}{4}\right)^2 = -\frac{31}{16}$
	A1	For a value of $-\frac{31}{16}$ with a conclusion $-\frac{31}{16} < 0$ so the quadratic has no real roots - The only intersection [root/solution] with the x-axis is at $x = \frac{2}{3}$
	Complex roots	
	The question states clearly ‘Show Algebraically’ so just quoting the complex roots is M0. If they go through the process of finding them fully, please send to REVIEW	



Question number	Scheme	Marks
2	$w(w+2)$ or $w+w+2+w+w+2$ oe	B1
	$w^2+2w>8$ or $4w+4<30$ oe	B1
	$w<\frac{13}{2}$ oe	B1
	$w^2+2w-8\{=0\}\Rightarrow(w-2)(w+4)\{=0\}\Rightarrow w=...$	M1
	cv: $w=2$	A1
	$2<w<\frac{13}{2}$ oe	B1ft
		[6]
	ALT Works in terms of the length where $l=w+2$ or $w=l-2$	
	$l(l-2)$ or $l+l-2+l+l-2$ oe	B1
	$l^2-2l>8$ or $4l-4<30$ oe	B1
	$\left(l<\frac{34}{4}\right)\Rightarrow w<\frac{13}{2}$ oe	B1
	$l^2-2l-8\{=0\}\Rightarrow(l-4)(l+2)\{=0\}\Rightarrow l=...$	M1
	$l=4$	A1
	$\left(4<l<\frac{17}{2}\right)\Rightarrow 2<w<\frac{13}{2}$ o.e. in terms of w only.	B1ft
		[6]
Total 6 marks		

Mark	Additional Guidance
	Allow working in other variables, such as x for all marks except the final A mark
B1	For either correct expression shown.
B1	For either correct inequality shown (can be implied by a fully correct answer).
B1	For $w < \frac{13}{2}$, allow $=$ or any inequality
M1	Correct method to solve their 3TQ and finds at least one critical value of w (see general guidance for minimally acceptable attempt to solve).
A1 (M1 on ePen)	The correct positive cv solution ($w = 2$ or $w > 2$ or $w < 2$) from a correct 3TQ inequality or 3TQ equation, could be implied by their correct final answer, ignore the negative critical value even if it is incorrect.
B1ft (A1 on ePen)	Correctly combine their solutions from their linear and quadratic inequalities. Follows through their $\frac{13}{2}$ with their 2, both w values must be positive. Must be in terms of w .
ALT works in terms of l	
B1	For either correct expression shown
B1	For either correct inequality shown (can be implied by a fully correct answer)
B1	For $\frac{13}{2}$ in terms of w only UNLESS the final inequality is given as $2 < w < \frac{13}{2}$ in which case score B1 here.
M1	Correct method to solve their 3TQ equation or inequality and finds at least one critical value of w (see general guidance for minimally acceptable attempt to solve).
A1 (M1 on ePen)	For a value of $l = 4$ coming from a correct 3TQ Ignore the negative value even if it is incorrect.
B1ft (A1 on ePen)	Correctly combine their solutions from their linear and quadratic inequalities. Follows through their $\frac{13}{2}$ with their 2, both w values must be positive. Must be in terms of w

Question number	Scheme	Marks
5 (a)	$2x + 3y = 24$, $y = 2x$, $3y = 2x - 12$	M1 A1A1 [3]
(b)		B1 [1]
(c)	Greatest value of F is at A , intersection of lines $y = 2x$ and $2x + 3y = 24$ Greatest value of F is at $(3, 6)$ Greatest value of $F = 2 \times 3 + 5 \times 6 = 36$	M1 A1 A1ft [3]
Total 7 marks		

(a)	M1	Attempt to find two points on any one of the straight lines. Look for two correct points (stated, plotted or implied by a correct line), or for any incorrect point from working which shows an appropriate substitution of one variable with an attempt to solve for the second variable.
	A1	For any two correct lines.
	A1	For three correct lines.
(b)	B1	Correct region, identified by shading in or outside the region. Label R is not required but it needs to be correct if it is shown.
(c)	M1	Identify A as the point with coordinates that give the greatest value of F , or find the value of F at A and B , ft their graph, or draw line $2x + 5y = k$ for at least one value of k .
	A1	Correct coordinates $(3, 6)$ for the point identified to give the greatest value of F . May be read from graph or implied by working.
	A1ft	$(F =) 36$ ft their coordinates for A . Note that this mark is shown as B1 on ePEN.

Question number	Scheme	Marks
4 (a) (i)	$4x + 8$ oe e.g. accept $x + (x + 4) + x + (x + 4)$	B1
(ii)	$x(x + 4)$ or $x^2 + 4x$	B1 [2]
(b)	$4x + 8 < 30$ or $x^2 + 4x > 12$ $x < 5.5$ oe $(x^2 + 4x - 12 > 0)$ $(x + 6)(x - 2)(> 0)$ $x = 2$ [$x = -6$] $2 < x < 5.5$	M1 B1 M1 A1 B1ft [5]

Part	Mark	Additional Guidance
(b)	Note: Do not accept $\leq$ or $\geq$ in place of $<$ and $>$	
	M1	Either their linear expression < 30 (seen or implied in later work) or their quadratic expression > 12 (seen or implied in later work).
	B1	$x < 5.5$ oe allow implied M1 if $(4x + 8)$ not seen in earlier work. Allow $x = 5.5$ for this mark.
	M1	A minimally acceptable (see general guidance) attempt to factorise, use the formula or complete the square to solve their 3TQ.
	A1	For the correct critical value $x = 2$. Ignore $x = -6$ for this mark. This may be given as an inequality. Award the mark for 2 seen even if it is embedded in a spurious inequality.
	B1ft	ft their values for x , provided the inequality is written correctly and is correct for their values. Must be in the form $a < x < b$ Both values of x must be positive. Their value of '2' must be less than their value from the linear solution. i.e $<$ their 5.5



Question number	Scheme	Marks
1 a	$9x < 6 \Rightarrow x < \dots$ $x < \frac{2}{3}$	M1 A1 (2)
b	$(3x+1)(x-3) < 0$ $x = -\frac{1}{3} \quad x = 3$ $-\frac{1}{3} < x < 3$	M1 A1 M1 A1 (4)
c	$-\frac{1}{3} < x < \frac{2}{3}$	B1ft (1)
Total 7 marks		

Part	Mark	Notes
(a)	M1	For a complete method to find a value for x They must obtain a value for x with at most one processing error. The inequality must be correct in this part of the question.
	A1	For $x < \frac{2}{3}$ Accept awrt 0.67
(b)	M1	For attempting to factorise or otherwise solve the given quadratic using any method. If there is no method, [use of a calculator] then both roots must be fully correct for evidence of this mark. See general guidance for the definition of an attempt. Accept $<$, $>$, $=$ or even no sign at all for this mark.
	A1	For both correct critical values. $x = -\frac{1}{3} \quad x = 3$ Accept awrt -0.33
	M1	For a correct inside region using their values ' $-\frac{1}{3}$ ' $< x < '3'$
	A1	For $-\frac{1}{3} < x < 3$
(c)	B1ft	For ' $-\frac{1}{3}$ ' $< x < '\frac{2}{3}'$ Ft their values from parts (a) and (b), providing they are inequalities. Do not follow through an equals sign given in part (a). Allow recovery for a fully correct answer seen.

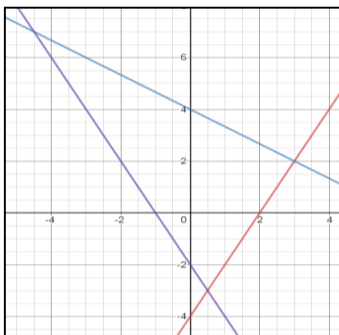
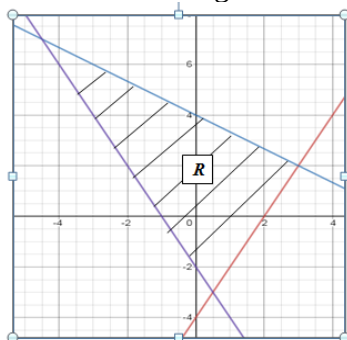
Question	Scheme	Marks																
5 (a)		B3																
(b)		B1																
(c)	<table><tr><td></td><td>A</td><td>B</td><td>C</td></tr><tr><td>x</td><td>9.4</td><td>-4</td><td>0.4</td></tr><tr><td>y</td><td>0.4</td><td>-3</td><td>5.8</td></tr><tr><td>P</td><td>$2 \times 9.4 - 5 \times 0.4$ $= 16.8$</td><td>$2 \times -4 - 5 \times -3$ $= 7$</td><td>$2 \times 0.4 - 5 \times 5.8$ $= -28.2$ [Exact value is] $\left[-\frac{365}{13} \approx -28.1 \right]$</td></tr></table> Allow ± 0.1 on the coordinates in each case.		A	B	C	x	9.4	-4	0.4	y	0.4	-3	5.8	P	$2 \times 9.4 - 5 \times 0.4$ $= 16.8$	$2 \times -4 - 5 \times -3$ $= 7$	$2 \times 0.4 - 5 \times 5.8$ $= -28.2$ [Exact value is] $\left[-\frac{365}{13} \approx -28.1 \right]$	M1 M1A1
	A	B	C															
x	9.4	-4	0.4															
y	0.4	-3	5.8															
P	$2 \times 9.4 - 5 \times 0.4$ $= 16.8$	$2 \times -4 - 5 \times -3$ $= 7$	$2 \times 0.4 - 5 \times 5.8$ $= -28.2$ [Exact value is] $\left[-\frac{365}{13} \approx -28.1 \right]$															
Total 7 marks																		

Part	Mark	Notes
The tolerance for all marks in this question is $\pm$ half a small square.		
(a)	B1	For one of the lines correctly drawn to within tolerance (as a minimum, examiners should check intersections with axes, candidates do not need to mark these).
	B1	For two of the lines drawn to within tolerance (as a minimum, examiners should check intersections with axes, candidates do not need to mark these).
	B1	For all three lines correctly drawn to within tolerance (as a minimum, examiners should check intersections with axes, candidates do not need to mark these).
As a minimum, lines must intersect with other for marks in (a) and (b) to be awarded		
(b)	B1ft	For the correct enclosed region shaded in or out or for R clearly labelled. The ft mark can only be awarded if 3 distinct lines have been drawn and it's clear they've shaded on the correct 'side' for each of their lines. If there's no labelling and it's not clear which line is which, this mark cannot be awarded.
Part c of this question states "using your graph"..... Therefore solutions which obviously use exact coordinates of intersection points having used a graphical calculator or from working algebraically can only score M0 M1 A0.		
(c)	M1	For reading from the graph at least one point of intersection using their lines. The pair used for this and the next method mark must be within the tolerance of ± 0.1 of the values shown in the table. Any solutions which work out the values algebraically will not gain this mark or the final accuracy mark, but may gain the next method mark. Occasionally, students are working out the non-integer coordinates from algebra or from a calculator but reading $(-4, -3)$ from the graph and using this to find the value of P . In this case, we can apply bod (benefit of the doubt) and this mark can be awarded if subbed in to find the value of P .
	M1	For a correct substitution to find the value of P from at least one set of their coordinates of the point of intersection. This is not a dependent mark, so they can use any one of their pairs of values even if it doesn't fall in tolerance.
	A1	For the correct least value of -28.2 Allow a value between -28.9 and -27.5 so long as this follows through from their values. Do not allow a value out of range to be rounded to a value within range.



Question number	Scheme	Marks
5	$(P \Rightarrow) 6x + 6$ $(A \Rightarrow) x(2x + 3)$ $6x + 6 > 10$ or $x(2x + 3) < 35$ $x > \frac{2}{3}$ oe $(2x^2 + 3x - 35 < 0)$ $(2x - 7)(x + 5)(< 0)$ (critical values are) $x = \frac{7}{2}, (x = -5)$ $\frac{2}{3} < x < \frac{7}{2}$	B1 B1 M1 A1 M1 A1 A1ft [7]
Total 7 marks		

Part	Mark	Additional Guidance
5 (a)	B1	For either correct expression.
	B1	For both correct expressions.
	M1	Either their linear expression > 10 (seen or implied in later work) or their quadratic expression < 35 (seen or implied in later work).
	A1	For the correct linear inequality.
	M1	For forming a $3\text{TQ} = 0$ or $3\text{TQ} < 0$ and for a minimally acceptable attempt at its solution leading to two values of x
	A1	For the value of $\frac{7}{2}$, and no other positive value.
	A1ft	For the answer shown, allow ft marks, so long as they have come from solution of a linear and 3 term quadratic. If the linear solution is out of range and cannot be incorporated into the final answer this is A0

Question	Scheme	Marks															
4 (a)	$y = 2x - 4$ Intersections with axes at $(0, -4)$ $(2, 0)$ $2x + 3y = 12$ Intersections with y axes at $(0, 4)$ and. $(6, 0)$ $y + 2x + 2 = 0$ Intersections with y axes at $(0, -2)$ and. $(-1, 0)$ 	B1 B1 B1 [3]															
(b)	For the correct region shaded in or out 	B1ft [1]															
(c)	Points of intersection are: <table border="1" data-bbox="290 1274 1005 1328"><tr><td>$(0.5, -3)$</td><td>$(3, 2)$</td><td>$(-4.5, 7)$</td></tr></table> <table border="1" data-bbox="290 1361 1232 1496"><tr><td>Vertex</td><td>$(0.5, -3)$</td><td>$(3, 2)$</td><td>$(-4.5, 7)$</td></tr><tr><td>$P = x - 2y$</td><td>6.5</td><td>-1</td><td>-18.5</td></tr><tr><td></td><td></td><td></td><td>Least</td></tr></table> For $P = -18.5$	$(0.5, -3)$	$(3, 2)$	$(-4.5, 7)$	Vertex	$(0.5, -3)$	$(3, 2)$	$(-4.5, 7)$	$P = x - 2y$	6.5	-1	-18.5				Least	M1 A1 dM1 A1 [4]
$(0.5, -3)$	$(3, 2)$	$(-4.5, 7)$															
Vertex	$(0.5, -3)$	$(3, 2)$	$(-4.5, 7)$														
$P = x - 2y$	6.5	-1	-18.5														
			Least														
ALT- objective line approach																	

	Slope of objective line is $\frac{1}{2}$ $\left(-\frac{9}{2}, 7\right)$ $P = -\frac{9}{2} - 2(7)$ For $P = -18.5$	[M1 A1 M1 A1]
Total 8 marks		

Part	Mark	Notes
(a)	B1B1B1	B1 for each line drawn correctly $y = 2x - 4$ Intersections with axes at $(0, -4)$ $(2, 0)$ $2x + 3y = 12$ Intersections with y axes at $(0, 4)$ and $(6, 0)$ $y + 2x + 2 = 0$ Intersections with y axes at $(0, -2)$ and $(-1, 0)$ Minimum length of line is 4 units horizontally or 4 units vertically.
(b)	B1ft	For the correct region marked - allow shaded in or out Ft for shading the closed region from their lines. Must be a closed region.
(c)	M1	For attempting to find the correct coordinates of at least one intersection either by reading values from their graphs or by solving simultaneous equations. If they are solving simultaneous equations, they must find a value for x and a corresponding value for y
	A1	For at least one correct point of intersection
	dM1	For substituting one point of intersection into the given P ft their coordinates
	A1	For identifying $P = -18.5$
ALT – objective line approach		
	M1	For attempt to use objective line approach. Identifies that the slope of objective line is $\frac{1}{2}$ Identifies the intersection of $2x + 3y = 12$ and $y + 2x + 2 = 0$ as the point where P is least.
	A1	For finding the correct coordinates $\left(-\frac{9}{2}, 7\right)$
	M1	For substituting their $\left(-\frac{9}{2}, 7\right)$ into P .
	A1	For $P = -18.5$

Question number	Scheme	Marks
4 (a)	$f(\pm 1)=0$ or $f(\pm 2)=-5$ $-1 + 1p + -1q + 7 = 0$ ($p - q + 6 = 0$) and $-8 + 4p + -2q + 7 = -5$ ($4p - 2q - 1 = -5$) $4(q - 6) - 2q = -4$ ($2q = 20$) $p = 4$ $q = 10$ ALT – polynomial division $(x^3 + px^2 + qx + 7) \div (x + 1) = x^2 + (p - 1)x + q - p + 1$ and comparison of final step of division with 7 to obtain an equation or $(x^3 + px^2 + qx + 7) \div (x + 2) =$ $x^2 + (p - 2)x + (q - 2p + 4)$ remainder -5 and comparison of final step of division with obtaining remainder -5 to obtain an equation $(x^3 + px^2 + qx + 7) \div (x + 1) = x^2 + (p - 1)x + q - p + 1$ and comparison of final step of division with 7 to identify $q - p + 1 = 7$ and $(x^3 + px^2 + qx + 7) \div (x + 2) =$ $x^2 + (p - 2)x + (q - 2p + 4)$ remainder -5 and comparison of final step of division with obtaining remainder -5 to identify $7 - 2(q - 2p + 4) = -5$ $(p + 6) - 2p = 2$ ($-p = -4$) $p = 4$ $q = 10$	M1 A1 dM1 A1 A1 [5] M1 A1 dM1 A1 A1
(b)	$\begin{array}{r} x^2 \quad (+3x + 7) \\ x+1 \overline{) x^3 + 4x^2 + 10x + 7} \\ \underline{x^3 + x^2} \\ 3x^2 \end{array}$ or $x^3 + 4x^2 + 10x + 7 \equiv (x + 1)(x^2 + Ax + B)$ $3^2 - 4(1)(7) = \text{a value}$ -19 and a conclusion drawn e.g. the discriminant is negative so only one real root ALT – use of completing the square $\begin{array}{r} x^2 \quad (+3x + 7) \\ x+1 \overline{) x^3 + 4x^2 + 10x + 7} \\ \underline{x^3 + x^2} \\ 3x^2 \end{array}$ or $x^3 + 4x^2 + 10x + 7 \equiv (x + 1)(x^2 + Ax + B)$	M1 dM1 A1 [3] M1

	$\left(x + \frac{3}{2}\right)^2 + \frac{19}{4} > 0$	dM1
	$\left(x + \frac{3}{2}\right)^2 + \frac{19}{4} > 0$ and a conclusion drawn e.g. the completed square form is always greater than 0 so only one real root	A1
Total 8 marks		

(a)	M1	Correct substitution of ± 2 or ± 1 into $f(x)$ and equating to appropriate value.
	A1	Two fully correct equations as shown, powers of -1 and -2 evaluated, need not be simplified Missing brackets must be recovered for the award of this mark.
	dM1	An attempt to solve algebraically. Minimum attempt is to correctly substitute an expression for p or q into the other equation and attempt to solve or to multiply each equation correctly to make the coefficients $\pm p$ or $\pm q$ the same in each equation and attempt to add or subtract the equations (consistent appropriate operation).
	A1	Correct p
	A1	Correct q
ALT – polynomial division		
	M1	Correct method for polynomial division together with comparison of the final step of the division with the required result (no remainder for division by $(x + 1)$ and remainder -5 for division by $(x + 2)$)
	A1	Two fully correct equations as shown, need not be simplified
	dM1	An attempt to solve algebraically. Minimum attempt is to correctly substitute an expression for p or q into the other equation and attempt to rearrange or to multiply each equation correctly to make the coefficients $\pm p$ or $\pm q$ the same in each equation and attempt to add or subtract the equations.
	A1	Correct p
	A1	Correct q
(b)	M1	Attempts long division. Minimally acceptable attempt is the division and correct working as written in the MS as shown for their p and their q . Or if comparing coefficients, a correct equation/comparison (for their p and their q) must be written followed by an attempt to find A or B . Must get a 3TQ.
	dM1	Correctly substitutes 1, their A and their B into the expression for the discriminant and reaches a value.
	A1	Correct value of -19 (or for stating $9 - 28 < 0$) from correct quadratic and draws a conclusion. Allow for showing that there are complex roots for the quadratic together with appropriate conclusion. There must be some conclusion drawn, but it can be as simple as writing ‘#’ or “shown”.
ALT – completing the square		
	M1	Attempts long division. Minimally acceptable attempt is the division and correct working as written in the MS as shown for their p and their q . Or if comparing coefficients, a correct equation/comparison (for their p and their q) must be written followed by an attempt to find A or B . Must get a 3TQ.

	dM1	Attempt at completing the square See general guidance for what constitutes an attempt at completing the square
	A1	Correct completed square form and draws a conclusion. There must be some conclusion drawn, but it can be as simple as writing ‘#’ or “shown”.

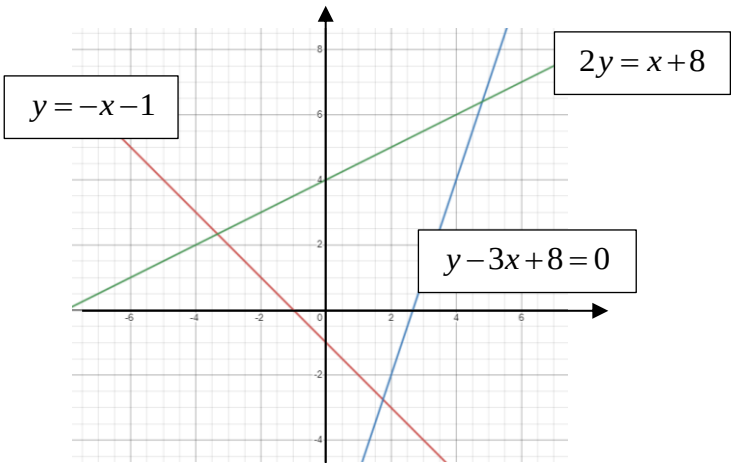
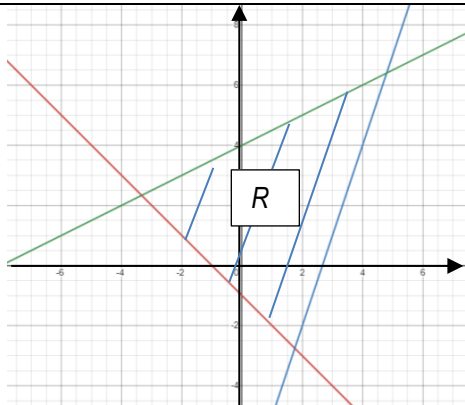


Question	Scheme	Marks
2(a)	$6 \times \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)q = 5 \Rightarrow \frac{3}{2} + \frac{q}{2} = 5 \Rightarrow q = 7$	M1A1
	$y = px + 9 \Rightarrow 7 = p \times -\frac{1}{2} + 9 \Rightarrow p = 4$	M1A1 [4]
	ALT	
	$6x^2 - x(px + 9) = 5 \Rightarrow \frac{3}{2} - \frac{p}{4} + \frac{9}{2} - 5 = 0 \Rightarrow p = 4$	[M1A1
	$y = px + 9 \Rightarrow q = 4 \times -\frac{1}{2} + 9 \Rightarrow q = 7$	M1A1]
(b)	ALT 2	
	$6x^2 - x(px + 9) - 5 = 0 \text{ and}$ $(2x+1)(ax+b) = 0 \Rightarrow 2ax^2 + x(2b+a) + b = 0$	
	$6 - p = 2a \text{ and } b = -5$ $-9 = 2b + a \Rightarrow -9 = 2 \times -5 + a \Rightarrow a = 1$ $\therefore 6 - p = 2 \Rightarrow p = 4$	[M1A1
	$q = 4 \times -\frac{1}{2} + 9 = 7$	M1A1]
(b)	$6x^2 - x(4x+9) = 5 \Rightarrow 6x^2 - 4x^2 - 9x - 5 = 0 \Rightarrow 2x^2 - 9x - 5 = 0$	M1
	$2x^2 - 9x - 5 = 0 \Rightarrow (2x+1)(x-5) = 0 \Rightarrow x = 5$	M1A1
	$y = 4 \times 5 + 9 = 29 \Rightarrow (5, 29)$	A1 [4]
Total 8 marks		

Part	Mark	Notes
(a)	M1	Substitutes $\left(-\frac{1}{2}\right)$ into the quadratic to find the value of q (or y)
	A1	For $q = 7$
	M1	For using the linear equation with $y =$ their q and $x = -\frac{1}{2}$ to find the value of p
	A1	For $p = 4$
	ALT	
	M1	Substitutes the linear expression for y and $\left(-\frac{1}{2}\right)$ into the quadratic to find the value of p
	A1	For $p = 4$
	M1	For using the linear equation with their p and $x = -\frac{1}{2}$ to find the value of q
	A1	For $q = 7$
	ALT 2	
	M1A1	For the complete method which must be shown: - Substituting the linear expression for y into the quadratic - Writing $(2x + 1)(ax + b) = 0$ and expanding - Comparing coefficients for form equations in a, b and p - Solving simultaneous equations to find a, b and p
	M1	For using the linear equation with their p and $x = -\frac{1}{2}$ to find the value of q
	A1	For $q = 7$
(b)	M1	For substituting the linear equation (with their value of p) into the quadratic equation to form a 3TQ in x
	M1	For solving their 3TQ, must be a 3TQ. Independent of the first M. See general guidance on what constitutes an attempt to solve a 3TQ.
	A1	For $x = 5$
	A1	For the other solution to the equations. Does not have to be given as coordinates.
	M1M1A1 may be awarded for correct work in (a) if ALT2 was used, then award the final A mark as per scheme.	

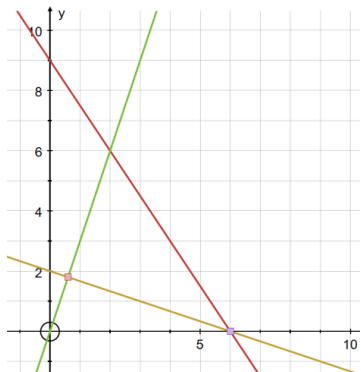
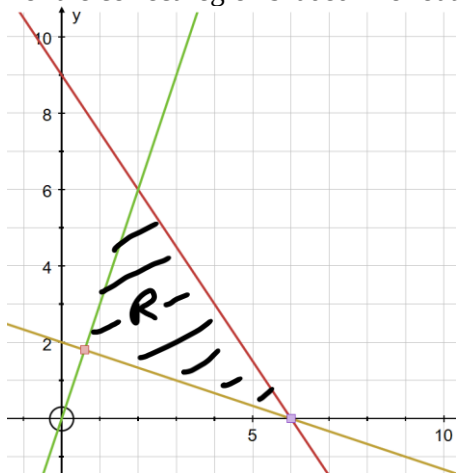
Question	Scheme	Marks
3(a)	$\left(y = \int (mx^2 - 10x - 37) dx = \frac{mx^3}{3} - \frac{10x^2}{2} - 37x + c\right)$ $[\text{At } (1, 20)]$ $20 = \frac{m \times 1^3}{3} - \frac{10 \times 1^2}{2} - 37 \times 1 + c \left(\Rightarrow 62 = \frac{m}{3} + c \right) \text{ oe}$ $[\text{Factor of } (x - 5)]$ $0 = \frac{m \times 5^3}{3} - \frac{10 \times 5^2}{2} - 37 \times 5 + c \left(\Rightarrow 310 = \frac{125m}{3} + c \right) \Rightarrow m = 6 \quad c = 60$ $\Rightarrow (g(x) =) 2x^3 - 5x^2 - 37x + 60 \quad *$	M1 dM1 M1 M1 A1cso [5]
ALT	3 rd and 4 th method marks $(x - 5) \left(\frac{m}{3} x^2 + bx + \frac{c}{5} \right) = \frac{m}{3} x^3 - 5x^2 - 37x + c \Rightarrow m = 6 \quad c = 60 \quad (b = 5)$	M1 M1
Special case up to SC3 – mark as M1dM1M1 M0A0 in Epen	Differentiates the given expression for $g(x)$ correctly to give $2(3x^2) - 5(2x) - 37 (= 6x^2 - 10x - 37)$ and concludes $m = 6$ Correctly substitutes $x = 5$ into the given function and shows $g(x) = 0$ $(g(5) =) 2 \times 5^3 - 5 \times 5^2 - 37 \times 5 + 60 = 0$ Correctly substitutes $x = 1$ into the given function and shows and $g(x) = 20$ $(g(1) =) 2 \times 1^3 - 5 \times 1^2 - 37 \times 1 + 60 = 20$	SC1 SC1 SC1
(b)	$\begin{array}{r} 2x^2 + 5x - 12 \\ x - 5 \overline{) 2x^3 - 5x^2 - 37x + 60} \end{array}$ $2x^2 + 5x - 12 = (2x - 3)(x + 4)$ $(g(x) = (x - 5)(2x - 3)(x + 4) = [0])$ $\Rightarrow x = 5, \quad \frac{3}{2}, \quad -4$	M1 M1 A1 [3]
Total 8 marks		

Part	Mark	Notes
(a)	M1	For a minimally acceptable attempt to integrate the given expression (see general guidance). No power of x to decrease. +c is not required for this mark
	dM1	Correctly substitutes $x = 1$ and $y = 20$ to form an equation – simplification is not required Dependent on 1 st method mark. A fully correct equation with no incorrect working can imply this mark. We will be lenient on not seeing the substitution of $x = 1$ as this is trivial, so long as the resulting equation is correct from their $g(1) = 20$. +c is not required for this mark
	M1	Uses the information correctly that $(x - 5)$ is a factor of $g(x)$ by correct substitution of $x = 5$ and $g(x) = 0$ into their $g(x)$. Their $g(x)$ must be a 4 term cubic expression. A fully correct equation with no incorrect working can imply this mark – but if the equation is incorrect we must see substitution of $x = 5$ such that $g(5) = 0$.
	M1	Uses a valid and complete method to solve their resulting simultaneous equations which must be in two variables. Allow up to 2 errors.
	A1 cso	For the correct function as shown or the correct expression. This is a show question. There must be no errors for the award of this mark.
	ALT	
(b)	3 rd M1	Uses the information correctly that $(x - 5)$ is a factor of $g(x)$ by writing a correct statement like the one shown for their $g(x)$. Their $g(x)$ must be a 4 term cubic expression.
	4 th M1	Uses a valid and complete method to solve their resulting simultaneous equations in three variables. Allow up to 2 errors. It is not necessary to see the value for “b”.
	M1	For an attempt at polynomial division or equate coefficients to reach a 3 term quotient. The minimally acceptable quotient is $2x^2 \pm 5x \pm k \quad k \neq 0$ where k is a constant. In the absence of any other working, $2x^2 \pm 5x \pm k \quad k \neq 0$ may be accepted as evidence of working.
	M1	For attempting to factorise their quotient which must be of the form $2x^2 \pm 5x \pm k \quad k \neq 0$ - we must at least see a quadratic factor to factorise. See General Guidance for the definition of an attempt. It would also be possible to see a minimal attempt to use the quadratic formula or completing the square to solve (see general guidance) if their quadratic, of the form $2x^2 + 5x \pm k \quad k \neq 0$ does not factorise. If using the formula the substitution of their a , b and c must be completely correct.
	A1	For the correct values of x
Candidates have clearly been directed to use algebra in the question and therefore 0 marks can be scored without seeing M1 M1		

Question	Scheme	Marks								
4(a)	 Intersections/coordinates at: $y = -x - 1 \Rightarrow (-1, 0)$ and $(0, -1)$ $2y = x + 8 \Rightarrow (-2, 3)$ and $(0, 4)$ $y - 3x + 8 = 0 \Rightarrow (3, 1)$ and $(2, -2)$	B3 [3]								
(b)		B1ft [1]								
(c)	<table border="1"><thead><tr><th>Intersection</th><td>$\left(-3\frac{1}{3}, 2\frac{1}{3}\right)$</td><td>$\left(1\frac{3}{4}, -2\frac{3}{4}\right)$</td><td>$\left(4\frac{4}{5}, 6\frac{2}{5}\right)$</td></tr></thead><tbody><tr><td>P</td><td>$\frac{44}{3}$</td><td>-10.75</td><td>-1.6</td></tr></tbody></table> $P_{\max} = \frac{44}{3}$ $P_{\min} = -10.75$	Intersection	$\left(-3\frac{1}{3}, 2\frac{1}{3}\right)$	$\left(1\frac{3}{4}, -2\frac{3}{4}\right)$	$\left(4\frac{4}{5}, 6\frac{2}{5}\right)$	P	$\frac{44}{3}$	-10.75	-1.6	M1A1 dM1A1 [4]
Intersection	$\left(-3\frac{1}{3}, 2\frac{1}{3}\right)$	$\left(1\frac{3}{4}, -2\frac{3}{4}\right)$	$\left(4\frac{4}{5}, 6\frac{2}{5}\right)$							
P	$\frac{44}{3}$	-10.75	-1.6							

Total 8 marks

Part	Mark	Notes
(a)	B1	For at least one line drawn correctly. Please check their intersections/coordinates carefully, these lines are to be drawn accurately.
	B1	For at least two lines drawn correctly
	B1	For all three lines drawn correctly.
(b)	B1ft	For an enclosed region shaded in or out. Ft their lines from (a) provided the region is closed. <i>R</i> does not need to be written in the region.
(c)	M1	For an attempt to find the coordinates of at least one point of intersection either by simultaneous equations or by reading off from their graph. The question does not specify 'using your graph' so either method is fine.
	A1	For at least one set of correct coordinates. If the candidate uses the graph, then allow all values awrt: $(-3.3 \pm 0.1, 2.3 \pm 0.1)$ for $\left(-3\frac{1}{3}, 2\frac{1}{3}\right)$ $(1.7 \pm 0.1, -2.7 \pm 0.1)$ for $\left(1\frac{3}{4}, -2\frac{3}{4}\right)$ $(4.8 \pm 0.1, 6.4 \pm 0.1)$ for $\left(4\frac{4}{5}, 6\frac{2}{5}\right)$
	dm1	For using at least one set their coordinates of their intersections to find any value of <i>P</i> Explicit substitution need not be seen provided it is clear candidates are using the expression for <i>P</i> . Accept awrt: 14.6 ± 0.1 or -10.7 ± 0.1 or -1.6 ± 0.1 Note this is dependent on the previous M mark If their values are incorrect and you see no working this is M0 If their values are correct, you see no working and no value of <i>P</i> is correct M0
	A1	For both the correct minimum and maximum values. $P_{\max} = 14.66$ accept awrt 14.7 ± 0.3 $P_{\min} = -10.75$ accept awrt -10.7 ± 0.3

Question number	Scheme	Marks															
4 (a)	$3x + 2y = 18$ Intersections with axes at (0,9) (6,0) $x + 3y - 6 = 0$ Intersections with axes at (0,2) (6,0) $y = 3x$ Coordinates at (0,0) (2, 6) 	B1 B1 B1 [3]															
(b)	For the correct region shaded in or out 	B1ft [1]															
(c)	Points of intersection are: <table><tr><td>(0.6,1.8)</td><td>(2,6)</td><td>(6,0)</td></tr></table> <table><tr><td>Vertex</td><td>(0.6,1.8)</td><td>(2,6)</td><td>(6,0)</td></tr><tr><td>$P = 2x - y$</td><td>-0.6</td><td>-2</td><td>12</td></tr><tr><td></td><td></td><td>Least</td><td></td></tr></table> For $P = -2$ ALT – objective line approach Slope of objective line is 2 (2,6) $P = 2 \times 2 - 6 = -2$ For $P = -2$	(0.6,1.8)	(2,6)	(6,0)	Vertex	(0.6,1.8)	(2,6)	(6,0)	$P = 2x - y$	-0.6	-2	12			Least		M1 A1 dM1 A1 [4] [M1 A1 dM1 A1]
(0.6,1.8)	(2,6)	(6,0)															
Vertex	(0.6,1.8)	(2,6)	(6,0)														
$P = 2x - y$	-0.6	-2	12														
		Least															
Total 8 marks																	

Part	Mark	Additional Guidance			
(a)	B1B1B1	B1 for one line correct, B1B1 for two lines correct, B1B1B1 for all three lines correct: $3x + 2y = 18$ Intersections with axes at (0,9) (6,0) $x + 3y - 6 = 0$ Intersections with axes at (0,2) (6,0) $y = 3x$ Intersections with axes at (0,0) (2, 6)			
(b)	B1ft	For the correct region marked – allow shaded in or out. Ft for shading the closed region from their three lines.			
(c)	M1	For attempting to find at least one coordinate pair of their intersections: Either by reading values from their graph (might be written on the graph), or by solving simultaneous equations leading $x = \dots y = \dots$			
	A1	Finds all three correct points of intersection: Allow point (0.6,1.8) to be $(0.6 \pm 0.1, 1.8 \pm 0.1)$ <table><tr><td>(0.6,1.8)</td><td>(2,6)</td><td>(6,0)</td></tr></table>	(0.6,1.8)	(2,6)	(6,0)
	(0.6,1.8)	(2,6)	(6,0)		
	dM1	For substituting any one point of intersection (could be from reading from the graph or solved by simultaneous equations) into the given P ft their coordinates. Any correct P value: $P = -2$, $P = -0.6$ or $P = 12$ implies this mark. Dependent on the previous M mark			
	A1	For $P = -2$ from testing at least two points, and all three points of intersection must be correct (0.6,1.8), (2,6) and (6,0)			
ALT – objective line approach					
	M1	For an attempt to use objective line approach. Identified that the slope of the objective line is 2. Identified that the intersection of $3x + 2y = 18$ and $y = 3x$ is where P is the least.			
	A1	For finding the correct coordinates (2,6)			
	dM1	For substituting their (2,6) into P .			
	A1	For identifying $P = -2$			

Part	Mark	Additional Guidance
(a)	M1	For correct substitution of -1 or 2 into $f(x)$
	M1	For correct substitution of -1 and 2 into $f(x)$
		For the first two marks: Where candidates employ simultaneous equations, mark the work for the substitution Accept eg -1 for $(-1)^3$ 4 for 2^2 Allow $-1 + c$ with no errors to imply substitution of -1 Allow $14 + c$ with no errors to imply substitution of 2
	ddM1	For $4[(-1)^3 + 1^2 + (-1) + c] = 2^3 + 2^2 + 2 + c$ or condone the 4 on the ‘wrong side’. $(-1)^3 + 1^2 + (-1) + c = 4[2^3 + 2^2 + 2 + c]$ Dependent on both previous method marks. For forming simultaneous equations eg $4R = 14 + c$ $R = -1 + c$ Condone $R = 14 + c$ $4R = -1 + c$ The work doesn’t have to be simplified for this mark.
	A1*cso	Correct solution, no errors or omissions, minimum steps as shown. The steps will need to be checked. Note there are different notations for long division.
		For a route that uses long division – examples in mark scheme. M1 For completing a correct long division, giving a correct remainder M1 for completing 2 correct long divisions, giving both correct remainders ddM1 for the correct equation or condone the 4 on the wrong side. A1 Correct solution, no errors or omissions, minimum steps as shown. The steps will need to be checked.
(b)	M1	For $[(x+2)](x^2 + Ax + 3)$ or $[(x+2)](x^2 - x + B)$ using any method – there must be a quadratic to award this mark. Mark the quadratic seen, not the method.
	A1	For $x^2 - x + 3$
	M1	For calculating the discriminant correctly for their 3 term quadratic. If working with the correct quadratic – 11 will imply this mark. If not, they must show at least partial substitution (their b^2 and $4ac$ evaluated will be fine) And condone/allow $(1)^2 - 4(1)(3) < 0$ since $(-1)^2 = (1)^2$ Or full correct completion of square and $-\frac{11}{4} < 0$ oe. The discriminant may be calculated in the quadratic formula.
	A1*cso	Fully correct solution and short conclusion State < 0 and (can be “no (real) roots”, $(x =)$ “ -2 is the only (real) root”, “shown” or #), no errors. Minimum steps as shown.
		Send any work on complex numbers to Review.

Question number	Scheme	Marks
5(a)	$f(-2) = 0, f(-3) = 21$ $a(-2)^3 + 5b(-2)^2 + 8a(-2) - 4b = 0$ $a(-3)^3 + 5b(-3)^2 + 8a(-3) - 4b = 21$ $2b = 3a$ $41b = 51a + 21$ $a = 2^*, b = 3$	M1 A1 M1 A1cso A1 [5]
(b)	$x+2 \overline{) 2x^3 + 15x^2 + 16x - 12}$ $\frac{2x^2 + 11x - 6}{x+2 \overline{) 2x^3 + 15x^2 + 16x - 12}}$ ALT $2x^3 + 15x^2 + 16x - 12 = (x+2)(Ax^2 + Bx + C) \Rightarrow (x+2)(2x^2 + 11x - 6)$ $2x^2 + 11x - 6 = (2x-1)(x+6)$ $(2x-1)(x+6) = 0 \Rightarrow 2x-1 = 0, x+6 = 0$ $x = -6, -2, \frac{1}{2}$	M1 M1 M1 A1 [4]
Total 9 marks		

Question	Marks	Scheme
(a)	M1	For attempting either $f(-2) = 0$ or $f(-3) = 21$ Allow $f(\pm 2) = 0$ or $f(\pm 3) = 21$ for this mark. Allow for $f(\pm 3) = 21$ with $a = 2$ assumed.
	A1	For both correct equations in terms of a and b $a(-2)^3 + 5b(-2)^2 + 8a(-2) - 4b = 0$ $a(-3)^3 + 5b(-3)^2 + 8a(-3) - 4b = 21$ Evaluation not required for this mark, just the correct substitution.
	M1	Attempts to solve their two linear simultaneous equation in a and b $2b = 3a$ $41b = 51a + 21$ Condoned one slip provided consistent addition or subtraction if using elimination.
	A1 cs0	For $a = 2^*$
	A1	For $b = 3$
(b)	M1	For attempting division of $f(x) = 2x^3 + 15x^2 + 16x - 12$ by $(x+2)$ getting as far as $2x^2 + \dots$ $x+2 \overline{) 2x^3 + 15x^2 + 16x - 12}$ $\frac{2x^2 + 11x - 6}{x+2 \overline{) 2x^3 + 15x^2 + 16x - 12}}$ ALT

		Equates coefficients to find the 3TQ factor $2x^3 + 15x^2 + 16x - 12 = (x + 2)(Ax^2 + Bx + C) \Rightarrow (x + 2)(2x^2 + 11x - 6)$ Must get as far as $(x + 2)(2x^2 + \dots)$ for the mark.
	M1	For attempting to factorise their 3TQ, but it must be a 3TQ $2x^2 + 11x - 6 = (2x - 1)(x + 6)$ Refer to general guidance for what constitutes an attempt to factorise.
	M1	An attempt to solve $f(x) = 0$
	A1	For $x = -6, -2, \frac{1}{2}$ Note: Correct answers with no working scores M0M0M0A0

Question number	Scheme	Marks
2 a	$(8x + y)(6x + y) [= 48x^2 + 14xy + y^2]$ oe	B1 (1)
b	$48x^2 + 14xy + y^2 - 2(9x^2 + 15x^2)$ oe $= 14xy + y^2$ *	M1 M1 A1 cso* (3)
ALT	eg $3(y \times 3x) + (y \times 5x) + y^2$ or $y(8x + y) + 2(3x \times y)$ or $y \times 3x + y \times 5x + y \times (6x + y)$ or $y(6x + y) + y(8x + y) - y^2$ $= 14xy + y^2$ *	M1 M1 A1 cso* (3)
c	$(14xy + y^2 = 1080, 48x^2 + 14xy + y^2 = 3432)$ $48x^2 + 1080 = 3432$ $(48x^2 = 2352)$ $x = 7$ $y^2 + 98y - 1080 (= 0)$ $(y - 10)(y + 108) (= 0)$ $y = 10$	M1 A1 M1 M1 A1 (5)
ALT	$(14xy + y^2 = 1080, 48x^2 + 14xy + y^2 = 3432)$ $x = \frac{1080 - y^2}{14y} \Rightarrow 48 \left(\frac{1080 - y^2}{14y} \right)^2 + \left(\frac{1080 - y^2}{14y} \right) 14y + y^2 = 3432$ $\Rightarrow y^4 - 2160y^2 + 1166400 = 0$	M1
	$(y^2 - 1164)(y^2 - 100) (= 0)$ $(\Rightarrow y^2 = 1164 \text{ or } y^2 = 100 \text{ or } y = \pm 108 \text{ or } y = \pm 10)$	M1 (A1 on ePen)
	$y = 10$	A1 (M1 on ePen)
	$14(10)x + 100 = 1080 \Rightarrow x =$	M1
	$x = 7$	A1
Total 9 marks		

Part	Marks	Notes
(a)	B1	For $(8x + y)(6x + y)$ oe
(b)	M1	For their " $(8x + y)(6x + y) - 2(9x^2 + 15x^2)$ " oe Neither of these expressions need to be in simplified form.
	M1	For correct expansion to give $48x^2 + 14xy + y^2$ - may be seen in part (a) or for $2(9x^2 + 15x^2)$ or $18x^2 + 30x^2$ or $48x^2$ written in any correct form – doesn't need to be simplified.
	A1 cso*	Obtains the given result with no errors seen.
	Note M0 M1 A0 is possible.	
ALT	M1	For an attempt to add or subtract separate components to give the area of the cross. Allow 1 omission or up to 2 errors. Examples are given in the MS, though these are not exhaustive.
	M1	All components present and correct and an attempt to simplify.
	A1 cso*	Obtains the given result with no errors seen.
	Note M0 M1 A0 is possible.	
(c)	M1	For solving simultaneously, using the correct equations, to obtain $48x^2 + 1080 = 3432$
	A1	For $x = 7$ (must select the positive solution)
	M1	For substituting their value of x to obtain a 3TQ in terms of y only.
	M1	For an attempt to solve their 3TQ – see general guidance for minimally acceptable attempt.
	A1	For $y = 10$ (must reject the other solution) final M1 A1 can be awarded for $y = 10$ with no working shown final M1 A0 can be awarded for $y = 10$ and $y = -108$.
ALT	M1	Correct rearrangement for x and correct substitution into $48x^2 + 14xy + y^2 = 3432$ reaching an equation of the form $ay^4 + by^2 + c = 0$
	M1 (A1 on ePen)	For an attempt to solve their equation of the form $ay^4 + by^2 + c = 0$ as a quadratic in y^2 (minimally acceptable attempt to solve, see general guidance) (A fully correct solution would give: $y^2 = 1164$ or $y^2 = 100$ or $y = \pm 108$ or $y = \pm 10$)
	A1 (M1 on ePen)	$y = 10$ (must clearly select this solution) 2 nd M1 A1 can be awarded if no working and $y = 10$ 2 nd M1 A0 can be awarded if no working and $y = \pm 108$ and $y = \pm 10$ seen
	M1	For substituting their value for y and solving a linear equation to arrive at $x =$
	A1	$x = 7$

Question number	Scheme	Marks
4 a Mark parts (i) and (ii) together	$(f(1)=)1^3 + p \times 1^2 + q \times 1 + 6 = 0$ oe or $(f(-1)=)(-1)^3 + p \times (-1)^2 + q \times (-1) + 6 = 8$ oe $p + q = -7$ oe $p - q = 3$ oe Solving simultaneously $2p = -4$ or $2q = -10$ $p = -2$ * and $q = -5$	M1 A1 A1 M1 A1 cso B1 (A1 on ePen) cso (6)
b	$(f(x) = x^3 - 2x^2 - 5x + 6 = 0)$ $(x-1)(x^2 - x - 6) = 0$ $[(x-1)](x+2)(x-3) = 0$ $x = 1, -2, 3$	M1 M1 A1 (3)
Total 9 marks		

$ \begin{array}{r} x-1 \overline{) x^3 + px^2 + qx + 6} \\ \underline{x^3 - x^2} \\ (p+1)x^2 + qx \\ \underline{(p+1)x^2 - (p+1)x} \\ (q+p+1)x + 6 \\ \underline{(q+p+1)x - (q+p+1)} \\ 6+q+p+1 \end{array} $	$ \begin{array}{r} x^2 \\ (p+1)x \\ q+p+1 \end{array} $
remainder $6+q+p+1=0$ $p+q=-7$	
$ \begin{array}{r} x+1 \overline{) x^3 + px^2 + qx + 6} \\ \underline{x^3 + x^2} \\ (p-1)x^2 + qx \\ \underline{(p-1)x^2 + (p-1)x} \\ (q-p+1)x + 6 \\ \underline{(q-p+1)x + (q-p+1)} \\ 6-q+p-1 \end{array} $	$ \begin{array}{r} x^2 \\ (p-1)x \\ q-p+1 \end{array} $
remainder $6-q+p-1=8$ $p-q=3$	

Part	Mark	Additional Guidance
(a)	M1	For a fully correct substitution of either 1 into $f(x) = 0$ OR -1 into $f(x) = 8$ as shown. The bracketing for the substitution of -1 must be correct, but may be recovered later. A correct equation can imply this mark.
	A1	For a fully correct substitution of 1 into $f(x) = 0$ AND -1 into $f(x) = 8$ as shown. The bracketing for the substitution of -1 must be correct, but may be recovered later. A correct equation can imply this mark.
	A1	For $p + q = -7$ and $p - q = 3$ or The equations do not need to be simplified.
		Note, if students choose to do long division, to be comparable with the main scheme, this must be fully correct and complete, to allow them to arrive at the correct equations to be awarded the method mark. See the example included. The accuracy marks can be then awarded as stated above. There is an example under the main mark scheme. If in any doubt, please send to review.
	M1	For an attempt to solve their equations simultaneously, allow one processing error. Must eliminate one variable and arrive at a value for p or q .
	A1 CSO	For $p = -2$
	B1 (A1 on ePen)	$q = -5$ Note this is an independent accuracy mark, allowing students to use the given value of p to find q .
(b)	M1	For $(x-1)(x^2 \pm Ax - 6) = 0$ $A \neq 0$ This mark can be awarded for sight of $x^2 \pm Ax - 6$ $A \neq 0$
	M1	For $[(x-1)](x+2)(x-3) = 0$ For this mark, for the correct or ft their quadratic: Allow any minimally acceptable attempt to solve the quadratic, by factorising, completing the square or use of the quadratic formula – see general guidance, leading to two values of x in addition to $x = 1$. It is not necessary to see $= 0$.
	A1	For $x = 1, -2, 3$ If 3 three correct solutions appear with no working M1 M1 A1 as the question has not stated show working or solve algebraically.

Question	Scheme	Marks
8	$\int (18x^2 - 2x + 13) \, dx = \frac{18x^3}{3} - \frac{2x^2}{2} + 13x(+c)$	M1A1
	$f\left(\frac{1}{2}\right) = 6 \times \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 13 \times \left(\frac{1}{2}\right) + c = 0 \Rightarrow c = -7$	M1A1
	$f(x) = 6x^3 - x^2 + 13x - 7$ oe	A1
	$\begin{array}{r} 3x^2 + x + 7 \\ 2x - 1 \overline{) 6x^3 - x^2 + 13x - 7} \end{array}$	M1A1
	$b^2 - 4ac = 1^2 - 4 \times 3 \times 7 = -83$	M1
	Conclusion: $b^2 - 4ac < 0$ so there is only one root/intersection with the x-axis $\left[\text{at } \left(\frac{1}{2}, 0\right) \right]$	A1cso
		[9]
Total 9 marks		

Mark	Notes
M1	For integrating the given $f'(x)$. $+c$ is not required for this mark This can be simplified or unsimplified. Accept $\frac{18x^3}{3} - \frac{2x^2}{2} + 13x(+c)$ or $6x^3 - x^2 + 13x(+c)$ At least two terms must be integrated correctly, with no differentiation.
A1	For the correct integrated expression. Ignore the absence of $+c$
M1	For substituting $x = \frac{1}{2}$ into their integrated expression (which must include $+c$, otherwise it is M0) and setting equal to 0. ALT Some candidates are completing this step using polynomial division. If they do not have $+c$ at this stage then it is M0. If it is clear they are attempting to find the value of c by finding a numerical value for c [even if it is incorrect], award both this mark and the next M mark provided they obtain a quotient of $3x^2 + x + k$



A1	For the correct value of $c = -7$
A1	For the fully correct $f(x) = 6x^3 - x^2 + 13x - 7$ all on one line. You do not need to see $f(x) = \dots$ Accept even when you see this as a complete dividend in the 'bus stop' or when equating coefficients.
M1	<u>Polynomial division</u> For dividing their $f(x)$ by $(2x - 1)$ to achieve as a minimum $3x^2 + x$. Award this for correct attempts even without either $+c$ or a value for c . $6x^2 + 2x (+1)$ Allow also: $x - \frac{1}{2} \overline{) 6x^3 - x^2 + 13x - 7}$ [the correct quotient is $6x^2 + 2x + 14$] <u>ALT</u> <u>Equating coefficients.</u> They must achieve a correct value for A and B for the award of this mark. $6x^3 - x^2 + 13x - 7 = (2x - 1)(Ax^2 + Bx + C) \Rightarrow A = 3, B = 1$ OR $6x^3 - x^2 + 13x - 7 = \left(x - \frac{1}{2}\right)(Ax^2 + Bx + C) \Rightarrow A = 6, B = 2$
A1	For the correct quotient/quadratic factor $3x^2 + x + 7$ or $6x^2 + 2x + 14$
M1	For using $b^2 - 4ac$ on their $f(x)$ which must be a 3TQ Accept this as part of the quadratic formula. Sight of the complex roots $\left(\frac{-1 \pm (\sqrt{83})i}{6} \right)$ is M1 Allow this mark even if they do not have $+c$ or find the correct cubic expression.
A1 CSO	For concluding that as $b^2 - 4ac < 0$ then there is only one root/intersection with the x -axis. Do not accept statements such as 'not possible' or 'will not factorise' without reference to a negative discriminant. Accept evidence of a statement such as $1^2 < 4 \times 3 \times 7 \Rightarrow b^2 < 4ac$ oe, or embedded in a formula without explicit evaluation. Complex roots must be followed by a comment that as the roots are not real, there are no intersections with the x -axis. Quoting the complex roots without a correct explanation is A0. If they have not found $+c$ at the start, this mark is not available as this solution is CSO.

Question number	Scheme	Marks
4 (a)	$f(2) = 2 \times 2^3 + a \times 2^2 + b \times 2 + 18 = 0$ $f'(x) = 6x^2 + 2ax + b \Rightarrow f'(2) = 6 \times 2^2 + 2 \times a \times 2 + b = 5$ $4a + 2b + 34 = 0$ $4a + b + 19 = 0$ $\Rightarrow b = -15, a = -1$	M1 M1M1 A1 M1A1 [6]
(b)	$\begin{array}{r} 2x^2 + 3x - 9 \\ x-2 \overline{) 2x^3 - x^2 - 15x + 18} \\ \underline{2x^3 - 4x^2 + 6x - 18} \\ 3x^2 - 11x + 36 \\ \underline{3x^2 - 6x - 18} \\ 5x + 54 \\ \underline{5x - 10} \\ 64 \end{array}$ $2x^2 + 3x - 9 = (x+3)(2x-3)$ $\Rightarrow (x-2)(x+3)(2x-3)$	M1 M1A1 [3]
(c)	$x = 2, -3, \frac{3}{2}$	B2ft [2]
Total 11 marks		
(a) M1 M1 M1 A1 M1 A1 (b) M1 M1 A1 (c) B2 ft	$f(2) = 0$ leading to an equation in a and b Attempt to differentiate $f'(2) = 5$ leading to an equation in a and b $4a + 2b + 34 = 0$ and $4a + b + 19 = 0$ Solving simultaneously $b = -15, a = -1$ Dividing by $x - 2$ to obtain a 3TQ Factorising the 3TQ All 3 terms correct $x = 2, -3, \frac{3}{2}$ (B1 for 2 correct)	

Question number	Scheme	Marks
2 (a)	$f(1) = 1 + p + q = -12$ $p + q = -13$ $f(4) = 64 + 4p + q = 30$ $4p + q = -34$ $3p = -21$ $p = -7$ and $q = -6$	M1 A1 M1 A1 M1 A1 (6)
(b)	$3^3 - 7 \times 3 - 6 = 27 - 21 - 6 = 0$ *	B1 cso (1)
(c)	$(x-3)(x^2 + 3x + 2)$ $(x-3)(x+2)(x+1)$	M1 M1 A1 (3)
(d)	$x = 3$ $x = -2$ $x = -1$	B1 ft (1)
	Notes	[11]
(a) M1 A1 M1 A1 M1 A1	For substitution of 1 into $f(x)$ For $p + q = -13$ oe For substitution of 4 into $f(x)$ For $4p + q = -34$ oe For solving simultaneously $p = -7$ and $q = -6$	
(b) B1cso	For substituting 3 into $f(x)$ and obtaining the given result	
(c) M1 M1 A1	For $(x-3)(x^2 + 3x + 2)$ For factorising the quadratic $(x-3)(x+2)(x+1)$	
(d) B1ft	For $x = 3$ $x = -2$ $x = -1$ or follow through part (c)	

Question	Scheme	Marks
4(a)	$f'(x) = 3px^2 + 2qx - 37$ $f'(-2) = 3p(-2)^2 + 2q(-2) - 37 = -33$ $f(-5) = p(-5)^3 + q(-5)^2 - 37(-5) - 12q = 0$ $\begin{cases} 12p - 4q = 4 \\ -125p + 13q = -185 \end{cases} \Rightarrow p = \dots q = \dots$ $p = 2, q = 5$	M1 M1 M1 dM1 A1A1 [6]
(b)	$\begin{array}{r} 2x^2 - 5x - 12 \\ x + 5 \overline{) 2x^3 + 5x^2 - 37x - 60} \end{array}$ OR $2x^3 + 5x^2 - 37x - 60 = (x + 5)(Ax^2 + Bx + C)$ $\Rightarrow A = 2, B = \dots, C = \dots$ $2x^2 - 5x - 12 = (2x + 3)(x - 4)$ $\{f(x)\} = (x + 5)(2x + 3)(x - 4)$	M1 M1 A1 [3]
(c)	$(x + 5)(2x + 3)(x - 4) = 0, \Rightarrow x = -5, 4, -\frac{3}{2}$	M1A1 [2]
Total 11 marks		

Part	Mark	Notes
(a) Mark i and ii together	M1	For differentiating the given expression in terms of p and q with respect to x , at least one term correct and no terms integrated.
	M1	Attempts $f'(\pm 2) = -33$
	M1	Attempts $f(\pm 5) = 0$
	dM1	For solving the two resulting simultaneous equations by any correct method (allow slips) providing there are two variables p and q in both, leading a value of p or q . Explicit method to solve must be seen. Depends on all previous method marks
	A1	For $p = 2$ or $q = 5$ from correct working in two correct equations
	A1	For both $p = 2$ and $q = 5$ from correct working
(b)	M1	For using polynomial division (or any other algebraic method) to find the quadratic factor. Division: Look for divisor of $(x+5)$, quotient of a quadratic $2x^2 - \text{their } qx + c$ ($c \neq 0$) Compare coefficients: minimum required is an identity of the correct form followed by values of $A = 2, B = -\text{their } q, C = \dots$ ($C \neq 0$)
	M1	Correct method of factorising their quadratic factor. (See General Principles)
	A1	Correct factorised expression written in full on one line.
(c)	M1	Correct method of solving their $f(x) = 0$. Their $f(x)$ must have 3 solutions, one of which must be $x = -5$, “=0” or their method may be implied by their x values. They must use their result in part b which $f(x)$ reaches 2 or 3 factors, if part b is not attempted, a full algebraic method of solving $f(x)=0$ must be seen in this part.
	A1	For all three correct values of x , no extra solutions.

(a) ALT I Algebraic Division

M1 Same as main scheme**M1** Their $f'(x)$ is in the form of a 3TQ, look for divisor of $(x+2)$, quotient of a linear expression leading to a constant remainder in p and q , sets $= -33$ **M1** For dividing $f(x)$, look for divisor of $(x+5)$, quotient of a 3TQ, leading to a constant remainder in p and q , sets $= 0$ **dM1A1** Same as main scheme

You may also see variations for their division, marking is similar as above, (if not sure, send to review)
e.g.

For $f'(x)$

$$\begin{array}{r}
 -2 \overline{) \begin{array}{rrr} 3p & 2q & -37 \\ & -6p & 12p-4q \\ \hline 3p & -7p+2q & 12p-4q-37 = -33 \end{array} }
 \end{array}$$

For $f(x)$

$$\begin{array}{r}
 -5 \overline{) \begin{array}{rrrr} p & q & -37 & -12q \\ & -5p & -5q+25p & 25q-125p+185 \\ \hline p & q-5p & -5+25p-37 & 13q-125p+185=0 \end{array} }
 \end{array}$$

(a) ALT II Compare coefficients

M1 Same as main scheme**M1** Sets their $f'(x)$ which is in the form of a 3TQ, equal to $(x+2)(Kx+C)-33$ ($K \neq 0, C \neq 0$), then attempts to equate the coefficients of x to form an equation in p and q **M1** Sets $f(x)=(x+5)[px^2 + Rx + S]$ ($R \neq 0, S \neq 0$), then attempts to equate their coefficients of x to form an equation in p and q .**dM1A1** Same as main scheme

For reference, the correct working for this method:

$$\begin{aligned}
 f'(x) &= 3px^2 + 2qx - 37 \\
 &= (x+2)(3px-2) - 33 \\
 \Rightarrow 6px - 2x &= 2qx \\
 6p - 2 &= 2q \\
 f(x) &= px^3 + qx^2 - 37x - 12q = (x+5) \left[px^2 + (q-5p)x - \frac{12q}{5} \right] \\
 \Rightarrow 5(q-5p)x - \frac{12q}{5}x &= -37x \\
 5(q-5p) - \frac{12q}{5} &= -37 \\
 \frac{13q}{5} - 25p &= -37
 \end{aligned}$$



Question number	Scheme	Marks
11 (a)	$f(x) = \int ax^2 - 14x - 10 \, dx = \frac{ax^3}{3} - \frac{14x^2}{2} - 10x + c$ $f(4) = \frac{a \times 4^3}{3} - \frac{14 \times 4^2}{2} - 10 \times 4 + c = 0 \Rightarrow \frac{64a}{3} - 152 + c = 0$ $f(-1) = \frac{a \times (-1)^3}{3} - \frac{14 \times (-1)^2}{2} - 10 \times (-1) + c = 25 \Rightarrow -\frac{a}{3} - 22 + c = 0$ $\frac{64a}{3} - 152 + c = 0$ $-\frac{a}{3} - 22 + c = 0$ $\Rightarrow 152 - \frac{64a}{3} = 22 + \frac{a}{3} \Rightarrow a = 6$	M1A1 M1 M1 M1A1 [6]
(b)	$c = 22 + \frac{6}{3} = 24$ $f(x) = 2x^3 - 7x^2 - 10x + 24$ $x - 4 \overline{) 2x^3 - 7x^2 - 10x + 24}$ $f(x) = (x - 4)(2x - 3)(x + 2) = 0$ $x = 4, \frac{3}{2}, -2$ ALT $2x^3 - 7x^2 - 10x + 24 = (x - 4)(ax^2 + bx + c)$ $a = 2, b = 1, c = -6$ $f(x) = (x - 4)(2x^2 + x - 6)$ $f(x) = (x - 4)(2x - 3)(x + 2) = 0$ $x = 4, \frac{3}{2}, -2$	B1 M1A1 dM1A1 A1 [6] [M1 A1 dM1A1 A1]
Total 12 marks		

Part	Mark	Notes
(a)	M1	For an attempt to integrate $f'(x) = ax^2 - 14x - 10$ See General Guidance on what constitutes an attempt to integrate.
	A1	For correctly integrating $f'(x)$ to obtain $f(x) = \frac{ax^3}{3} - \frac{14x^2}{2} - 10x + c$ Must include $+c$ for this mark.
	M1	For substituting $x = \pm 4$ in their $f(x)$ and setting $=0$
	M1	For substituting $x = \pm 1$ in their $f(x)$ and setting $=25$

	M1	For correct method to solve the equations simultaneously to find a A correct intermediate step is required e.g. $65a = 390$
	A1 cs0	For fully correct working leading to $a = 6$
(b)	B1	For correctly identifying $c = 24$
	M1	For attempt to divide $f(x)$ by $x - 4$ Must get as far as $2x^2 + \dots$
	A1	For correct division of $f(x)$ by $x - 4$ to obtain $2x^2 + x - 6$
	dM1	For an attempt to factorise their 3TQ which must come from a cubic. See General Guidance on what constitutes an attempt to factorise.
	A1	For obtaining correct factorisation of the cubic: $(x - 4)(2x - 3)(x + 2)$
	A1	For all three correct solutions of the equation: $x = 4, \frac{3}{2}, -2$
Alternative method		
	M1	For an attempt to find the quadratic factor that multiplies $x - 4$ to give $f(x)$ Must get as far as $f(x) = (x - 4)(2x^2 + bx + c)$
	A1	For correctly comparing coefficients to obtain $2x^2 + x - 6$
	dM1	For an attempt to factorise their 3TQ which must come from a cubic. See General Guidance on what constitutes an attempt to factorise.
	A1	For obtaining correct factorisation of the cubic: $(x - 4)(2x - 3)(x + 2)$
	A1	For all three correct solutions of the equation: $x = 4, \frac{3}{2}, -2$
Note: Correct solution seen with no working scores B0M0A0M0A0A0		

Question	Scheme	Marks
5(a)	$f(-2) = -16a + 4 + 2b + 3a = 0$ and $f(1) = 2a + 1 - b + 3a = 0$ $-13a + 4 + 2b = 0$ and $5a + 1 - b = 0 \Rightarrow a = 2^*$ $b = 11$	M1M1 M1A1cso A1 [5]
(b)	$f(x) = 4x^3 + x^2 - 11x + 6 = (4x - 3)(x + 2)(x - 1)$	M1A1 [2]
(c)	$h(y) = 2^{(3y+2)} + 2^{2y} - 11(2^y) + 6 \Rightarrow h(y) = 4(2^y)^3 + (2^y)^2 - 11(2^y) + 6$ $x = 2^y \Rightarrow 2^y - 1 = 0, 2^y + 2 = 0, 4(2^y) - 3 = 0$ $2^y = 1 \Rightarrow y = 0$ $2^y = \frac{3}{4} \Rightarrow y = \log_2 \frac{3}{4} = -0.4150... \Rightarrow y = -0.415$ $[2^y = -2 \text{ no solution}]$	M1 M1 B1 M1A1 [5]
Total 12 marks		

Question	Notes	Marks
(a)	For substituting ± 2 into $f(x) = 0$ Allow sign errors	M1
	For substituting ± 1 into $f(x) = 0$ Allow sign errors	M1
	For solving the resulting pair of simultaneous equations. $-13a + 4 + 2b = 0$ $5a + 1 - b = 0$	M1
	For $a = 2^*$	A1 cso
	For $b = 11$	A1 [5]
	ALT uses polynomial division.	
	Multiplies out the product of the two factors to give: $(x + 2)(x - 1) = x^2 + x - 2$ This must be correct.	M1

	Uses polynomial division to give as a minimum: $2ax + (1 - 2M)$ where M is a constant. $\begin{array}{r} 2ax + (1 - 2a) \\ x^2 + x - 2 \overline{) 2ax^3 + x^2 - bx + 3a} \end{array}$	M1
	Equates coefficients: $(x^2 + x - 2)(2ax + (1 - 2a)) = 2ax^3 + x^2 - bx + 3a$ $\Rightarrow 2ax^3 + x^2 + x(1 - 6a) + (-2 + 4a) = 2ax^3 + x^2 - bx + 3a$ $3a = 4a - 2$ $1 - 6a = -b$	M1
	For $a = 2$ *	A1 cso
	For $b = 11$	A1
(b)	Divides $f(x) = 4x^3 + x^2 - 11x + 6$ by either $[x^2 + x - 2]$ or $x + 2$ or $x - 1$ Either, $\begin{array}{r} 4x + k \\ x^2 + x - 2 \overline{) 4x^3 + x^2 - 11x + 6} \end{array}$ $\begin{array}{r} 4x^2 - 7x \pm k \\ x + 2 \overline{) 4x^3 + x^2 - 11x + 6} \end{array}$ $\begin{array}{r} 4x^2 + 5x \pm k \\ x - 1 \overline{) 4x^3 + x^2 - 11x + 6} \end{array}$	M1
	For the correct factorisation of $f(x)$ $4x^2 - 7x + 3 = (4x - 3)(x - 1) \Rightarrow f(x) = (4x - 3)(x - 1)(x + 2)$	A1 [2]
(c)	For manipulating the indices to achieve $\Rightarrow h(y) = 4(2^y)^3 + (2^y)^2 - 11(2^y) + 6$ OR Uses the factorised expression to obtain: $h(y) = (2^y + 2)(2^y - 1)(4(2^y) - 3)$	M1
	Substitutes $x = 2^y$ into the factorised $f(x)$ to find either $2^y - 1 = 0$ or $2^y + 2 = 0$ or $4(2^y) - 3 = 0$	M1
	For $2^y = 1 \Rightarrow y = 0$	B1
	For $4(2^y) = 3 \Rightarrow 2^y = \frac{3}{4} \Rightarrow y = \log_2 \frac{3}{4}$	M1
	For awrt $y = -0.4150... \Rightarrow y \approx -0.415$ [No solution for $2^y = -2$ - must reject]	A1 [5]
Total 12 marks		

Question number	Scheme	Marks
6 (a)	$f(3) = 27 + 9p + 9 - 30 + q = 0$ $9p + q + 6 = 0$ *	M1 A1 A1 cso (3)
(b)	$f(-p) = -p^3 + p^2(p+1) + 10p + q = 0$ $p^2 + 10p + q = 0$ *	M1 A1 A1 cso (3)
(c)	$p^2 + 10p - 9p - 6 = 0$ $p^2 + p - 6 = 0$ $(p+3)(p-2) = 0$ $p = 2$ $q = -24$	M1 A1 M1 A1 A1 (5)
(d)	$(x+a)(x-3)(x+2)$ So $-3 \times 2 \times a = -24$ $a = 4$ $(x+4)(x-3)(x+2)$	M1 A1 (2)
Total 13 marks		

Part	Mark	Guidance
General guidance for marking parts (a) and (b) - For the award of full marks in parts (a) and/or (b) you must see $= 0$ used in a line of working before the final answer. - If a candidate does not use $= 0$ in either parts (a) or (b) [except in the final line – which is a given answer] deduct the M mark (and the subsequent A marks) in only the first occurrence of the absence.		
(a)	M1	For using $f(\pm 3) = 0$ in the given equation set $= 0$
	A1	For obtaining the correct unsimplified expression: $27 + 9p + 9 - 30 + q = 0$
	A1 cso	For obtaining the given equation $9p + q + 6 = 0$ * Note: This is a show question. There must be no errors seen.
(b)	M1	For use of $f(\pm p) = 0$ in the given equation set $= 0$
	A1	For obtaining the correct unsimplified expression: $-p^3 + p^2(p+1) + 10p + q = 0$

	A1 CSO	For obtaining the correct given equation $p^2 + 10p + q = 0$ * Note: This is a show question. There must be no errors seen.
(c)	M1	For attempting to solve the given two equations simultaneously to achieve a 3TQ in either p or q only. E.g. substitutes $q = \mp 9p \mp 6$ or $\left[p = \frac{\mp q \mp 6}{9} \text{ and } p^2 = \frac{(\mp q \mp 6)^2}{81} \right]$ into $p^2 + 10p + q = 0$ This mark may be implied by the correct 3TQ
	A1	For the correct 3TQ $p^2 + p - 6 = 0$ or $q^2 + 3q - 504 = 0$
	M1	For an attempt to solve their 3TQ in either p or q using factorisation, use of the formula or completing the square. See general guidance for the definition of an attempt. For example: $(p+3)(p-2) = 0 \Rightarrow p = \dots$ or $(q+24)(q-21) = 0 \Rightarrow q = \dots, \dots$ If a candidate uses their calculator to solve their 3TQ, the final values must be correct for the award of this mark unless a valid method is seen.
	A1	For either the correct value of p OR the correct value of q $p = 2$ or $q = -24$ Condone the presence $p = -3$, and/or $q = 21$
	A1	For both the correct value of p ($= 2$) AND the correct value of q ($= -24$) Must reject $p = -3$, and/or $q = 21$ if seen.
(d)	M1	$[f(x) = (x+a)(x-3)(x+2)]$ For attempting to find the value of a $-3 \times 2 \times a = -24 \Rightarrow a = \dots$ OR For an attempt using division with their values of p and q $x^2 - x - 6 \overline{) x^3 + 3x^2 - 10x - 24} \quad \text{or} \quad x - 3 \overline{) x^3 + 3x^2 - 10x - 24}$ Allow a quotient of $x + b$ or $x^2 + 6x + b$ where b is a constant.
	A1	For the correct factorised expression $(x+4)(x-3)(x+2)$ which must be written out in full on one line.

Question	Scheme	Marks
5(a)	$f(1) = 2 \times 1^3 + a \times 1^2 - 14 \times 1 + b = 0 \Rightarrow a + b = 12$ $f(4) = 2 \times 4^3 + a \times 4^2 - 14 \times 4 + b = 39 \Rightarrow 16a + b = -33$ $\Rightarrow 15a = -45 \Rightarrow a = -3^*$ $\Rightarrow b = 15$	M1 M1 M1A1 cso B1 [5]
	ALT – polynomial division $(2x^3 + ax^2 - 14x + b) \div (x - 1) = 2x^2 + (a + 2)x - 14 + a + 2$ and comparison of final step of division with b to obtain an equation or $(2x^3 + ax^2 - 14x + b) \div (x - 4) = 2x^2 + (a + 8)x + (18 + 4a)$ r 39 and comparison of final step of division with obtaining remainder 39 to obtain an equation $(2x^3 + ax^2 - 14x + b) \div (x - 1) = 2x^2 + (a + 2)x - 14 + a + 2$ and comparison of final step of division with b to obtain an equation and $(2x^3 + ax^2 - 14x + b) \div (x - 4) = 2x^2 + (a + 8)x + (18 + 4a)$ r 39 and comparison of final step of division with obtaining remainder 39 to obtain an equation $b = 12 - a$ $b + 4(18 + 4a) = 39$ $a = -3^*$ $b = 15$	 [M1 M1 M1 A1cso B1]
(b)	$(x-1) \overline{) 2x^3 - 3x^2 - 14x + 15}$ $f(x) = (x-1)(2x+5)(x-3)$	M1A1 dM1A1 [4]
(c)	$p = -\frac{5}{2} \quad q = 1 \quad r = 3$	B1ft B1ft B1ft [3]



(d)	$A = \left \int_1^3 (2x^3 - 3x^2 - 14x + 15) \, dx \right + \int_{-\frac{5}{2}}^1 (2x^3 - 3x^2 - 14x + 15) \, dx$	M1
	$A = \left[\left[\frac{2x^4}{4} - \frac{3x^3}{3} - \frac{14x^2}{2} + 15x \right]_1^3 \right] + \left[\frac{2x^4}{4} - \frac{3x^3}{3} - \frac{14x^2}{2} + 15x \right]_{-\frac{5}{2}}^1$	M1*
	$A = \left[\left[\left(\frac{3^4}{2} - 3^3 - 7 \times 3^2 + 15 \times 3 \right) - \left(\frac{1^4}{2} - 1^3 - 7 \times 1^2 + 15 \times 1 \right) \right] \right]$ $+ \left[\left(\frac{1^4}{2} - 1^3 - 7 \times 1^2 + 15 \times 1 \right) - \left(\frac{\left(-\frac{5}{2} \right)^4}{2} - \left(-\frac{5}{2} \right)^3 - 7 \times \left(-\frac{5}{2} \right)^2 + 15 \times \left(-\frac{5}{2} \right) \right) \right]$	dM1
	$A = 12 + \frac{1715}{32} = \frac{2099}{32} \quad \text{or} \quad 65.59375$	A1 [4]
Total 16 marks		

Part	Mark	Notes
(a)	M1	For use of $f(1) = 0$ or use of $f(4) = 39$
	M1	For use of $f(1) = 0$ and use of $f(4) = 39$
	M1	For solving the simultaneous equations in a and b Must see at least one line of working before a or b obtained. Must see an equation in just one variable.
	A1cso	For showing $a = -3^*$ Must see expression only in terms of a
	B1	For the correct value of $b = 15$ seen. Note: this is an A mark in epen.
	ALT – polynomial division	
	M1	Correct method for forming one equation by polynomial division together with comparison of the final step of the division with the required result (no remainder for division by $(x - 1)$ and remainder 39 for division by $(x - 4)$)
	M1	Correct method for forming both equations by polynomial division together with comparison of the final step of the division with the required result (no remainder for division by $(x - 1)$ and remainder 39 for division by $(x - 4)$)
	M1	For solving the simultaneous equations in a and b Must see at least one line of working before a or b obtained.
	A1cso	For showing $a = -3^*$ Must see expression only in terms of a ie $ka = -3k$ before $a = -3$
	B1	For the correct value of $b = 15$ seen. Note: this is an A mark in epen.
	Assuming $a = -3$ and solving to find $b = 15$ scores maximum M1M0M0A0A1	
(b)	Condone (b) completed in (c)	
	M1	For carrying out polynomial division/comparing coefficients to find the quadratic factor. Must reach $2x^2 \pm \dots$. Allow statement of quadratic factor if polynomial division by $(x - 1)$ used in (a).
	A1	For the correct quadratic factor.
	dM1	For factorising their 3TQ See general guidance for what constitutes an attempt to factorise a 3TQ.
	A1	For the correct factorisation of $f(x)$ on one line. Condone omission of $f(x) =$
	Note: complete factorisation seen with no method scores M0A0M0A0.	
(c)	B1ft	For one correct value for any of p, r or q FT their linear factors in (b). Values need not be identified as p, r, q , but if they are labelled these should be $p < q < r$.
	B1ft	For two correct values. FT their linear factors in (b). Values need not be identified as p, r, q , but if they are labelled these should be $p < q < r$.
	B1ft	For all three correct values. FT their linear factors in (b), but 1 must be one of the values . Values need not be identified as p, r, q , but if they are labelled these should be $p < q < r$.

(d)	M1	For a correct statement for the area of the shaded regions with their limits correctly used. Minimum required is: $\int_p^q (2x^3 - 3x^2 - 14x + 15)dx - \int_q^r (2x^3 - 3x^2 - 14x + 15)dx$ with $p < q < r$ shown as numerical limits or $\int_p^q (2x^3 - 3x^2 - 14x + 15)dx + \left \int_q^r (2x^3 - 3x^2 - 14x + 15)dx \right $ with $p < q < r$ shown as numerical limits
	M1*	For an attempt to integrate their expression. Minimum required is: • two powers of terms to increase by 1 • no powers of terms to decrease Condone $+c$
	dM1	For substituting their limits correctly to evaluate the integrated expression. Must show the substitution if their integral or their limits are incorrect. Correct exact answer following correct integral with correct limits implies this mark. Condone $+c$ as long as this is cancelled out during the substitution.
	A1	For the correct area. Exact area required.

Question number	Scheme	Marks
7 (a)	$64\left(\frac{1}{4}\right)^3 - 64\left(\frac{1}{4}\right)^2 + 3 = 0$ so $(4x - 1)$ is a factor	M1 A1 [2]
(b)	$\begin{array}{r} 16x^2 - 12x + 3 \\ 4x-1 \overline{) 64x^3 - 64x^2 + 0x + 3} \\ \underline{64x^3 - 16x^2} \\ -48x^2 \\ \underline{-48x^2} \\ 0 \end{array}$ or $64x^3 - 64x^2 + 3 = (4x - 1)(16x^2 - 12x + C)$ $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(16)(-3)}}{2 \times 16}$ $x = \frac{1}{4} \quad \text{oe}$ $x = \frac{3 \pm \sqrt{21}}{8} \quad \text{oe}$	M1 dM1 B1 A1 [4]
(c)	$ar^2 = 9$ or $\frac{a}{1-r} = 192$ oe $(192 - 192r)r^2 = 9$ oe $192r^2 - 192r^3 = 9 \Rightarrow 64r^3 - 64r^2 + 3 = 0$	B1 M1 A1*cso [3]
(d)	$r = \frac{1}{4}$	B1 [1]
(e)	$a\left(\frac{1}{4}\right)^2 = 9$ or $\frac{a}{1-\frac{1}{4}} = 192 \Rightarrow a = \dots$ $a = 144$	M1 A1*cso [2]
(f)	$\frac{144(1-0.25^n)}{1-0.25} > 191.9$ $1-0.25^n > \frac{1919}{1920}$ $0.25^n < 0.00052083\dots$ $n > \frac{\log 0.00052083}{\log 0.25}$ $n > 5.45\dots \Rightarrow n = 6$	M1 dM1 M1 A1 [4]
Total 16 marks		

Part	Mark	Additional Guidance
(a)	M1	For $64\left(\pm\frac{1}{4}\right)^3 - 64\left(\pm\frac{1}{4}\right)^2 + 3$
	A1	For = 0 and a conclusion stated, no errors seen. Accept minimal conclusion.
(b)	M1	Attempts long division. Minimally acceptable attempt is the division and correct working as written in the scheme. Must get a 3TQ factor. If comparing coefficients, a correct equation/comparison must be written followed by an attempt to find C. Must get a 3TQ factor.
	dM1	For a fully correct method to solve their 3TQ to find x, correct solutions from a correct 3TQ implies this mark.
	B1	For correct root $\frac{1}{4}$
	A1	For correct roots $\frac{3 \pm \sqrt{21}}{8}$
(c)	B1	For either correct equation.
	M1	For both correct equations and any correct step to eliminate q
	A1*cso	For a fully correct solution, no errors, with at least one correct intermediate line leading to $64r^3 - 64r^2 + 3 = 0$
(d)	B1 (A1 on ePen)	For the correct value of r
(e)	M1	For correct substitution of their value for r in either of their equations to find a
	A1	For $a = 144$
(f)	M1	Allow working in equation or any inequality for all M marks Correct use of sum to n terms formula with $a = 144$ or their a and their r to set up an equation or inequality
	dM1	Attempts to rearrange and obtains $\pm(\text{"their } r\text{"})^n \dots p$
	M1	Correct use of logs and attempt to evaluate the log. (Can be implied by awrt 5.5 only)
	A1	$n = 6$ (must come from solving inequalities with no errors in processing the inequalities. If from solving equations only, award A0)